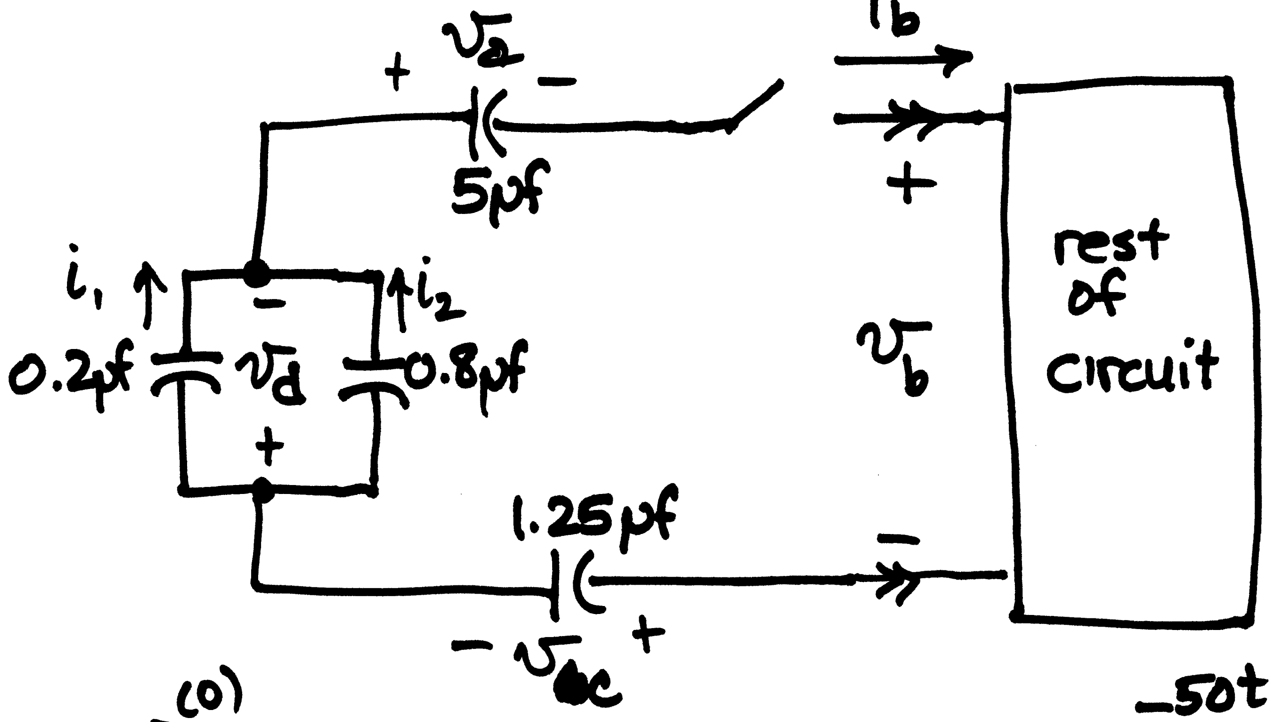
INITIAL CONDITIONS

EXAMPLE 7.27

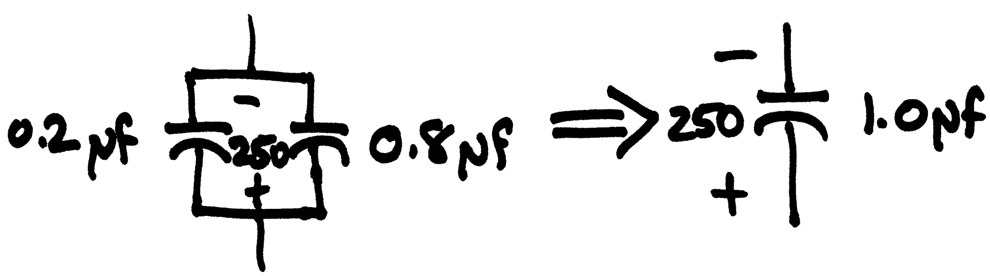


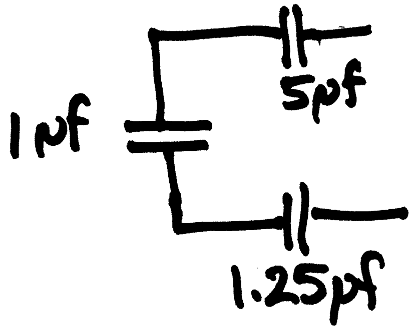
$v_a(0) = -20$

$v_c(0) = -30$

$v_d(0) = +250$

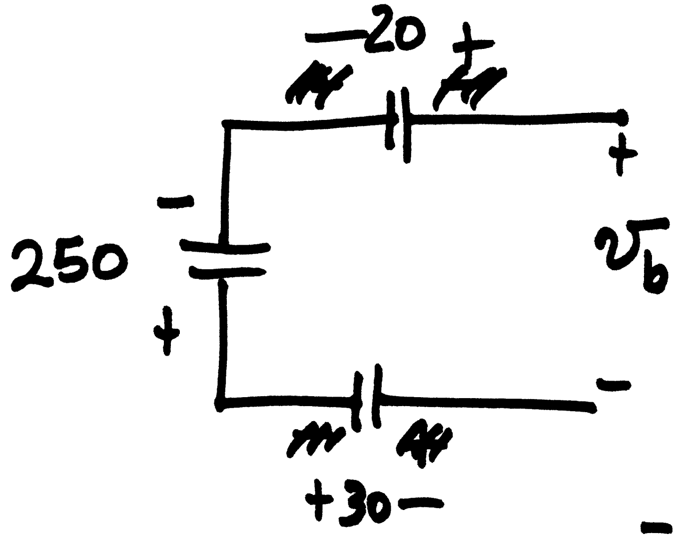
measure $i_b(t) = -5e^{-50t}$ mA





$$\frac{1}{C_{EQ}} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25}$$

$$C_{EQ} = 0.5\text{pf}$$



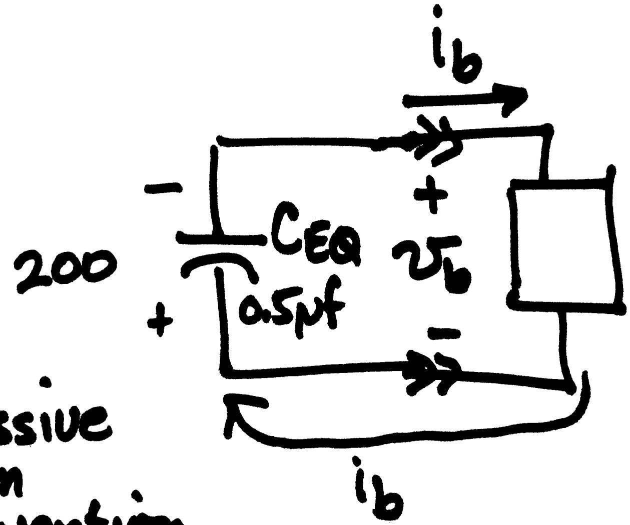
$t > 0$

@ $t = 0$

$-30 + 250 - 20 + v_b = 0$

$200 + v_b = 0$

