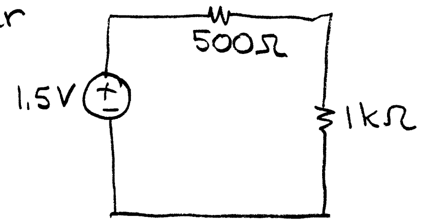
How do you assign reference marks?

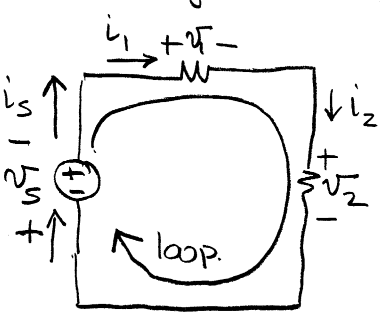
optional:
1. Draw currents from + to − nodes of voltage sources or aligned with current sources if possible.
2. Align element currents with loop currents.

REQUIRED:
3. Follow passive sign convention
4. When in doubt just do (3)

Consider

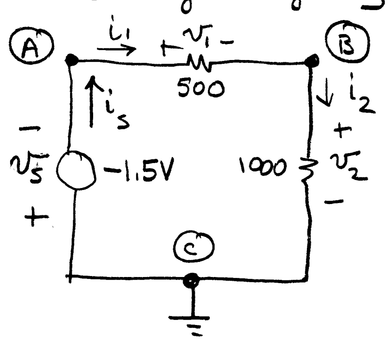


Draw loop current from + to −. Follow with passive sign convention for elements.



Note source current was aligned with that of loop. This requires v_s to be in opposite direction to given polarity.

Finalize by assigning nodes and reference (ground).



This can now be solved.

$$v_s = -1.5$$
$$v_1 = 500 i_1$$
$$v_2 = 1000 i_2$$

(element equations)

Connection equations

KCL @ A $\quad \sum_{in} i = 0 \quad +i_s - i_1 = 0$

KCL @ B $\quad \sum_{in} i = 0 \quad +i_1 - i_2 = 0$

KVL $\quad \sum_{\circlearrowright} v = 0 \quad +v_s + v_1 + v_2 = 0$

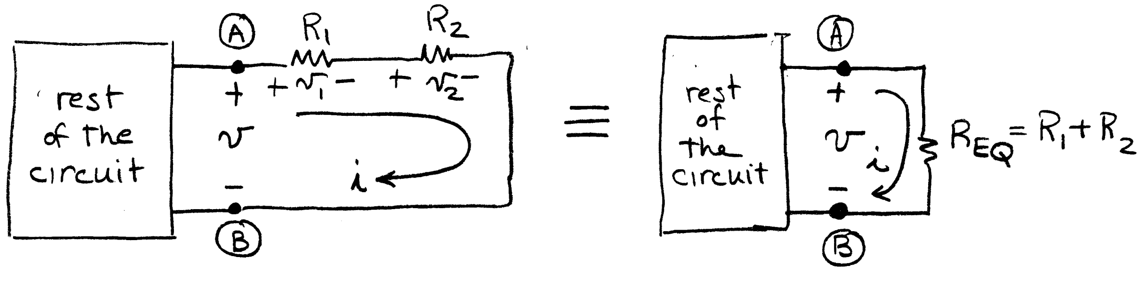
EQUIVALENT CIRCUITS

As circuits get more complex we want to replace parts of the circuit with equivalent but simpler circuits

Circuits are equivalent if they have the same i-v characteristics at a specified pair of terminals.

Equivalent Resistances
Source Transformations

Equivalent resistance (series)



KVL from Ⓐ to Ⓑ

$\sum_{CW} v_i \qquad -v + v_1 + v_2 = 0$

$v = v_1 + v_2$

but $i_1 = i_2 = i$

$v = i_1 R_1 + i_2 R_2$

$v = i R_1 + i R_2$

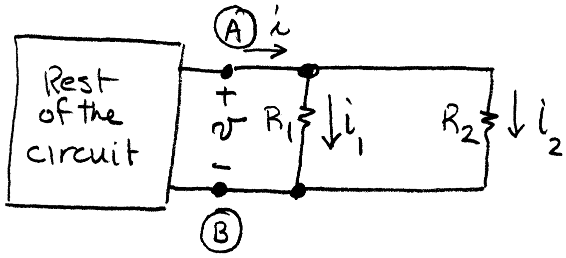
$v = i\,(R_1 + R_2)$

For this circuit we simply use Ohm's Law

$v = i R_{EQ}$

These are identical if $R_{EQ} = R_1 + R_2$

Equivalent resistance (parallel)



