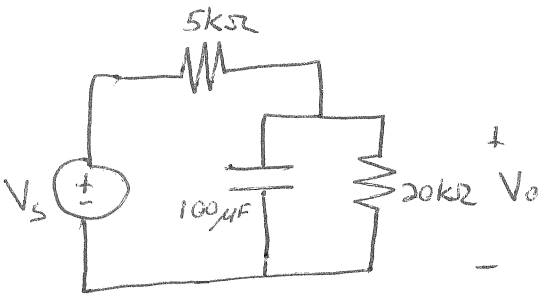
ENGR 210 HW 12 Solutions Spring '05

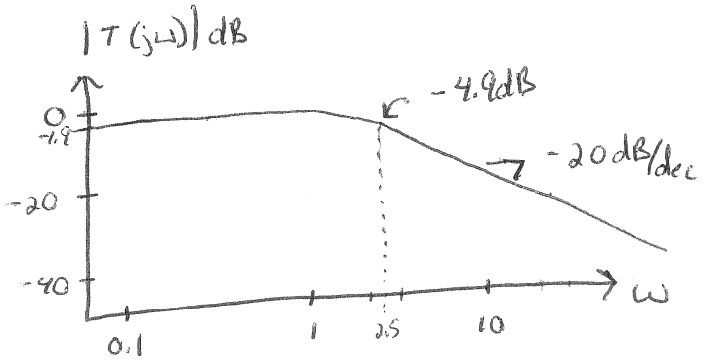
#1 a) $T(j\omega) = \frac{V_o(j\omega)}{V_s(j\omega)} = ?$



Voltage Divider: $V_o = V_s \dfrac{20k \,//\, \frac{1}{j\omega C(100\mu F)}}{\left[20k \,//\, \frac{1}{j\omega(100\mu F)}\right] + 5k}$

$$T(j\omega) = \frac{\frac{20k}{1+j2\omega}}{\frac{20k}{1+j2\omega} + 5k} = \frac{20k}{25k + j(10k)\omega} = \boxed{\frac{0.8}{1 + j\frac{\omega}{5}}}$$

$x_{dB} = 20 \log x$

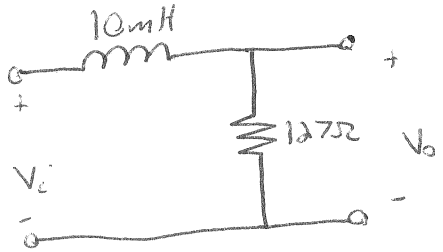


At $\omega = 0$ $|T(j\omega)| = 0.8 \rightarrow -1.9$dB

$\omega_c = \frac{5}{3}\frac{rad}{s} = 2.5$ (when denom $= 1 + j$)

#2 Voltage Divider:

$$T(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{127}{127 + j\omega(10m)} = \boxed{\frac{1}{1 + j\omega(7.87\times10^{-5})}}$$



a) $f_c = ? = \frac{\omega_c}{2\pi}$ $1 + j\omega_c(7.87\times10^{-5}) = 1 + j$

$\omega_c = \frac{1}{7.87\times10^{-5}} = 12.7k\frac{rad}{s}$

$\boxed{f_c = 2021\ Hz}$

$\boxed{\text{Low-Pass}}$ (when $\omega = 0$ $|T(j\omega)| > 0$; when $\omega \rightarrow \infty$ $|T(j\omega)| \rightarrow 0$)

b) $T(j(0.2\omega_c)) = (0.96 - j0.192) = 0.98\, e^{-j11.3°}$

$T(j\omega_c) = (0.5 - j0.5) = 0.707\, e^{-j45°}$

$T(j(5\omega_c)) = (0.038 - j0.192) = 0.196\, e^{-j78.7°}$

#2 cont...

c) $v_i = 10\cos(\omega t)$

$\omega = 0.2\omega_c$

$$V_o = T(j\omega)V_i(j\omega) = 9.8\cos(\omega t - 11.3^\circ)$$

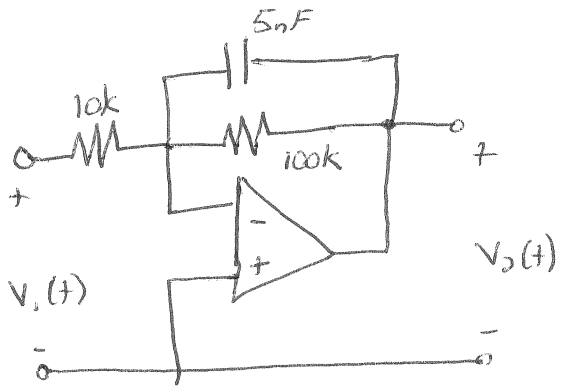
$\omega = \omega_c$

$$V_o(t) = 7.07\cos(\omega t - 45^\circ)$$

$\omega = 5\omega_c$

$$V_o(t) = 1.96\cos(\omega t - 78.7^\circ)$$

#3



$T(j\omega) = ?$

Inverting Amp: $V_o = \frac{-Z_F}{Z_{in}} V_{in}$

$$T(j\omega) = \frac{V_o}{V_i} = \frac{-(100k \,//\, \frac{1}{j\omega 5n})}{10k} = \frac{\frac{-100k}{1+j\omega(5\times10^{-4})}}{10k}$$