$\underline{v_b(t=0) = -200 \text{ volts.}}$



passive sign convention is ok.

$i_b = C_{EQ} \frac{dv}{dt} = C_{EQ} \frac{d}{dt}(-v_b)$

$i_b = -C_{EQ} \frac{dv_b}{dt}$

$$\int_0^t -\frac{1}{C_{EQ}} i_b \, \underline{\underline{dt}} = \int_0^t dv_b$$

$$-\frac{1}{C_{EQ}} \int_0^t \left(-5\times10^{-3} e^{-50x} \, dx\right) = v_b(t) - \underline{\underline{v_b(0)}}$$

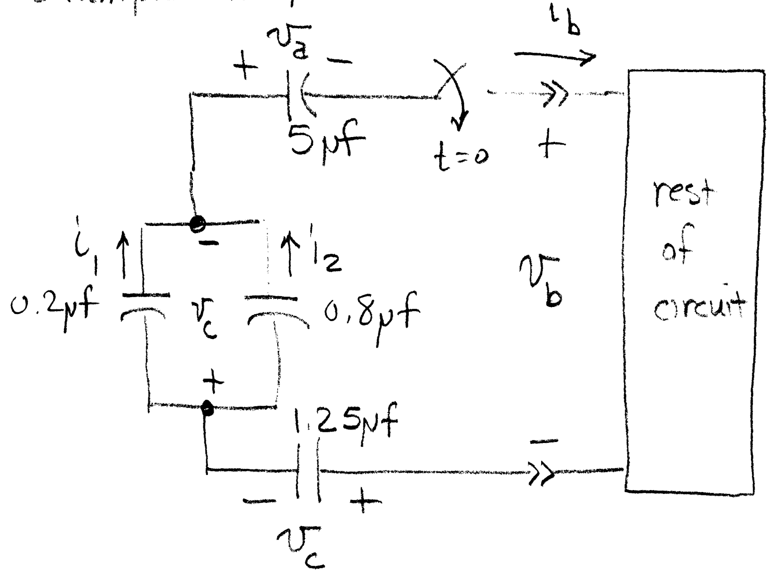
$$\frac{5\times10^{-3}}{.5\times10^{-6}} \frac{e^{-50x}}{-50} \Big|_0^t = v_b(t) - (-200)$$

