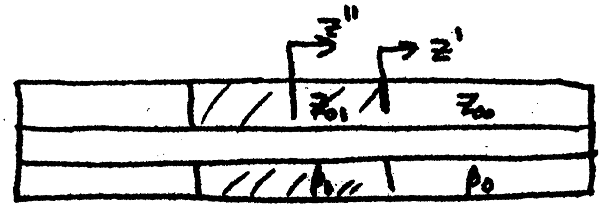
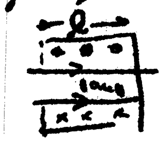
1.



the impedance of a coaxial cable, $Z_0 = \sqrt{\frac{\mu}{\epsilon}}\,\frac{\ln b/a}{2\pi}$

which can be found as follows $Z_0 = \sqrt{L/C}$ L, C per unit length →



$L = \frac{\text{flux}}{\text{amp.}} \times \frac{1}{l}$ let 1 amp flow in center conductor then

$B = \mu H = \frac{\mu I}{2\pi r}$ $a < r < b$

$\Phi = \int B \cdot da = \frac{\mu I l}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu I l}{2\pi}\ln b/a$ so $L = \frac{\mu}{2\pi}\ln b/a$

$C = Q/_{\text{volt}} \times \frac{1}{l}$ for which one should know the solution for the transverse potential $\phi = \phi_0 \ln r/a$ or $V_0 \ln r/a$

the transverse (E_r) field $\underline{E} = A/r\, a_r = -\phi_0/r\, a_r$
$= -V_0/r\, a_r$

from which we find the surface charge (on outer conductor) $\sigma = +V_0/b\,\epsilon$

and thus $Q = \sigma \times 2\pi b l$, $C = 2\pi\epsilon\, V_0 / \underbrace{V_0 \ln b/a}_{\text{pot. at outer cylinder}} = 2\pi\epsilon/\ln b/a$

$$\boxed{\text{so } Z_{0} = \sqrt{\frac{\mu}{\epsilon}}\,\frac{\ln b/a}{2\pi}}$$

[I did not expect you to derive this but you only need have determined the resonant freq in terms of Z_0 (not explicitly stated).]

call impedance of air section Z_{00}, dielectric section Z_{01}

$$Z' = j Z_{00}\tan\beta_0 l/2 \qquad Z'' = Z_{01}\left[\frac{j Z_{00}\tan\beta_0 l/2 \cos\beta_1 l/2 + j Z_{01}\sin\beta_1 l/2}{Z_{01}\cos\beta_1 l/2 + j(Z_{00}\tan\beta_0 l/2)\sin\beta_1 l/2}\right]$$

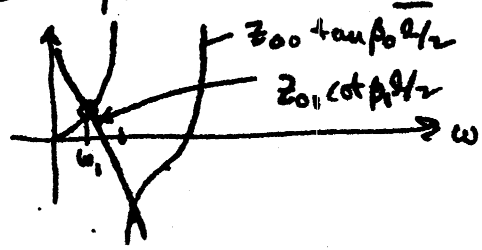
lowest resonance is when $Z'' \to \infty$ for first time (effective $1/2\,\lambda$ between ends)

thus $Z_{01}\cos\beta_1 l/2 - Z_{00}\tan\beta_0 l/2 \sin\beta_1 l/2 = 0$

or $Z_{01}/Z_{00} = \tan\beta_0 l/2 \tan\beta_1 l/2$ ← ANS

$\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$

$\beta_1 = \omega\sqrt{\mu_0\epsilon}$



1. $Q = \omega_0$ energy stored / Power diss.

$$Q = \frac{\epsilon' + \epsilon_0}{\epsilon''}$$

energy stored $= 2\left[\underset{\text{dielectric region}}{\int \frac{1}{4}\epsilon' |\hat{E}|^2 dv} + \underset{\text{air region}}{\frac{1}{4}\int \epsilon_0 |\hat{E}_r|^2 dv}\right]$

power dissipated/vol. $= \frac{1}{2} \hat{J}_r \cdot \hat{E}_r = \frac{1}{2}\omega\epsilon''|E_r|^2$

