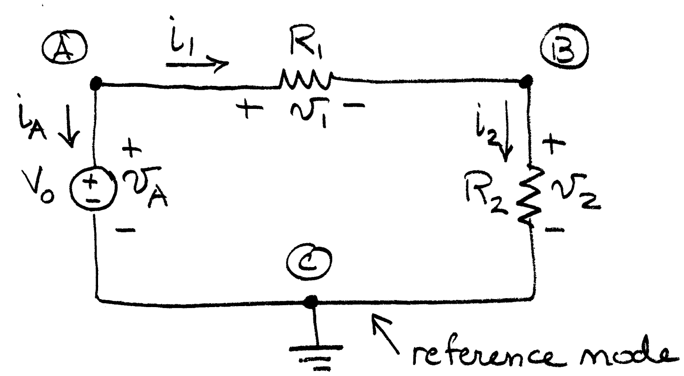


V_0 is the known

R_1, R_2 also given

I. Do element equations

$$v_A = V_0 \qquad \text{given source} \qquad (1)$$

$$v_1 = i_1 R_1 \qquad \text{Ohm's Law} \qquad (2)$$

$$v_2 = i_2 R_2 \qquad \text{Ohm's Law} \qquad (3)$$

} we will shortly start assuming these.

II. Do connection equations: KCL@ each mode, KVL for loop.

KCL@A: $\sum_{+in} i = 0 \qquad -i_A - i_1 = 0$ (4)

KCL@B $\sum_{+in} i = 0 \qquad i_1 - i_2 = 0$ (5)

KVL $\sum_{\circlearrowleft} v = 0 \qquad -v_A + v_1 + v_2 = 0$ (6)

III. Substitute element equations into connection equations

Using (6) $\quad -v_A + v_1 + v_2 = 0$

$$-(V_0) + (i_1 R_1) + (i_2 R_2) = 0$$

from (1) from (2) from (3)

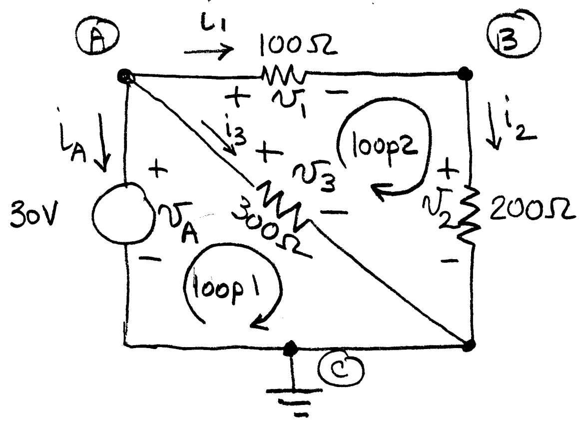
Now use (5) to reduce this to one unknown

$$-V_0 + i_1 R_1 + i_1 R_2 = 0$$

$$i_1 = \frac{V_0}{R_1 + R_2}$$

Given V_0, R_1, R_2 all variables can now be found.

Exercise 2-6



Given: 30V, 100Ω, 200Ω, 300Ω

(a) write the complete set of element equations.

$v_A = 30$ (1)

$v_1 = 100 i_1$ (2)

$v_2 = 200 i_2$ (3)

$v_3 = 300 i_3$ (4)

(b) Write the complete set of connection equations
It is two nodes and two loops.

KCL@ node A $\quad \sum_{+in} i = 0 \quad -i_A - i_1 = 0$ (5)

KCL@ node B $\quad \sum_{+in} i = 0 \quad +i_1 - i_2 = 0$ (6)

KVL@ loop 1 $\quad \sum_{\circlearrowright} v = 0 \quad -v_A + v_3 = 0$ (7)

KVL@ loop 2 $\quad \sum_{\circlearrowright} v = 0 \quad -v_3 + v_1 + v_2 = 0$ (8)

(c) Solve these equations.
Substitute everything into (8)

$$-(v_A) + (100 i_1) + (200 i_2) = 0$$

↑ from (7) ↑ from (2) ↑ from (3)

$$-30 + 100 i_1 + 200 i_2 = 0$$

From (6) $i_1 = i_2 \Rightarrow -30 + 100 i_1 + 200 i_1 = 0$

$$\therefore i_1 = \frac{30}{100 + 200} = \frac{30}{300} = 0.1\,A = 100\,mA$$

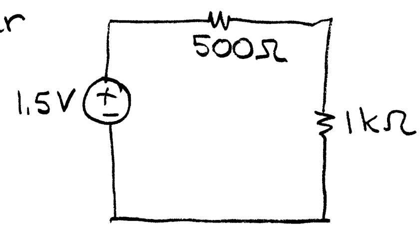
All other variables can now be solved for.

How do you assign reference marks?