$$v_b(t) + 200 = \underbrace{\frac{5\times10^{-3}}{.5\times10^{-6}(-50)}}_{-200}\left[e^{-50t} - \cancelto{1}{e^{0}}\right]$$

$$v_b(t) + \cancel{200} = -200\,e^{-50t} + \cancel{200}$$

$$v_b(t) = -200\,e^{-50t} \text{ volts.}$$

Example 7.27



Given the initial conditions we determine that: $i_b(t) = -5e^{-50t}$ mA for $t \geq 0$

Determine the equivalent circuit for the capacitances, the voltage v_b, and the power being delivered to the rest of the circuit.

$v_a(0) = -20$

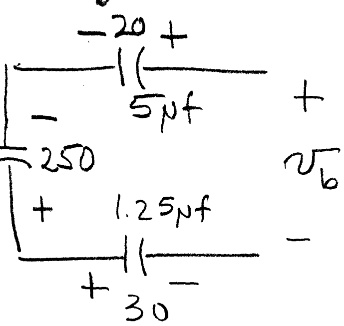
$v_c(0) = -30$

$v_d(0) = 250$

We measure $i_b = -5e^{-50t}$ mA

@t=0

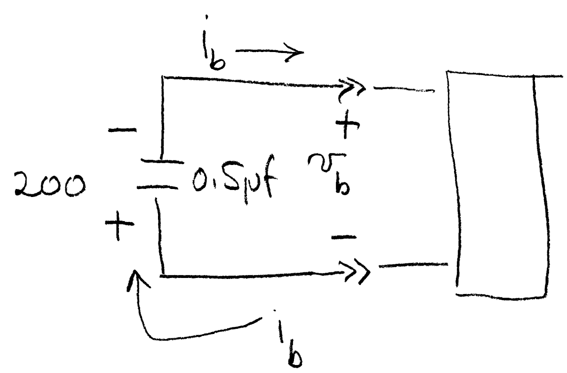
0.2μf + 0.8μf = 1μf



at t=0 $\quad -30 + 250 - 20 + v_b(0) = 0$

$v_b(0) = -200$ volts

$$\frac{1}{C_{EQ}} = \frac{1}{1.25} + \frac{1}{1} + \frac{1}{5} = \frac{10}{5} \Rightarrow C_{EQ} = 0.5\mu f.$$



$$i_b = C_{EQ}\frac{d(-v_b)}{dt}$$

$$\int_0^t \frac{1}{C_{EQ}} i_b\,dt = -\int_0^t dv_b$$

$$\frac{1}{0.5\times10^{-6}}\int_0^t -5\times10^{-3}e^{-50x}dx = -v_b(t)+v_b(0)$$

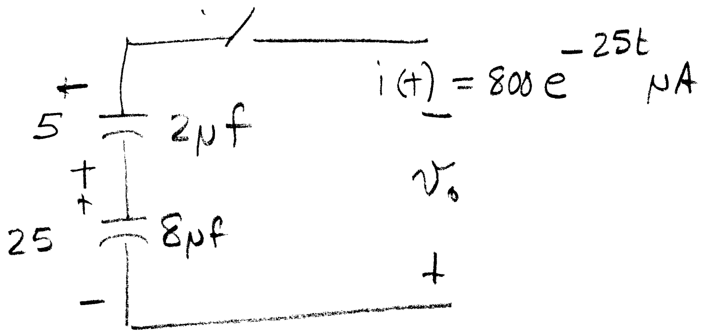
$$\frac{-5\times10^{-3}}{0.5\times10^{-6}}\int_0^t e^{-50x}dx = -v_b(t)+(-200) = -v_b(t)-200$$

