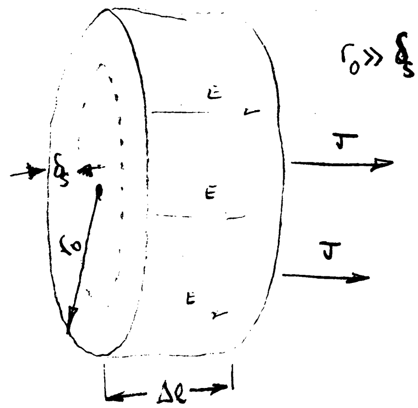
Francis L. Merat
ENGR 339
2/21/72

2.14.



A

the ~~i~~ intrinsic impedance of the material is Z_s

the impedance of the strip is $Z_{ac} = Z_s \dfrac{\Delta l}{2\pi r_o}$

the d.c. resistance of the strip is $R_{d.c.} = \dfrac{1}{\sigma} \dfrac{\Delta l}{\pi r_o^2}$

$r = \rho \dfrac{l}{A}$

$$\boxed{\frac{Z_{ac}}{R_{dc}} = \frac{\sigma r_o}{2} Z_s} \qquad \frac{Z_{ac}}{R_{dc}} = \frac{Z_s \cancel{\Delta l}}{2\cancel{\pi r_o}} \cdot \frac{\sigma \cancel{\pi} r_o^{\cancel{2}}}{\cancel{\Delta l}} = \frac{\sigma r_o}{2} Z_s$$

✓

$f = 10^6$ cycles

$\sigma = 5.8 \times 10^7 \ \mho/m$

$r_o = 0.1$ cm

$\mu = \mu_o$

$$Z_s = \frac{1+j}{\sigma \delta}$$

$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{\cancel{2}}{\cancel{2}(3.14)(10^6)(4\pi \times 10^{-7})(5.8 \times 10^7)}}$$

$$= \sqrt{\frac{1}{229 \times 10^6}}$$

$$\delta_s = \frac{1}{\sqrt{2.29 \times 10^8}} = \frac{1}{1.514 \times 10^4}$$

$$\delta_s = 0.660 \times 10^{-4} \text{ m.}$$

$$\delta_s = 6.6 \times 10^{-3} \text{ cm}$$

$$\frac{Z_{ac}}{R_{dc}} = \frac{(\cancel{5.8 \times 10^7})(10^{-3} m)}{2} \frac{(1+j)}{(\cancel{5.8 \times 10^7})(6.6 \times 10^{-5})}$$

$$= \frac{(1+j)}{2(6.6)} \times 10^2 = 0.0757 \times 10^2 \ (1+j)$$

$$\therefore \boxed{\frac{Z_{ac}}{R_{dc}} = 7.57\ (1+j)}$$

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ENGR 339
2/21/72

3.5 air-filled coaxial line

show minimum attenuation occurs when $x \ln x = 1 + x$ where $x \triangleq \frac{b}{a}$

also determine Z_c.

let $\nabla^2 \Phi = 0$ in the line

then $\frac{\partial}{\partial r}\left(r \frac{\partial \Phi}{\partial r}\right) = 0$ which requires that $\Phi = C_1 \ln r + C_2$

letting boundary conditions be: $\Phi(a) = 0 \quad \Phi(b) = V^+ \quad \Rightarrow \Phi = V^+ \frac{\ln\left(\frac{r}{a}\right)}{\ln\left(\frac{b}{a}\right)}$

$$\bar{e} = -\nabla_t \Phi = -\bar{\iota}_r \frac{V^+}{r \ln\left(\frac{b}{a}\right)}$$

$$\bar{h} = -\frac{\bar{a}_z \times \bar{e}}{Z} = \bar{\iota}_\phi \frac{V^+}{r Z \ln\left(\frac{b}{a}\right)} \qquad \text{where } Z = \sqrt{\frac{\mu}{\epsilon}}$$

power flowing down the line $\triangleq P$

$$P = \mathrm{Re} \int_S \tfrac{1}{2}(\bar{E} \times \bar{H}^*) \cdot \bar{\iota}_z \, dS = \mathrm{Re}\, \tfrac{1}{2} \int_S \left[\frac{(V^+)^2}{r^2 Z \ln^2\left(\frac{b}{a}\right)} - \frac{(V^-)^2}{r^2 Z \ln^2\left(\frac{b}{a}\right)}\right] dS$$

$$= \tfrac{1}{2} \mathrm{Re} \int_a^b \int_0^{2\pi} \frac{(V^+)^2 - (V^-)^2}{Z \ln^2\left(\frac{b}{a}\right)} \frac{r \, d\phi \, dr}{r^2} = \frac{(V^+)^2 - (V^-)^2}{2 Z \ln^2\left(\frac{b}{a}\right)} \int_a^b \int_0^{2\pi} \frac{d\phi \, dr}{r}$$

but $\int_a^b \int_0^{2\pi} \frac{d\phi\, dr}{r} = 2\pi \int_a^b \frac{dr}{r} = 2\pi \ln\left(\frac{b}{a}\right)$

$$P = \frac{(V^+)^2 - (V^-)^2}{2 Z \ln^2\left(\frac{b}{a}\right)} 2\pi \ln\left(\frac{b}{a}\right)$$

$$P = \frac{\left[(V^+)^2 - (V^-)^2\right]\pi}{Z \ln\left(\frac{b}{a}\right)}$$

power loss due to imperfect conductor $\triangleq P_\ell$

$$P_\ell = \mathrm{Re} \oint_{S_1 + S_2} \frac{Z_m}{2} |J_s|^2 \, d\ell$$

$$\bar{J}_s = \bar{n} \times \bar{h} = -\bar{\iota}_r \times \bar{\iota}_\phi \left[\frac{V^+ - V^-}{r Z \ln \frac{b}{a}}\right] = -\bar{\iota}_z \frac{V^+ - V^-}{r Z \ln \frac{b}{a}}$$

$$P_\ell = \mathrm{Re} \oint_{S_1 + S_2} \frac{1}{2} \frac{1+j}{\sigma \delta_s} \frac{(V^- - V^-)^2}{r^2 Z^2 \ln^2\left(\frac{b}{a}\right)} r \, d\phi$$

Use a wave going in one direction only with $J_s J_s^* \propto |V^+|^2$ to calculate α from $\alpha = \frac{P_\ell}{2P}$, otherwise you can not say $-\frac{dP}{dz} = 2\alpha P$

$$P_e = \frac{R_m (V^+ - V^-)^2}{Z^2 \ln^2(\frac{b}{a})} \left(\frac{1}{b}+\frac{1}{a}\right) \pi \qquad \text{where } R_m \triangleq \frac{1}{\sigma \delta_s}$$

$$\alpha = \frac{P_e}{2P} = \frac{R_m\left[(V^+) - (V^-)\right]^2 \left(\frac{a+b}{ab}\right)\pi}{\dfrac{\pi\left[(V^+)^2-(V^-)^2\right]^2}{Z \ln(\frac{b}{a})}} = \frac{R_m}{2Z\ln(\frac{b}{a})}\left(\frac{a+b}{ab}\right)\frac{(V^+-V^-)^2}{(V^+)^2-(V^-)^2}$$

Note: I think the factors $(V^+ - V^-)^2$ & $(V^+)^2 - (V^-)^2$ should cancel out as I do not think that $\alpha = f(V^+, V^-)$. This would be a non-linear system.

$$\frac{d\alpha}{da} = \frac{d}{da}(a+b)\left(ab\ln\frac{b}{a}\right)^{-1} = 0$$

$$\frac{1}{ab\ln\frac{b}{a}} + \frac{a+b}{a^2b^2\ln^2(\frac{b}{a})}\left[b\ln(\tfrac{b}{a}) - b\right] = 0$$

$$\frac{ab\ln\cancel{(\frac{b}{a})} + ab\cancel{\ln}(\frac{b}{a}) - b^2\ln(\frac{b}{a}) + ab + b^2}{ab} = 0$$

$$-\frac{b}{a}\ln(\tfrac{b}{a}) + 1 + \frac{b}{a} = 0$$

$$1 + \frac{b}{a} = \frac{b}{a}\ln\left(\frac{b}{a}\right)$$

if $x \triangleq \frac{b}{a}$

$$1 + x = x\ln x \quad \rightarrow \quad x \approx 3.6$$

A

$$Z_c = \frac{V}{I} = \frac{V}{\dfrac{V 2\pi}{Z\ln(\frac{b}{a})}} = \frac{Z\ln(\frac{b}{a})}{2\pi}$$

if α is minimized this becomes

$$Z_c = \frac{Z}{2\pi}\left(\frac{1+\frac{b}{a}}{\frac{b}{a}}\right) = \frac{Z}{2\pi}\left(1+\frac{a}{b}\right) \approx 77\Omega$$

