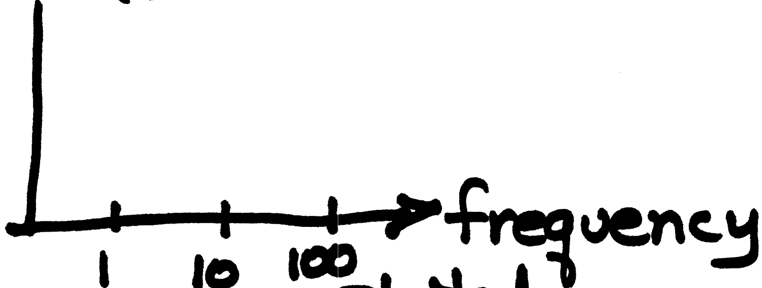
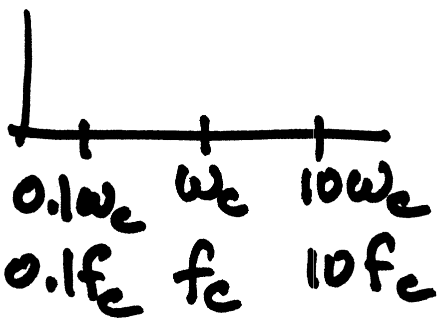
Frequency response (Bode plot)

Horizontal:



plotted
as f or ω
in decades (powers of 10)
octaves



Vertical:

plot gain $\quad \underline{T}(j\omega) = \dfrac{\underline{V}_2(j\omega)}{\underline{V}_1(j\omega)}$

in phasor domain
(for sinusoids)

phasor - complex number

$$T(j\omega) = \underline{\underline{\underline{|T(j\omega)|}}} \angle T(j\omega)$$

usually plot the magnitude

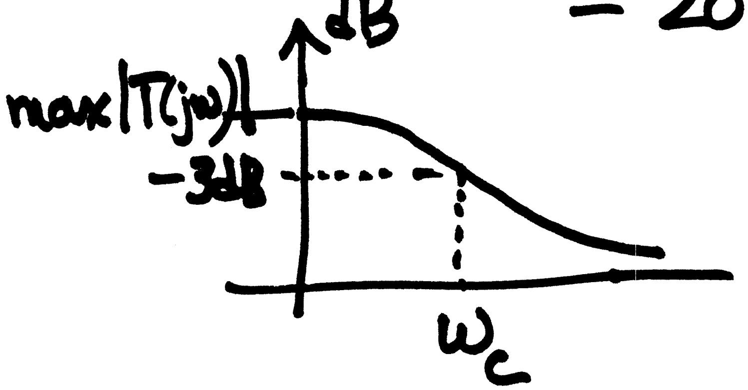
decibels plot gain (vertical axis) in decibels

$$\left|T(j\omega)\right|_{dB} = 20\log_{10}\left|T(j\omega)\right|$$

for cutoff frequency $\omega_c \Rightarrow \left|\frac{T(j\omega)}{\sqrt{2}}\right|$

$$20\log_{10}\left|\frac{T(j\omega)}{\sqrt{2}}\right| = 20\log_{10}\left|T(j\omega)\right| - 20\log_{10}\sqrt{2}$$

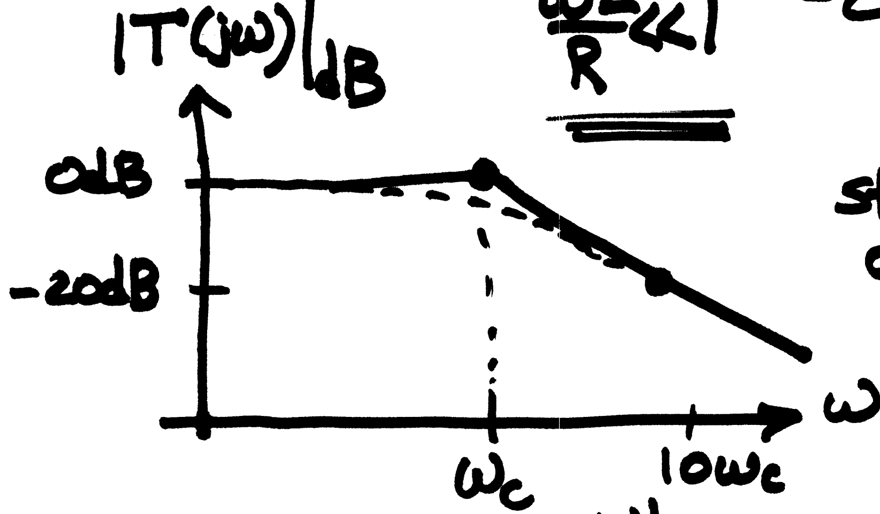
$$= 20\log_{10}\left|T(j\omega)\right| - 3\,dB$$