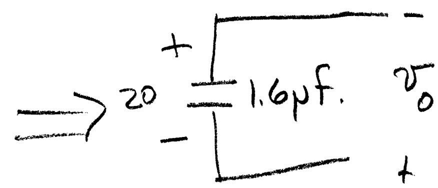
$$-v_b(t)-200 = -10^4\left.\frac{e^{-50x}}{-50}\right|_0^t = \frac{-10^4}{-50}\left(e^{-50t}-1\right) = 200e^{-50t}-200$$

$$v_b(t) = -200e^{-50t}$$

$$p = v_b(t)i_b(t) = (-5e^{-50t}\times10^{-3})(-200e^{-50t})$$

$$= +e^{-100t} \text{ watts}$$





(a) $\frac{1}{C_{EQ}} = \frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$ $\quad C_{EQ} = \frac{8}{5} = 1.6\mu f.$

$v_{EQ}(0) = 25 - 5 = 20$

$-20 - v_0 = 0$

$v_0 = -20$

$i = C\frac{dv}{dt}$

$\frac{1}{C} i\,dt = dv_0$

$\frac{1}{C}\int_0^t i(x)dx = v_0(t) - v_0(0)$

$\frac{1}{1.6\times10^{-6}}\int_0^t 800\times10^{-6} e^{-25x}\,dx = v_0(t) - (-20)$

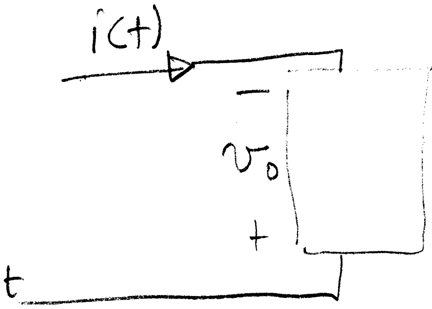
$\frac{800}{1.6} \left.\frac{e^{-25x}}{-25}\right|_0^t = v_0(t) + 20$

$\frac{500}{-25}\left(e^{-25t} - 1\right) = v_0(t) + 20$

$-20e^{-25t} + \cancel{20} = v_0(t) + \cancel{20}$

$v_0(t) = -20e^{-25t}$

(b) $v_1(t) = -16e^{-25t} + 21$



(c) $v_2(t) = -4e^{-25t} - 21$

(d) $p = -v_0 i = 16\times10^{-3} e^{-50t}$ watts

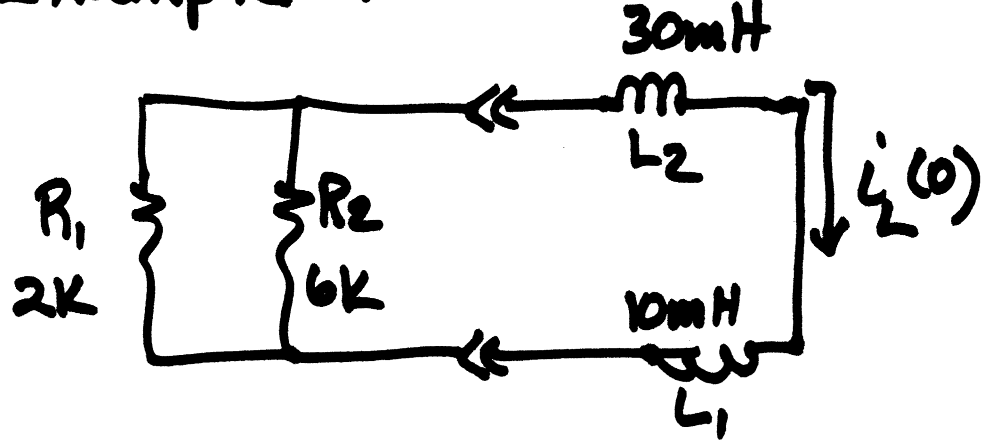
↑ passive sign convention

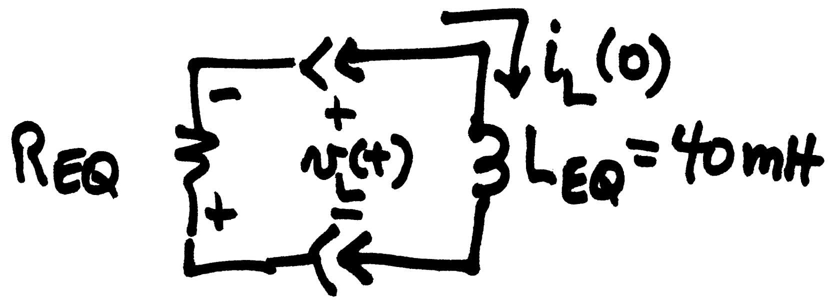
$$w = \int_0^{\infty} p(x)\,dx = 16\times10^{-3}\int_0^{\infty} e^{-50x}\,dx = 16\times10^{-3}\left.\frac{e^{-50x}}{-50}\right|_0^{\infty} = 320\times10^{-6}\,\text{J}.$$

(e) $w(0) = \frac{1}{2}C\,v^2(0) + \frac{1}{2}C\,v^2(0) = 2525\ \text{J}.$

what is trapped energy?

