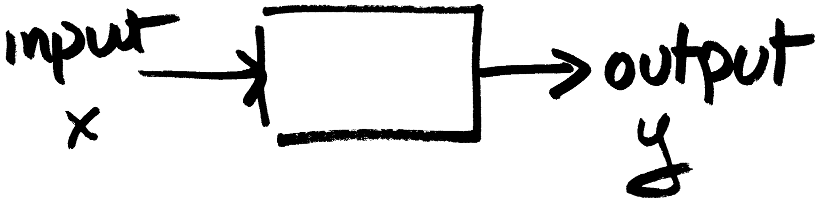
Linearity

input
output



linear

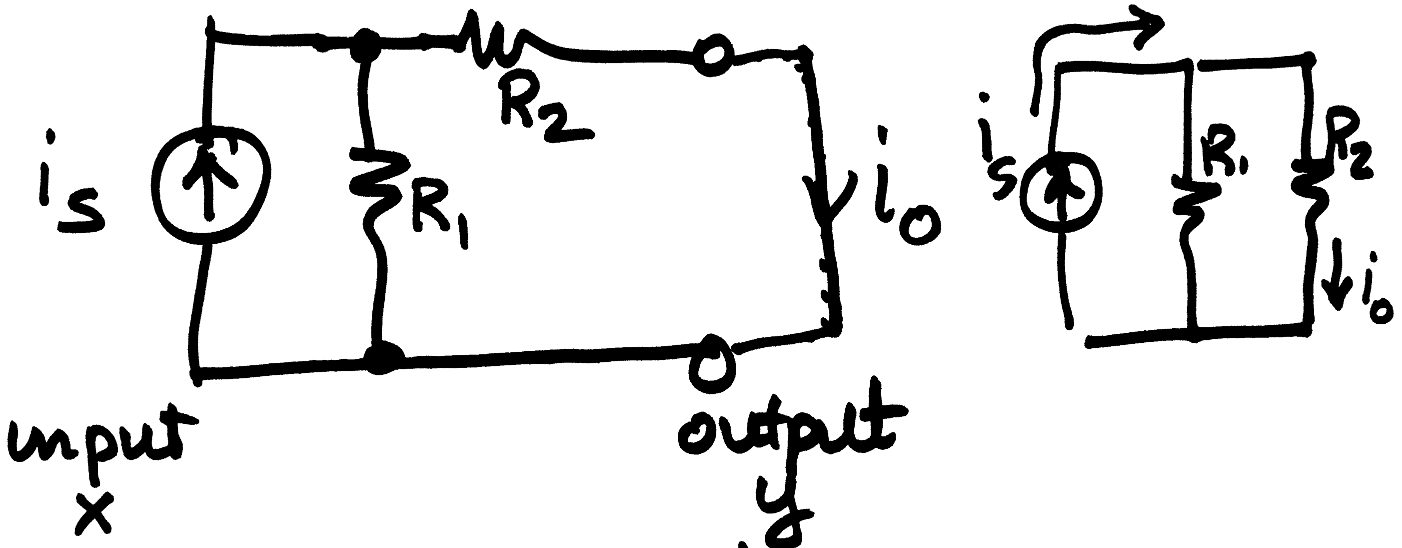
$$\text{linear} \longrightarrow \quad y = \underset{\substack{\uparrow \\ \text{gain}}}{K}\, x$$

$$\text{superposition} \longrightarrow \quad \begin{aligned} y_1 &= K x_1 \\ y_2 &= K x_2 \end{aligned}$$

$$(y_1 + y_2) = K(x_1 + x_2)$$

find the contribution to the output due to each source and add up the individual outputs to get the total output

Example



i_s — electrically — i_o

analyze using a current divider $i_o = \frac{R_1}{R_1+R_2} i_s$

output

input

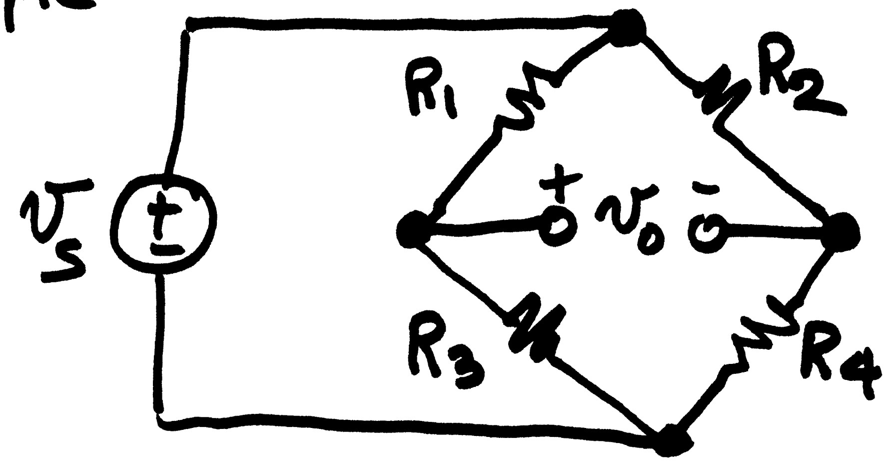
<u>constant of proportionality</u>

<u>gain</u>

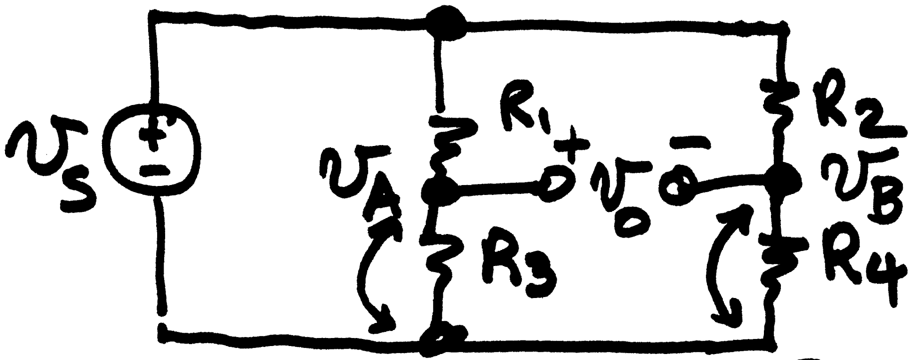
amplifier gain > 1

attenuator gain < 1

Example



strain gage circuit
v_0 - output
v_s - input



by voltage divider $v_A = \frac{R_3}{R_1+R_3} v_s$

$$v_B = \frac{R_4}{R_2+R_4} v_s$$

$$v_0 = v_A - v_B = \frac{R_3}{R_1+R_3} v_s - \frac{R_4}{R_2+R_4} v_s$$

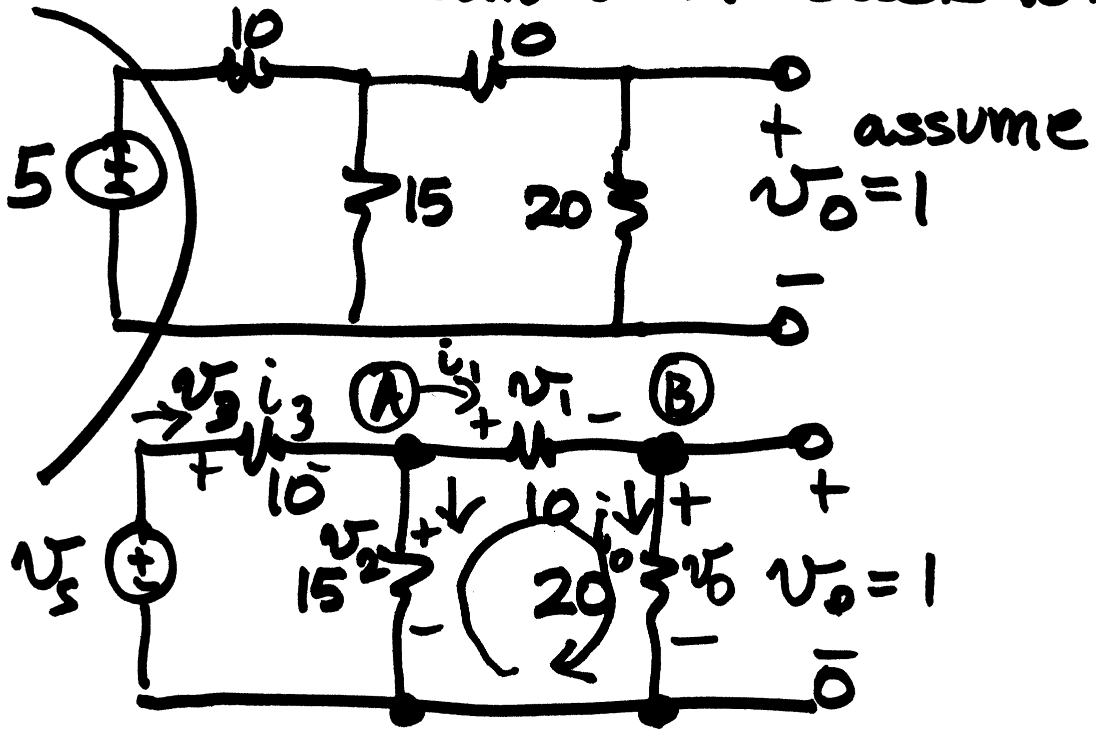
$$\underset{\text{output}}{v_0} = \underbrace{\left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right)}_{\substack{\text{gain} \\ \text{constant of proportionality}}} \underset{\text{input}}{v_s}$$

