$$P_{diss} = \int_{\text{surface of guide}} \tfrac{1}{2} R_s |H_{tan}|^2 \, dS = \Delta z \int \tfrac{1}{2} R_s |H_{tan}|^2 \, dl$$

$$2\alpha = \frac{\int R_s |H_{tan}|^2 \, dl}{\int Re\,(\hat{e}_t \times \hat{h}_t)\cdot \underline{a_z}\, dA}$$ ← power dissipated / ← power transmitted.

$$P = \tfrac{1}{2} E \times H^*$$

T.M

$$E_z = A J_n(k_c r) \begin{cases} \cos n\phi \\ \sin n\phi \end{cases}$$

$$H_r = -j \frac{n f}{k_c \eta r f_c} A J_n(k_c r) \begin{cases} \sin n\phi \\ -\cos n\phi \end{cases}$$

$$H_\phi = -j \frac{f}{f_c \eta} A J_n'(k_c r) \begin{cases} \cos n\phi \\ \sin n\phi \end{cases}$$

$$E_\phi = -H_r Z_{TM}$$

$$E_r = H_\phi Z_{TM}$$

TE.

$$H_z = B J_n(k_c r) \begin{cases} \cos n\phi \\ \sin n\phi \end{cases}$$

$$E_r = j \frac{n \eta f}{k_c r f_c} B J_n(k_c r) \begin{cases} \sin n\phi \\ -\cos n\phi \end{cases}$$

$$E_\phi = j\eta \frac{f}{f_c} B J_n'(k_c r) \begin{cases} \cos n\phi \\ \sin n\phi \end{cases}$$

$$H_\phi = \frac{E_r}{Z_{TE}}$$

$$H_r = -\frac{E_\phi}{Z_{TE}}$$

for  cavity γ used boundary condition $n \cdot \nabla h$ on endwalls.

$$\gamma = j \frac{\omega}{v} \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{\frac{1}{2}}$$

$$Z_{TM} = \eta \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{\frac{1}{2}}$$

$$Z_{TE} = \eta \left[1 - \left(\frac{f_c}{f}\right)^2\right]^{-\frac{1}{2}}$$

The factor $e^{j\omega t - \gamma z}$ is assumed for negatively traveling waves ($e^{j\omega t \mp \gamma z}$) understood. Z_{TM} and Z_{TE} reverse sign.

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \; ; \; J = \sigma E \; ; \; \nabla \cdot E = \frac{\rho}{\epsilon_0} \quad \nabla \cdot B = 0 \quad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times \vec{H}) = \nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H}$$

$$\nabla \times \vec{H} = a_x \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] + a_y \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right] + \underline{a_z} \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

solve for E use $\nabla \cdot E$ & $\nabla \times E$ then use $\nabla \times H$ to get H.

$$P \triangleq \tfrac{1}{2} \hat{E} \times \hat{H}^*$$

$$-\nabla \cdot \frac{(\hat{E} \times \hat{H}^*)}{2} = \frac{\hat{E} \cdot \hat{J}^*}{2} + 2j\omega \left[\tfrac{1}{4}\mu |\hat{H}|^2 - \tfrac{1}{4}\epsilon |\hat{E}|^2\right]$$

$$v_g = \frac{d\omega}{d\beta} \quad ; \quad v_p = \frac{\omega}{\beta}$$

Power loss $= \frac{1}{2}\hat{J}\cdot\hat{E} = \frac{1}{2}\omega\epsilon''|E|^2$

because

$$J = j\omega(-j\epsilon'')E = \omega\epsilon'' E$$

if $\frac{\beta}{\omega}$ and $\frac{\partial\beta}{\partial\omega}$ differ in sign we have a backward wave

A backward traveling wave has the sign of the space component reversed from that of a normal foreward traveling wave.

$k^2 = \omega^2\mu\epsilon$

$\bar{J} = \bar{n} \times \bar{H}$ n is the normal to the surface.

