

MU QUIZ #3

OLIN 408
3:30 TODAY

Two methods to solve transient problems

1. write equations in state variable

$$L \rightarrow i$$

$$C \rightarrow v$$

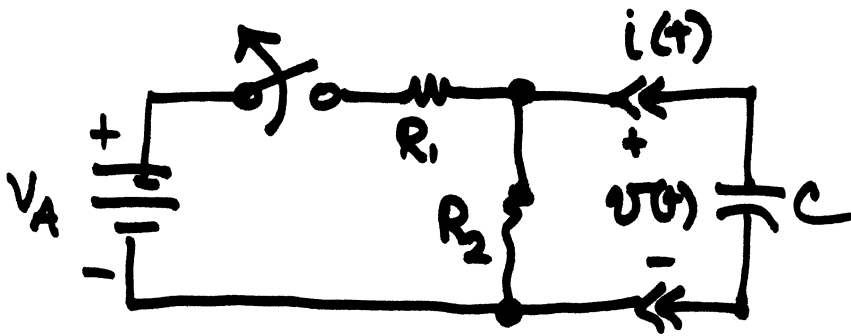
2. initial-final value theorem

$$(IV - FV)e^{-\frac{t}{T_c}} + FV$$

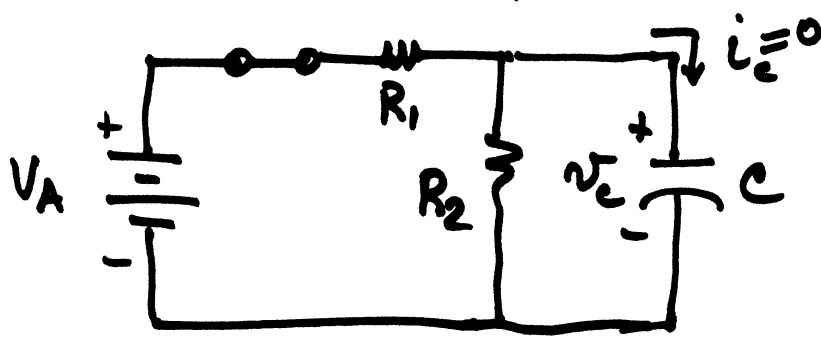
IV - initial value

FV - final value

T_c - time constant



1. initial condition value
dc or steady state value before
switch opens

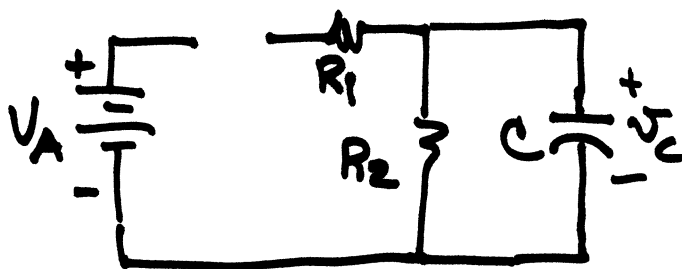


$i_C = C \frac{dv_C}{dt}$
capacitor
acting like
an open

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_A$$

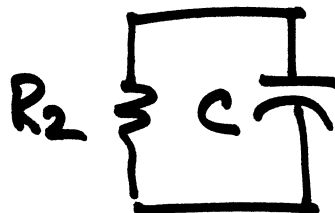
$$v_C(0) = V_{R_2} = \frac{R_2}{R_1 + R_2} V_A$$

