1. Prove or give a counterexample: (i) If \( P(A|B,C) = P(B|A,C) \) then \( P(A|C) = P(B|C) \). (ii) If \( P(A|B) = P(A) \) then \( P(A|B,C) = P(A|C) \). \( A, B, C \) are arbitrary random variables. (10 points)

2. A game has two players, \( E \) and \( O \). Each turn, a fair 6-sided die is rolled. If it shows an even number, \( E \) gets a point, else \( O \) gets a point. The first player to get 7 points wins. Suppose at an intermediate stage the score is 4-2 (\( E \) has 4 points). What is the probability that \( E \) will win this game? What about the general case when the score is \( e-o \) for arbitrary \( e \) and \( o \) (\( 0 \leq e, o < 7 \))? (10 points)

3. A coin shows head with probability \( x \) when flipped. (i) Suppose you know \( x \). Are successive flips of the coin independent? (ii) Suppose you do NOT know \( x \). Are successive flips independent now? Explain carefully why in each case. (15 points)

4. There are three coins \( C_1, C_2 \) and \( C_3 \) that have probability of showing heads 0.25, 0.5 and 0.75. One of these is picked uniformly at random and flipped three times, the outcome variables are \( X_1, X_2 \) and \( X_3 \). (i) Draw a Bayesian network with appropriate CPTs to reflect this experiment. (ii) If the observed outcomes are two tails and one head, determine which coin was most likely to have been drawn. (15 points)

5. Prove that any 3SAT problem can be reduced to exact inference in a corresponding Bayesian network. Hence show that exact inference in a BN is NP-hard. (Consider a BN that has nodes for the propositional symbols, the clauses and the conjunction.) (15 points)

6. Write down specifications for learning systems that learn to (a) understand a language and (b) play tennis. Do these tasks fit well into the supervised learning framework we have been studying? Explain your answer. (10 points)

7. This question is best answered with the help of a mathematics package such as \( R \), \( Matlab \) or \( Mathematica \). Look at Figure 20.1 in the book. The data used for Figure 20.1 was generated by hypothesis \( h_5 \). For hypotheses \( h_3 \) and \( h_4 \), generate one dataset each of size 100 and plot graphs for: (i) \( P(h_i|d_1, ... d_N), \ i=1,..,5 \), (ii) \( P(D_{N+1}=\text{lime}|d_1, ... d_N) \), (iii) \( P(D_{N+1}=\text{lime}|h_{\text{MAP}}) \) and (iv) \( P(D_{N+1}=\text{lime}|h_{\text{ML}}) \) as \( N \) varies from 1 to 100. For (iii) and (iv), pick \( h_{\text{MAP}} / h_{\text{ML}} \) just from \( h_3 \) and \( h_4 \) and assume the prior probability of \( h_3 \) to be 0.9. Put the lines for (i) in one graph and (ii), (iii)
and (iv) in a second graph, so you generate four graphs in total, two for $h_3$ and two for $h_4$. Attach any code you used to generate these graphs to your answer. (25 points)

8. Suppose that Alice’s utilities for cherry and lime candies are $c_A$ and $l_A$, while Bob’s are $c_B$ and $l_B$. Alice has a bag of $x$ candies generated by one of the $h_3$, $h_4$ or $h_5$ hypotheses above (Alice does not know which one). At each point, she can either unwrap a candy and eat it, or she can sell the remaining bag of candies to Bob. Bob will pay the “true” price (i.e. the value according to him once he unwraps all the remaining candies). Determine a good selling criterion for Alice. (10 points)