3.6 $$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Given: $b = 3a = 1$ cm

$f = 10^9$ Hz

$\epsilon = (2.56 - j0.001)\epsilon_0$

Show: $\Im m(Z_c) \ll \Re e(Z_c)$

and $Z_c \approx \sqrt{\frac{L}{C}}$

to equate transmission line to equivalent circuit use formulas presented in text & use fields for coaxial line, Furthermore assume that $\mu \approx \mu_0$.

$$L = \frac{\mu}{|I_0|^2} \int_S |H|^2 ds = \frac{4\,\mu_0}{|I_0|^2} \int_a^b \int_0^{2\pi} \frac{|V_0|^2}{r^2 Z^2 \ln^2(\frac{b}{a})} r\,d\phi\,dr = \frac{2\pi\mu_0}{\ln(\frac{b}{a})} \left[\frac{|V_0|^2}{Z^2 |I_0|^2}\right]$$

ERRATA

$$Z_c^2 = \frac{Z^2 \ln^2 \frac{b}{a}}{(2\pi)^2}$$

which should change all following figures

using identity

$$\frac{|V_0|^2}{|I_0|^2} = Z_c^2 = \frac{Z^2 \ln \frac{b}{a}}{2\pi} \quad \text{for a coaxial line}$$

$$\Rightarrow \frac{|V_0|^2}{|I_0|^2 Z^2} = \frac{\ln(\frac{b}{a})}{2\pi} \qquad (1)$$

$$\Rightarrow L = \frac{2\pi\mu_0}{\ln(\frac{b}{a})} \frac{\ln(\frac{b}{a})}{2\pi} = \mu_0 = 4\pi \times 10^{-7} = 1.26 \times 10^{-6}$$

$$C = \frac{\epsilon'}{|V_0|^2} \int_S |E|^2 ds = \frac{\epsilon'}{|V_0|^2} \int_a^b \int_0^{2\pi} \frac{|V_0|^2}{r^2 \ln^2(\frac{b}{a})} r\,dr\,d\phi = \frac{\epsilon'\,2\pi}{\ln(\frac{b}{a})}$$

$$= \frac{2.56 \times 10^{-9}}{36\pi} \cdot \frac{2\pi}{\ln(3)} \approx 1.29 \times 10^{-10}$$

$$R = \frac{R_m}{|I_0|^2} \oint_{S_1+S_2} |H|^2 dl = \frac{R_m}{|I_0|^2} \frac{|V_0|^2}{Z^2 \ln^2(\frac{b}{a})} \int_0^{2\pi} \left(\frac{1}{a} + \frac{1}{b}\right) d\phi = \frac{R_m\,2\pi}{\ln^2(\frac{b}{a})} \left[\frac{1}{a} + \frac{1}{b}\right] \frac{|V_0|^2}{Z^2 |I_0|^2}$$

$$= \frac{R_m}{\ln(\frac{b}{a})} \left(\frac{1}{a} + \frac{1}{b}\right) \text{ by (1)}$$

for metals $\sigma \geq 10^7$

$$R_m = \sqrt{\frac{\omega\mu}{2\sigma}} \leq \sqrt{\frac{2\pi \cdot 10^9\ 4\pi \times 10^{-7}}{2 \times 10^7}}$$

$R \leq 2.41 \times 10^{-3}$

$R_m \leq 1.9[illegible] \times 10^{-3}$

$$G = \frac{\omega\epsilon''}{|V_0|^2}\int_S |E|^2 ds = \frac{\omega\epsilon''}{\cancel{|V_0|^2}}\frac{\cancel{|V_0|^2}}{\ln^2(\frac{b}{a})} 2\pi \ln\left(\frac{b}{a}\right) = \frac{2\pi\omega\epsilon''}{\ln(\frac{b}{a})}$$

$$G = 2\pi\left(\frac{10^{-9}}{36\pi}\right)\frac{(2\pi\times10^9)(-10^{-3})}{\ln(3)} \approx -3.15\times10^{-4}$$

$$Z_c = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{(RG+\omega^2LC)+j(\omega LG-\omega RC)}{G^2+\omega^2C^2}}$$

$$Z_c = \sqrt{\frac{(-7.6\times10^{-7}+6.4\times10^{2})+j(-2.99-1.61\times10^{-3})}{9.92\times10^{-8}+65.6\times10^{-2}}} \qquad (1)$$

$$Z_c \approx \sqrt{\frac{6.4\times10^2 - j2.99}{0.656}} = \sqrt{975-j4.55}$$

$$Z_c \approx \sqrt{\sqrt{(95000+21)}\,e^{-j(.046)}}$$

$$Z_c \approx (95000+21)^{\frac{1}{4}}\,e^{-j0.023}$$

$$e^{-j0.023} \approx 1 - j0.023$$

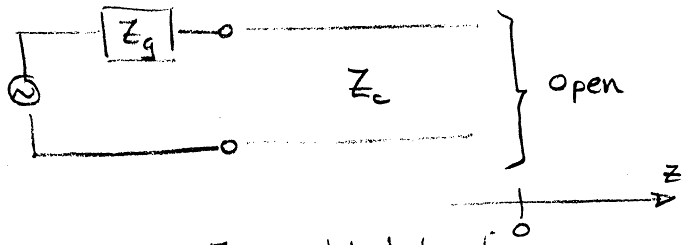
$$\Rightarrow \operatorname{Re} Z_c \gg \operatorname{Im} Z_c$$

A

in addition inspection of (1) reveals that

$$Z_c \approx \sqrt{\frac{\omega^2LC}{\omega^2C^2}} = \sqrt{\frac{L}{C}}$$

3.7



Assume Z_g matched to line

$$V = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$$

$$I = \frac{V^+}{Z_c} e^{-j\beta z} - \frac{V^-}{Z_c} e^{+j\beta z}$$

boundary condition that @ $z=0$ $I=0$

$$\Rightarrow \frac{V^+}{Z_c} - \frac{V^-}{Z_c} = 0$$

$$\Rightarrow V^+ = V^-$$

then $V = V^+(e^{-j\beta z} + e^{+j\beta z}) = 2V^+ \cos\beta z$

$$I = \frac{V^+}{Z_c}(e^{-j\beta z} - e^{+j\beta z}) = -j\frac{2V^+}{Z_c} \sin\beta z$$

A

3.8

$Z_c = 50\Omega$

$Z_L = 25 + j25$

(1) find Γ_L

$$\bar{Z}_L = \frac{Z_L}{Z_c} = \frac{25+j25}{50} = \frac{1}{2} + j\frac{1}{2}$$

$$\Gamma_L = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} = \frac{2j-1}{5}$$

(2) find SWR

$$S = \frac{1+\rho}{1-\rho} \quad \text{where} \quad \rho = |\Gamma| = \frac{\sqrt{2}}{5} = 0.2828$$

$$S = \frac{1+0.283}{1-0.283} \approx 1.790$$

(3) power transferred to load $\triangleq P_L$

total current on line $= I_L$

$$P_L = \mathrm{Re}\,\tfrac{1}{2} V I^* = \mathrm{Re}\,\tfrac{1}{2}(V^+ + \Gamma V^+)(I^+ - I^-)^* \qquad I_L = \frac{V^+ - \Gamma V^+}{Z_c}$$

$$= \mathrm{Re}\,\tfrac{1}{2}(V^+ + \Gamma V^+)\left(\frac{V^+ - \Gamma V^+}{Z_L}\right)^* \qquad = \frac{V^+ + \Gamma V^+}{Z_L} = \frac{V_L}{Z_L}$$

$$= \mathrm{Re}\,\frac{\tfrac{1}{2} V^+ (1+\Gamma)(V^+)^* (1-\Gamma^*) \; Z_L}{Z_L^* Z_L}$$

$$= \frac{1}{2}\frac{|V^+|^2}{|Z_L|^2}\left(1 - |\Gamma|^2\right) \mathrm{Re}\, Z_L$$

$$P_L = (1-|\Gamma|^2) P_{inc}$$

$$P_L = \frac{|V^+|^2 \left(1 - \frac{2}{25}\right)}{2(50)^2} \mathrm{Re}\,[25 + j25]$$