2. final value $t \rightarrow \infty$



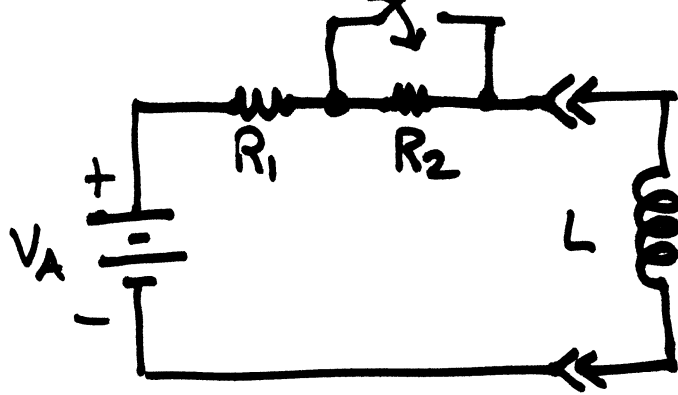
$$v_C(\infty) = 0$$

3. Time constant $t \geq 0$

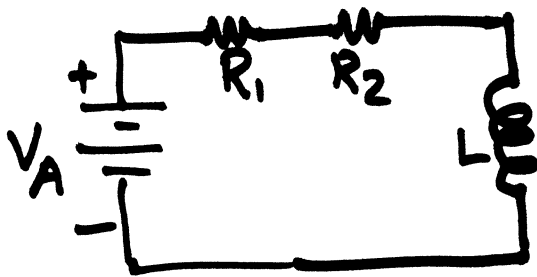


$$\tau_C = \frac{1}{R_2 C}$$

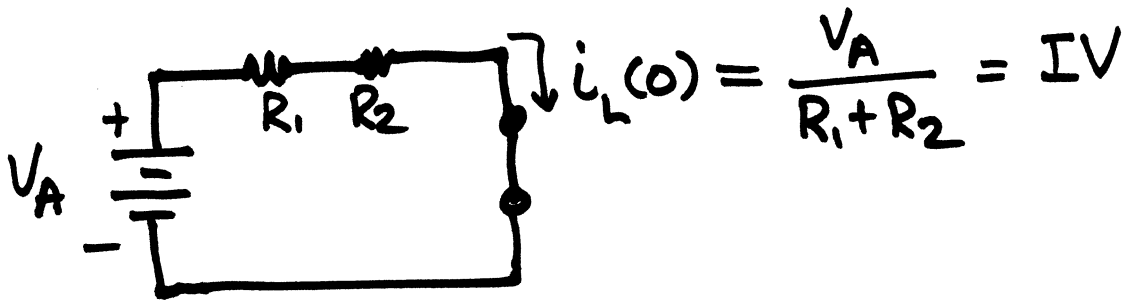
Example 7-8 $t=0$



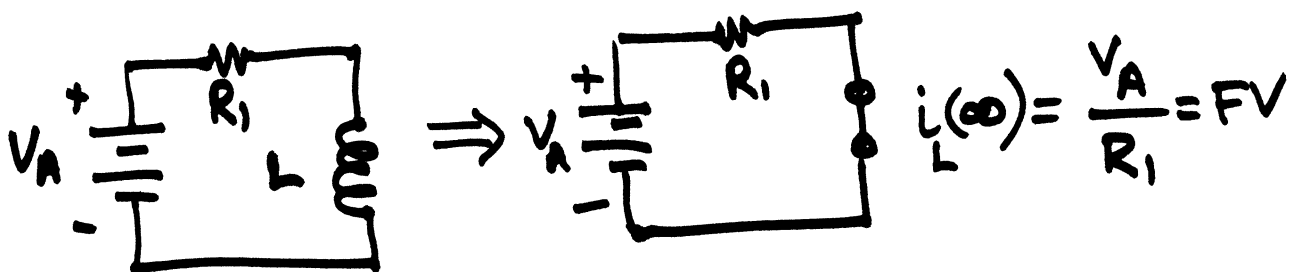
1. initial value $t < 0$ steady state



$$v_L = L \frac{di_L}{dt} = 0$$



2. Final value $t \rightarrow \infty$



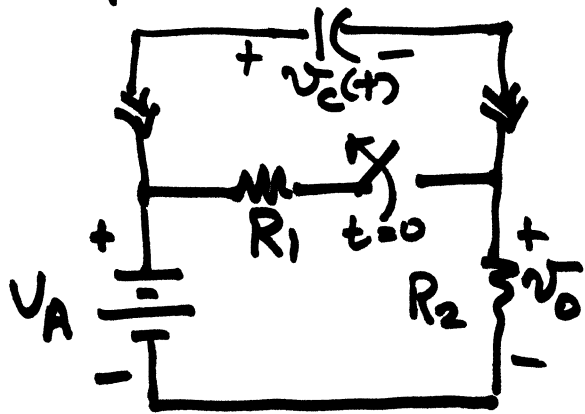
3. Time constant $t \geq 0$

$$T_c = \frac{L}{R_1}$$

$$\text{Final solution } i_L(t) = \left[\frac{V_A}{R_1 + R_2} - \frac{V_A}{R_1} \right] e^{-\frac{t}{L/R_1}} + \frac{V_A}{R_1}$$



Example 7-9



want $v_O(t)$

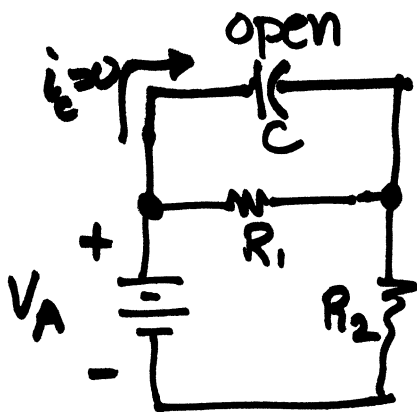
0. Recognize that we will find $v_C(t)$ first, state variable

