ENGR 210 HW 11 Solutions Spring '05

#1 a) $V_1 = 10 + j40$, $\omega = 10$

change to polar form $Ae^{j\phi}$

$Re\{e^{j\theta}\} = \cos(\theta)$

$A = \sqrt{10^2 + 40^2} = 10\sqrt{17} = 41.23$

$\phi = \tan^{-1}\left(\frac{40}{10}\right) = 75.96°$

$\Big\} \; V_1 = 41.23\, e^{j75.96°}$

$v_1(t) = Re\{V_1 e^{j\omega t}\} = \boxed{41.23 \cos(10t + 75.96°)}$

b) $V_2 = (8 - j3)5e^{-j60°}$ $\omega = 20$

change polar part ($5e^{-j60°}$) into rect ($a + jb$)

$a = 5\cos(-60°) = 2.5$

$b = 5\sin(-60°) = -4.33$

$V_2 = (8-j3)(2.5 - j4.33) = 7 - j42.14 = 42.7e^{-j80.56°}$

$v_2(t) = Re\{V_2 e^{j\omega t}\} = \boxed{42.7 \cos(20t - 80.56°)}$

c) $I_1 = 8 - j3 + \frac{3}{j}$, $\omega = 300$

$\frac{1}{j} = -j$ $\quad I_1 = 8 - j3 - j3 = 8 - j6$ put into polar form.

$I_1 = 10e^{-j36.87°}$

$i_1(t) = Re\{I_1 e^{j\omega t}\} = Re\{10e^{-j36.87°} e^{j300t}\} = Re\{10e^{j(300t - 36.87°)}\}$

$\boxed{i_1(t) = 10\cos(300t - 36.87°)}$

d) $I_2 = \frac{3+j}{1-3j}$ $\quad \omega = 50$

We want this in the form $a+jb$ so multiply top and bottom by complex conjugate of denominator.

$I_2 = \frac{3+j}{1-3j}\frac{(1+3j)}{(1+3j)} = \frac{3+9j-3+j}{1+9} = j \rightarrow 1e^{j90°}$

$i_2(t) = Re\{I_2 e^{j\omega t}\} = Re\{e^{j90°}e^{j50t}\}$

$\boxed{i_2(t) = \cos(50t + 90°)} = -\sin(50t)$

#2 $v_1(t) = 50\cos(\omega t - 45°) = Re\{50e^{-j45°}e^{j\omega t}\} \rightarrow V_1 = 50e^{-j45°}$

$v_2(t) = 25\sin(\omega t) = 25\cos(\omega t - 90°) = Re\{25e^{-j90°}e^{j\omega t}\} \rightarrow V_2 = 25e^{-j90°}$

$v_3(t) = ?$ $\quad v_1 + v_2 + v_3 = 0$

Put $v_1(t)$ and $v_2(t)$ into Phasors,

$V_3 = -V_2 - V_1$ Phasors add (or subtract) as long as ω is the same for each function.

$V_3 = -50e^{-j45°} - 25e^{-j90°}$ ← You can do this in your calc, but hard to do by hand. To do by hand, put into rect. form.

$= (-35.36 + j35.36) + (0 + j25)$

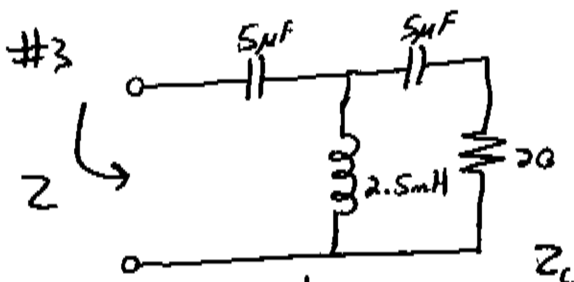
$V_3 = (-35.36 + 60.36)$ ← Add real parts and add imag. parts.

$V_3 = 69.95\, e^{j120.36°}$ ← Put in Polar form

$v_3(t) = Re\{V_3 e^{j\omega t}\}$ ← Return the phasor to the time domain

$$\boxed{v_3(t) = 69.95\cos(\omega t + 120.36°)}$$

#3

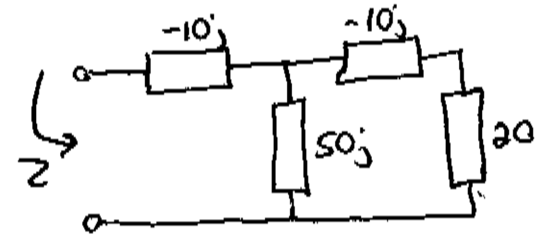


$\omega = 20\,\frac{krad}{s}$

Make everything impedance $\longrightarrow$

$Z_C = \frac{1}{j\omega C} = -10j$

$Z_L = j\omega L = 50j$



Impedances act like resistors so combine the devices (series or parallel) as if they were resistors.