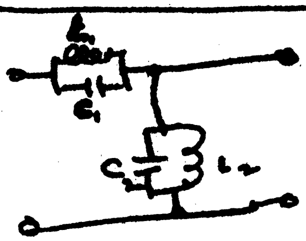
$$Q = \frac{\int \frac{1}{2}\epsilon |E_r|^2 dv + \frac{1}{2}\int \epsilon_0 |E|^2 dv}{\frac{1}{2}\int \omega\epsilon'' |E_r|^2 dv}$$

since $J_r = j\omega(-j\epsilon'') E_r = \omega\epsilon'' E_r$

thus $Q = \omega_1 \left[\frac{\epsilon'}{\omega_1\epsilon''} + \frac{\epsilon_0 \int |E_r^2| dv_{\text{air volume}}}{\omega_1\epsilon'' \int_{\text{dielectric volume}} E_r^2 dV}\right]$ note E_r varies with z as in a standing wave

and the exact spatial dependence depends upon β_0 & β_1. The radial integrals ($E_r \sim \frac{1}{r} f(z)$) for both cancel out since we divide one vol. integral by other.

2.



$\gamma = \sqrt{ZY}$

$$Z = \frac{1}{\frac{1}{j\omega L_1} + j\omega C_1} = \frac{j\omega L_1}{1 - \omega^2 L_1 C_1} = \frac{j\omega L_1}{1 - \omega^2/\omega_1^2}$$

where $\omega_1^2 = \frac{1}{L_1 C_1}$

$$Y^{-1} = j\omega C_2\left(1 - \omega^2 L_2 C_2\right) = j\omega C_2\left(1 - \frac{\omega^2}{\omega_2^2}\right)$$

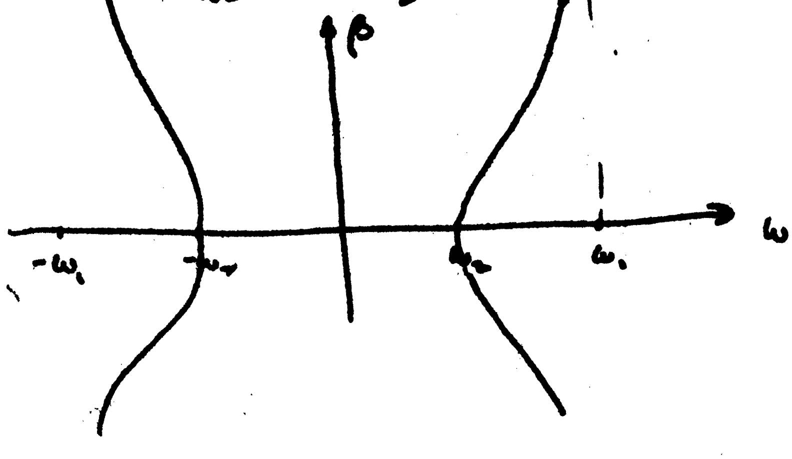
where $\omega_2^2 = \frac{1}{L_2 C_2}$

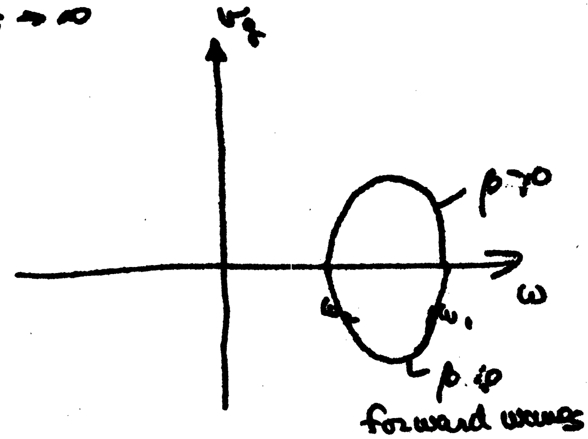
so $\gamma^2 = -\omega^2 L_1 C_1 \left(1 - \omega_2^2/\omega^2\right)\left(1 - \omega^2/\omega_1^2\right)^{-1}$

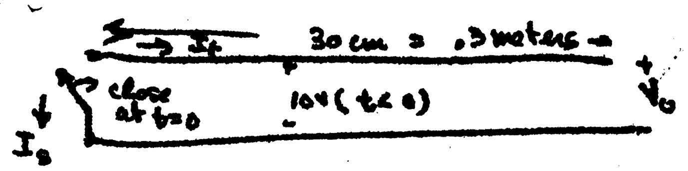
case a $\omega_1 > \omega_2$

then for $\omega_2 < \omega < \omega_1$ $\gamma^2 < 0$ hence $\gamma = \pm j\beta$