Example 7-2





$R_{EQ} = \frac{2 \cdot 6}{2+6} = \frac{3}{2}$k or 1500Ω

$\frac{1}{R_{EQ}} = \frac{1}{2K} + \frac{1}{6K}$

KVL gives

$i_L R_{EQ} + v_L = 0$ $v_L = L\frac{di_L}{dt}$

$1500\, i_L + L\frac{di_L}{dt} = 0$

$40 \times 10^{-3} \frac{di_L}{dt} + 1500\, i_L = 0$

let $i_L(t) = Ke^{st}$

$40 \times 10^{-3} s\,(Ke^{st}) + 1500\, Ke^{st} = 0$

$40 \times 10^{-3} s + 1500 = 0$

$s = -37500$

$$i_L(t) = Ke^{-37500t} \qquad t \geq 0$$

what is the time constant T_c?

$$T_c = \frac{1}{37500}$$

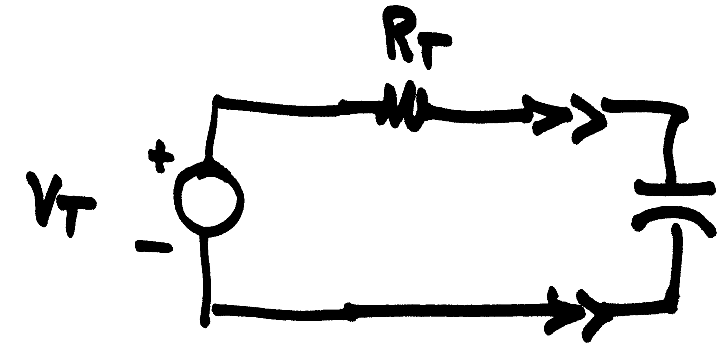
$$i_L(t) = Ke^{-\frac{t}{26.7\mu sec}}$$

to get K use $i_L(0) = 0.1$ Amps.

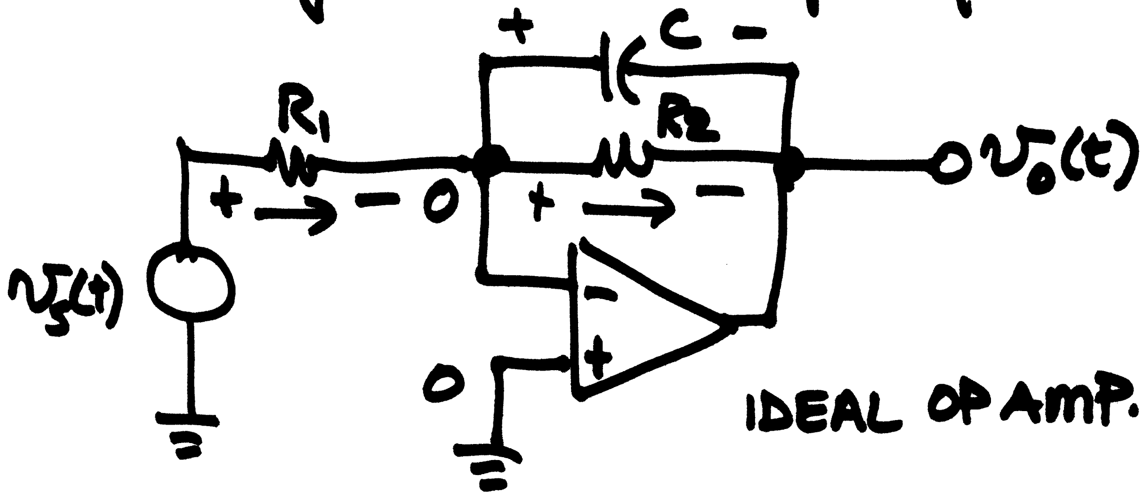
$$i_L(0) = Ke^0 = 0.1 \text{ amps}.$$

$$K = 0.1$$

$$i_L(t) = 0.1e^{-\frac{t}{26.7\mu sec}} \qquad t \geq 0$$



What do you do with an op amp circuit?