1. initial value

$t < 0$

steady state

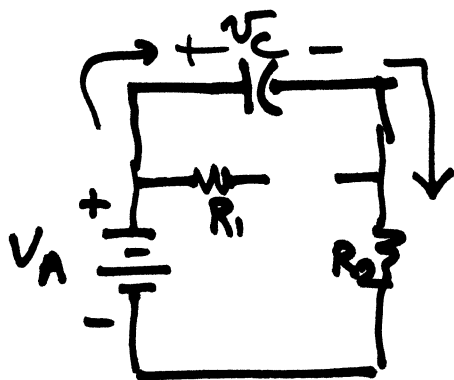
$$i_C = C \frac{dv_C}{dt} = 0$$



$$v_C(0) = \frac{R_1}{R_1 + R_2} V_A = IV$$

voltage divider

2. Final value $t \rightarrow \infty$

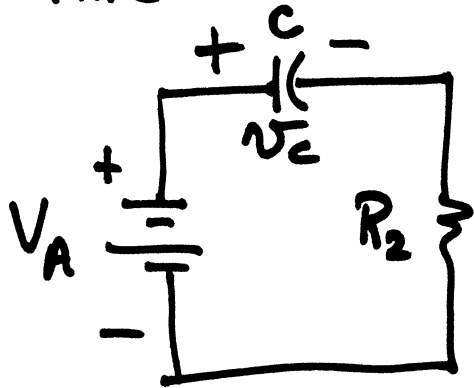


capacitor is fully charged to V_A

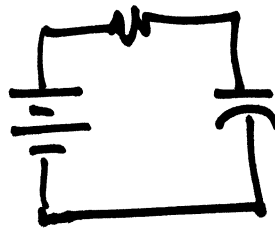
$$-V_A + v_C(\infty) + v_{R_2}(\infty) = 0$$

$$v_C(\infty) = V_A$$

3. Time constant

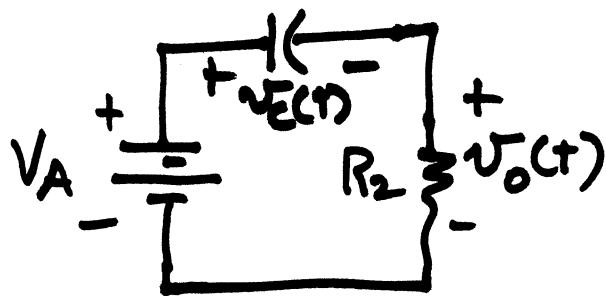


$$\tau_c = R_2 C$$



$$\begin{aligned} v_C(t) &= (IV - FV) e^{-\frac{t}{\tau_c}} + FV \quad t \geq 0 \\ &= \left(\frac{R_1}{R_1 + R_2} V_A - V_A \right) e^{-\frac{t}{R_2 C}} + V_A \end{aligned}$$

4. find $v_o(t)$ in terms of $v_C(t)$ $t \geq 0$

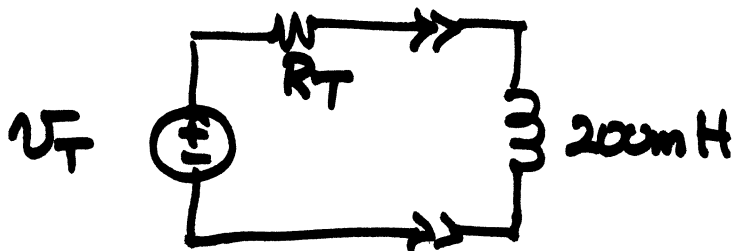


KVL:

$$\begin{aligned} -V_A + v_C(t) + v_o(t) &= 0 \\ v_o(t) &= V_A - v_C(t) \end{aligned}$$

Example 7-10

$$i_L(t) = 50 + 100e^{-5000t} \text{ mA}$$



(a) identify forced and natural responses

natural solution always of form Ke^{st}

$$i_{\text{NATURAL}} = 100e^{-5000t}$$

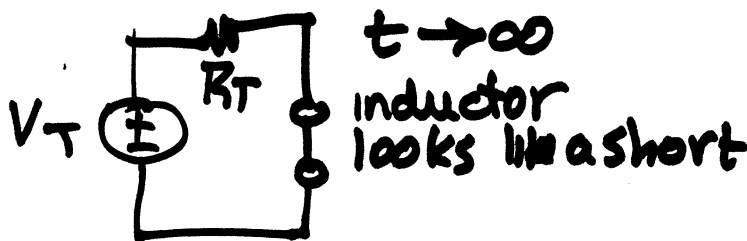
$$i_{\text{TOTAL}} = i_{\text{NATURAL}} + i_{\text{DC FORCED}}$$

$$i_{\text{FORCED}} = 50$$

(b) identify time constant $T_c = \frac{L}{R} = 0.2\text{ms}$

(c) identify Thevenin equivalent circuit.

$$T_c = \frac{L}{R_T} \quad R_T = \frac{L}{T_c} = \frac{200\text{mH}}{0.2 \times 10^{-3} \text{ seconds}} = 1000 \Omega$$



$$\begin{aligned} i_L(\infty) &= 50\text{mA} \\ V_T &= R_T i_L(\infty) \\ &= (1000)(50\text{mA}) \\ &= 50 \text{ VOLTS} \end{aligned}$$