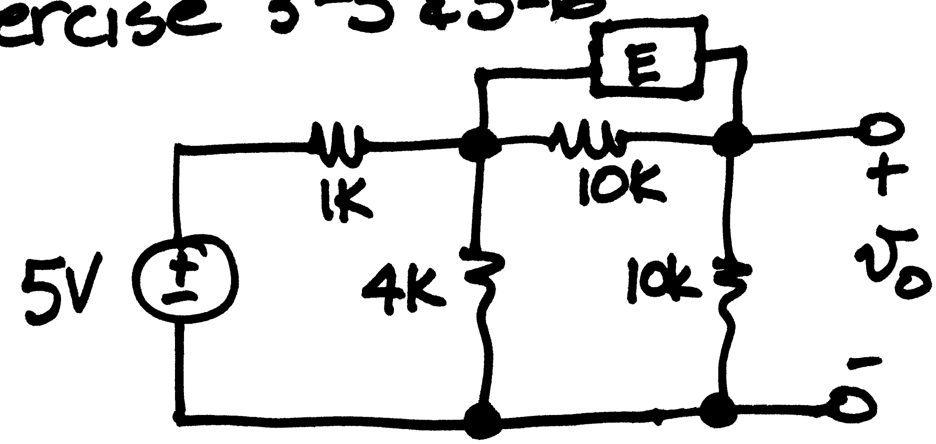
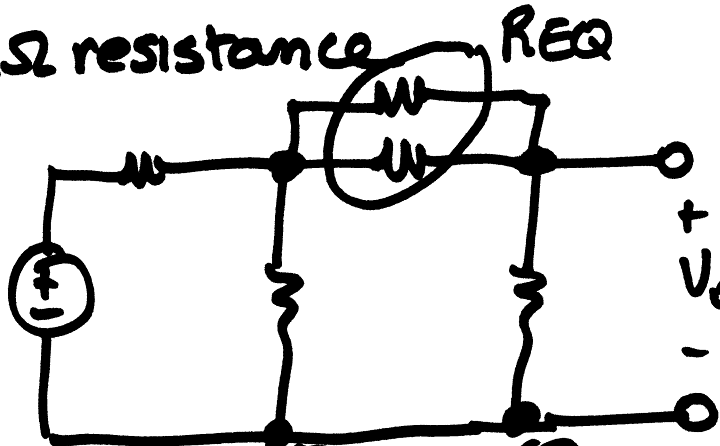
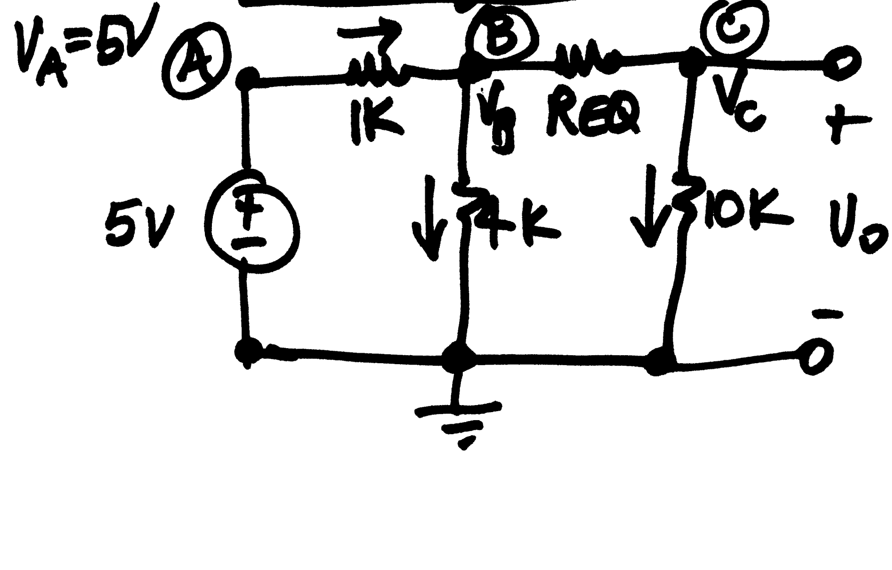
NODE VOLTAGE