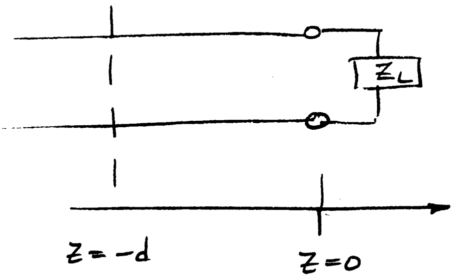
$$P_L = 0.0092\, |V_+|^2$$

C

3.9 Verify $\bar{Z}_{in} = \dfrac{Z_{in}}{Z_c} = \dfrac{Z_L + jZ_c \tan\beta l}{Z_c + jZ_L \tan\beta l}$

compute Z_{in} for $l = \dfrac{\lambda_0}{4}$ from termination

use line given in # 3.8



$$V(z) = V^+ \left(e^{-j\beta z} + \Gamma e^{+j\beta z} \right)$$

$$I(z) = \frac{V^+}{Z_c} \left(e^{-j\beta z} - \Gamma e^{+j\beta z} \right)$$

$$Z_{in}(d) = \frac{V(-d)}{I(-d)} = \frac{V^+}{\frac{V^+}{Z_c}} \left(\frac{e^{+j\beta d} + \Gamma e^{-j\beta d}}{e^{+j\beta d} - \Gamma e^{-j\beta d}} \right) \qquad \Gamma = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$Z_{in}(d) = \frac{Z_L e^{j\beta d} + Z_L e^{-j\beta d} + Z_c e^{j\beta d} - Z_c e^{-j\beta d}}{Z_L e^{j\beta d} + Z_c e^{j\beta d} - Z_L e^{-j\beta d} + Z_c e^{-j\beta d}} \; Z_c$$

$$\frac{Z_{in}(d)}{Z_c} = \frac{2Z_L \cos\beta d + 2Z_c j \sin\beta d}{2Z_c \cos\beta d + 2Z_L j \sin\beta d} \qquad (1)$$

$$\bar{Z}_{in}(d) = \frac{Z_{in}(d)}{Z_c} = \frac{Z_L + jZ_c \tan\beta d}{Z_c + jZ_L \tan\beta d}$$

A

if $d = \dfrac{\lambda_0}{4}$ $\quad \beta d = \dfrac{2\pi}{\lambda_0} \dfrac{\lambda_0}{4} = \dfrac{\pi}{2}$

using (1)

$$\bar{Z}_{in}(d) = \frac{Z_{in}(d)}{Z_c} = \frac{(25 + j25) \cdot 0 + j\,50\,(1)}{50 \cdot 0 + j\,(25 + j25)(1)} = \underline{1 - j}$$

$$Z_{in}\left(\frac{\lambda_0}{4}\right) = Z_c(1 - j) = 50(1 - j)\,\Omega$$

3.10

$Z_c = 50\Omega$

$V(z = -0.4\lambda_0) = 4 + j2$

$I(z = -0.4\lambda_0) = -2$

find $\bar{Z}_L$

$$Z_{in}(0.4\lambda_0) = \frac{V(-0.4\lambda_0)}{I(-0.4\lambda_0)} = \frac{\frac{Z_L}{Z_c}\cos\beta d + j\sin\beta d}{\cos\beta d + j\frac{Z_L}{Z_c}\sin\beta d}$$

$$\beta d = \frac{2}{5}\lambda_0 \cdot \frac{2\pi}{\lambda_0} = \frac{4\pi}{5} = 144^\circ$$

$\sin(144^\circ) = 0.588$
$\cos(144^\circ) = -0.809$

$$\frac{4+j2}{-2} = -2-j = \frac{(x)(-.809) + j(.588)}{-.809 + j(x)(.588)} \qquad \text{where } x \triangleq \bar{Z}_L = \frac{Z_L}{Z_c}$$

$$-j(1.176)x + 1.618 + 0.588x + j0.809 = -.809x + j(0.588)$$

$$(1.397 - j1.176)\,x = -1.618 - j\,0.221$$

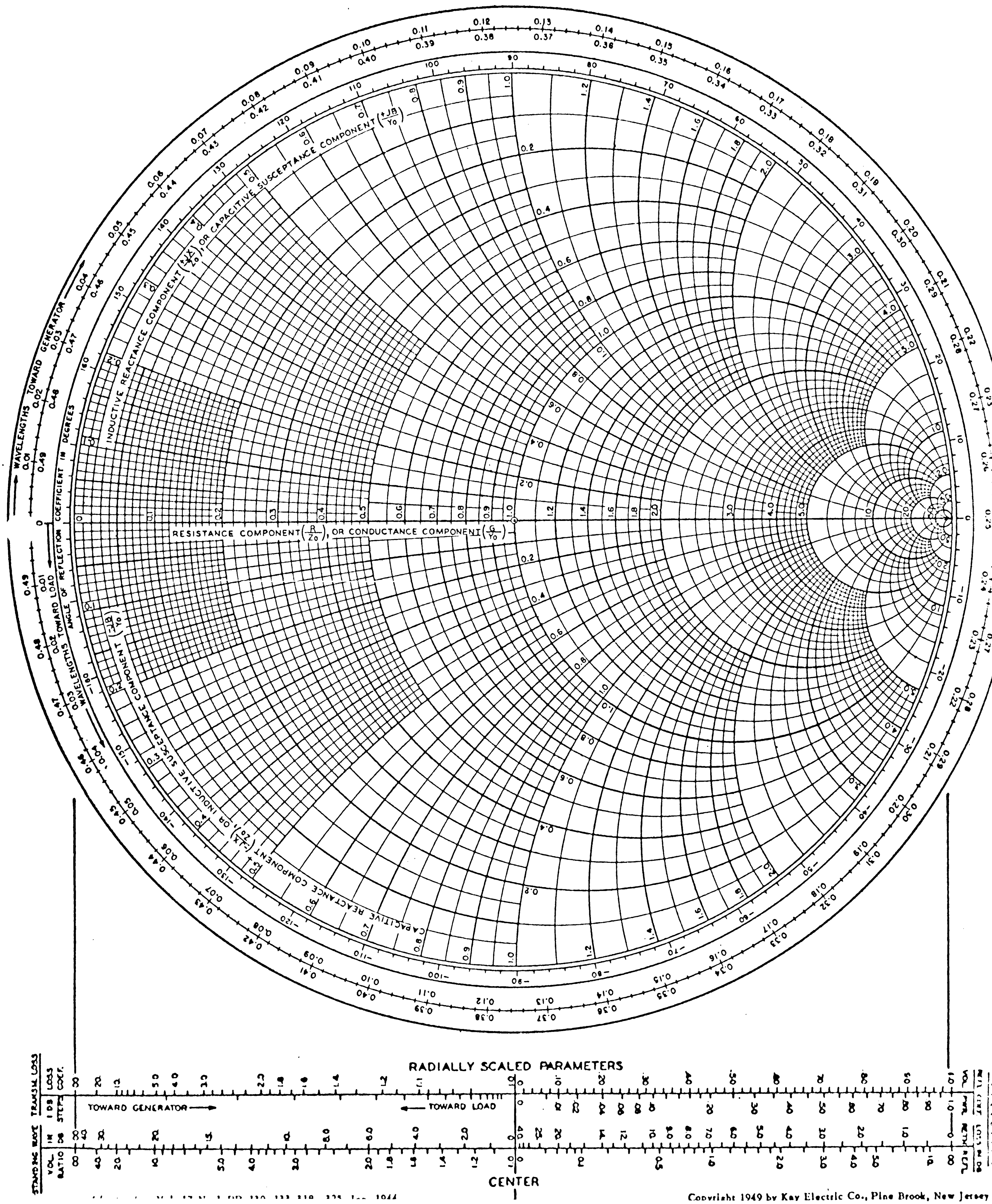
$$\bar{Z}_L = \frac{-1.618 - j\,0.221}{1.397 - j\,1.176}$$

$$\bar{Z}_L = -0.6 - j\,0.664$$

A

NAME	TITLE	DWG. NO.
SMITH CHART Form 756-N	GENERAL RADIO COMPANY, WEST CONCORD, MASSACHUSETTS	DATE

IMPEDANCE OR ADMITTANCE COORDINATES

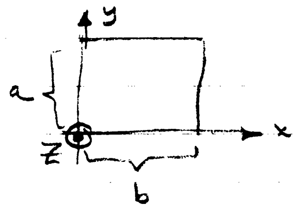


... Vol. 17, No. 1, pp. 130-133, 318-325, Jan. 1944

FRANCIS L. MERAT
ENGR 339
3/6/72

3.1B.