for $\omega = \omega_2$ $\gamma = 0$ | $\omega = \omega_1$ $\gamma \to \infty$





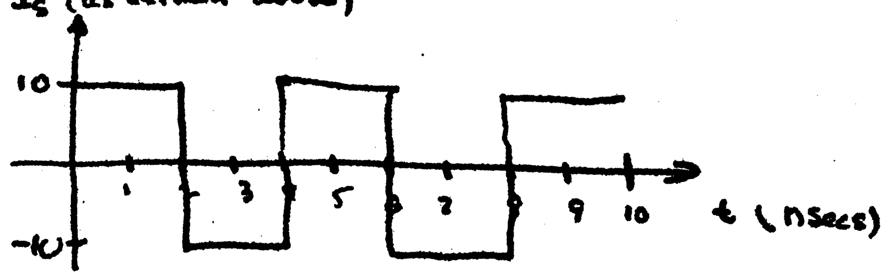


$T = .3/3\times10^{10} = 10^{-9}$ Sec.

at $t=0$ switch closes and $I_+ = -10/Z_0 = -10$ amps

$V_+ = -10V$

I_s (as defined above)



at open end

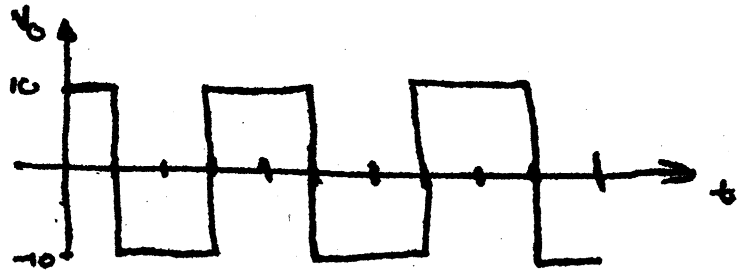
τ = transmission coeff = 2

ρ = refl coeff = +1

at shorted end

$\tau = 0$

$\rho = -1$



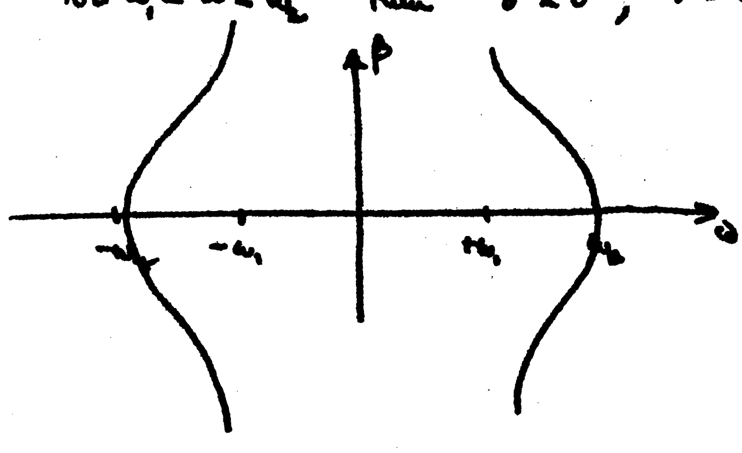
I forgot case b from prob 2

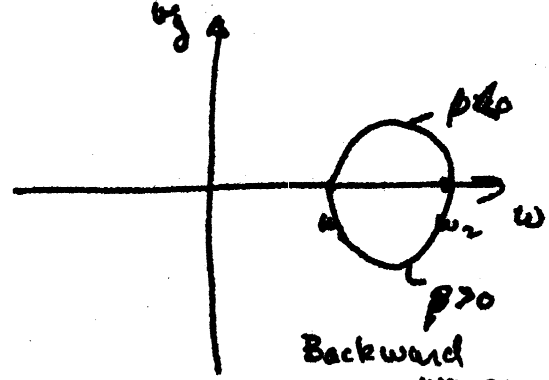
$\omega_1 < \omega_2$

$\gamma^2 = -\omega^2 L_1 C_2 (1 - \omega_2^2/\omega^2)(1 - \omega^2/\omega_1^2)^{-1}$

for $\omega_1 < \omega < \omega_2$ then $\gamma^2 < 0$, $\gamma = \pm j\beta$, $\beta \to \infty$ at $\omega = \omega_1$