cutoff frequency is often called the 3dB frequency numerator: $20\log_{10}(1) \rightarrow 0\text{dB}$

denominator: $\frac{\omega L}{R} \gg 1 \quad -20\log_{10}\frac{\omega L}{R}$

$\frac{\omega L}{R} \ll 1 \quad -20\log_{10}(1) \rightarrow 0\text{dB}$



straight-line approximation

$\frac{\omega L}{R} = 1$

$\omega_c = \frac{R}{L}$

always occurs when denom. is 1+j

$-20\log_{10}\frac{\omega L}{R}$

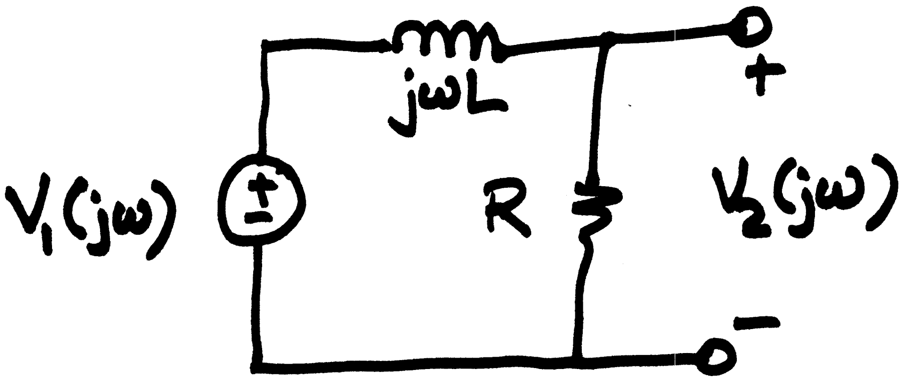
suppose $\omega \rightarrow 10\omega$

$-20\log_{10}\frac{10\omega L}{R} \rightarrow -20\log_{10}10 - 20\log_{10}\frac{\omega L}{R}$

$-20 - 20\log_{10}\frac{\omega L}{R}$

-20dB/decade

Example 12-2



voltage divider $V_2(j\omega) = \frac{R}{R+j\omega L} V_1(j\omega)$

$$T(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{R}{R+j\omega L} = \frac{1}{1+j\frac{\omega L}{R}}$$

divide by R
to get a leading 1
this is "standard" form

gain

$$20\log_{10}|T(j\omega)| = 20\log_{10}\left|\frac{1}{1+j\frac{\omega L}{R}}\right|$$