TE$_{10}$ mode rectangular waveguide $\Rightarrow e_z \equiv 0$



1) find h_z

$$\nabla_t^2 h_z + k_c^2 h_z = 0$$

$$\frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} + k_c^2 h_z = 0$$

let $h_z = f(x) g(y)$

$$g \frac{d^2 f}{dx^2} + f \frac{d^2 g}{dx^2} + k_c^2 fg = 0$$

$$\frac{1}{f}\frac{d^2 f}{dx^2} + \frac{1}{g}\frac{d^2 g}{dx^2} = -k_c^2$$

let $k_x^2 + k_y^2 = k_c^2$ then

$$\frac{1}{f}\frac{d^2 f}{dx^2} = -k_x^2 \qquad \frac{1}{g}\frac{d^2 g}{dy^2} = -k_y^2$$

$$\frac{d^2 f}{dx^2} + k_x^2 f = 0 \qquad \frac{d^2 g}{dy^2} + k_y^2 g = 0$$

$$f = A\cos k_x x + B \sin k_x x \qquad g = C\cos k_y y + D \sin k_y y$$

boundary conditions require that normal component of $\overline{h}$ vanish at the walls

as $\overline{h} = -\frac{j\beta}{k_c^2} \nabla_t h_z$

this requires that $\overline{n}\cdot\overline{h} = -\frac{j\beta}{k_c^2}\, \overline{n}\cdot\nabla_t h_z = 0$

or that $\overline{n}\cdot\nabla_t h_z = 0$

A

thus $\frac{\partial h_z}{\partial x} = 0$ @ $x = 0, b$

$\frac{\partial h_z}{\partial y} = 0$ @ $y = 0, a$

$$h_z = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)$$
$$\frac{\partial h_z}{\partial x} = (-Ak_x \sin k_x x + Bk_x \cos k_x x)(C\cos k_y y + D\sin k_y y)$$
$$\frac{\partial h_z}{\partial y} = (A\cos k_x x + B\sin k_x x)(-Ck_y \sin k_y y + Dk_y \cos k_y y)$$

3.18. (cont)

$$h_z = A_{10} \cos \frac{\pi}{b} x$$

$$\bar{h} = \bar{a}_x \frac{j\beta_{10}}{k_{c,10}^2} A_{10} \frac{\pi}{b} \sin \frac{\pi}{b} x$$

$$\bar{J}_s = \bar{n} \times \bar{H}$$

$$\bar{H} = A_{10} \cos \frac{\pi}{b} x \, \bar{a}_z + \bar{a}_x \frac{j\beta_{10}}{k_{c,10}^2} A_{10} \frac{\pi}{b} \sin \frac{\pi}{b} x$$

as n changes for each wall — n is the inward normal on the wall

on the wall $x=0$ $\quad \bar{n} = \bar{a}_x$

$$\bar{a}_x \times \bar{a}_x = 0 \Rightarrow \bar{J}_s = \bar{a}_x \times \bar{a}_z \cdot A_{10} \cos \frac{\pi}{b} x$$

$$= -\bar{a}_y A_{10} \cos \frac{\pi}{b} x$$

$$= -\bar{a}_y A_{10}$$

on the wall $x = b$ $\quad \bar{n} = -\bar{a}_x$

$$\bar{a}_x \times (-\bar{a}_x) = 0 \Rightarrow \bar{J}_s = -\bar{a}_x \times \bar{a}_z A_{10} \cos \frac{\pi}{b} x$$

$$\bar{J}_s = +\bar{a}_y A_{10} \cos \pi$$

$$\bar{J}_s = -\bar{a}_y A_{10}$$

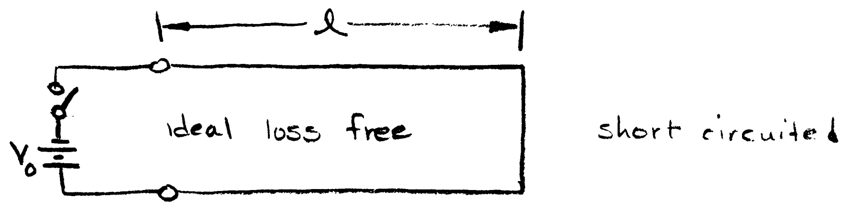
on the wall $y=0$ $\quad \bar{n} = \bar{a}_y$

$$J_s = \bar{a}_x A_{10} \cos \frac{\pi}{b} x + \bar{a}_z \frac{j\beta_{10}}{k_{c,10}^2} A_{10} \frac{\pi}{b} \sin \frac{\pi}{b} x$$

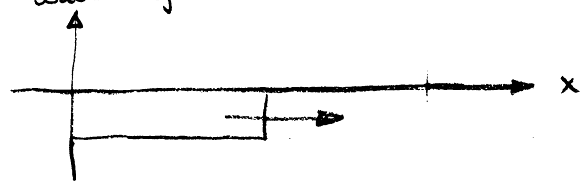
on the wall $y = a$ $\quad \bar{n} = -\bar{a}_y$

$$J_s = -\bar{a}_x A_{10} \cos \frac{\pi}{b} x - \bar{a}_z \frac{j\beta_{10}}{k_{c,10}^2} A_{10} \frac{\pi}{b} \sin \frac{\pi}{b} x$$

3.30.

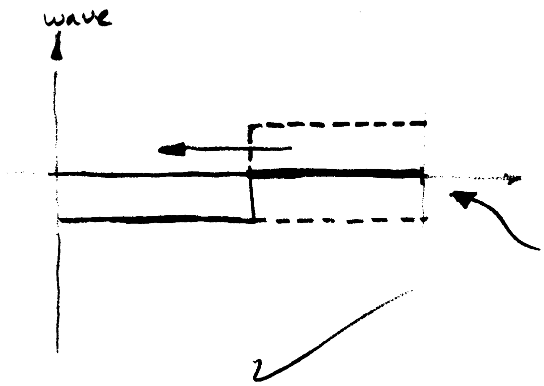


$0 < t < \frac{l}{c}$



a wave of $-V_0$ volts traveling in the $+x$ direction with a velocity c $\left(\frac{1}{\sqrt{\mu_0 \epsilon_0}}\right)$.

$\frac{l}{c} < t < \frac{2l}{c}$



B.C. is that the voltage is zero
this causes a wave of magnitude $+V_0$ to propagate in the $-x$ direction

A

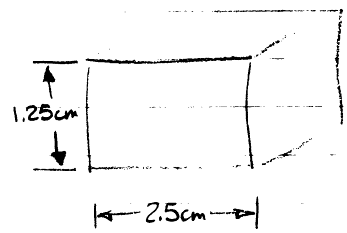
The overall effect is that we see a ~~the~~ boundary between $-V_0$ volts and 0 volts (the $+V_0$ & $-V_0$ waves cancel out) Propagate in the $-x$ direction.

$\frac{2l}{c} < t < \frac{3l}{c}$

B.C. at source is that $V(0) = -V_0 \;\forall t$
this causes a wave of magnitude $-V_0$ to propagate in the $+x$ direction

(same as $0 < t < \frac{l}{c}$)

3.34.



amplitude modulated signal $(1+m\cos\omega_m t)\cos\omega_c t$ is transmitted through the guide.
if $f_m = 20\,\text{kHz}$ and $f_c = 10{,}000\,\text{MHz}$, how long must the waveguide be before the upper & lower side bands undergo a relative phase shift of π radians.

$f_o(t) = f_i(t) * Z_w(t)$ where Z_w is the wave impedance

Fourier transforming

$$F_o(\omega) = F_i(\omega)\,Z_w(\omega)$$

$$Z_{w,nm} = \frac{k_0}{\beta_{nm}} Z_0 \qquad \text{where } \beta_{nm}^2 = k_0^2 - k_{c,nm}^2$$

$\Rightarrow$ the propagating mode must be determined

1) is 10^{10} Hz a propagating mode
at cutoff $\beta_{nm}=0$ or $k_0^2 = k_c^2$ or $k_0 = k_c$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c}$$