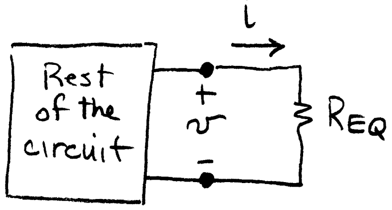
KCL @ upper node $\sum_{+in} i = 0$ $\qquad +i - i_1 - i_2 = 0$

$$i = i_1 + i_2$$

using Ohm's Law $i = \frac{V}{R}$ $\qquad i = \frac{V_1}{R_1} + \frac{v_2}{R_2}$

but $v = v_1 = v_2$ since these are in parallel

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



For this equivalent circuit

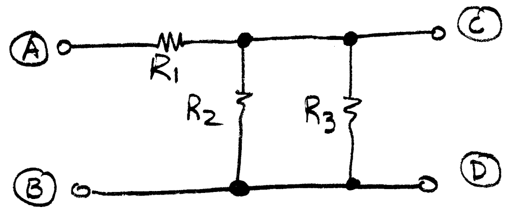
$$i = \frac{v}{R_{EQ}}$$

These circuits will be equivalent if $\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$

This can be put in a more common form by simply inverting

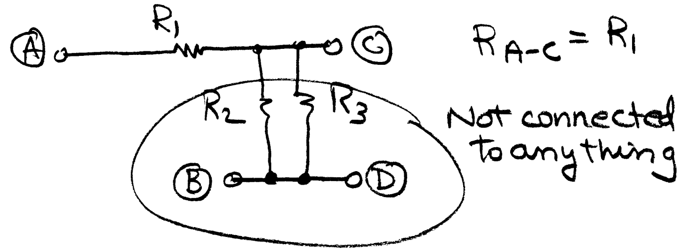
$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Example: between two terminals



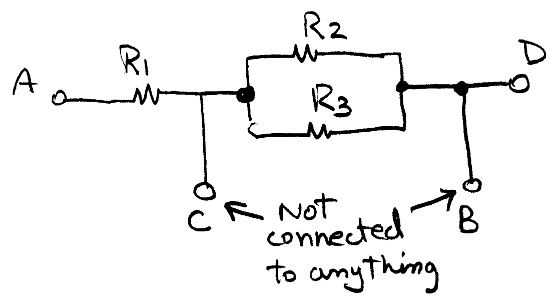
We can derive equivalent resistances between any two pairs of terminals — assuming nothing is connected to the others

between (A) & (C)



$R_{A\text{-}C} = R_1$

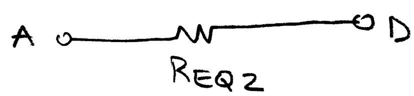
between (A) & (D)



This is two equivalent resistances

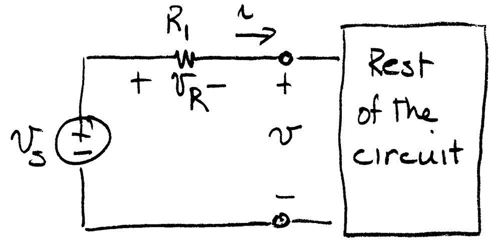
$$R_{EQ1} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{EQ1} = R_2 \parallel R_3$$

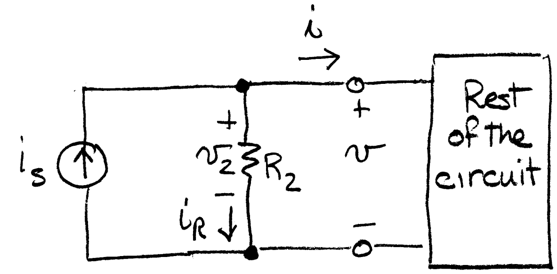


$$R_{EQ2} = R_1 + R_{EQ1}$$

EQUIVALENT SOURCES



Circuit A



Circuit B

Circuit A and circuit B will be indistinguishable if their i-v characteristics are the same.

Use KVL $\sum_{\circlearrowleft} v = 0$

$$-v_S + v_R + v = 0$$

Use Ohm's Law

$$v_R = i R_1$$

Combine and rearrange

$$-v_S + i R_1 + v = 0$$

$$i R_1 = -v + v_S$$

$$i = -\frac{v}{R_1} + \frac{v_S}{R_1}$$

Use KCL $\sum_{\text{in}} i = 0$

$$+i_S - i_R - i = 0$$

Use Ohm's Law

$$i_R = \frac{v_2}{R_2} = \frac{v}{R_2}$$

$$i = -i_R + i_S$$

$$i = -\frac{v}{R_2} + i_S$$

these will be identical if $R_1 = R_2 = R$

and $i_S = \frac{v_S}{R_1}$