Exercise 3-5 & 3-6

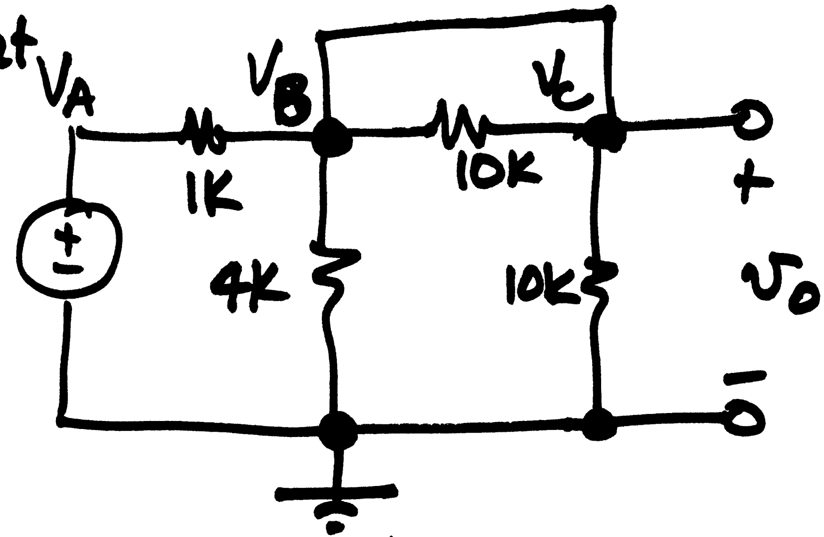


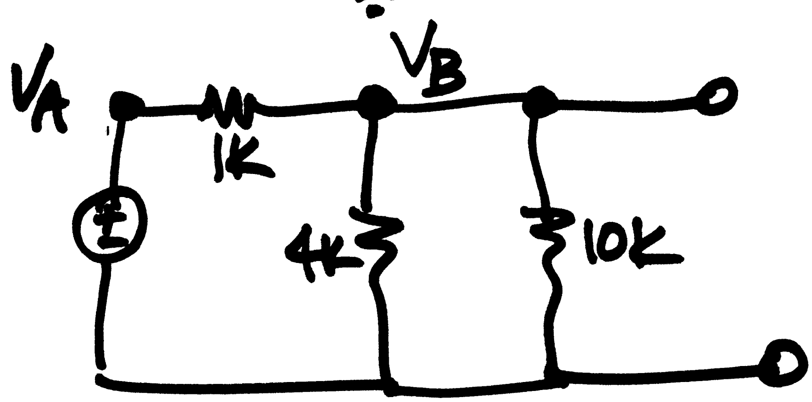
E: 10kΩ resistance



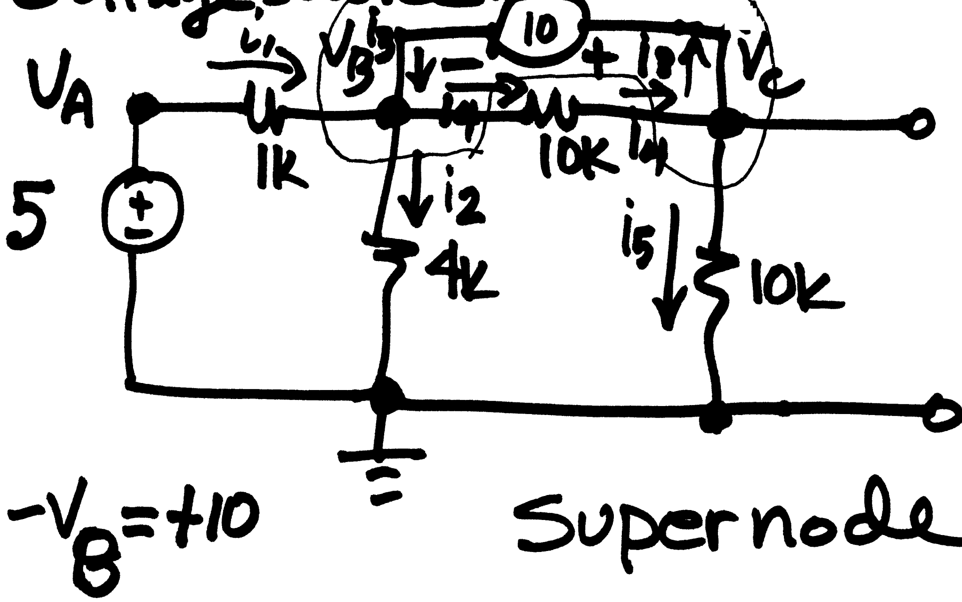


(c) short



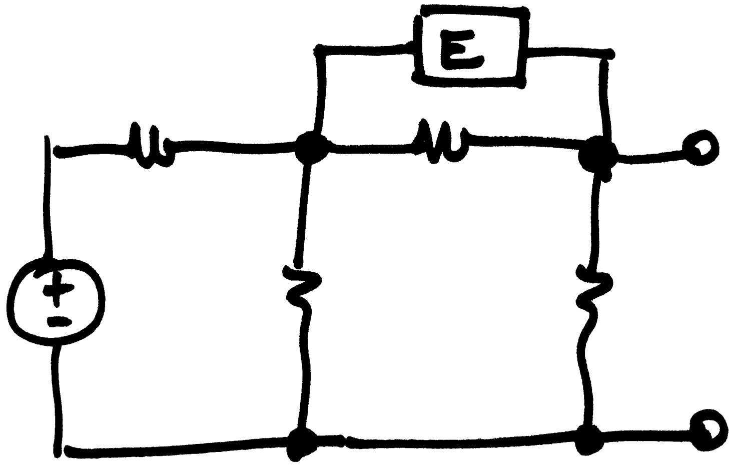