B

$$\therefore \quad k_c^2 = k_x^2 + k_y^2$$

from prob. 3.18 $k_c^2 = k_x^2 + k_y^2 = \dfrac{n^2\pi^2}{b^2} + \dfrac{m^2\pi^2}{a^2}$

if $b = 2a \Rightarrow a = \frac{b}{2}$

$$k_c^2 = \frac{n^2\pi^2}{b^2} + \frac{4m^2\pi^2}{b^2} = \frac{\pi^2}{b^2}(n^2+4m^2)$$

$$k_c = \frac{\pi}{b}\sqrt{n^2+4m^2}$$

$$\therefore \quad f_c = \frac{c}{2\pi}k_c = \frac{c}{2\pi}\frac{\pi}{b}\sqrt{n^2+4m^2}$$

$$= \frac{c}{2b}\sqrt{n^2+4m^2}$$

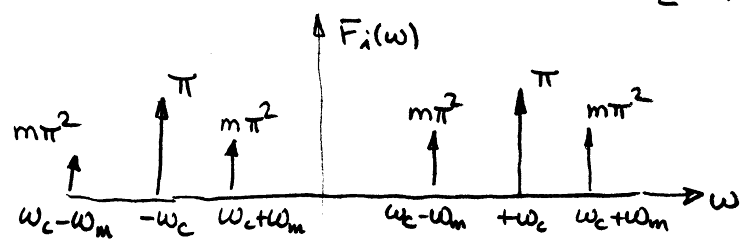
$$f_i(t) = (1+m\cos\omega_m t)\cos\omega_c t$$

$$F_i(\omega) = \mathfrak{F}[1+m\cos\omega_m t] * \mathfrak{F}[\cos\omega_c t]$$

$$= \left\{\mathfrak{F}[1] + \mathfrak{F}[m\cos\omega_m t]\right\} * \mathfrak{F}[\cos\omega_c t]$$

$$= \left\{\delta(\omega) + m\pi\left\{\delta(\omega-\omega_m)+\delta(\omega+\omega_m)\right\}\right\} *^{\pi}\left\{\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right\}$$

$$= \pi\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right] + m\pi^2\left[\delta(\omega-\omega_c-\omega_m)+\delta(\omega-\omega_c+\omega_m) + \delta(\omega+\omega_c-\omega_m)+\delta(\omega+\omega_c+\omega_m)\right]$$



$F_o(\omega)$ will consist of these impulses multiplied by the wave impedance at that frequency $\left[120\pi\omega\left(\omega^2-\frac{c^2\pi^2}{6.25}\right)\right]$

$$v_p^{(\omega)} = \frac{\omega}{\beta}$$

but $$v_p(\omega) = v_p(\omega_c) + \left.\frac{dv_p}{d\omega}\right|_{\omega_c}(\omega-\omega_c) + \frac{1}{2}\frac{d^2v_p}{d\omega^2}(\omega-\omega_c)^2$$

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\left(\frac{\omega^2}{c^2}-\frac{\pi^2}{6.25}\right)^{\frac{1}{2}}}$$

$$\frac{dv_p}{d\omega} = \omega\left(-\frac{1}{\cancel{2}}\right)\frac{1}{\left(\frac{\omega^2}{c^2}-\frac{\pi^2}{6.25}\right)^{\frac{3}{2}}}\cancel{2}\omega/c^2 + \frac{1}{\left(\frac{\omega^2}{c^2}-\frac{\pi^2}{6.25}\right)^{\frac{1}{2}}}$$

$$= \frac{-\omega^2/c^2 + \omega^2/c^2 - \pi^2/6.25}{\left(\frac{\omega^2}{c^2}-\frac{\pi^2}{6.25}\right)^{3/2}} = -\frac{\pi^2}{6.25}\frac{1}{\left(\frac{\omega^2}{c^2}-\frac{\pi^2}{6.25}\right)^{3/2}}$$

let l be the point at which the sidebands are π radians out of phase. At that point

$$f(t-\beta_0' z)\cos(\underbrace{\omega_0 t-\beta_0 z}_{\text{phase factor}})$$

$$(\omega_0 t-\beta_0 z)_{\omega_c+\omega_m}-(\omega_0 t-\beta_0 z)_{\omega_c-\omega_m}=\pi$$

$$(\omega_c+\omega_m)t-\beta(\omega_c+\omega_m)l-(\omega_c-\omega_m)t+\beta_0(\omega_c-\omega_m)l=\pi$$

$$t-\frac{\beta(\omega_c+\omega_m)}{\omega_c+\omega_m}l-\left(\frac{\omega_c-\omega_m}{\omega_c+\omega_m}\right)t+\frac{\beta_0(\omega_c-\omega_m)l}{\omega_c+\omega_m}=\frac{\pi}{\omega_c+\omega_m}$$

$$\left[\frac{\cancel{\omega_c}+\omega_m-\cancel{\omega_c}+\omega_m}{\omega_c+\omega_m}\right]t+\frac{\beta_0(\omega_c-\omega_m)l}{\omega_c+\omega_m}-\frac{\beta(\omega_c+\omega_m)}{\omega_c+\omega_m}l=\frac{\pi}{\omega_c+\omega_m}$$

$$\omega_c \gg \omega_m \rightarrow \frac{2\omega_m}{\omega_c+\omega_m}\approx 0$$

$$\frac{\pi}{\omega_c+\omega_m}\approx 0$$

and $\omega_c+\omega_m \approx \omega_c$

$$\rightarrow \frac{\beta_0(\omega_c-\omega_m)l}{\cancel{\omega_c}}-\frac{\beta(\omega_c+\omega_m)}{\cancel{\omega_c}}l=\frac{\pi}{\cancel{\omega_c}}$$

$$l=\frac{\pi}{\beta(\omega_c-\omega_m)-\beta(\omega_c+\omega_m)}$$

$$\beta_{10} = \left(\frac{\omega^2}{c^2} - \frac{\pi^2}{6.25}\right)^{\frac{1}{2}}$$

let $\beta_{10}(\omega_c \pm \omega_m) = \beta_{10}(\omega_c) + \left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_c}(\omega_c - \omega_m)$

$$\beta_{10}(\omega_c - \omega_m) - \beta_{10}(\omega_c + \omega_m) = \cancel{\beta_{10}(\omega_c)} + \left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_c}(\omega_c - \omega_m) - \cancel{\beta_{10}(\omega_c)} - \left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_c}(\omega_c + \omega_m)$$

$$= \left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_c}(\omega_c - \omega_m - \omega_c + \omega_m)$$

$$= \left.\frac{\partial\beta}{\partial\omega}\right|_{\omega=\omega_c}(-2\omega_m)$$

$$\frac{\partial\beta_{10}}{\partial\omega} = \frac{1}{\cancel{2}}\left(\frac{\omega^2}{c^2} - \frac{\pi^2}{6.25}\right)^{-\frac{1}{2}} \frac{\cancel{2}\omega}{c^2} = \frac{\omega/c^2}{\left(\frac{\omega^2}{c^2} - \frac{\pi^2}{6.25}\right)^{\frac{1}{2}}}$$

$$\left.\frac{\partial\beta_{10}}{\partial\omega}\right|_{\omega_c=2\pi\times10^{10}} = \frac{\frac{6.28\times10^{10}}{9\times10^{20}} = 6.97\times10^{-10}}{\left[\frac{(6.28)^2\times\cancel{10^{20}}}{9\times\cancel{10^{20}}} - \frac{(3.14)^2}{6.25}\right]^{\frac{1}{2}}} = \frac{6.97\times10^{-10}}{(4.38-1.58)^{\frac{1}{2}}}$$

$$\frac{(6.28)^2}{9} = 4.38 \qquad \frac{(3.14)^2}{6.25} = \frac{9.88}{6.25} = 1.58 \qquad (4.38-1.58)^{\frac{1}{2}} = \sqrt{2.80}$$

$\omega_c = 6.28\times10^{10}$

$$\left.\frac{\partial\beta_{10}}{\partial\omega}\right|_{\omega_c=2\pi\times10^{10}} = \frac{6.97\times10^{-10}}{1.67} = 4.17\times10^{-10}$$

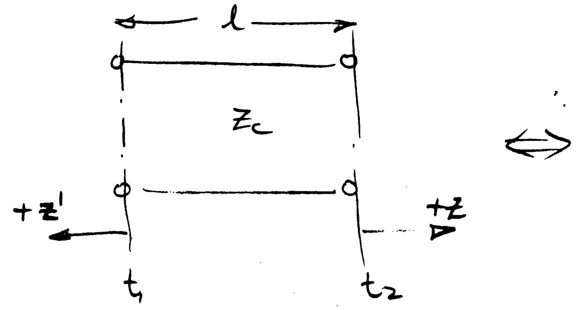
$$\therefore \quad l = \frac{\pi}{2\omega_m \left.\frac{\partial\beta_{10}}{\partial\omega}\right|_{\omega_c}} = \frac{\cancel{3.14}}{2\cdot\frac{\cancel{(6.28)}}{2}(2\times10^4)(4.17\times10^{-10})} \quad \frac{1}{8(4.17)\times10^{-6}} \text{ cm}$$