$$Z = \underbrace{-10j + \overbrace{50j \,//\, (\overbrace{-10j + 20}^{\text{series}})}^{\text{parallel}}}_{\text{series}} = -10j + \frac{50j(20 + j(-10))}{50j + (20 - j10)} = -10j + \frac{500 + j1000}{20 + 40j}\,\frac{(20-40j)}{(20-40j)}$$

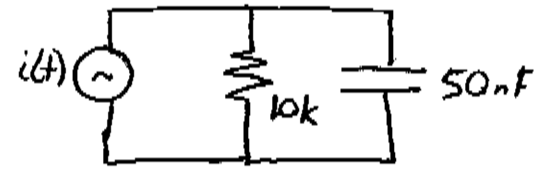
$$= -10j + \frac{50000}{2000} = -10j + 25$$

$$\boxed{Z = 25 - j10}\,\Omega$$

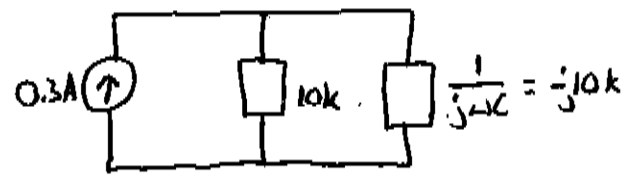
#4 $i(t) = 300\cos(2000t)\ \underline{\underline{mA}}$ $\qquad \underline{\underline{\omega = 2000}}$

$= 0.3\cos(2000t)\ A$

$I = 0.3\, e^{j0} = 0.3 A$ ← phasor of input



↓ change into impedance



Since impedance acts like resistors you can do a current divider to find $i_R(t)$ and $i_c(t)$

$$I_R = \frac{-j10k}{-j10k + 10k}(0.3) = (0.15 - j0.15)\ A$$

$$= \underset{\sim\sim\sim}{0.212\, e^{-j45°}}\ A$$

phasor of $i_R(t)$

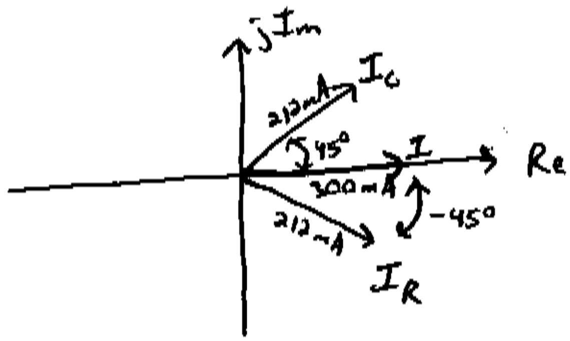
$$i_R(t) = \text{Re}\{I_R\, e^{j\omega t}\} = \boxed{0.212\cos(2000t - 45°)\ \underline{\underline{A}}}$$

$$I_c = \frac{10k}{10k - j10k}(0.3) = (0.15 + j0.15)A = 0.212\, e^{j45°} A$$

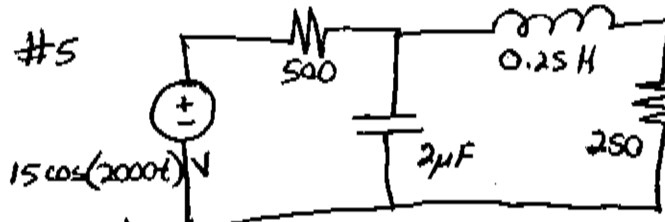
$$i_c(t) = \text{Re}\{I_c\, e^{j\omega t}\} = \boxed{0.212\cos(2000t + 45°)\ A}$$

Phasor Diagram

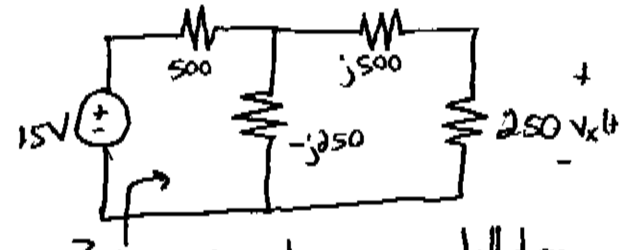
Plot I, I_R and I_c on the complex plane



#5



Change into impedance and phasors →



(All impedances modelled as resistors)

$\omega = 2000$ $\quad Z_C = \frac{1}{j\omega C} = -j250$

$Z_L = j\omega L = j500$

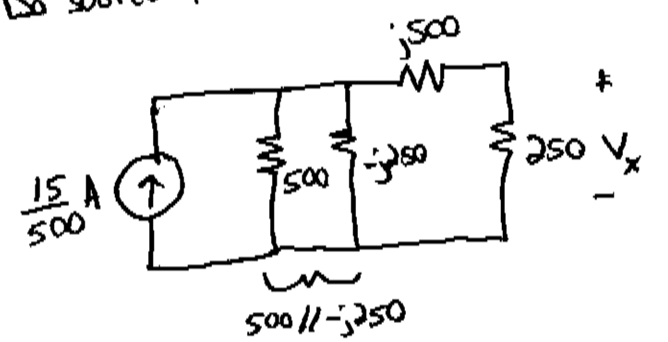
To find Z_{IN}, find the Z_{eq} of all ~~resist~~ impedances.