$\beta = 0$ at $\omega = \omega_2$



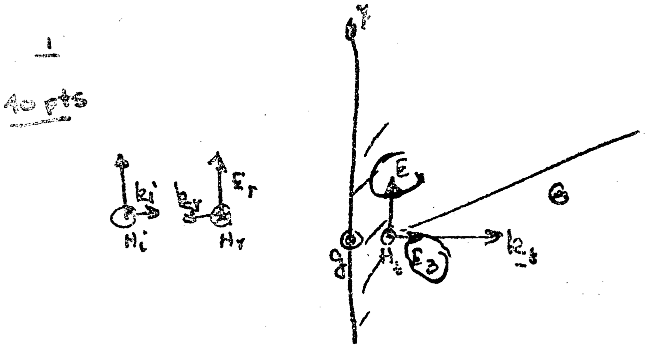


Backward waves

Solutions to final Exam

1

10 pts



(a) since $k_0 \sin\theta = k_t \sin\theta_t$ and $\theta = 0$, $\theta_t = 0$ thus $\underline{k_t} = k_t a_z$

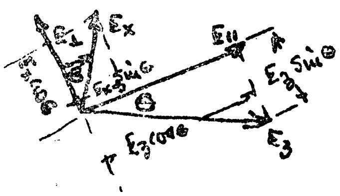
(b) $E_i + E_r = E_x$

$H_i - H_r = H_t$ but $-j\underline{k_t} \times (E_x a_x + E_z a_z) = -jk_t E_x \, a_y = -j\omega\mu_0 H_t a_y$

or $\sqrt{\frac{\epsilon_0}{\mu_0}}(E_i - E_r) = \frac{k_t E_x}{\omega\mu_0}$; $\frac{k_t}{\omega\mu_0}\sqrt{\frac{\mu_0}{\epsilon_0}} E_x = E_i - E_r$

thus $E_x = \dfrac{2E_i}{1 + k_t/k_0}$ **can have E_x & E_z**

(c)



from the geometry

$E_\perp = E_x \cos\theta - E_z \sin\theta$

$E_\parallel = E_z \cos\theta + E_x \sin\theta$

(d) we use $\nabla \cdot D \Rightarrow -j\underline{k_t} \cdot (\epsilon_\perp \underline{E_\perp} + \epsilon_\parallel \underline{E_\parallel}) = 0$

$\underline{k_t} = k_t \cos\theta \, \underline{i_\parallel} - k_t \sin\theta \, \underline{i_\perp}$ where $i_\parallel$ & $i_\perp$ are unit

vectors along and $\perp$ to the optic axis thus

$0 = -k_t \epsilon_\perp E_\perp \sin\theta + k_t \epsilon_\parallel E_\parallel \cos\theta = -k_t \epsilon_\perp (E_x \cos\theta - E_z \sin\theta)\sin\theta + k_t \epsilon_\parallel (E_z \cos\theta + E_x \sin\theta)\cos\theta$

so $+\epsilon_\perp E_z \sin^2\theta + \epsilon_\parallel E_z \cos^2\theta = \epsilon_\perp E_x \cos\theta \sin\theta - \epsilon_\parallel E_x \cos\theta \sin\theta$

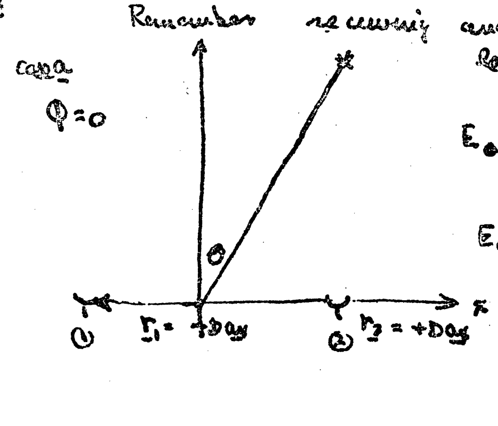
$$E_z = E_x \frac{(K_\perp - K_\parallel)\cos\theta\sin\theta}{K_\perp \sin^2\theta + K_\parallel \cos^2\theta}$$

2

Remember receiving and transmitting patterns are identical

case a

$\phi = 0$



let position of star be given by θ, r ($\phi = 0$ for case a)

$$E_{\theta_1} = K\frac{e^{-jkr}}{4\pi r} e^{-j\underline{k}\,\underline{a_r} \cdot (-D\underline{a_x}) - j\pi/4} = K\frac{e^{-jkr}}{4\pi r} e^{+j(kD\sin\theta\cos\phi - \pi/4)}$$

$$E_{\theta_2} = K\frac{e^{-jkr}}{4\pi r} e^{-jk\underline{a_r} \cdot (D\underline{a_x}) + j\pi/4} = K\frac{e^{-jkr}}{4\pi r} e^{-j(kD\sin\theta\cos\phi - \pi/4)}$$

thus $E_\theta = E_{\theta_1} + E_{\theta_2} = \frac{k e^{-jkr}}{4\pi r}\left[e^{+j(kD\sin\theta\cos\phi - \pi/4)} + e^{-j(kD\sin\theta\cos\phi - \pi/4)}\right]$

