

**EECS 391: Introduction to AI (Spring 2012) Written Homework 3 (Max Points: 100)**

**Assigned Tuesday February 14, due 5pm Tuesday February 21. Write your answers neatly and remember to show all relevant work. Before turning in your work, staple your answer sheets together and write your name(s) and Case ID(s) on the front page.**

1. Prove the following assertions: (a)  $\alpha$  is valid iff  $\text{TRUE} \models \alpha$ . (b) For any  $\alpha$ ,  $\text{FALSE} \not\models \alpha$ . (10 points)
2. Prove the deduction theorem in propositional logic. (10 points)
3. Suppose a propositional language has only four symbols A, B, C and D. How many models are there for the following sentences? (10 points)
  - (a)  $(A \wedge B) \vee (B \wedge C)$
  - (b)  $A \vee B$
  - (c)  $A \Leftrightarrow B \Leftrightarrow C$
4. From the following assertions:  
“If the Unicorn is mythical, then it is immortal, but if it is not mythical, it is a mortal mammal. If the Unicorn is either immortal or a mammal, then it is horned. The Unicorn is magical if it is horned.”  
Can you prove that the Unicorn is (a) mythical, (b) magical, (c) horned? (10 points)
5. Any propositional logic sentence is logically equivalent to the assertion that each possible world in which it would be false is not the case. From this observation, prove that any sentence in propositional logic can be written in CNF. (10 points)
6. Is a random 4CNF formula more likely to be satisfiable than a random 3CNF formula, both consisting of  $m$  clauses over  $n$  symbols? Explain your answer. (10 points)
7. Suppose an FOL KB has only two sentences:  $P(a)$  and  $P(b)$ . Does this KB entail  $\forall x P(x)$ ? (10 points)
8. Write down a FOL statement such that every world in which it is true has exactly one object. (10 points)
9. Write FOL statements to represent the following. Define your own sensible vocabulary. (10 points)
  - (a) There is a barber who shaves all men in town who do not shave themselves.
  - (b) A person born outside the country of Guilder, one of whose parents is a Guilder citizen by birth, is a Guilder citizen by descent.
  - (c) Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they cannot fool all of the people all of the time.
10. Explain what is wrong with this definition of adjacent squares in the Wumpus world:  $\forall x, y$   
 $\text{Adjacent}([x, y], [x+1, y]) \wedge \text{Adjacent}([x, y], [x, y+1])$ . (10 points)