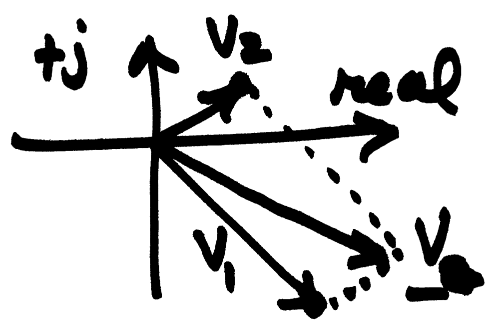
phasors — vector notation for representing sinusoids

$$\underline{v(t) = v_1(t) + v_2(t)}$$

$$v_1 = 10\cos(1000t - 45^\circ)$$

$$v_2 = \underbrace{5\cos}_{\text{cosines}}(\underbrace{1000}_{\omega}t + 30^\circ)$$



$$\underline{V_1} = 10\angle -45^\circ$$

$$\underline{V_2} = 5\angle +30^\circ$$

$$\underline{V} = \underline{V_1} + \underline{V_2} = \underbrace{10\angle -45^\circ + 5\angle +30^\circ}_{\text{polar represention}}$$

$$= 10\cos 45^\circ - j10\sin 45^\circ + 5\cos 30^\circ + j5\sin 30^\circ$$

$$\underline{V} = (10\cos 45^\circ + 5\cos 30^\circ) + j(-10\sin 45^\circ + 5\sin 30^\circ)$$

$$= 7.07 - j7.07 + 4.33 + j2.5$$

$$\underline{V} = 11.4 - j4.57 = 12.28\angle -21.8^\circ$$

$$v(t) = 12.28\cos(1000t - 21.8^\circ)$$

integration and differentiation

$$\frac{d}{dt}\left[\underbrace{15\cos(200t-30^\circ)}_{15\angle -30^\circ}\right]$$

$$\boxed{15e^{-j30^\circ}\underline{e^{j200t}}}$$

usually we don't write this

$$\frac{d}{dt}\,\mathrm{Re}\left\{15e^{-j30^\circ}e^{j200t}\right\}$$

$$\mathrm{Re}\left\{\frac{d}{dt}\left(15e^{-j30^\circ}e^{j200t}\right)\right\}$$

$$\mathrm{Re}\left\{j200\ 15e^{-j30^\circ}e^{j200t}\right\}$$

in general $\underbrace{j\omega\ \underline{\text{Phasor}}}_{\text{derivative}}$

integral $\left.\dfrac{\underline{\text{Phasor}}}{j\omega}\right\}$ integral

$$\frac{d}{dt}\left[15\cos(200t - 30^\circ)\right]$$

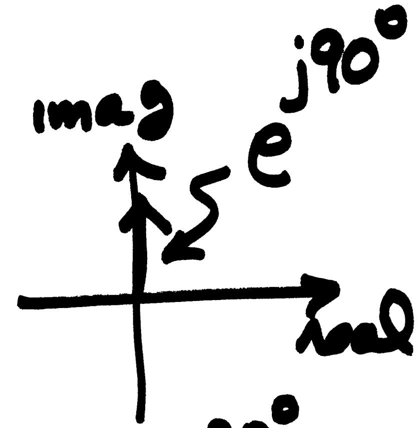
$$\frac{d}{dt}\left[15\angle -30^\circ\right]$$

$$\frac{d}{dt}\left[15e^{-j30^\circ}\right]$$

$$\underset{\underset{200}{\uparrow}}{j\omega}\, 15e^{-j30^\circ}$$

$$j(200)(15)e^{-j30^\circ}$$

$$e^{j90^\circ}\, 3000\, e^{-j30^\circ}$$



$$j = e^{j90^\circ}$$

$$3000\, e^{+j60^\circ}$$

phasor derivative

$$3000\cos(200t + 60^\circ)$$

Phasor circuit analysis

resistors

$$v_R = R i_R$$

$$\underline{V}_R = R\,\underline{I}_R$$

inductor

$$v_L = L \frac{di_L}{dt}$$

$$\underline{V}_L = L\, j\omega\, \underline{I}_L = \underline{\underline{j\omega L}}\, \underline{I}_L$$

impedance
(generalized resistance)

capacitor

$$i_c = C \frac{dv_c}{dt}$$

$$\underline{I}_c = C\, j\omega\, \underline{V}_c = j\omega C\, \underline{V}_c$$

$$\underline{V}_c = \frac{1}{j\omega C}\, \underline{I}_c$$

impedance

do circuit analysis

convert

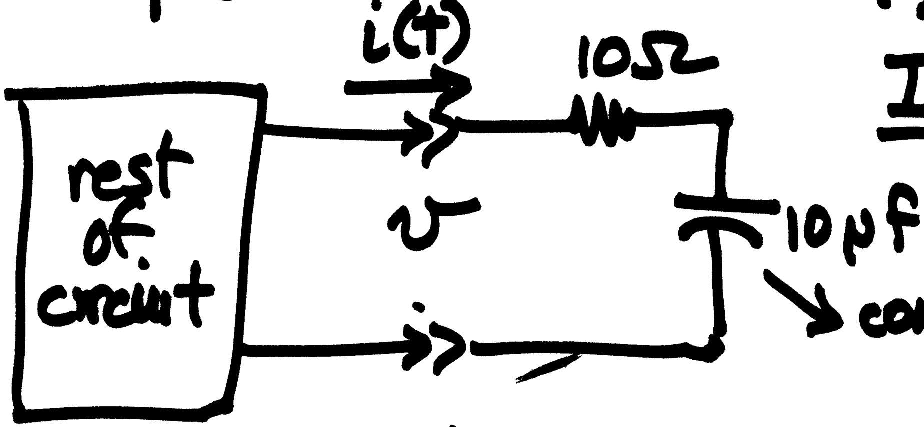
$$\left.\begin{array}{l} R \rightarrow R \\ L \rightarrow j\omega L \\ C \rightarrow \dfrac{1}{j\omega C} \end{array}\right\} \text{behave algebraically like resistors}$$

all sources
currents
voltages

Example 8-5

$i(t) = 4\cos 5000t$

$\underline{I} = 4\angle 0° = 4 + j0$



convert impedance

$$v_R = iR \quad \Big| \quad 4(10) = 40 = \underline{V}_R$$

real | phasors

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j(5000)(10\times10^{-6})} = -j20$$

$$\underline{V}_c = \underline{I}\,\underline{Z}_c = (4)(-j20) = -j80$$

$$v(t) = v_R + v_c$$

$$\underline{V} = \underline{V}_R + \underline{V}_c = 40 - j80 = 89.44\angle -64°$$

$$v(t) = 89.44\cos(5000t - 64°)$$