Shunt resistance of a rectangular waveguide

$$G = \frac{2W_L}{(E_o b)^2}$$

b is the vertical separation of the plates

$$G(\theta,\phi) = \frac{P_r(\theta,\phi)}{\frac{1}{4\pi}\int P_r(\theta,\phi)\,d\Omega}$$

Area $A(\theta,\phi) = \frac{W_{MAX\ AT\ RCVR}}{P_r(\theta,\phi)}$

$$A_r(\theta,\phi) = \frac{\lambda^2}{4\pi} G_r(\theta,\phi)$$

$F(\theta) = \frac{60 I_m}{r} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$ $\frac{\lambda}{2}$ dipole pattern

$$Q = \frac{2\omega_o\langle u_e\rangle}{\langle P_d\rangle}$$

$\langle u_e\rangle = \frac{1}{4}\epsilon_o \int_{\text{vol of cavity}} |E|^2\,dv$; $\langle P_d\rangle = \frac{1}{2}R_s \int_{\text{wall surfaces}} |H_{tan}|^2\,ds$

$R_s = \frac{1}{\sigma\delta}$ $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

$$E_{total} = \frac{f(\theta,\phi)}{4\pi r}\sum_{i=0}^{N} e^{+jk(a_r\cdot a_x x_i)}$$

$$k = \frac{2\pi}{\lambda}$$

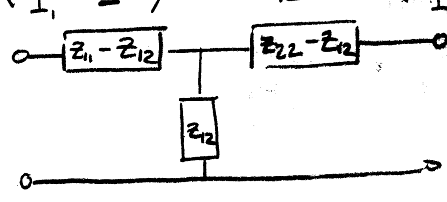
Bessel functions:

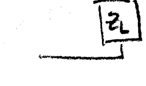
$$\frac{2n}{k_c r} J_n(k_c r) = J_{n+1}(k_c r) + J_{n-1}(k_c r)$$

$Z_{11} \triangleq 2_0(k_0 h)^2 + \frac{2_0 h^2}{jk_0 r^3}$ $Z_{21} \triangleq -\left(\frac{\underline{E_1}}{I_1}\cdot \underline{L_2}\, h\right)$ vector vector $Z_{12} \triangleq -\left(\frac{\underline{E_2}}{I_2}\cdot i_1 h\right)$

$P_2 = \frac{1}{2}[I_2 Z_{22} + Z_{21} I_1] I_2^*$

$P_1 = \frac{1}{2}[I_1 Z_{11} + Z_{12} I_2] I_1^*$





max power transfer when $Z_L = Z_{11}^*$

continuous tan E.

perfect conductors are equipotentials and have surface currents.

normal J is cont.

$$\oint_S D \cdot dS = \int_V \rho \, dV$$

$$\oint_S B \cdot dS = 0$$

$$\oint E \cdot dl = -\frac{\partial}{\partial t}\int_S B \cdot dS$$

$$\oint H \cdot dl = \int_S J \cdot dS + \frac{\partial}{\partial t}\int_S D \cdot dS$$

$\sigma = \infty$ $\Rightarrow$ no tangential $\vec{E}$

$\nabla \cdot J = \sigma \nabla \cdot E = -\frac{\partial \rho}{\partial t}$

$$n \cdot (J_1 - J_2) = 0$$

$$n \times (\vec{H}_1 - \vec{H}_2) = K$$

E $\oplus \rightarrow \ominus$

$w_m = w_e$ in a plane wave.