$= 2\frac{k e^{-jkr}}{4\pi r}\cos(kD\sin\theta\cos\phi - \pi/4)$ $\qquad k = \frac{2\pi f_0}{c}$

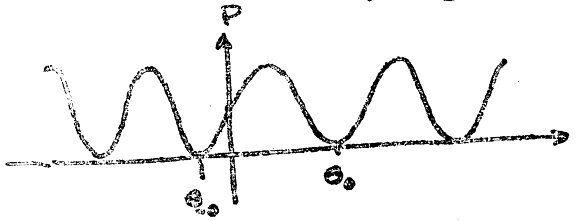
and Power density $= 4\sqrt{\frac{\mu_0}{\epsilon_0}}\,K^2\frac{1}{4\pi r^2}\cos^2(kD\sin\theta\cos\phi - \pi/4)$

for case a $\phi = 0$

b $\phi = \pi/2$

to find 1st zero $kD\sin\theta = \pi/4 + \pi/2$ if $D/\lambda >> 1$ then $\theta << 1$

$2\pi(\frac{D}{\lambda})\theta_0 \approx \pi/4 + \pi/2$



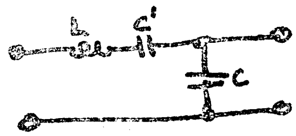
$\theta_0 \approx 3\pi/4 \cdot \lambda/2\pi D = \frac{3\lambda}{8D}$

$\theta_{-0} = -\pi/4\,\frac{\lambda}{2\pi D} = -\frac{\lambda}{8D}$

$\Delta\theta = \theta_0 - \theta_{-0} = \frac{\lambda}{2D} << 1$

case b $\phi = \pi/2$ thus $\cos\phi = 0$ and the pattern is independent of the stars position. Since the rate of change of θ with time is fixed and known two crossed antenna arrays can determine the azimuthal angle ϕ.

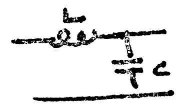
3



as $f >> f_c$ $\quad \gamma^2 \to -k_0^2 = -\omega^2\mu_0\epsilon_0$

and $Z_{TM} \to \sqrt{\frac{\mu_0}{\epsilon_0}}$ and

as $f >> f_c$ the circuit becomes



thus $\boxed{L = \mu_0 \text{ and } C = \epsilon_0}$

and at $f = f_c$ the series inductive reactance becomes capacitive that is

$j2\pi f_c L = -\frac{1}{j2\pi f_c C'}$ so $\boxed{C' = \frac{1}{(2\pi)^2 f_c^2 L} = \frac{1}{(2\pi f_c)^2\mu_0}}$

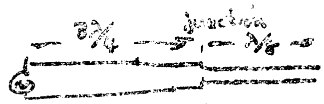
This is the easy way, the other way would be to set $-k_0^2(1 - f_c^2/f^2)^{1/2} = ZY$

and $Z_{TM} = \sqrt{\frac{Z}{Y}}$, $Z_{TM}^2 = Z/Y$ thus $\frac{-k_0^2(1 - f_c^2/f^2)^{1/2}}{Z_{TM}^2} = Y^2 = -\omega^2\epsilon_0$

$Y = j\omega\epsilon_0$, so $C = \epsilon_0$

then $(Z_{TM}^2)(-k_0^2)(1 - f_c^2/f^2)^{1/2} = Z^2$ and solve., $Z^2 = (j\omega L + \frac{1}{j\omega C'})^2$ etc

6)



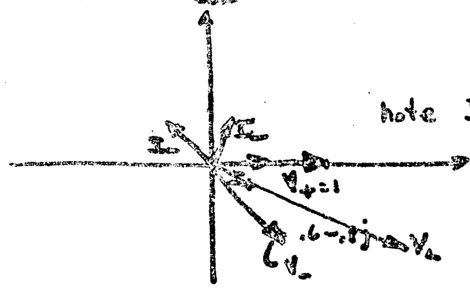
$$\vec{Z}_L = -jZ_0' \operatorname{ctn}\beta l = -2j \operatorname{ctn}\tfrac{\pi}{4} = -2j$$

