Two methods to solve transient problems:

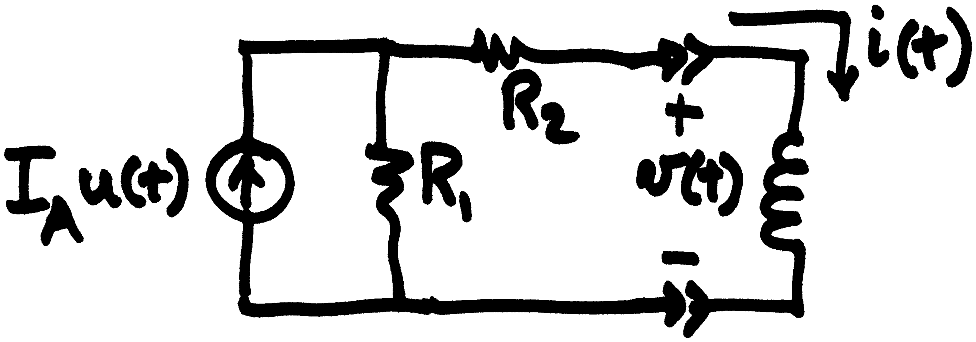
1. writing a differential equation in the state variable

   capacitor — voltage
   inductor — current

2. initial - final value theorem

   for first order
   (one reactive element)

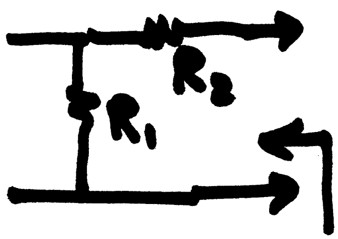
Example 7-5



Nortonize

Step 1: simplify source (left hand side)

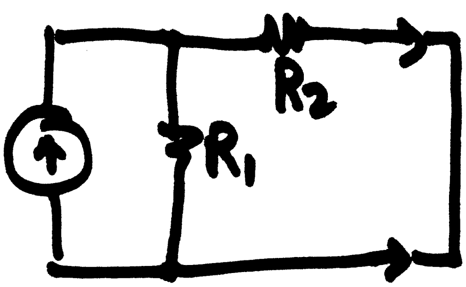
turn off sources to get R_N



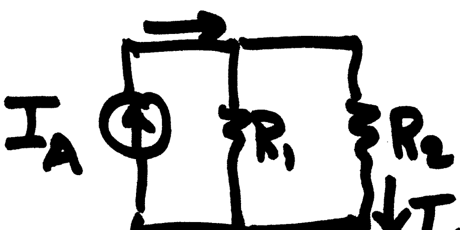
$R_N = R_1 + R_2$

short circuit

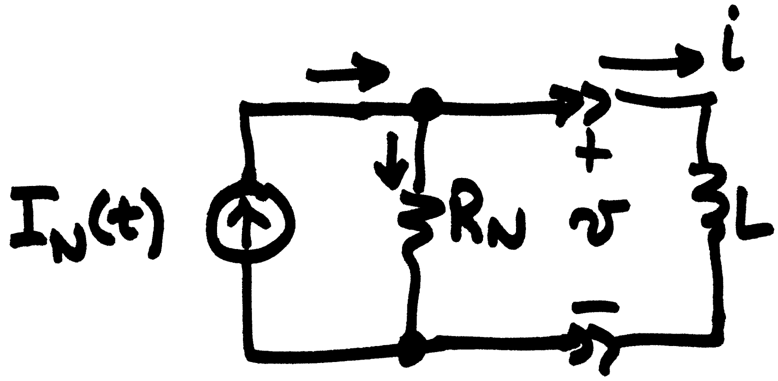
$t \geq 0$ I_A



$I_{sc} = I_N$



$$I_{sc} = \frac{R_1}{R_1 + R_2} I_A$$



$t \geq 0$

<u>step 2:</u> find D.E. in the state variable

use KCL $\sum_{+\text{in}} i = 0$

use constraint eqn.

$$+I_N - \frac{v}{R_N} - i = 0 \qquad v = L\frac{di}{dt}$$

$$I_N - \frac{L}{R_N}\frac{di}{dt} - i = 0$$

$$\frac{L}{R_N}\frac{di}{dt} + i = I_N$$

<u>step 3</u> solve the equation

$$i(t) = i_{\text{NATURAL}} + i_{\text{FORCED}}$$

steady-state DC

$i_{NATURAL}$

$$\frac{L}{R_N}\frac{di}{dt}+i=0$$

let $i_N = Ke^{st}$

$$\frac{L}{R_N} s Ke^{st} + Ke^{st} = 0$$

$$Ke^{st}\Big(\underbrace{\frac{L}{R_N}s+1}_{\text{require this to be zero}}\Big)=0$$

$$s = -\frac{R_N}{L}$$

$$i_N(t) = Ke^{-\frac{R_N}{L}t}$$

$$= Ke^{-\frac{t}{L/R_N}}$$

i_{Forced} $\quad \frac{d}{dt} \to 0$ $\quad \cancel{\frac{L}{R_N}\frac{di}{dt}}^{\,0} + i = I_N$

$$\Rightarrow i_F = I_N$$

total solution

$$i(t) = i_N + i_F = Ke^{-\frac{t}{L/R_N}} + I_N$$

get K from initial condition

initial condition $i(0) = I_0$

$$i(0) = I_0 = Ke^{0} + I_N = Ke^{0} + \frac{R_1}{R_1+R_2} I_A$$

solve for $K = I_0 - \frac{R_1 I_A}{R_1+R_2}$

Example 7-6

state variable response of a first-order RC circuit is

$$v_c(t) = 20e^{-200t} - 10 \qquad t \geq 0$$

(a) What is the time constant?

$$e^{-\frac{t}{1/200}} \qquad T_c = \frac{1}{200} = 5\,\text{msec.}$$

(b) What is the initial voltage across the capacitor?

$$v_c(0) = 20e^{0} - 10 = 20 - 10 = +10 \text{ volts.}$$

(c) What is the amplitude of the forced response?

$$\lim_{t\to\infty} v_c(t) = -10$$

(d) At what time is $v_c(t) = 0$

$$0 = 20e^{-200t} - 10$$

$$\ln \frac{10}{20} = \ln e^{-200t}$$

$$t = \frac{\ln(1/2)}{-200} = 3.46\,\text{msec.}$$