(d) voltage source

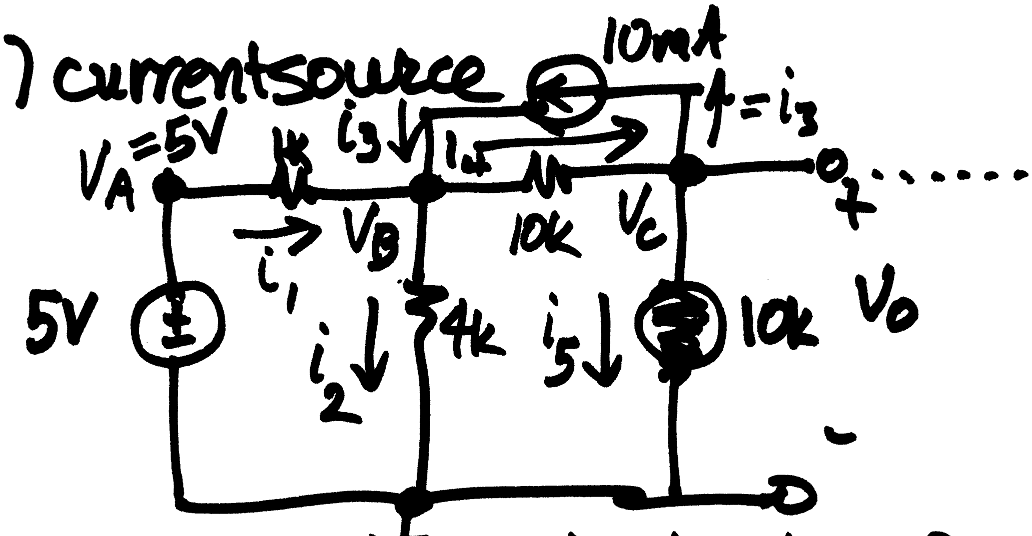


$V_C - V_B = +10$ Supernode

supernode $\sum_{+in} i = 0$ $\quad +i_1 - i_2 - \cancel{i_4} + \cancel{i_4} - i_5 = 0$



