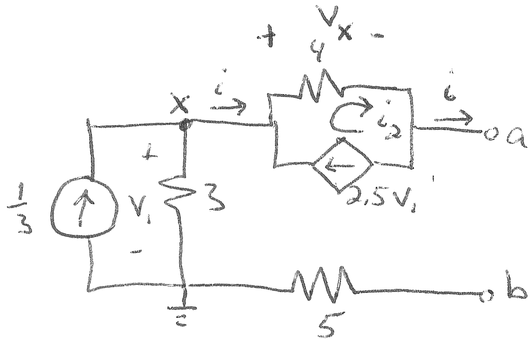
ENGR 210 Spring '05 HW 7 Solutions



#1 Find Norton Equivalent

Find V_{oc}: $i = 0$

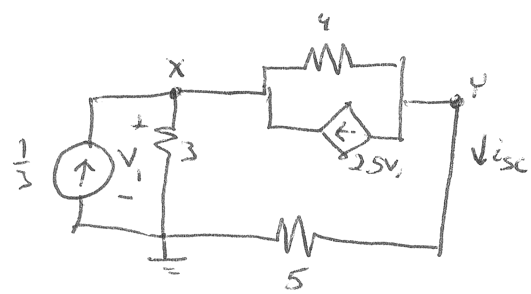
KCL @ X $\frac{1}{3} = \frac{V_1}{3}$

$V_1 = 1V$

Even though $i = 0$ there is current in the top loop.

$i_2 = 2.5V_1$ $V_x = (2.5V_1)4 = 10V_1 = 10V$

$V_{oc} = V_{ab} = V_a = V_1 - V_x = -9V$



Find i_{sc}! KCL @ X $\frac{1}{3} + 2.5V_1 = \frac{V_1}{3} + \frac{V_1 - V_y}{4}$

KCL @ Y $\frac{V_1 - V_y}{4} = 2.5V_1 + \frac{V_y}{5}$

Remove Fractions: $4 + 30V_1 = 4V_1 + 3V_1 - 3V_y$

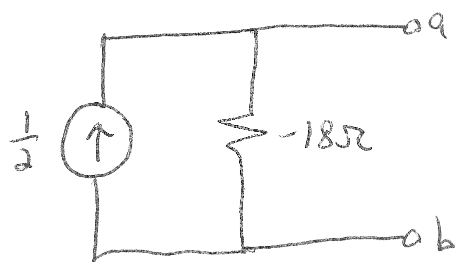
$23V_1 + 3V_y = -4$ $V_1 = -\frac{1}{2}$

$5V_1 - 5V_y = 50V_1 + 4V_y$

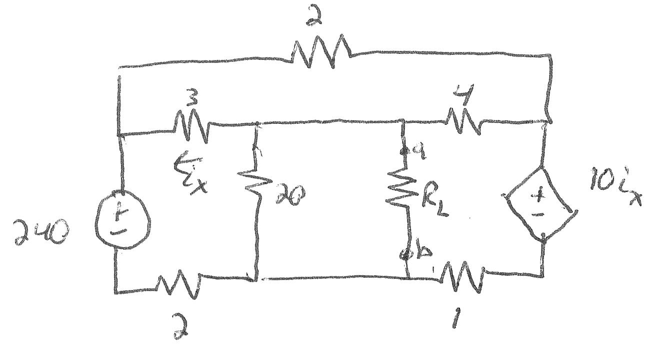
$V_y = -5V_1$ $V_y = 2.5$

$i_{sc} = \frac{V_y}{5} = \frac{1}{2} = I_N$

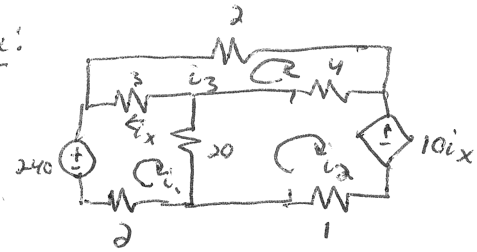
$R_N = \frac{V_{oc}}{i_{sc}} = \frac{-9}{1/2} = -18\Omega$