$$Z_{IN} = 500 + (-j250)//(250+j500) = 500 + \frac{(-j250)(250+j500)}{-j250+(250+j500)}$$

$$\boxed{Z_{IN} = 625 - j375\ \Omega}$$

To find $v_x(t)$ we first should find V_x (phasor)

Do source transformations (or any other ckt technique we've learned)



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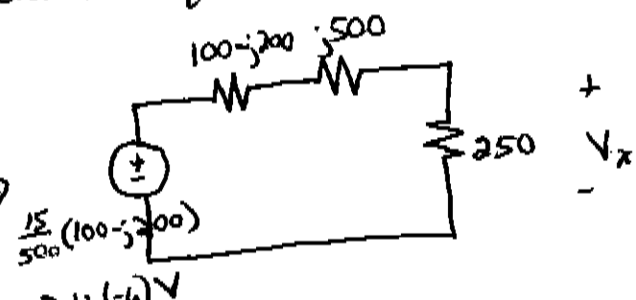


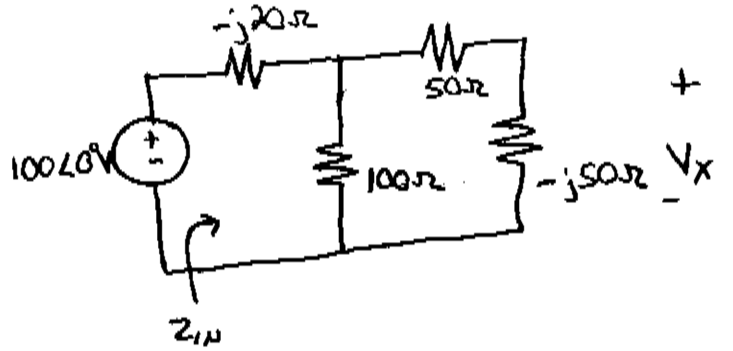
Use a Voltage Divider:

$$V_x = \frac{250}{250+j500+(100-j200)}(3-j6) = -0.88 - j3.53\ V = 3.64e^{-j104°}\ V$$

Transform back into time domain

$$v_x(t) = Re\{V_x e^{j\omega t}\} = \boxed{3.64\cos(2000t - 104°)V}$$

#6

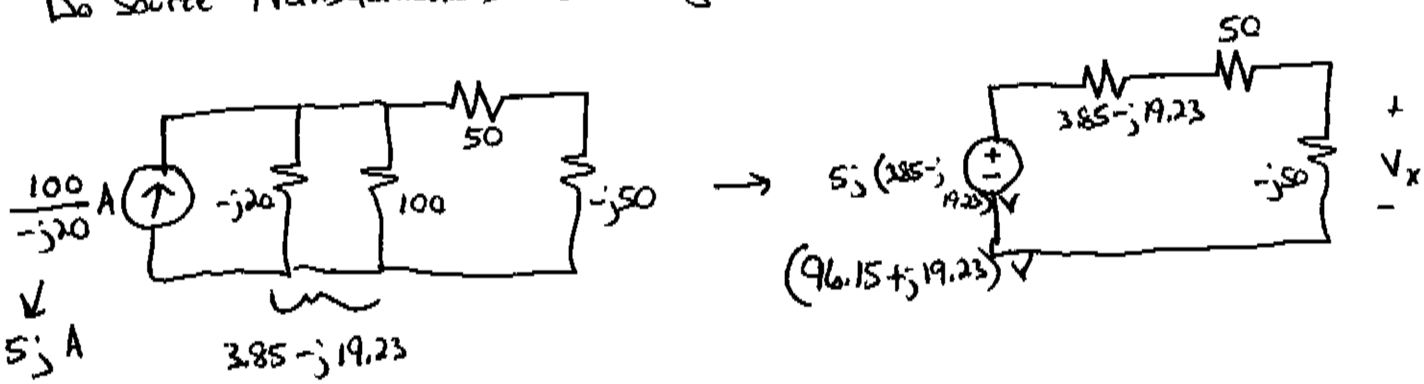


ckt is already in impedance and phasor form.

$$Z_{IN} = -j20 + 100 \,//\, (50 - j50) = -j20 + \frac{100\,(50 - j50)}{100 + 50 - j50}$$

$$\boxed{Z_{IN} = 40 - j40\ \Omega}$$

Do Source Transformations and Voltage divider again.



$$V_x = \frac{-j50}{-j50 + 50 + (3.85 - j19.23)}\,(96.15 + j19.23)\,V$$

$$\boxed{V_x = 50 - j25\ V} \quad \text{or} \quad 55.9\, e^{-j26.6^\circ}\ V$$