$$= 20\log_{10}|1| - 20\log_{10}\left|1+j\frac{\omega L}{R}\right|$$

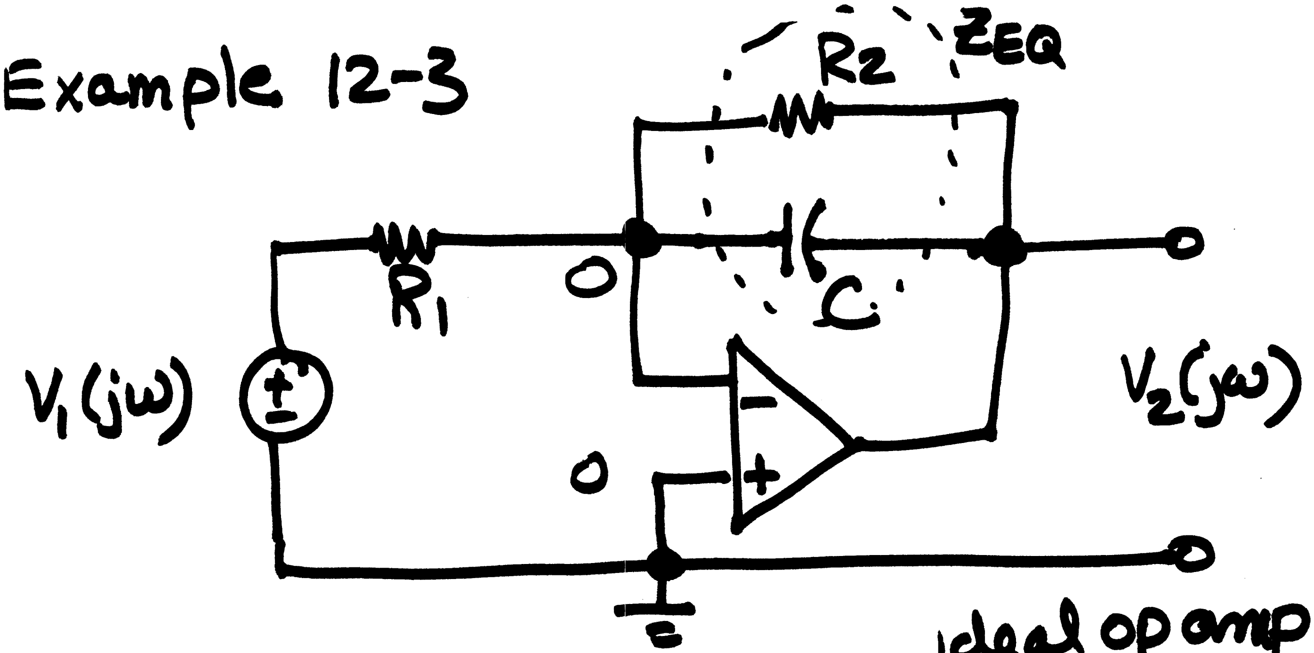
0dB

$\frac{\omega L}{R} \ll 1$ $-20\log_{10}|1|$
0dB

$\frac{\omega L}{R} \gg 1$ $-20\log_{10}\left|j\frac{\omega L}{R}\right|$

$-20\log_{10}\frac{\omega L}{R}$

Example 12-3

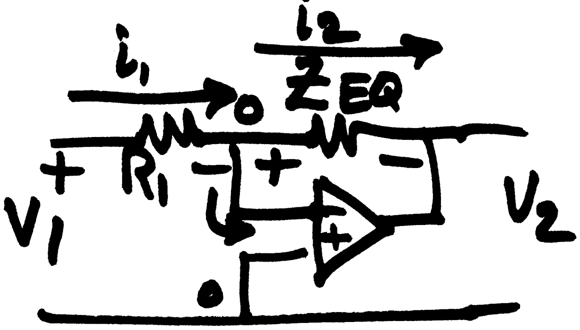


Find the gain $\dfrac{V_2(j\omega)}{V_1(j\omega)}$

$$Z_{EQ} = R_2 \parallel \frac{1}{j\omega C} \qquad \frac{1}{Z_{EQ}} = \frac{1}{R_2} + \frac{1}{\frac{1}{j\omega C}}$$

$$= \frac{R_2\left(\frac{1}{j\omega C}\right)}{R_2 + \frac{1}{j\omega C}}$$

$$Z_{EQ} = \frac{R_2}{j\omega R_2 C + 1}$$



$$i_1 = \frac{V_1(j\omega) - 0}{R_1} \qquad i_2 = \frac{0 - V_2(j\omega)}{Z_{EQ}}$$

$i_1 = i_2$ for ideal op-amp since $i_{IN} = 0$

$$\frac{V_1(j\omega)}{R_1} = -\frac{V_2(j\omega)}{Z_{EQ}}$$

$$\frac{V_2(j\omega)}{V_1(j\omega)} = -\frac{Z_{EQ}}{R_1} = -\frac{\frac{R_2}{1 + j\omega R_2 C}}{R_1}$$

Plot $\dfrac{V_2(j\omega)}{V_1(j\omega)} = -\dfrac{R_2/R_1}{1 + j\omega R_2 C}$

$$20\log_{10}\left|\frac{-\frac{R_2}{R_1}}{1+j\omega R_2 C}\right| = 20\log_{10}\left(\frac{R_2}{R_1}\right) - 20\log_{10}\left(\underline{\underline{1+j\omega R_2 C}}\right)$$

$\omega R_2 C = 1$

$\omega_c = \dfrac{1}{R_2 C}$