unit output method output = K input

find k

Start at output let output = 1

and work back to input



$$i_0 = \frac{v_0}{20} = \frac{1}{20} = .05$$

KCL @ B $\sum_{+in} i = 0$ $+i_1 - i_0 = 0$

$i_1 = i_0 = .05$ Amps.

$v_1 = i_1 R_1 = (.05)(10) = 0.5$ volts

KVL ↻

$-v_2 + v_1 + v_0 = 0$

$-v_2 + 0.5 + 1 = 0$

$v_2 = 1.5$ volts

$$i_2 = \frac{v_2}{R_2} = \frac{1.5 \text{ volts}}{15} = 0.1 \text{ A.}$$

KCL @ A $\sum_{+in} i = 0$ $+i_3 - i_2 - i_1 = 0$

$$i_3 - 0.1 - 0.05 = 0$$

$$i_3 = +0.15 \text{ Amps.}$$

$$v_3 = i_3 R_3 = (0.15)(10) = 1.5$$

loop $-v_s + v_3 + v_2 = 0$

$$-v_s + 1.5 + 1.5 = 0$$

$$v_s = 3$$

input	output
$v_s = 3$	$v_o = 1$

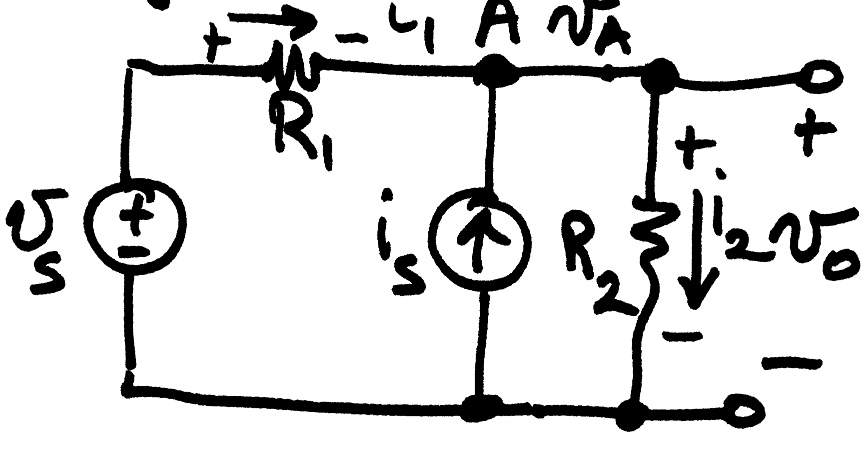
$$K = \frac{\text{output}}{\text{input}} = \frac{1}{3}$$

If The input is 5 volts, then the output

$$V_o = K v_s = \frac{1}{3} \cdot 5 = \frac{5}{3} \text{ volts.}$$

Superposition

$$y = k_1 x_1 + k_2 x_2$$



$$v_0 = v_A$$

$$v_0 = k_1 v_S + k i_S$$

KCL@node A $\sum_{+in} i@A=0$ $+i_1+i_S-i_2=0$

$$\left(\frac{v_S - v_A}{R_1}\right) + i_S - \left(\frac{v_A - 0}{R_2}\right) = 0$$

$$\frac{v_S}{R_1} - \frac{v_A}{R_1} + i_S - \frac{v_A}{R_2} = 0$$

$$\frac{v_S}{R_1} + \underline{i_S} = v_A\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \underline{v_0}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$$= \underline{v_0}\left(\frac{R_1+R_2}{R_1 R_2}\right)$$

$$v_0 = \frac{R_1 R_2}{R_1+R_2}\frac{1}{R_1} v_S + \frac{R_1 R_2}{R_1+R_2} i_S$$

output $=$ k_1 input$_1$ + k_2 input$_2$

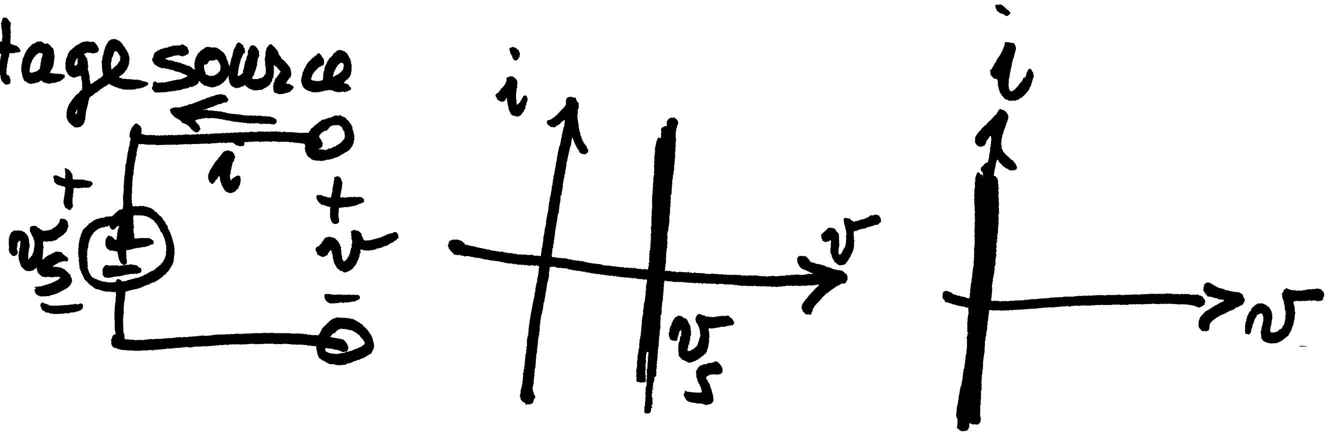
Superposition

→ turn off all sources but one and find the contribution from that source

repeat for all sources

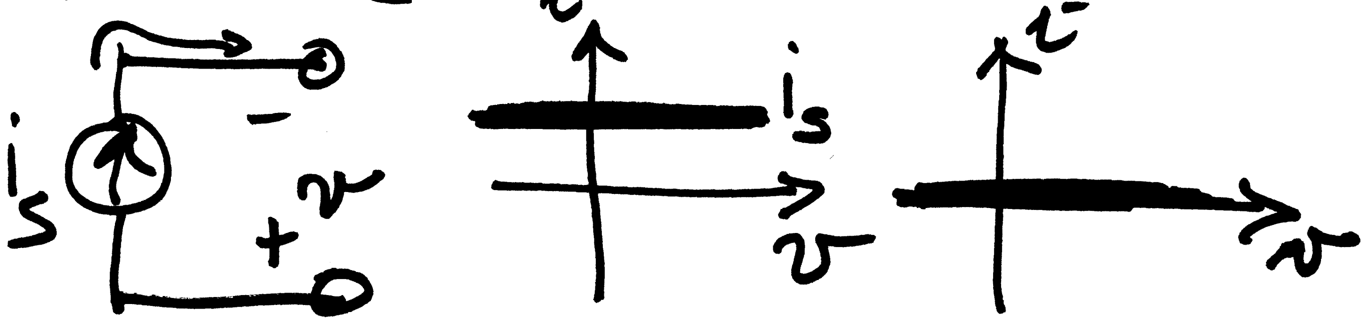
turn off a source

voltage source



short

current source



open