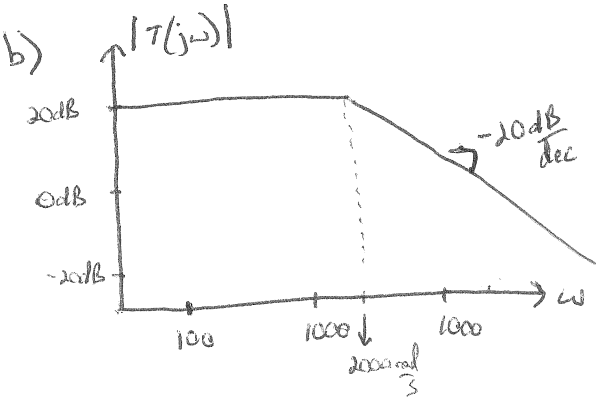
$$T(j\omega) = \frac{-10}{1+j\omega(5\times10^{-4})}$$

a) DC gain : -10 $(\omega = 0)$

$\omega_c = 2000\ \frac{rad}{s} = 318\ Hz$

Low pass

b)



c) $V_o = T(j\omega)V_i$

$\omega = 0.5\omega_c$

$$V_o(t)_{Amp} = 50$$

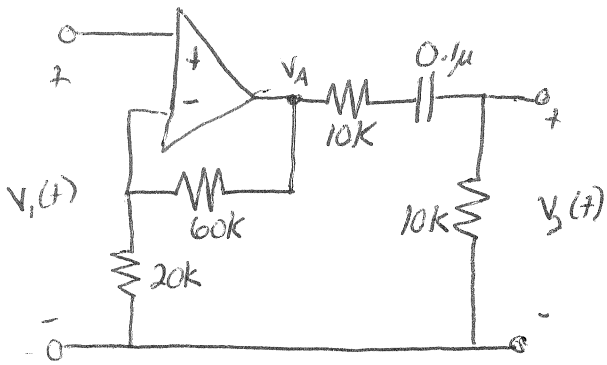
$\omega = \omega_c$

$$V_{o\,Amp} = 50$$

$\omega = 2\omega_c$

$$V_{o\,Amp} = 25$$

#4 Find transfer function $T(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)}$



$$T(j\omega) = \frac{V_2}{V_A} \cdot \frac{V_A}{V_i} = \underbrace{\frac{10k}{10k + \frac{1}{j\omega(0.1\mu)}}}_{\text{Voltage Divider}} \cdot \underbrace{\left(1 + \frac{60k}{20k}\right)}_{\text{Non-inverting Amp.}}$$

$$T(j\omega) = \frac{40k\,(j\omega(0.1\mu))}{1 + j\omega(10k)(0.1\mu)} = \frac{j\omega(2)}{j\omega + 500} = \frac{j\omega(4m)}{1 + j\omega(2m)}$$

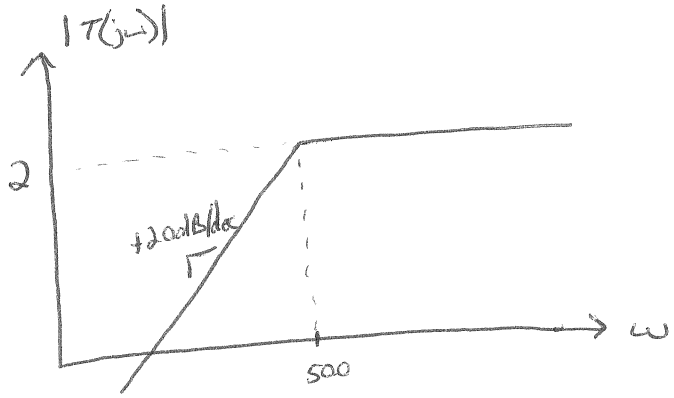
a) DC gain ($\omega = 0$) : 0

f_∞ gain : 2

ω_c : 500 $\frac{rad}{s}$ = 79.6 Hz

b) **High pass** ($|T(j\omega)| = 0$ @ low freq. and is flat at high freq.)

c)



#5 $$T(j\omega) = \frac{10}{10^2 + \frac{j\omega}{20}} = \frac{\frac{1}{10}}{1 + \frac{j\omega}{2000}}$$

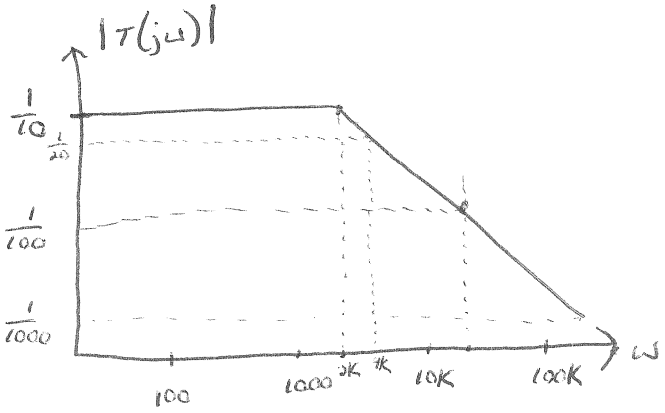
a) Low pass

$\omega_c = 2000 \frac{rad}{s} = 318\ Hz$

Passband Gain: $\frac{1}{10}$

$\omega = 0$ for low pass passband

b)



c)

$|T(j(0.5\omega_c))| = \frac{1}{10}$

$|T(j\omega_c)| = \frac{1}{10}$

$|T(j(2\omega_c))| = \frac{1}{20}$