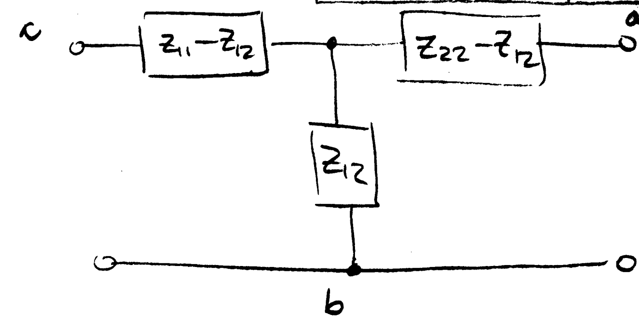
$$l = \frac{1}{33.3}\times10^6 \text{ cm} = 3\times10^4 \cancel{\text{cm}} \times \frac{10^{-2}\text{ m}}{\cancel{\text{cm}}} = 300\text{ m}$$

$$\boxed{l = 300 \text{ meters}}$$

FRANCIS MERAT
ENGR 339
MARCH 17, 1972

4.9





at $z' = -l$

$$Z(l) = \frac{Z(0) + j Z_c \tan\beta l}{Z_c + j Z(0) \tan\beta l} Z_c$$

let $Z(z'=0) = \infty$

then $Z(z'=-l) = \dfrac{Z_c}{j\tan\beta l} = -jZ_c \cot\beta l$

in the T-network model this is equivalent to ~~letting~~ the a-b junction open and examing the impedance $Z_{a'-b}$

$$Z_{a'-b} = Z_{22} - Z_{12} + Z_{12} = Z_{22}$$

but $Z_{a'-b} = -jZ_c \cot\beta l$

A

$$\Rightarrow \boxed{Z_{22} = -jZ_c \cot\beta l}$$

at $z = -l$

$$Z(l) = \frac{Z(z=0) + j Z_c \tan\beta l}{Z_c + j Z(z=0) \tan\beta l} Z_c$$

again let the line be open at $z = 0$ i.e. $Z(z=0) = \infty$

$$Z(l) = \frac{Z_c}{j\tan\beta l} = -jZ_c \cot\beta l$$

in the T-network model this is equivalent to letting $Z_{a'-b} = \infty$ and measuring Z_{a-b}

$$Z_{a-b} = Z_{11} - Z_{12} + Z_{12} = Z_{11}$$

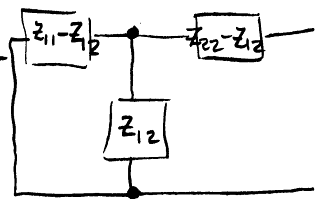
but $Z_{a-b} = -jZ_c \cot\beta l$

$$\Rightarrow \boxed{Z_{11} = -jZ_c \cot\beta l}$$

to get Z_{12} use reference plane t_1 again

let $Z(z=0) = 0$

i.e. consider



by Z-transformation of transmission line

$$Z_{a'-b} = Z_c \frac{Z(z=0) + jZ_c \tan\beta l}{Z_c + jZ(z=0)\tan\beta l} = jZ_c \tan\beta l$$

from T-network

$$Z_{a'b} = Z_{22} - Z_{12} + \frac{(Z_{11}-Z_{12})Z_{12}}{Z_{11} - \cancel{Z_{12}} + \cancel{Z_{12}}}$$

$$= Z_{22} - Z_{12} + \frac{Z_{12}}{Z_{11}}(Z_{11}-Z_{12})$$

$$= \frac{Z_{22}Z_{11} - \cancel{Z_{12}Z_{11}} + \cancel{Z_{12}Z_{11}} - Z_{12}^2}{Z_{11}} = \frac{Z_{22}Z_{11} - Z_{12}^2}{Z_{11}}$$

but $Z_{11} = Z_{22}$

$$Z_{a'-b} = \frac{Z_{11}^2 - Z_{12}^2}{Z_{11}}$$

solving for Z_{12}^2 $\quad Z_{11}Z_{a'-b} = Z_{11}^2 - Z_{12}^2$

$$Z_{12}^2 = Z_{11}^2 - Z_{11}Z_{a'-b}$$

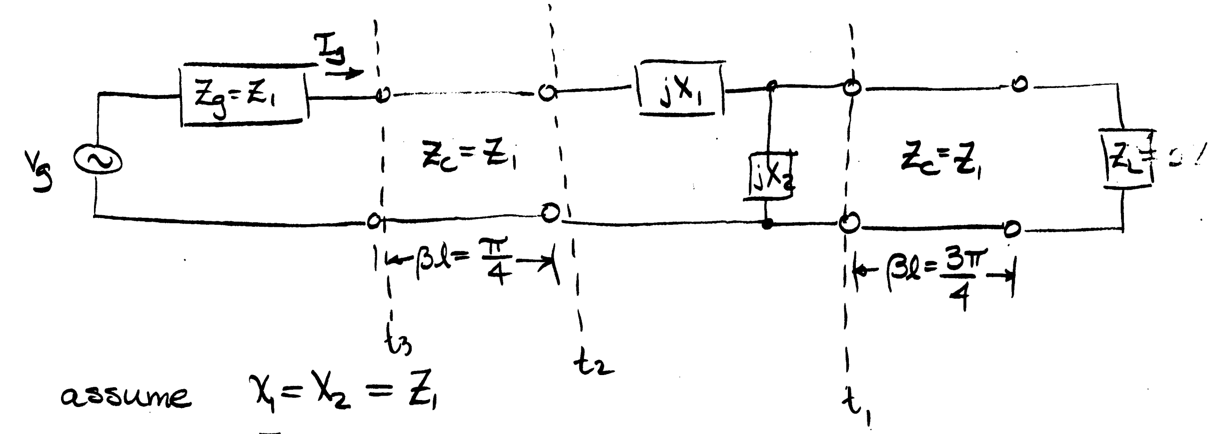
$$= j^2 Z_c^2 \cot^2\beta l - (-jZ_c\cot\beta l)(jZ_c\tan\beta l)$$

$$= -Z_c^2\cot^2\beta l - Z_c^2$$

$$= -Z_c^2(\cot^2\beta l + 1) = -Z_c^2 \csc^2\beta l$$

$$\Rightarrow \boxed{Z_{12} = \pm jZ_c \csc\beta l}$$

4.11



assume $X_1 = X_2 = Z_1$

$Z_L = 2Z_1$

$V_g = 5$ volts (peak)

transform Z_L to plane t_3 using intermediate planes t_1 and t_2

at plane t_1 $\quad Z_L' = Z_c \left(\dfrac{Z_c + jZ_L \tan\beta l}{Z_L + jZ_c \tan\beta l} \right)^{-1}$ $\quad Z_L' = Z_c \dfrac{Z_L + jZ_c \tan\beta l}{Z_c + jZ_L \tan\beta l}$

$$= Z_1 \frac{1 + j2\tan\beta l}{2 + j\tan\beta l}$$

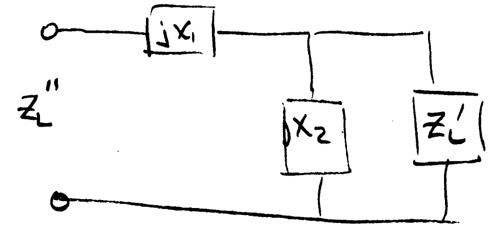
Procedure OK except for this error

See your Prob. 4.9

but $\tan\beta l = \tan\frac{3\pi}{4} = -\tan\frac{\pi}{4} = -1$

$$Z_L' = Z_1 \frac{1+2j}{2-j}\frac{(2+j)}{(2+j)} = Z_1 \frac{4-j3}{5}$$

at t_2



A

$$Z_L'' = jX_1 + \frac{jX_2 Z_L'}{jX_2 + Z_L'} = jZ_1 + \frac{jZ_1\left(\frac{4-j3}{5}\right)\cancel{Z_1}}{j\cancel{Z_1} + \cancel{Z_1}\left(\frac{4-j3}{5}\right)}$$

$$= jZ_1 + \left(\frac{j+2}{2}\right)Z_1 = Z_1\left(\frac{2j+j+2}{2}\right) = Z_1\left(\frac{3j+2}{2}\right)$$

$\tan\frac{\pi}{4} = +1$

at t_3 $\quad Z_L''' = Z_c \dfrac{Z_c + jZ_L \tan\beta l}{Z_L + jZ_c \tan\beta l} = Z_1 \dfrac{Z_1 + j\left(\frac{3j+2}{2}\right)Z_1}{\left(\frac{3j+2}{2}\right)Z_1 + jZ_1} = Z_1\left(\dfrac{8+9j}{25}\right)$

$$I_g = \frac{V_g}{Z_g + Z_L''} = \frac{\sqrt{5}}{Z_1 + Z_1\left(\frac{8+9j}{25}\right)} = \frac{125}{(33+9j)\,Z_1}$$

power delivered to Z_L'' is

$$P = \mathrm{Re}\,\tfrac{1}{2} V_g I_g^* = \tfrac{1}{2}(5)\,\frac{125}{(33-9j)\,Z_1^*}\;\frac{33+9j}{33+9j}$$

$$= \mathrm{Re}\,\frac{625}{2}\;\frac{33+9j}{1089+81}\;\frac{1}{Z_1^*} = \mathrm{Re}\,\frac{625}{2}\;\frac{33}{1170}\;\frac{1}{Z_1^*}$$

$$= \frac{125\cdot 33}{2(234)\,|Z_1|^2}\;\mathrm{Re}\,Z_1$$

$$= \frac{4125}{508\,|Z_1|^2}\;\mathrm{Re}\,Z_1$$

assume no losses in the system this is also the power delivered to the load.

