I. Do element equations
\[
\begin{align*}
\mathcal{V}_A &= V_0 \quad \text{given source} \\
\mathcal{V}_1 &= i_1 R_1 \quad \text{Ohm's Law} \\
\mathcal{V}_2 &= i_2 R_2 \quad \text{Ohm's Law}
\end{align*}
\]
we will shortly start assuming these.

II. Do connection equations: KCL @ each node, KVL for loop.
\[
\begin{align*}
\text{KCL @ A:} & \quad \sum i = 0 \quad -i_A - i_1 = 0 \\
\text{KCL @ B:} & \quad \sum i = 0 \quad i_1 - i_2 = 0 \\
\text{KVL} & \quad \sum \mathcal{V} = 0 \quad -\mathcal{V}_A + \mathcal{V}_1 + \mathcal{V}_2 = 0
\end{align*}
\]

III. Substitute element equations into connection equations
Using
\[
-\mathcal{V}_A + \mathcal{V}_1 + \mathcal{V}_2 = 0
\]
\[
-(V_0) + (i_1 R_1) + (i_2 R_2) = 0
\]
from (1) \quad \uparrow \quad \text{from (2)} \quad \text{from (3)}

Now use (5) to reduce this to one unknown
\[-V_0 + i_1 R_1 + i_1 R_2 = 0 \quad i_1 = \frac{V_0}{R_1 + R_2}
\]

Given $V_0, R_1, R_2$ all variables can now be found.
Exercise 2-6

Given: 30V, 100Ω, 200Ω, 300Ω

(a) Write the complete set of element equations.

1. \( \sum I = 0 \)
   \[ 30 = 100i_1 \]
   \[ 200i_2 = 300i_3 \]

(b) Write the complete set of connection equations.

It is two nodes and two loops.

KCL @ node A

\[ \sum I = 0 \quad -i_A - i_1 = 0 \]

KCL @ node B

\[ \sum I = 0 \quad +i_1 - i_2 = 0 \]

KVL @ loop 1

\[ \sum V = 0 \quad -v_A + v_3 = 0 \]

KVL @ loop 2

\[ \sum V = 0 \quad -v_3 + v_1 + v_2 = 0 \]

(c) Solve these equations.

Substitute everything into (8)

\[ -(v_A) + (100i_1) + (200i_2) = 0 \]

From (1), from (2), from (3)

\[-30 + 100i_1 + 200i_2 = 0 \]

From (6) \( i_1 = i_2 \implies -30 + 100i_1 + 200i_1 = 0 \)

\[ i_1 = \frac{30}{100 + 200} = \frac{30}{300} = 0.1A = 100mA \]

All other variables can now be solved for.
How do you assign reference marks?

1. Draw currents from + to - nodes of voltage sources or aligned with current sources if possible.

2. Align element currents with loop currents.

**REQUIRED**

3. Follow passive sign convention

4. When in doubt just do (3)

Consider

![Circuit Diagram]

Draw loop current from + to -. Follow with passive sign convention for elements.

Note source current was aligned with that of loop. This requires $v_s$ to be in opposite direction to given polarity.

Finalize by assigning nodes and reference (ground).

![Circuit Diagram]

This can now be solved:

\[
\begin{align*}
v_s &= -1.5 \\
v_1 &= 500i_1 \\
v_2 &= 1000i_2
\end{align*}
\]

Element equations

Connection equations

\[
\begin{align*}
\text{KCL @ A:} & \quad \sum i = 0 \quad +i_s - i_1 = 0 \\
\text{KCL @ B:} & \quad \sum i = 0 \quad +i_1 - i_2 = 0 \\
\text{KVL} & \quad \sum v = 0 \quad +v_s + v_1 + v_2 = 0
\end{align*}
\]
EQUIVALENT CIRCUITS
As circuits get more complex we want to replace parts of the circuit with equivalent but simpler circuits.

Circuits are equivalent if they have the same i-v characteristics at a specified pair of terminals.

Equivalent Resistances
Source Transformations

Equivalent resistance (series)

\[ \begin{align*}
\text{rest of the circuit} & \quad \begin{array}{c}
\text{A} \quad R_1 \\
+ \quad + \quad + \\
\text{B} \quad - \quad + \quad + \\
\text{rest of the circuit}
\end{array} \\
\text{v} & \quad = \quad i
\end{align*} \]

\[ R_{EQ} = R_1 + R_2 \]

KVL from A to B

\[ \sum V_x - V + V_1 + V_2 = 0 \]

\[ V = V_1 + V_2 \]

but \[ i_1 = i_2 = i \]

\[ v = i_1 R_1 + i_2 R_2 \]

\[ v = i R_1 + i R_2 \]

\[ v = i (R_1 + R_2) \]

For this circuit we simply use Ohm's Law

\[ v = i R_{EQ} \]

These are identical if \[ R_{EQ} = R_1 + R_2 \]
Equivalent resistance (parallel)

KCL @ upper node: \( \sum i = 0 \)

\[ i - i_1 - i_2 = 0 \]

\[ i = i_1 + i_2 \]

Using Ohm’s Law:

\[ i = \frac{V_1}{R_1} + \frac{V_2}{R_2} \]

But \( V = V_1 = V_2 \) since these are in parallel

\[ i = \frac{V}{R_1} + \frac{V}{R_2} = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

For this equivalent circuit:

\[ i = \frac{V}{R_{eq}} \]

These circuits will be equivalent if

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

This can be put in a more common form by simply inverting:

\[ R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \]