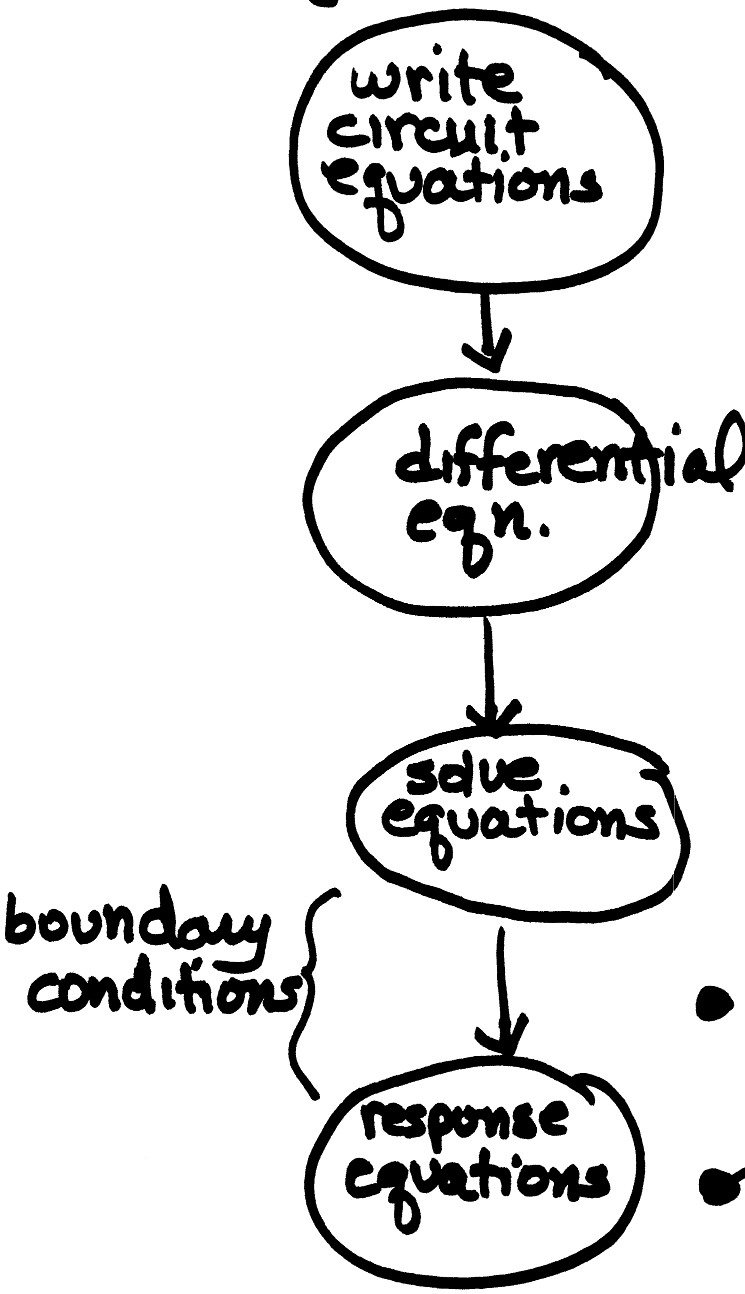
Circuits with <u>one</u> reactive element

Analyze such a circuit



i_c, v_c, i_L, v_L

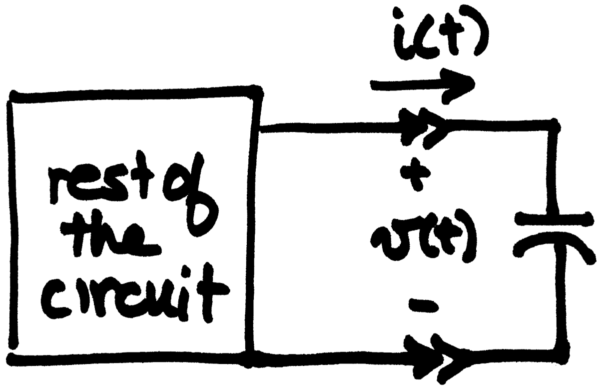
put in component constraints

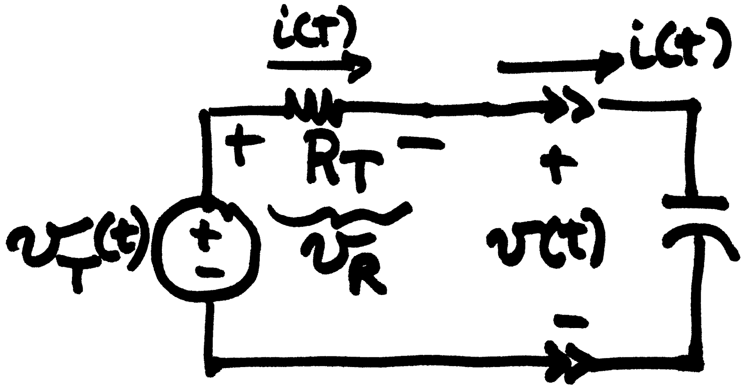
$$i_c = C\frac{dv_c}{dt}$$

$$v_L = L\frac{di_L}{dt}$$

use a <u>few</u> techniques to solve equations

- method of undetermined coefficients
- ~~Laplace~~
- phasor





use KVL

$$-v_T(t) + i(t)R_T + v(t) = 0$$

$$i(t) = C\frac{dv(t)}{dt}$$

$$-v_T(t) + R_T C\frac{dv(t)}{dt} + v(t) = 0$$

$$R_T C\frac{dv}{dt} + v = \underbrace{v_T}$$

$v(t)$ is the response function

state variable

input forcing function

switch (step)

signal generator (sinusoid)

7.4 Forced sinusoidal response

$$R_T C \frac{dv}{dt} + v = v_T$$

$$R_T C \frac{dv_F(t)}{dt} + v_F(t) = V_A \cos\omega t$$

forced response

(steady state response to signal generator)

method of undetermined coefficients

Assume $v_F(t) = a\cos\omega t + b\sin\omega t$

$$R_T C \frac{d}{dt}\left(a\cos\omega t + b\sin\omega t\right) + \left(a\cos\omega t + b\sin\omega t\right) = V_A \cos\omega t$$

$$R_T C\left(-a\omega \underline{\sin\omega t} + b\omega\cos\omega t\right) + \left(a\cos\omega t + \underline{b\sin\omega t}\right) = V_A\cos\omega t$$

Must always be true

$$-a\omega R_T C \sin\cancel{\omega t} + b\sin\cancel{\omega t} = 0$$

$$+b\omega R_T C \cos\cancel{\omega t} + a\cos\cancel{\omega t} = V_A \cos\cancel{\omega t}$$

$$-a\omega R_T C + b = 0$$

$$+b\omega R_T C + a = V_A$$

After some algebra

$$a = \frac{V_A}{1+(\omega R_T C)^2}$$

$$b = \frac{\omega R_T C V_A}{1+(\omega R_T C)^2}$$

$$v_F(t) = \frac{V_A}{1+(\omega R_T C)^2}\left[\cos\omega t + \omega R_T C \sin\omega t\right]$$

$$v_F(t) = \underline{\underline{\frac{V_A}{\sqrt{1+(\omega R_T C)^2}}\cos(\omega t+\theta)}}$$

$$\theta = \mathrm{Tan}^{-1}(-\omega R_T C)$$

Electrical engineers use a <u>simple</u> algebraic technique

$\rightarrow$ phasors

better technique to manipulate

$$v(t) = V_A \cos(\omega t + \phi)$$

Euler identity

$$e^{j\theta} = \cos\theta + j\sin\theta \qquad j = \sqrt{-1}$$

Then use Euler identity to write cosines as exponentials

$$\mathrm{Re}\{1e^{j\theta}\} = 1\cos\theta$$

$$\mathrm{Re}\{\cos\theta + j\sin\theta\} = \cos\theta \leftarrow \text{use this}$$

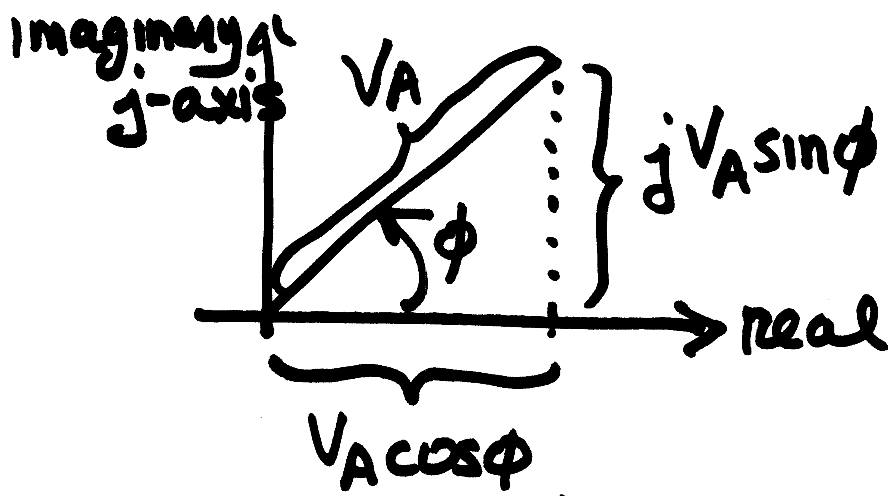
$$\mathrm{Im}\{\cos\theta + j\sin\theta\} = \sin\theta \quad \text{not too useful}$$

$$V_A \cos(\omega t + \phi) = \mathrm{Re}\left\{V_A e^{j(\omega t + \phi)}\right\}$$

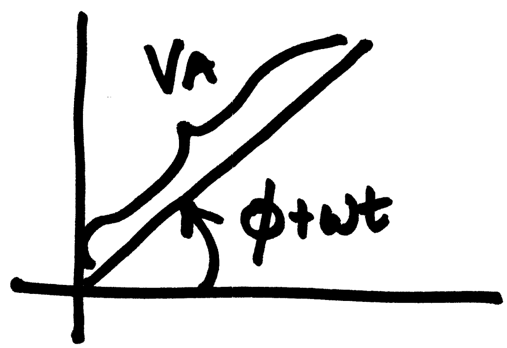
$$= \mathrm{Re}\left\{\underbrace{V_A e^{j\phi}}_{\text{phasor}}\ \underbrace{e^{j\omega t}}_{\text{assume and never write it}}\right\}$$

$$\underline{V} = V_A e^{j\phi}$$

$\underline{V}$ is a vector



why do we ignore $e^{j\omega t}$



Example 8-1

$$v_1(t) = 10\cos(\underline{\underline{1000t}} - 45^\circ)$$

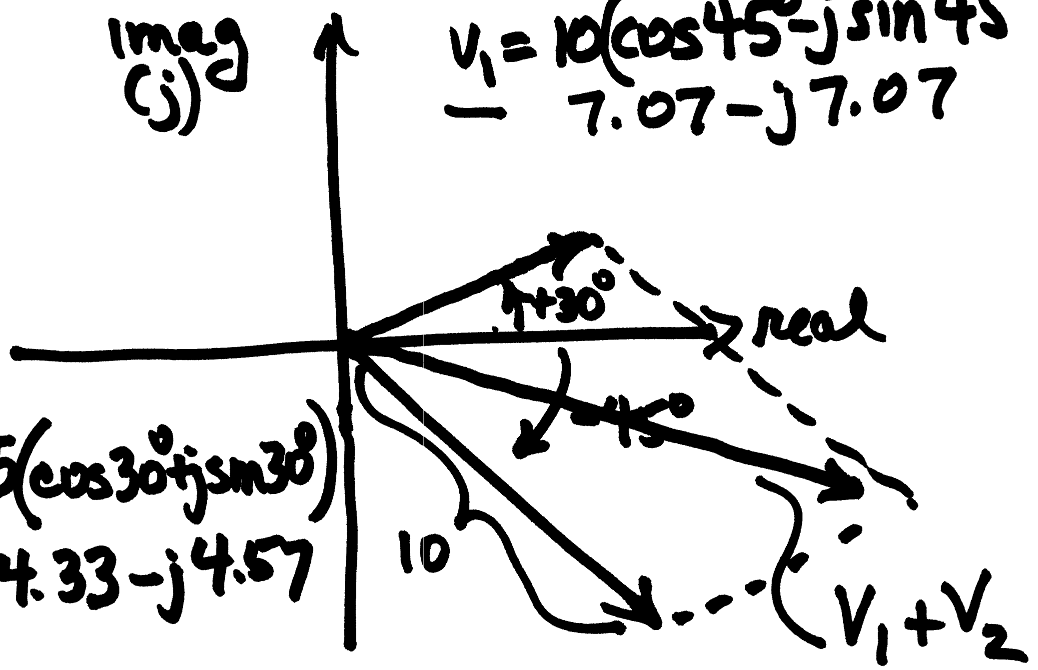
$$v_2(t) = 5\cos(1000t + 30^\circ)$$

$$v(t) = v_1(t) + v_2(t)$$

phasor for $v_1(t)$: $\underline{V_1} = 10\,\underline{e^{-j45^\circ}}$

for $v_2(t)$: $\underline{V_2} = 5\,\underline{e^{+j30^\circ}}$

$$\underline{V_1} = 10(\cos 45^\circ - j\sin 45^\circ)$$
$$= 7.07 - j7.07$$



$$\underline{V_2} = 5(\cos 30^\circ + j\sin 30^\circ)$$
$$= 4.33 - j4.57$$

$$\underline{V_1} + \underline{V_2} = 11.4 - j4.57$$