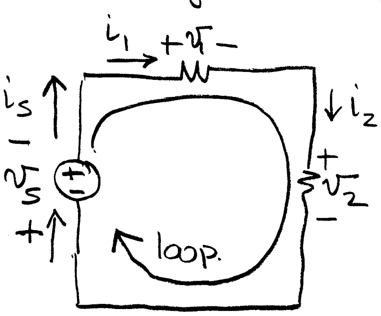
optional {
① Draw currents from + to − nodes of voltage sources or aligned with current sources if possible.
② Align element currents with loop currents.

REQUIRED ③ Follow passive sign convention
④ When in doubt just do ③

Consider

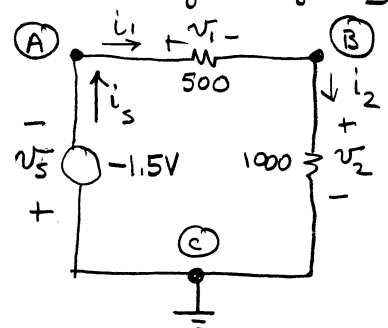


Draw loop current from + to −. Follow with passive sign convention for elements.



Note source current was aligned with that of loop. This requires v_s to be in opposite direction to given polarity.

Finalize by assigning nodes and reference (ground).



This can now be solved.

$$v_s = -1.5$$
$$v_1 = 500 i_1$$
$$v_2 = 1000 i_2$$
} element equations

Connection equations

KCL @ A $\quad \sum_{+in} i = 0 \quad +i_s - i_1 = 0$

KCL @ B $\quad \sum_{+in} i = 0 \quad +i_1 - i_2 = 0$

KVL $\quad \sum_{\circlearrowright} v = 0 \quad +v_s + v_1 + v_2 = 0$

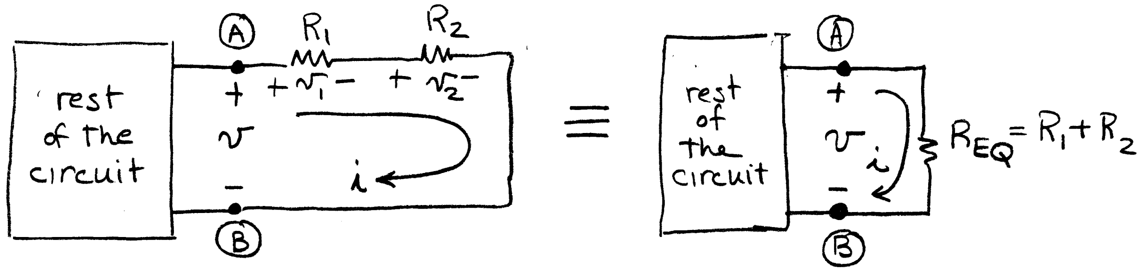
EQUIVALENT CIRCUITS

As circuits get more complex we want to replace parts of the circuit with equivalent but simpler circuits

Circuits are equivalent if they have the same i-v characteristics at a specified pair of terminals.

- Equivalent Resistances
- Source Transformations

Equivalent resistance (series)



KVL from (A) to (B)

$$\sum_{CW} v_i \qquad -v + v_1 + v_2 = 0$$

$$v = v_1 + v_2$$

but $i_1 = i_2 = i$

$$v = i_1 R_1 + i_2 R_2$$

$$v = i R_1 + i R_2$$

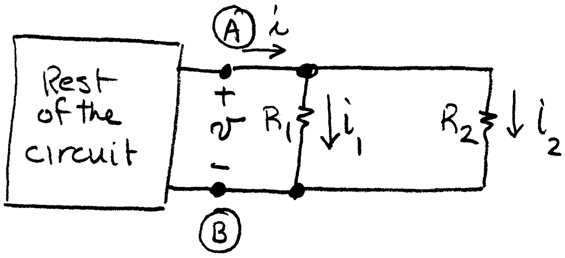
$$v = i(R_1 + R_2)$$

For this circuit we simply use Ohm's Law

$$v = i R_{EQ}$$

These are identical if $R_{EQ} = R_1 + R_2$

Equivalent resistance (parallel)



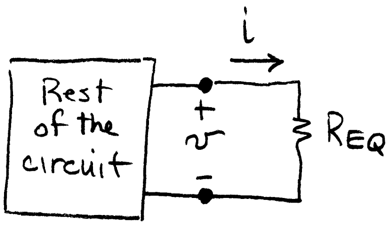
KCL @ upper node $\sum_{\text{+in}} i = 0$ $\quad +i - i_1 - i_2 = 0$

$$i = i_1 + i_2$$

using Ohm's Law $i = \frac{V}{R}$ $\quad i = \frac{v_1}{R_1} + \frac{v_2}{R_2}$

but $v = v_1 = v_2$ since these are in parallel

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$



For this equivalent circuit

$$i = \frac{v}{R_{EQ}}$$

These circuits will be equivalent if $\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$

This can be put in a more common form by simply inverting

$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$