(b) current source



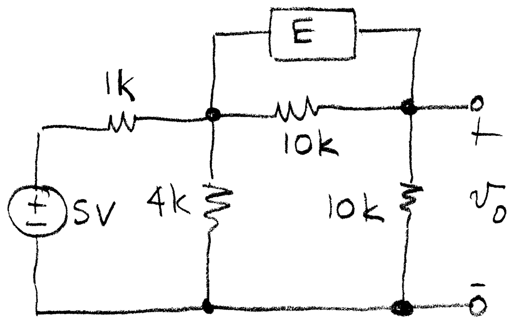
KCL@B $\sum_{+in} i$ $\quad +i_1 - i_2 + i_3 - i_4 = 0$

$$\left(\frac{5-V_B}{1\times10^3}\right) - \left(\frac{V_B-0}{4\times10^3}\right) + (10\times10^{-3}) - \left(\frac{V_B-V_C}{10\times10^3}\right) = 0$$

KCL@C $\sum_{+in} i$ $\quad -i_3 + i_4 - i_5 = 0$

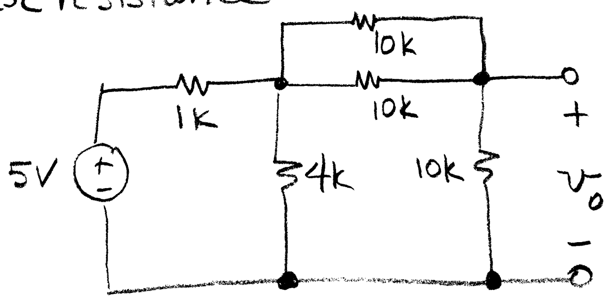
$$-(10\times10^{-3}) + \left(\frac{V_B-V_C}{10\times10^3}\right) - \left(\frac{V_C-0}{10\times10^3}\right) = 0$$

Exercise 3-5

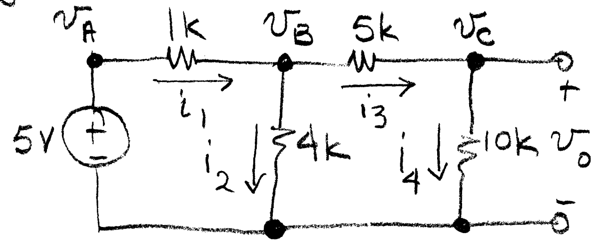


Find v_o when the element E is

(a) a $10k\Omega$ resistance.



Simply combine the two parallel resistors $R_{EQ} = \frac{(10k)(10k)}{10k + 10k} = 5k$



label nodes. By inspection $v_A = 5$ $v_C = v_o$

$\Rightarrow$ We only have two unknowns v_B and v_C and only need two equations.

$\sum_{+in} i$ @ B $\quad +i_1 - i_2 - i_3 = 0$

$\sum_{+in} i$ @ C $\quad +i_3 - i_4 = 0$ $\quad$ No current to output terminals!

Currents are $i_1 = \frac{v_A - v_B}{1K} = \frac{5 - v_B}{1K}$

$i_2 = \frac{v_B - 0}{4k} = \frac{v_B}{4k}$

$i_3 = \frac{v_B - v_C}{5K}$

$i_4 = \frac{v_C - 0}{10k} = \frac{v_C}{10k}$.

Substituting

$$\frac{5 - v_B}{1K} - \frac{v_B}{4K} - \frac{v_B - v_C}{5K} = 0$$

$$\left(\frac{v_B - v_C}{5k}\right) - \left(\frac{v_C}{10k}\right) = 0$$

$$\left(\frac{1}{1K} + \frac{1}{4K} + \frac{1}{5K}\right) v_B - \frac{1}{5K} v_C = \frac{5}{1K}$$

$$\left(\frac{1}{5k}\right) v_B - \left(\frac{1}{5K} + \frac{1}{10k}\right) v_C = 0$$

$$(20+5+4)\, v_B - 4 v_C = 100$$

$$2\, v_B - (2+1)\, v_C = 0$$

$$29 v_B - 4 v_C = 100$$

$$2 v_B - 3 v_C = 0$$

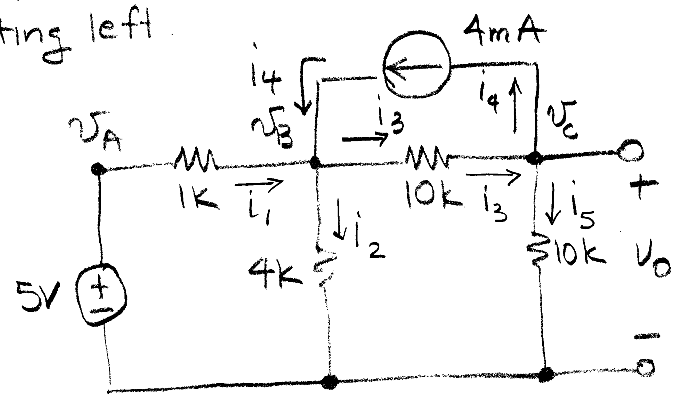
$$29\left(\frac{3}{2} v_C\right) - 4 v_C = 100$$