$$\tau = \frac{-2j(2)}{-2j+1} = \frac{-4j}{5}(1+2j) = 1.6 - .8j$$

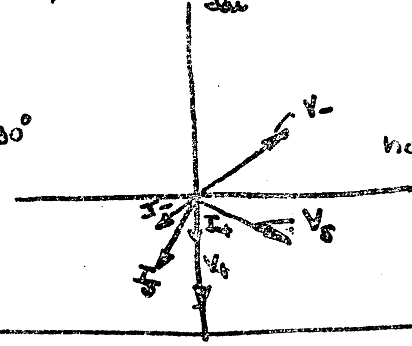
$$\rho = \frac{-2j-1}{-2j+1} = \frac{(-2j-1)(+2j+1)}{5} = \frac{3-4j}{5} = .6-.8j$$

at junction

at source

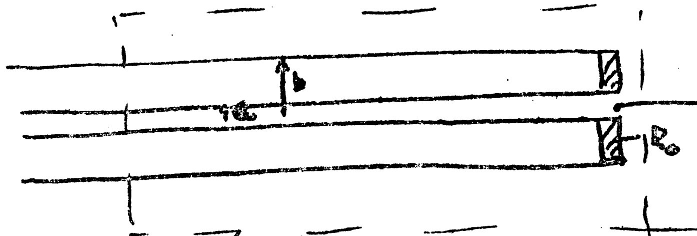


note I_L lead V_L by $90°$



note V_S lead I_S by $90°$

7)



inner radius, a
outer radius, b

Problem can be worked either in S-S-S-S or for generalization function

S-S-S-S

$$-\oint \hat{P}\cdot d\vec{s} = \frac{1}{2}\int_{vol} \hat{J}\cdot\hat{E}^* dv + 2j\omega \int_{vol}\left[\tfrac{1}{4}\mu_0|H|^2 - \tfrac{1}{4}\epsilon_0|E|^2\right]dv$$

$\hat{P} = \frac{1}{2}\,\hat{E}\times\hat{H}^*$ for a coaxial cable with a matched load

$$\hat{E} = \underline{a}_r \frac{V_0}{r} e^{-jk_0 z}$$

$$\hat{H} = \underline{a}_\phi \sqrt{\frac{\epsilon_0}{\mu_0}}\frac{V_0}{r} e^{-jk_0 z}$$

thus $\hat{P} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\frac{V_0^2}{r^2}\,\underline{a}_z$ (all real) $a < r < b$

and $-\oint \hat{P}\cdot d\underline{s} = \pi\sqrt{\frac{\epsilon_0}{\mu_0}}\, V_0^2 \int_a^b \frac{dr}{r} = \pi\sqrt{\frac{\epsilon_0}{\mu_0}}\, V_0^2 \ln(b/a)$

$d\underline{s} = -\underline{a}_z\, r\,dr\,d\phi$

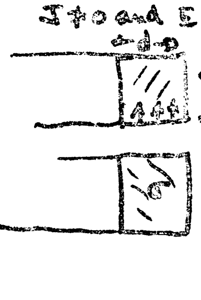
$\frac{1}{4}\epsilon_0|E|^2 = \frac{1}{4}\frac{V_0^2}{r^2}\epsilon_0$

$\frac{1}{4}\mu_0|H|^2 = \frac{1}{4}\mu_0\frac{\epsilon_0}{\mu_0}\frac{V_0^2}{r^2} = \frac{1}{4}\epsilon_0|E|^2$

thus $W_m - W_e = 0$

and $\int_{vol}\left(\frac{1}{4}\mu_0|H|^2 - \frac{1}{4}\epsilon_0|E|^2\right)dv = 0$

So it remains for us to evaluate $\frac{1}{2}\int \hat{J}\cdot\hat{E}^* dv$ which only is non-zero at load where $J \neq 0$ and $E \neq 0$. assume E constant in load



$$V = \int_a^b E\,dl = V_0 \ln b/a$$

$$\frac{1}{2}\int \hat{J}\cdot\hat{E}^* dv = \frac{1}{2}\sigma V_0^2 \int_a^b\int_0^d \frac{2\pi}{r}\,dr\,dz = \pi\sigma V_0^2 d \ln b/a$$

$$= \pi V_0^2 \sqrt{\frac{\epsilon_0}{\mu_0}} \ln b/a$$

which checks

$$I = \int J\cdot ds = \sigma E(a)\cdot 2\pi a d$$

$$= \sigma V_0\, 2\pi d$$

hence $Z_L = V/I = \frac{\ln b/a}{2\pi\sigma d} = Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}\frac{\ln b/a}{2\pi}$, by definition, thus $\sigma d = \sqrt{\frac{\epsilon_0}{\mu_0}}$