#2 a) Find thevenin Equivalent



Find V_{oc}:



KVL i_1: $-240 + (i_1 - i_3)3 + (i_1 - i_2)20 + i_1(2) = 0$

i_2: $(i_2 - i_1)20 + (i_2 - i_3)4 + 10i_x + i_2(1) = 0$

i_3: $(i_3 - i_1)3 + i_3(2) + (i_3 - i_2)4 = 0$

$i_3 - i_1 = i_x$

$\Rightarrow$

$25i_1 - 20i_2 - 3i_3 = 240$
$-20i_1 + 25i_2 - 4i_3 + 10i_x = 0$
$-3i_1 - 4i_2 + 9i_3 = 0$
$-i_1 + i_3 - i_x = 0$

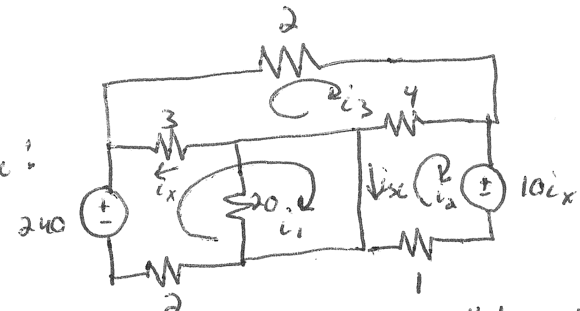
Plug into calculator to solve

$i_1 = 99.6$ $i_3 = 78$
$i_2 = 100.8$ $i_x = -21.6$

$V_{oc} = V_{20} = (i_1 - i_2)20$

$V_{oc} = -24V$

Find i_{sc}:



neglect 20Ω because it's in parallel w/ a short circuit

$20 \| 0 = 0$

KVL i_1: $-240 + (i_1 - i_3)3 + 2i_1 = 0$

i_2: $(i_2 - i_3)4 + 10i_x + i_2 = 0$

i_3: $(i_3 - i_1)3 + 2i_3 + (i_3 - i_2)4 = 0$

$i_3 - i_1 = i_x$

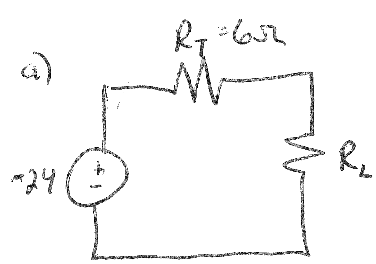
$\Rightarrow$

$5i_1 - 3i_3 = 240$
$5i_2 - 4i_3 + 10i_x = 0$
$-3i_1 - 4i_2 + 9i_3 = 0$
$-i_1 + i_3 - i_x = 0$

$i_1 = 92$
$i_2 = 96$
$i_3 = 73.33$
$i_4 = -18.67$

$i_{sc} = i_1 - i_2 = -4A$ $\Rightarrow R_T = \frac{V_{oc}}{i_{sc}} = 6\Omega$

a)



b) max power when $R_L = R_T$

$R_L = 6\Omega$

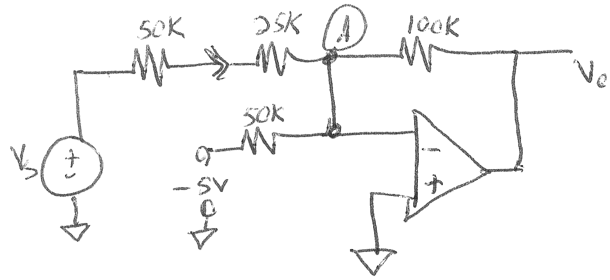
c) $P_{max} = \frac{V_T^2}{4R_L} = \frac{(-24)^2}{4(6)}$