$$87 v_C - 8 v_C = 200$$

$$79 v_C = 200$$

$$v_C = +2.53 \text{ volts.}$$

(b) a 4mA independent current source with reference arrow pointing left



by inspection $v_A = 5$ volts

$\sum_{+\text{in}} i @ B \quad +i_1 - i_2 - i_3 + i_4 = 0$

$\sum_{+\text{in}} i @ C \quad +i_3 - i_4 - i_5 = 0$

currents are $i_1 = \dfrac{v_A - v_B}{1K}$

$i_2 = \dfrac{v_B - 0}{4K}$

$i_3 = \dfrac{v_B - v_C}{10K}$

$i_4 = 4mA$

$i_5 = \dfrac{v_C - 0}{10K}$

Substituting and multiplying

$$\left(\frac{v_A - v_B}{1K} - \frac{v_B}{4K} - \frac{v_B - v_C}{10K} + 4mA = 0\right) \times 10 \times 10^3$$

$$\left(\frac{v_B - v_C}{10K} - 4mA - \frac{v_C}{10K} = 0\right) \times 10 \times 10^3$$

$v_A = +5$

$$50 - 10v_B - 2.5v_B - v_B + v_C + 40 = 0$$

$$v_B - v_C - 40 - v_C = 0$$

Rearranging

$$(-13.5\,v_B + v_C = -90) \times 2$$

$$v_B - 2v_C = +40$$

$$-27v_B + 2v_C = -180$$

$$v_B + 2v_C = +40$$

$$-26v_B = -140$$

$$v_B = +5.385$$

$$-13.5v_B + v_c = -90$$

$$-13.5(5.385) + v_c = -90$$

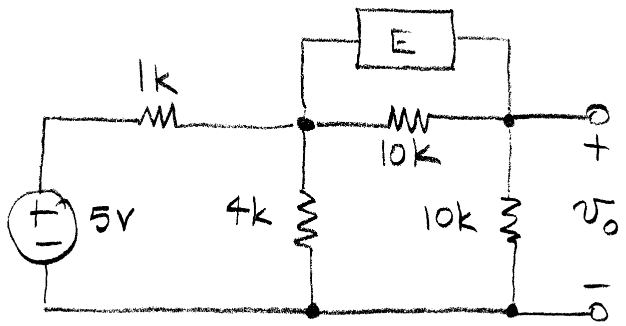
$$-72.70 + v_c = -90$$

$$v_c = -17.31 \text{ volts.}$$

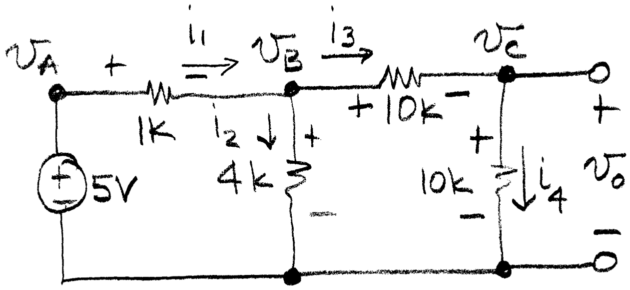
$$\therefore v_o = -17.3 \text{ volts.}$$