standing wave ratios

@ t_2

$$\Gamma = \frac{Z_L'' - Z_c}{Z_L'' + Z_c} = \frac{\frac{3j+2}{2} - 1}{\frac{3j+2}{2} + 1} = \frac{3j+2-2}{3j+2+2} = \frac{3j}{\cancel{3j+4}}\;\frac{4-j3}{4-j3}$$

$$= \frac{12j+9}{25}$$

$$|\Gamma| = \sqrt{\frac{144+81}{(25)^2}} = \frac{\sqrt{225}}{25} = \frac{15}{25} = \frac{3}{5}$$

✓

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{3}{5}}{1-\frac{3}{5}} = \frac{8}{2} = 4$$

$$\boxed{S \text{ at plane } t_2 = 4}$$

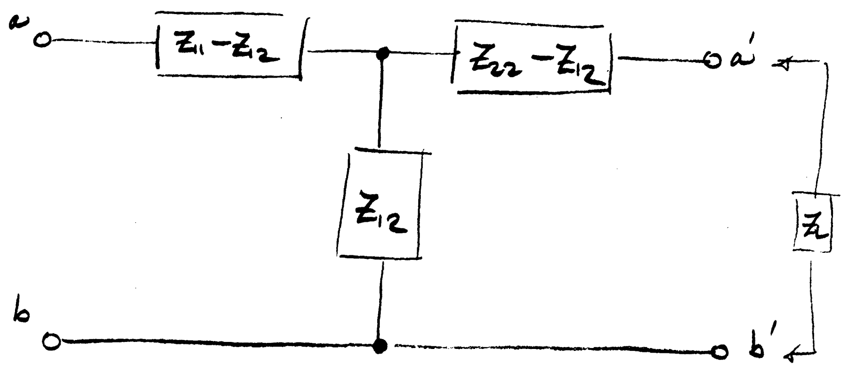
at plane t_1

$$\Gamma = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{2Z_1 - Z_1}{2Z_1 + Z_1} = \frac{1}{3}$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \frac{4}{2} = 2$$

$$\boxed{s = 2}$$

4.12.

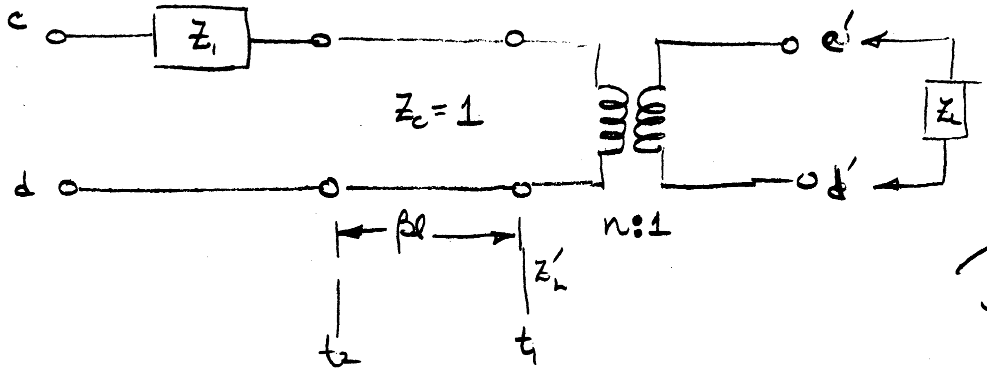


$Z_{11} = j2$

$Z_{12} = \frac{j}{\sqrt{2}}$

$Z_{22} = -\frac{j}{4}$

transform to



B

$@t_1 \quad Z_L' = n^2 Z_L$

let $Z_L = \infty$ then $Z_{a'b'} = Z_{c'd'} = \infty$

$$Z_{ab} = Z_{11} - Z_{12} + Z_{12} = Z_{11}$$

$$Z_L' = n^2 Z_L = \infty$$

$$@t_2 \quad Z_L'' = Z_c \frac{Z_L' + jZ_c \tan\beta l}{Z_c + jZ_L' \tan\beta l} = \frac{Z_L' + j\tan\beta l}{1 + jZ_L' \tan\beta l}$$

$$= \frac{1}{j\tan\beta l} = -j\cot\beta l$$

$\Rightarrow \quad Z_{cd} = Z_1 + Z_L'' = Z_1 - j\cot\beta l$

equating Z_{cd} & Z_{ab}

$$Z_{11} = Z_1 - j\cot\beta l$$

let $Q \triangleq \cot\beta l$

$$Z_{11} = Z_1 - jQ \qquad (1)$$

let $Z_L = 0$ then

$$Z_{a'b'} = Z_{c'd'} = 0$$

$$Z_{ab} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22} - Z_{12})}{Z_{22} - Z_{12} + Z_{12}} = Z_{11} - Z_{12} + \frac{Z_{12}Z_{22} - Z_{12}^2}{Z_{22}}$$

$$Z_{ab} = \frac{Z_{11}Z_{22} - Z_{12}Z_{22} + Z_{12}Z_{22} - Z_{12}^2}{Z_{22}}$$

$$Z_{ab} = \frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{22}} \triangleq K = \frac{(j2)(-\frac{j}{4}) - \frac{j^2}{2}}{-\frac{j}{4}} = \frac{+j^2 4}{+j} = 4j \checkmark$$

$$Z_L' = n^2 Z_L = 0$$

$$Z_L'' = Z_c \frac{Z_L' + jZ_c \tan\beta l}{Z_c + jZ_L' \tan\beta l} = \frac{Z_L' + j\tan\beta l}{1 + jZ_L' \tan\beta l}$$

$$= j\tan\beta l = \frac{j}{Q}$$

$$Z_{cd} = Z_1 + Z_L'' = Z_1 + \frac{j}{Q}$$

but $Z_{cd} = Z_{ab}$

$$\Rightarrow K = Z_1 + \frac{j}{Q} \checkmark \qquad (2)$$

at this point we have 2 equations (1) & (2) in 2 unknows [Z_1 and Q]

$$Z_{11} = Z_1 - jQ$$

$$K = Z_1 + \frac{j}{Q}$$

$$Z_{11} - K = -jQ - \frac{j}{Q}$$

$$\frac{(Z_{11} - K)Q}{-j} = \frac{-jQ^2 - j}{-j}$$

$$Q^2 + 1 = j(Z_{11} - K)Q$$

$$Q^2+1 = j(Z_{11}-K)Q$$

$$Q^2 - j(Z_{11}-K)Q + 1 = 0$$

$$Q = \frac{j(Z_{11}-K) \pm \sqrt{-(Z_{11}-K)^2+4}}{2}$$

$$\boxed{\cot\beta l = Q = \frac{j(Z_{11}-K) \pm \sqrt{4-(Z_{11}-K)^2}}{2}}$$

$$Q = \frac{j(j2-4j) \pm \sqrt{4-(j2-4j)^2}}{2}$$

$$= 1 \pm \sqrt{2}$$

solving second equation for Q

$$K = Z_1 + \frac{j}{Q}$$

$$K - Z_1 = \frac{j}{Q}$$

$$Q = \frac{j}{K-Z_1}$$

$Z_1 = Z_{11} + jQ$ (from previous page)