$P_{max} = 24\ W$

#3

For OpAmp $V_p = V_n$

$i_p = i_n = 0$



$V_p = 0 = V_n$

KCL @ A:
$$\frac{V_s - V_A}{75K} = \frac{V_A - -5}{50K} + \frac{V_A - V_o}{100K}$$

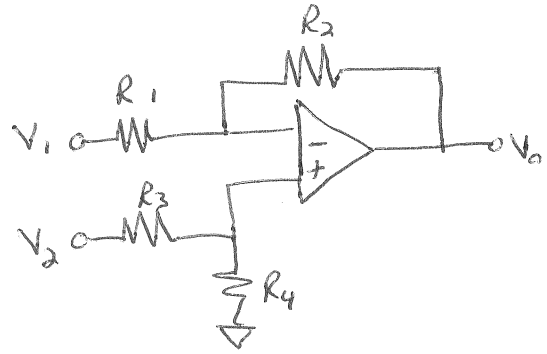
$$V_A = V_n = 0$$

$$V_o = -100K\left(\frac{V_s}{75K} - \frac{5}{50K}\right)$$

$$\boxed{V_o = 10 - \frac{4}{3}V_s}$$

#4

a) Show
$$V_o = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{R_3}{R_4}} V_2 - \frac{R_2}{R_1} V_1$$



$$V_p = V_n = V_2 \frac{R_4}{R_3 + R_4}$$

$$\frac{V_1 - V_p}{R_1} = \frac{V_p - V_o}{R_2} = \frac{V_1 - V_2 \frac{R_4}{R_3 + R_4}}{R_1} = \frac{V_2 \frac{R_4}{R_3 + R_4} - V_o}{R_2}$$

$$-V_o = \frac{R_2}{R_1} V_1 - \frac{R_2}{R_1} \frac{R_4}{R_3 + R_4} V_2 - \frac{R_4}{R_3 + R_4} V_2$$
) Factor out $\frac{R_4}{R_3 + R_4}$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)V_2 - \frac{R_2}{R_1} V_1$$
) divide top and bottom of $\frac{R_4}{R_3 + R_4}$ by R_4 $= \frac{1}{\frac{R_3}{R} + 1}$

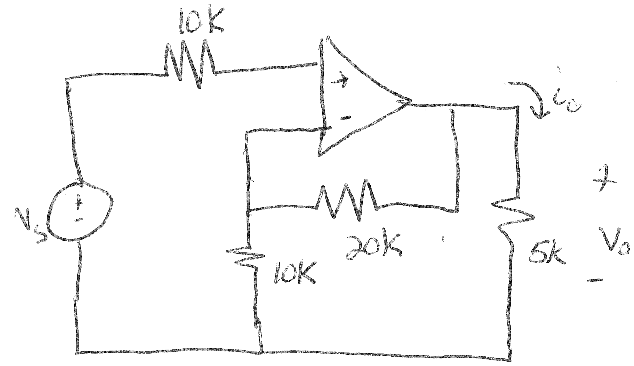
$$\boxed{V_o = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left(1 + \frac{R_3}{R_4}\right)} V_2 - \frac{R_2}{R_1} V_1}$$

b) $V_o = 4V_2 - 11V_1$

$$\boxed{\frac{R_2}{R_1} = 11} \qquad \frac{1 + 11}{1 + \frac{R_3}{R_4}} = 4 \qquad \boxed{\frac{R_3}{R_4} = 2}$$

One Design: (Many Possible) $\boxed{R_1 = 1k\Omega \; R_2 = 11k\Omega \quad R_4 = 1k\Omega \; R_3 = 2k\Omega}$

#5 a) $i_p = i_n = 0$ so $\underline{V_p = V_s} = V_n$



$V_n = V_o \dfrac{10k}{10k+20k}$ Voltage Divider

$V_o\left(\frac{1}{3}\right) = V_s$

$\boxed{V_o = 3V_s}$

b) $V_s = 1.5V$

$V_o = 4.5V$

$\boxed{i_o = \dfrac{V_o}{5K} = \dfrac{4.5V}{5K} = 900\mu A}$