Don't Thevenize. $\sum_{\text{in}} i = 0$

$$+\frac{v_s - 0}{R_1} - \frac{0 - v_0(t)}{R_2} - C\frac{d}{dt}\left(0 - v_0(t)\right) = 0$$

$$\frac{v_s}{R_1} + \frac{v_0}{R_2} + C\frac{dv_0}{dt} = 0$$

$$R_2 C\frac{dv_0}{dt} + v_0 = -\frac{R_2}{R_1}v_s$$

What is forced response?

$$v_T = A\,u(t)$$

$$R_T C \frac{dv}{dt} + v = v_T(t) \quad A \qquad t \geq 0$$

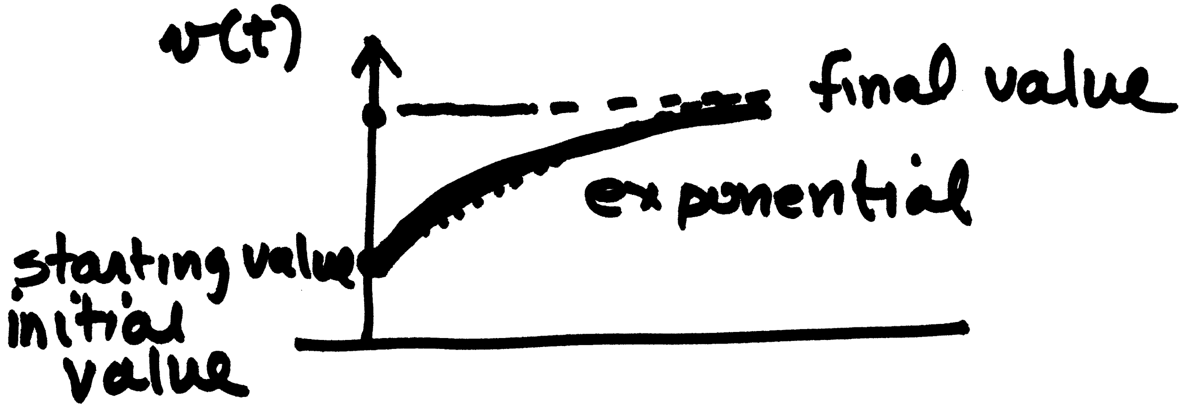
D.c. or steady state $\Rightarrow \frac{d}{dt} \to 0$

$$v = A$$

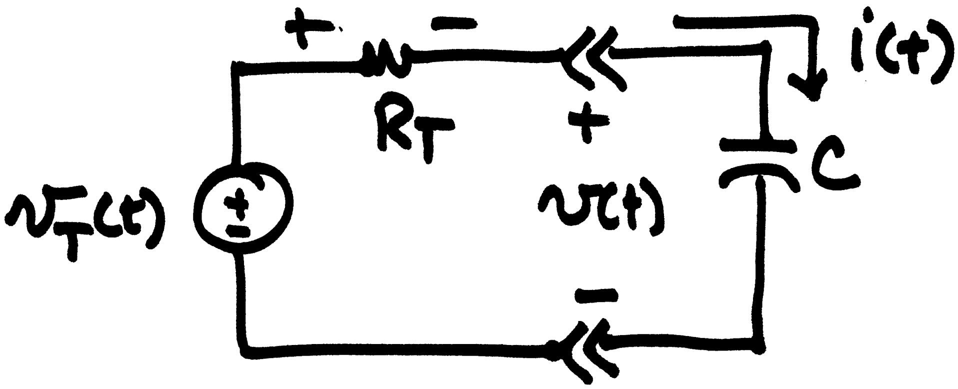
Total solution

$$v(t) = v_N(t) + v_F(t)$$

$$= Ke^{st} + A$$



First order step response



KVL $\quad -v_T(t) + i(t)R_T + v(t) = 0$

constraint for capacitor $i(t) = C\dfrac{dv}{dt}$

$$-v_T(t) + R_TC\frac{dv}{dt} + v(t) = 0$$

$$R_TC\frac{dv}{dt} + v = v_T(t)$$

Two solutions

zero-input response
natural response
$v_N(t)$

forced response
$v_F(t)$
(DC or steady state)

Superposition:

$$v(t) = v_N(t) + v_F(t)$$