perfect conductor:

$$w_m = \frac{1}{2}\mu_0 H^2$$

$$w_e = \frac{1}{2}\epsilon_0 E^2$$

waveguides: $\gamma \triangleq$

$$-\underline{a_z} \times \nabla_t \hat{e}_z - \gamma \underline{a_z} \times \hat{\underline{e}}_t = -j\omega\mu \hat{\underline{h}}_t$$

$$\nabla_t \times \hat{\underline{e}}_t = -j\omega\mu \hat{\underline{h}}_z$$

$$-\underline{a_z} \times \nabla_t \hat{\underline{h}}_z - \gamma \underline{a_z} \times \hat{\underline{h}}_t = j\omega\epsilon \hat{\underline{e}}_t$$

$$\nabla_t \times \hat{\underline{h}}_t = j\omega\epsilon \hat{\underline{e}}_z$$

$$\nabla_t \cdot \hat{\underline{e}}_t = \gamma \hat{\underline{e}}_z$$

$$\nabla_t \cdot \hat{\underline{h}}_t = \gamma \hat{h}_z$$

cut-off when $\beta = 0$

TE $e_z = 0$

TM $h_z = 0$

$$\nabla^2 H + \omega^2\mu\epsilon H = 0$$

$$(\nabla_t^2 + \gamma^2)\hat{h}_t + \omega^2\mu\epsilon \hat{\underline{h}}_t = 0$$

$$(\nabla_t^2 + \gamma^2)\hat{h}_z + \omega^2\mu\epsilon \hat{h}_z = 0$$

to get dispersion relationship use fact that $\gamma^2 + \omega^2\mu\epsilon = k_c^2 = \frac{p_{mn}'^2}{a^2}$ from b.c.

$k_c a = p'_{mn}$

$$\frac{1}{2}\hat{V}\hat{I}^* = \langle P_d \rangle + 2j\omega\left[\langle \mathcal{U}_m \rangle - \langle \mathcal{U}_e \rangle\right]$$

remember $\frac{1}{4}$ for time average

$$\hat{Y} = \frac{\hat{I}}{\hat{V}}$$

for parallel

$$\langle \mathcal{U}_m \rangle = \frac{1}{4}L[I_e]^2 = \frac{1}{4}L\left|\frac{\hat{V}}{j\omega L}\right|^2 = \frac{1}{4}\frac{|\hat{V}|^2}{\omega^2 L}$$

$$\langle \mathcal{U}_e \rangle = \frac{1}{4}C|\hat{V}|^2$$

$$\frac{d^2f}{dr^2} + \frac{1}{r}\frac{df}{dr} + \left(k_c^2 - \frac{n^2}{r^2}\right)f = 0$$

soln is $f = B_1 J_n(k_c r) + B_2 Y_n(k_c r)$

TE $\frac{\partial h_z}{\partial r} = 0$ $r = a$ — normal component of magnetic field vanishes at a conducting surface.

TM tangential E field along the conducting boundary ~~must vanish~~

$e_z = 0$ $r = a$

the tangential component of E is continuous

difference in normal component of E $= \sigma_s$

normal component of B is continuous

tangent component of H differs by K_t. surface current density.

perfect conductor: tangential E & H must be continuous

$E_t = H_t = 0$ in media #2

perfect dielectric: Tangential E & H must be continuous (magnitude & phase)

in a good conductor $\frac{\hat{E}}{\hat{H}} = \frac{1+j}{\sigma\delta} = (1+j)R_s$

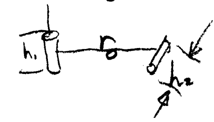
$$\left.\begin{array}{l}\text{tan E}\\ \text{tan H}\\ \text{nor B}\\ \text{nor D}\end{array}\right\}\text{cont.}$$

Poisson's equation $\nabla^2 \Phi = 0$ iff $\nabla \cdot E = 0$ & $\nabla \times E = 0$

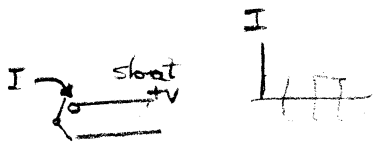
for cylindrical wave guide for transverse electric or H waves the required boundary condition is that the normal derivative of H_z be zero at all conducting surfaces. i.e. $J_n'(k_c a) = 0$