Exercise 3-6

Find v_o in the circuit below when the element E is



(a) an open circuit



by inspection $v_A = +5$ volts

$\sum_{+in} i$ @ B $\quad +i_1 - i_2 - i_3 = 0$

$\sum_{+in} i$ @ C $\quad +i_3 - i_4 = 0$

$$i_1 = \frac{v_A - v_B}{1K} = \frac{5 - v_B}{1K}$$

$$i_2 = \frac{v_B - 0}{4K}$$

$$i_3 = \frac{v_B - v_C}{10K}$$

$$i_4 = \frac{v_C - 0}{10K}$$

substituting $\quad \dfrac{5 - v_B}{1K} - \dfrac{v_B}{4K} - \dfrac{v_B - v_C}{10K} = 0$

$$\frac{v_B - v_C}{10K} - \frac{v_C}{10K} = 0$$

multiplying by 10×10^3

$$50 - 10v_B - 2.5v_B - v_B + v_C = 0$$

$$v_B - v_C - v_C = 0$$

rearranging

$$-13.5v_B + v_C = -50$$

$$v_B - 2v_C = 0$$

multiplying by 2

$$-27v_B + 2v_C = -100$$

$$v_B - 2v_C = 0 \quad (1)$$

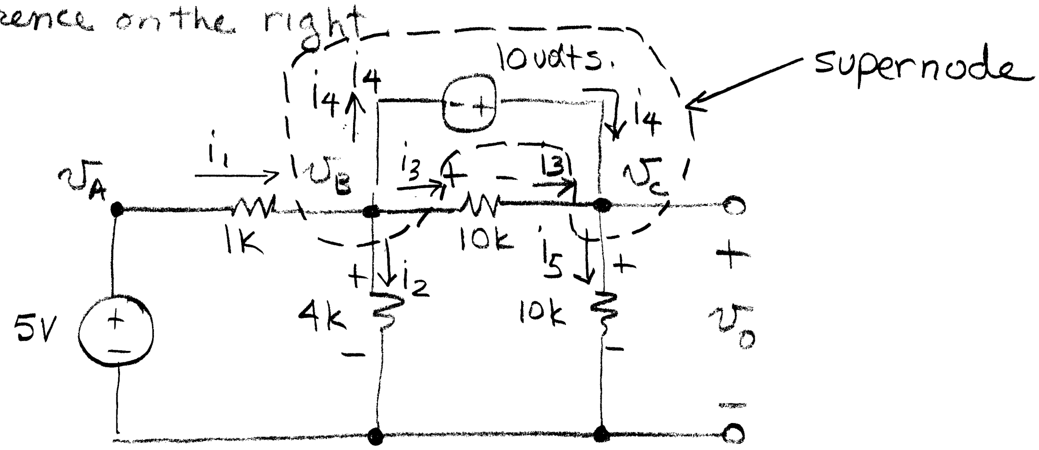
adding together

$$-26v_B = -100$$

$$v_B = +3.846 \text{ volts.}$$

From (1) $v_C = \frac{v_B}{2} = \frac{3.846}{2} = +1.923$ volts

(b) A 10-volt independent voltage source with the plus reference on the right



We have to use a supernode since we do not know i_4.

Define the supernode shown above

@supernode $\sum_{+in} i = 0$ $\quad +i_1 - i_2 - i_3 + i_3 - i_5 = 0$

(these cancel: $-i_3 + i_3$)

i_4 does not appear since it is inside the supernode.

The other equation is simply that of the voltage source.

$$v_C - v_B = +10$$

$$i_1 = \frac{v_A - v_B}{1K} = \frac{5 - v_B}{1K}$$

$$i_2 = \frac{v_B - 0}{4K}$$

$$i_3 = \frac{v_B - v_C}{10K}$$

$$i_4 = ?$$

$$i_5 = \frac{v_C - 0}{10K}$$

Substituting:

$$\frac{5 - v_B}{1K} - \frac{v_B}{4K} - \frac{v_C}{10K} = 0$$

$$+v_C - v_B = +10$$

Multiply top equation by 10×10^3

$$50 - 10v_B - 2.5v_B - v_C = 0$$
$$v_C - v_B = 10$$

Rearranging

$$-12.5v_B - v_C = -50$$
$$-v_B + v_C = 10 \quad (1)$$

Adding

$$-13.5v_B = -40$$
$$v_B = +2.963 \text{ volts.}$$

substituting into (1): $-2.963 + v_C = 10$

$$v_C = 10 + 2.963 = +12.963 \text{ volts}$$

$$v_o = v_C = +12.963 \text{ volts}$$