How do you assign reference marks?

1. Draw currents from + to - nodes or voltage sources or aligned with current sources if possible.

2. Align element currents with loop currents.

REQUIRED

3. Follow passive sign convention

4. When in doubt just do (3)

Consider

Draw loop current from + to -. Follow with passive sign convention for elements.

Note source current was aligned with that of loop.
This requires $v_s$ to be in opposite direction to given polarity.

Finalize by assigning nodes and reference (ground).

This can now be solved,

\[
\begin{align*}
\mathbf{V} &= \begin{bmatrix} v_s \\ v_i \\ v_2 \end{bmatrix} \\
\mathbf{I} &= \begin{bmatrix} i_s \\ i_1 \\ i_2 \end{bmatrix} \\
\mathbf{A} &= \begin{bmatrix} 500 & 1 & 0 \\ 1 & 500 & 0 \\ 0 & 1 & 500 \end{bmatrix} \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{A} \mathbf{V} &= \mathbf{I} \\
\mathbf{V}_s &= -1.5 \\
v_i = 500i_1 \\
v_2 &= 1000i_2 \\
\end{align*}
\]

Element equations

\[
\begin{align*}
\mathbf{KCL}_A : \sum i &= 0 \quad +i_s - i_1 = 0 \\
\mathbf{KCL}_B : \sum i &= 0 \quad +i_1 - i_2 = 0 \\
\mathbf{KVL} : \sum v &= 0 \quad +v_s + v_i + v_2 = 0 \\
\end{align*}
\]
As circuits get more complex, we want to replace parts of the circuit with equivalent but simpler circuits. Circuits are equivalent if they have the same i-v characteristics at a specified pair of terminals.

**Equivalent Resistances**

**Source Transformations**

Equivalent resistance (series)

\[ \text{rest of the circuit} \]

\[ \text{rest of the circuit} \]

\[ R_{\text{EQ}} = R_1 + R_2 \]

KVL from A to B:

\[ \Sigma V = -V + V_1 + V_2 = 0 \]

\[ V = V_1 + V_2 \]

but \( i_1 = i_2 = i \)

\[ V = i_1 R_1 + i_2 R_2 \]

\[ V = i R_1 + i R_2 \]

\[ V = i (R_1 + R_2) \]

For this circuit we simply use Ohm's Law:

\[ V = i R_{\text{EQ}} \]

These are identical if \( R_{\text{EQ}} = R_1 + R_2 \)
Equivalent resistance (parallel)

\begin{align*}
\text{KCL at upper node:} & \quad \sum i = 0 \quad \Rightarrow \quad i - i_1 - i_2 = 0 \\
& \quad i = i_1 + i_2
\end{align*}

\text{using Ohm's Law:} \quad i = \frac{V}{R} \quad \Rightarrow \quad i = \frac{V_1}{R_1} + \frac{V_2}{R_2}

\text{but:} \quad V = V_1 = V_2 \quad \text{since these are in parallel}

\begin{align*}
i &= \frac{V}{R_1 + R_2} = \frac{V}{R_{\text{eq}}} \\
\text{For this equivalent circuit:} \\
\frac{1}{R_{\text{eq}}} &= \frac{1}{R_1} + \frac{1}{R_2}
\end{align*}

These circuits will be equivalent if \( \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \)

This can be put in a more common form by simply inverting

\begin{align*}
R_{\text{eq}} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \\
&= \frac{R_1 R_2}{R_1 + R_2}
\end{align*}
Example: between two terminals

![Diagram showing circuit configuration between terminals A and C, and A and D.]

We can derive equivalent resistances between any two pairs of terminals — assuming nothing is connected to the others.

- Between A & C: \( R_{A-C} = R_1 \)
- Between A & D:

This is two equivalent resistances:

- \( R_{EQ_1} = \frac{R_2 R_3}{R_2 + R_3} \)
- \( R_{EQ_2} = R_2 \parallel R_3 \)
- \( R_{EQ_2} = R_1 + R_{EQ_1} \)
Equivalent Sources

Circuit A

Use KVL \[ \sum \nu = 0 \]
\[-\nu_s + \nu_R + \nu = 0 \]

Use Ohm's law \[ \nu_R = i R_1 \]

Combine and rearrange
\[-\nu_s + i R_1 + \nu = 0 \]
\[i R_1 = -\nu + \nu_s \]
\[i = -\frac{\nu}{R_1} + \frac{\nu_s}{R_1} \]

Circuit B

Use KCL \[ \sum i = 0 \]
\[+i_s - i_R - i = 0 \]

Use Ohm's law
\[ i_R = \frac{\nu_2}{R_2} = \frac{\nu}{R_2} \]

\[i = -i_R + i_s \]
\[i = -\frac{\nu}{R_2} + i_s \]

These will be identical if \( R_1 = R_2 = R \)
and \( i_s = \frac{\nu_s}{R_1} \)

\[ \text{slope} = -\frac{1}{R} \]

\[ \nu = i_s R = \nu_s \]