EECS 391: Introduction to Artificial Intelligence (Spring 2012) Written Homework 5 (Max points: 100)

Assigned Tuesday March 20, due 5pm Tuesday March 27. Write your answers neatly and remember to show all relevant work. Before turning in your work, staple your answer sheets together and write your name (or names if working as a pair) on each page.

1. Prove or give a counterexample: (i) If $P(A|B,C)=P(B|A,C)$ then $P(A|C)=P(B|C)$. (ii) If $P(A|B)=P(A)$ then $P(A|B,C)=P(A|C)$. A, B, C are arbitrary random variables. (10 points)

2. From the full joint distribution in the book in Figure 13.3, calculate $P(Cavity| Toothache=true \vee Catch=true)$. (10 points)

3. A game has two players, E and O. Each turn, a fair 6-sided die is rolled. If it shows an even number, E gets a point, else O gets a point. The first player to get 7 points wins. Suppose at an intermediate stage the score is 4-2 (E has 4 points). What is the probability that E will win this game? What about the general case when the score is e-o for arbitrary e and o ($0 \leq e, o < 7$)? (10 points)

4. I would like to transmit an n-bit message to my friend. Each bit can be uniformly, independently corrupted with a probability of ε. I attach k “parity” bits to the message that can correct at most k bit errors. What is the maximum length of a message I can send to my friend so that the correct message is received with probability $1-\delta$? Calculate this for ε=0.001, δ=0.01 and k=1,2,3. (15 points)

5. A coin shows head with probability x when flipped. (i) Suppose you know x. Are successive flips of the coin independent? (ii) Suppose you do NOT know x. Are successive flips independent now? Explain carefully why in each case. (15 points)

6. For arbitrary random variables A, B, C prove the following: $P(B|A,C)= P(A|B,C)P(B|C)/P(A|C)$. (10 points)

7. There are three coins C_1, C_2 and C_3 that have probability of showing heads 0.25, 0.5 and 0.75. One of these is picked uniformly at random and flipped three times, the outcome variables are X_1, X_2 and X_3. (i) Draw a Bayesian network with appropriate CPTs to reflect this experiment. (ii) If the observed outcomes are two tails and one head, determine which coin was most likely to have been drawn. (15 points)

8. Prove that any 3SAT problem can be reduced to exact inference in a corresponding Bayesian network. Hence show that exact inference in a BN is NP-hard. (Consider a BN that has nodes for the propositional symbols, the clauses and the conjunction.) (15 points)