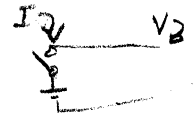
$$\oint_S \hat{P}\cdot ds = \int_S \tfrac{1}{2}(\hat{E}_1 \times \hat{H}_1^*)\cdot dS \Big\} P_1$$

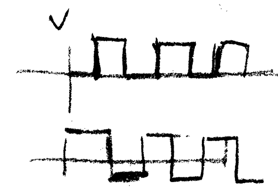
$$+ \int_S \tfrac{1}{2}(E_1 \times H_2^*)\cdot dS$$



$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \tau = \frac{2Z_L}{Z_L + Z_0}$$







$$Z_i = Z_0 \left[\frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right]$$

Z_i (short circuit) $= Z_0\, j \tan\beta l$

Z_i (open circuit) $= Z_0 - j\cot\beta l$

$$\bar{P} = \frac{1}{2}\sqrt{\frac{\mu}{\epsilon}}\left(\frac{Ih}{2\lambda r}\right)^2 \sin^2\theta\ \underline{a_r}$$

where $\lambda = \dfrac{1}{f\sqrt{\mu\epsilon}}$

$$G(\theta,\phi) = \frac{\hat{P}_r(r,\theta,\phi)}{\int_0^{\pi}\int_0^{2\pi} \hat{P}_r\, r^2 \sin\theta\, d\theta\, d\phi}\, 4\pi r^2$$

$$\hat{P} = \frac{1}{2}\left(\frac{I_0 h k^2}{4\pi}\right)^2 \sqrt{\frac{\mu}{\epsilon}} \begin{cases} \underline{a_r} \sin^2\theta \left[\frac{1}{k^2r^2} + \frac{1}{j(kr)^5}\right] \\ \underline{a_\theta}\, j \sin 2\theta \left[\frac{1}{(kr)^3} + \frac{1}{(kr)^5}\right] \end{cases}$$

wave propagating into an imperfect conductor

$$\nabla^2 \hat{E} = j\omega\mu_0\sigma \hat{E} \qquad \text{where } \sigma \gg \omega\epsilon_0$$

$$\gamma^2 = j\omega\mu_0\sigma$$

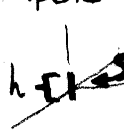
$$\sqrt{j\omega\mu_0\sigma} = \pm\sqrt{\omega\mu_0\sigma}\,\frac{1+j}{\sqrt{2}} = \pm\sqrt{\frac{\omega\mu_0\sigma}{2}}\,(1+j)$$

$$\int \sin^3\theta\, d\theta = -\frac{1}{3}\cos\theta\left[\sin^2\theta + 2\right] \qquad = \pm\frac{1+j}{\delta}$$

wave is of form $\hat{E}_x e^{\pm\left(\frac{1+j}{\delta}\right)z}$

dipole fields;

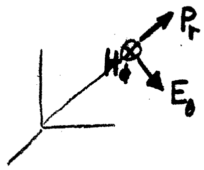
$$E_r = -\frac{I_0 h k^2}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\cos\theta\left[\frac{1}{(jkr)^2} + \frac{1}{(jkr)^3}\right]e^{-jkr}$$



the length of the antenna

$$E_\theta = -I_0\frac{hk^2}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\sin\theta\left[\frac{1}{jkr} + \frac{1}{(jkr)^2} + \frac{1}{(jkr)^3}\right]e^{-jkr}$$

$$H_\phi = -\frac{I_0 h k^2}{4\pi}\sin\theta\left[\frac{1}{jkr} + \frac{1}{(jkr)^2}\right]e^{-jkr}$$



$$P_r = \frac{E_\theta H_\phi^*}{2} = \frac{Z_r}{2}|H_\phi|^2$$

$$Z_r \triangleq \frac{E_\theta}{H_\phi} \qquad Z_p \triangleq -\frac{E_r}{H_\phi}$$