$= 2j + j \pm j\sqrt{2}$

$= (3 \pm \sqrt{2})j$

$\Rightarrow$ subst. in first equation

$$Z_{11} = Z_1 - j\left(\frac{j}{K-Z_1}\right)$$

$$Z_{11} = Z_1 + \frac{1}{K-Z_1}$$

$$Z_{11}(K-Z_1) = Z_1^2+1 \;?$$

$$Z_{11}(K-Z_1) = Z_1(K-Z_1)+1$$

$$= Z_1K - Z_1^2 + 1$$

$$Z_{11}K - Z_{11}Z_1 = Z_1^2+1$$

$$Z_1^2 + Z_{11}Z_1 + (1-Z_{11}K) = 0$$

$$\boxed{Z_1 = \frac{-Z_{11} \pm \sqrt{Z_{11}^2 - 4(1-Z_{11}K)}}{2}}$$

$$Z_1 = \frac{-j2 \pm \sqrt{(j2)^2 - 4(1-(j2)(j4))}}{2}$$

$$= -j \pm j\sqrt{10} = j(-1 \pm \sqrt{10})$$

to determine n

let $\quad Z_L = Z_{12}$

then
$$Z_{ab} = Z_{11} - Z_{12} + \frac{Z_{12} Z_{22}}{Z_{12} + Z_{22}}$$

$$Z_{ab} = \frac{Z_{11} Z_{12} - Z_{12}^2 + Z_{11} Z_{22} - \cancel{Z_{12} Z_{22}} + \cancel{Z_{12} Z_{22}}}{Z_{12} + Z_{22}}$$

$$Z_{ab} = \frac{Z_{11}(Z_{12} + Z_{22}) - Z_{12}^2}{Z_{12} + Z_{22}}$$

$$= Z_{11} - \frac{Z_{12}^2}{Z_{12} + Z_{22}}$$

$$Z_L' = n^2 Z_L = n^2 Z_{12}$$

$$Z_L'' = Z_c \frac{Z_L' + j Z_c \tan \beta l}{Z_c + j Z_L' \tan \beta l} = \frac{Z_L' + j \tan \beta l}{1 + j Z_L' \tan \beta l} = \frac{Z_L' Q + j}{Q + j Z_L'}$$

where Q is as defined previously

$$Z_{cd} = Z_1 + Z_L'' = Z_1 + \frac{Z_L' Q + j}{Q + j Z_L'} = Z_1 + \frac{n^2 Z_{12} Q + j}{Q + j n^2 Z_{12}}$$

$$= Z_{11} - \frac{Z_{12}^2}{Z_{12} + Z_{22}}$$

$$\frac{Z_{11} Z_{12} + Z_{11} Z_{22} - Z_{12}^2}{Z_{11} + Z_{22}} = \frac{Z_1 Q + j n^2 Z_1 Z_{12} + n^2 Z_{12} Q + j}{Q + j n^2 Z_{12}}$$

$$(Z_{11} Z_{12} + Z_{11} Z_{22} - Z_{12}^2) Q + (Z_{11} Z_{12} + Z_{11} Z_{22} - Z_{12}^2) j n^2 Z_{12} = (Z_1 Q + j)(Z_{11} + Z_{22}) + n^2 (j Z_1 Z_{12} + Z_{12} Q)(Z_{11} + Z_{22})$$

$$\boxed{n^2 = \frac{-(Z_{11} Z_{12} + Z_{11} Z_{22} - Z_{12}^2) Q - (Z_1 Q + j)(Z_{11} + Z_{22})}{(j Z_1 Z_{12} + Z_{12} Q)(Z_{11} + Z_{22}) - j Z_{12}(Z_{11} Z_{12} + Z_{11} Z_{22} - Z_{12}^2)}}$$

where Q & Z_1 are known.

$$Z_1 + \frac{n^2 Z_{12} Q + j}{Q + jn^2 Z_{12}} = Z_{11} - \frac{Z_{12}^2}{Z_{12}+Z_{22}}$$

$$j(-1\pm\sqrt{10}) + \frac{n^2 \frac{j}{\sqrt{2}}(1\pm\sqrt{2}) + j}{1\pm\sqrt{2} + jn^2\frac{j}{\sqrt{2}}} = j2 - \frac{\frac{j^2}{2}}{j2 - \frac{j}{4}}$$

$$(-1\pm\sqrt{10}) + \frac{n^2\left(\frac{1\pm\sqrt{2}}{\sqrt{2}}\right)+1}{(1\pm\sqrt{2}) - \frac{n^2}{\sqrt{2}}} = 2 - \frac{\frac{j}{2}}{j2-\frac{j}{4}} = 2 - \frac{\frac{1}{2}}{2-\frac{1}{4}} = \frac{12}{7}$$

$$(-1\pm\sqrt{10})(1\pm\sqrt{2}) - n^2\frac{(-1\pm\sqrt{10})}{\sqrt{2}} + n^2\left(\frac{1\pm\sqrt{2}}{2}\right) + 1 = \frac{12}{7}$$

$$n^2\left[\frac{-1\pm\sqrt{10}}{\sqrt{2}} - \frac{1\pm\sqrt{2}}{2}\right] = (-1\pm\sqrt{10})(1\pm\sqrt{2}) - \frac{5}{7}$$

$$n^2 = \frac{\left[(-1\pm\sqrt{10})(1\pm\sqrt{2}) - \frac{5}{7}\right]2\sqrt{2}}{2(-1\pm\sqrt{10}) - \sqrt{2}(1\pm\sqrt{2})}$$

if choose $-1+\sqrt{10}$ and $1-\sqrt{2}$ $\quad n^2 < 0$

$-1+\sqrt{10}$ and $1+\sqrt{2}$ $\quad n^2 > 0$

$-1-\sqrt{10}$ and $1-\sqrt{2}$ $\quad n^2 < 0$

$-1-\sqrt{10}$ and $1+\sqrt{2}$ $\quad n^2 > 0$

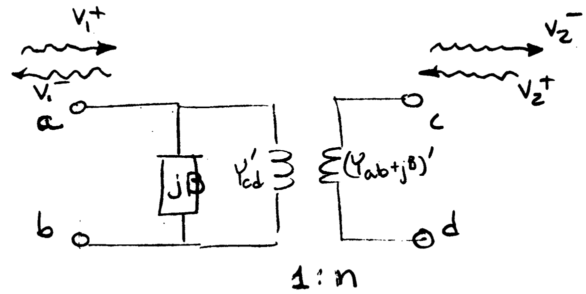
Procedure is OK but your value for Z_1 is not correct.

$\cot \beta l = 1+\sqrt{2}$ or $\boxed{\beta l = \text{Arccot}(1+\sqrt{2})}$

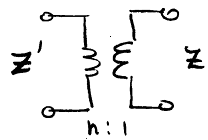
$$\boxed{\text{if } \quad Z_1 = -1+\sqrt{10} \qquad n^2 = \frac{-1+\sqrt{10}-\sqrt{2}+\sqrt{20}-\frac{5}{7}}{2\sqrt{10}-4-\sqrt{2}}\cdot 2\sqrt{2}}$$

$$\boxed{\text{if } \quad Z_1 = -1-\sqrt{10} \qquad n^2 = \frac{-1-\sqrt{2}-\sqrt{10}-\sqrt{20}-\frac{5}{7}}{-4-\sqrt{2}-\sqrt{10}}\, 2\sqrt{2}}$$

4.25



A



$z' = n^2 z$

$\frac{1}{Y'} = n^2 \frac{1}{Y} \Rightarrow Y = n^2 Y'$

if the turns ratio is reversed $Y = \frac{1}{n^2} Y'$ or $Y' = n^2 Y$

let us assume that c-d connect to a waveguide of $Y_c = 1$

furthermore, let us match the junction to the wave guide i.e. the admittance looking into the junction from terminals c,d is Y_c.

if $Y_{cd} = 1$

Y_{cd} appears as $n^2 Y_c$ or n^2 on the other side of the transformer.

then Y_{in} as seen at ab is given by

$$Y_{IN} = jB + Y_{cd}' = jB + n^2$$

if a waveguide of characteristic $Y_c = 1$ is connected to ab we have

$$\Gamma_{ab} = \frac{Y_c - Y_{IN}}{Y_c + Y_{IN}} = \frac{1-(jB+n^2)}{1+(jB+n^2)}$$

but if the network matches Y_c at cd then $\Gamma_{cd} = 0 = \frac{V_2^-}{V_2^+}$

$\Rightarrow V_2^- = 0$

but $V_1^- = S_{11} V_1^+ + S_{12} V_2^+$

$= S_{11} V_1^+$

or $S_{11} = \frac{V_1^-}{V_1^+} = \Gamma_{ab} = \frac{1-(jB+n^2)}{1+(jB+n^2)}$ ✓

thus $(jB + n^2) = \dfrac{1 - S_{11}}{1 + S_{11}}$

but $S_{11} = \dfrac{1-j}{3+j}$

$$\frac{1 - S_{11}}{1 + S_{11}} = \frac{3+j-1+j}{3+j+1-j} = \frac{2+2j}{4} = \frac{1}{2}(1+j)$$

$$jB + n^2 = \frac{1}{2}(1+j)$$

as n^2 must be real

$$n^2 = \frac{1}{2}$$

$$jB = j\frac{1}{2}$$

$$\rightarrow \quad B = \frac{1}{2}$$