These voltage and current source representations of the source are identical.

If each is open, i.e., remove load

\[ V_{oc} = V_T \quad \quad V_{oc} = i_N R_N \]

If each is shorted, i.e., replace the load by a short

\[ I_{sc} = \frac{V_T}{R_T} \quad \quad I_{sc} = i_N \]

Everywhere in between they are equivalent. We can show this by comparing \( V-i \) equations.

- By KVL \( \sum V = 0 \)
  \[ -V_T + iR_T + V_T = 0 \]
  \[ V = V_T - iR_T \]

- By KCL \( \sum I = 0 \)
  \[ +i_N - \frac{V_T}{R_N} - i = 0 \]
  \[ i_N = \frac{V_T}{R_N} - i \]

Rearranging \( V = i_N R_N - i R_N \)

If we set \( R_T = R_N \)
and \( V_T = i_N R_N \)

These expressions for Norton & Thevenin are identical.
Example 3-13

Find Thevenin equivalent
1. superposition always works.
2. circuit reductions (ladder networks)

This is a ladder network.

(a) Source transformation

\[ i = \frac{15V}{3k} = 5mA \]

(b) Combine

(c) Source transform

\[ v = iR = (3mA)(2k) = 6V \]

6V

Load 1

\[ i = \frac{6V}{9k+10k} = 0.3158mA \]

\[ P = i^2R = (0.3158mA)^2(10k) = .9973mW \]

Load 2

\[ i = \frac{6V}{9k+15k} = 0.1111mA \]

\[ P = i^2R = (0.1111mA)^2(15k) = .5555mW \]
Example 3-14.

(a) \[ \begin{array}{c}
\text{by inspection } i_C = -2A, \quad i_B = i \\
A: -40 + 60i_A - 60i_C + 180i_A - 180i_B = 0 \\
B: +180i_B - 180i_A + 15i_B - 15i_C + v = 0 \\
\end{array} \]

\[ -40 + 240i_A + 120 - 180i = 0 \]
\[ 195i_A - 180i_A + 30 + v = 0 \]
\[ 240i_A - 180i = -80 \]
\[ -180i_A + 195i = -30 - v \]

Using determinants
\[ \begin{vmatrix}
240 & -80 \\
195 & -30 - v \\
\end{vmatrix} = \frac{-21600 - 2400}{14400} \]

\[ i = -1.5 - \frac{v}{60} \]

Compare this with equation of Norton source.

Apply KCL: \[ \sum i = 0 \]
\[ +i_N - \frac{v}{R_N} - i = 0 \]
\[ i = +i_N - \frac{v}{R_N} \]

\[ i = -1.5 \text{ Amps} \]

(b) \[ \begin{array}{c}
\text{Given } 5 \text{ Watts find } i \\
\text{find } i \text{ for } v = 5 \text{v} \\
i = -1.5 - \frac{(5L)}{60} \\
60i^2 = -90i - 5 \\
12i^2 + 18i + 1 = 0 \\
\end{array} \]

\[ i = -1.442 \text{ A}, \]

\[ R_N = 60 \Omega \]
Derivation of Thévenin's Theorem

1. Start with typical source/load

2. Apply a current source $i_{\text{TEST}}$
   - We know $i_{\text{TEST}}$ and let $v = V_{\text{TEST}}$

3. Turn $i_{\text{TEST}}$ off but leave all sources on.

4. Turn $i_{\text{TEST}}$ on but turn all sources off.

Using superposition, the sum of (1) and (2) must be $V_{\text{TEST}}$

$$V_{\text{TEST}} = V_{\text{TEST}1} + V_{\text{TEST}2}$$

$$V_{\text{TEST}} = V_{\text{OC}} - i_{\text{TEST}} \cdot R_{\text{EQ}}$$

$$v = v_T - i \cdot R_T$$

Let $\Rightarrow V_{\text{TEST}} = v$

$$V_{\text{OC}} = v_T$$

$$R_T = R_{\text{EQ}}$$

$$i_{\text{TEST}} = i$$

This gives us a great method to determine $R_T$ simply turn off all sources and determine $R_{\text{EQ}}$. 

Looks exactly like KVL for Thévenin, which we got on first page.
Exercise 3-16(a)

Now use what we just learned to determine Thévenin & Norton.

If we eliminate all sources we can find $R_T$

\[ R_T = \frac{20 \cdot 30}{20 + 30} = \frac{600}{50} = 12 \, \Omega \]

\[ R_T = 12 + 60 = 72 \, \Omega \]

We then find open circuit voltage.

\[ V_{OC} = -30 \text{ volts}, \]

\[ i = \frac{E}{R} = \frac{50}{50} = 1 \, \text{A} \]

\[ V = iR = (1 \, \text{A})(30 \, \Omega) = 30 \text{ volts} \]

Since they are related, \[ i_N = \frac{V_T}{R_T} = \frac{-30 \text{V}}{72 \, \Omega} = -0.417 \, \text{mA} \]