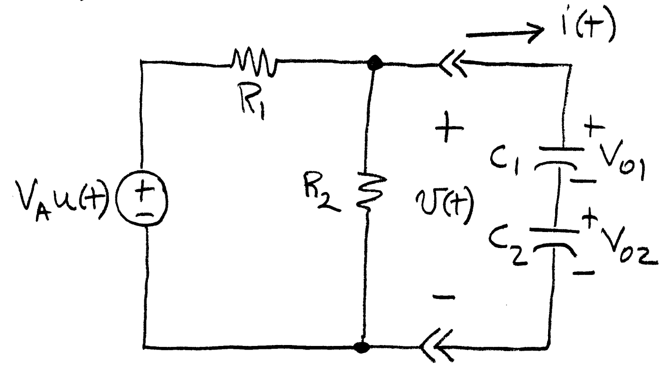
Example 7-4 Find the response of the given RC circuit.



The two capacitors can be replaced by a single equivalent capacitor

$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$V_A = 100\text{ V}$

at $t=0$ $V_{01} = 5\text{V}$

$V_{02} = 10\text{V}$

$C_1 = 0.1\,\mu f$

$C_2 = 0.5\,\mu f$

$R_1 = 30k$

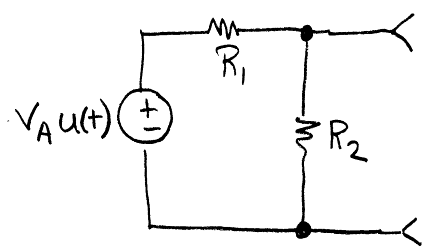
$R_2 = 10k$

$$C_{EQ} = \frac{(0.1)(0.5)}{0.1 + 0.5} = 0.0833\,\mu f$$

The initial voltage on C_{EQ} is the sum of the initial voltages on C_1 and C_2

$$V_0 = V_{01} + V_{02} = 5 + 10 = 15 \text{ volts.}$$

We next find the Thevenin equivalent seen by C_{EQ}.



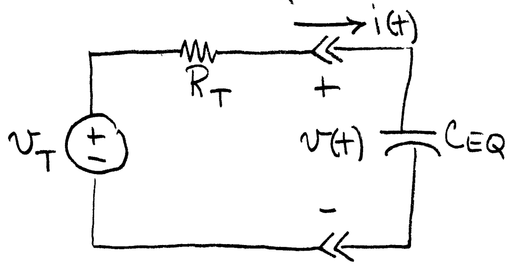
$$v_T = v_{oc} = \frac{R_2}{R_1 + R_2} V_A u(t) = \frac{10k}{30k + 10k} 100\, u(t)$$

$$v_T(t) = 25\, u(t)$$

Replacing the voltage source by a short we see that

$$R_T = R_1 \| R_2 = \frac{(30k)(10k)}{30k + 10k} = 7.5k$$

$$T_c = R_T C_{EQ} = (7.5 \times 10^{3})(0.0833 \times 10^{-6}) = 0.625\text{ ms}$$



The circuit equation is

$$-v_T(t) + i(t) R_T + v(t) = 0$$

but $i(t) = C_{EQ} \dfrac{dv(t)}{dt}$

$$-v_T(t) + R_T C_{EQ} \frac{dv}{dt} + v = 0$$

$$R_T C_{EQ} \frac{dv}{dt} + v = v_T(t)$$

The complete solution is $v(t) = v_N(t) + v_F(t)$

$$R_T C_{EQ} s + 1 = 0$$

$$s = -\frac{1}{R_T C_{EQ}} \qquad T_c = R_T C_{EQ} = 0.625\,ms$$

The forced solution is $v_F(t) = 25$ since $v_T = 25\,u(t)$

The natural solution is

$$v_N(t) = Ke^{-\frac{t}{T_c}} = Ke^{-\frac{t}{.625\,ms}}$$

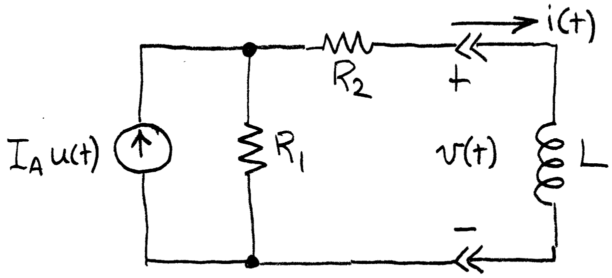
K can be found using the initial condition $v(t=0) = 15$

$$v(t=0) = 15 = Ke^{0} + 25$$

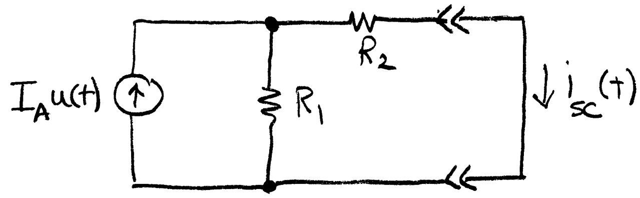
$$K = 15 - 25 = -10 \text{ volts.}$$

$$v(t) = 25 - 10e^{-\frac{t}{0.625\,ms}} \qquad t \geq 0.$$

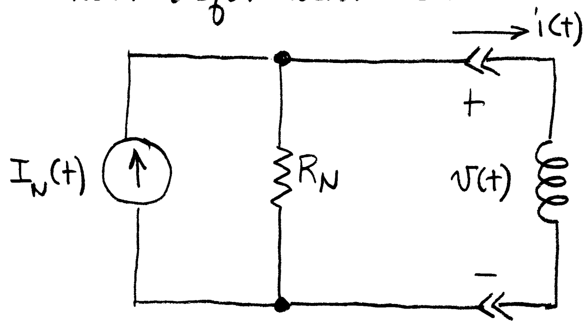
Example 7-5 Find the step response of the RL circuit given below. The initial condition is $i(0) = I_0$



The short circuit current is

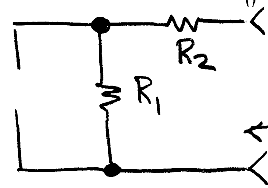


Norton equivalent circuit



$$I_N(t) = i_{sc}(t) = \frac{R_1}{R_1+R_2} I_A u(t)$$

The Norton equivalent resistance



$$R_N = R_1 + R_2$$

Do KCL @ node A $\sum_{+in} i = 0$

$$+I_N - \frac{v}{R_N} - i = 0$$

$$I_N - \frac{1}{R_N} L \frac{di}{dt} - i = 0$$

$$\frac{L}{R_N} \frac{di}{dt} + i = I_N(t)$$

The forced solution is $i_F(t) = i_{sc}(0) = \dfrac{R_1 I_A}{R_1 + R_2}$

The natural solution is $i_N(t) = K e^{-\frac{t}{L/R_N}}$

The total solution is $i(t) = i_F(t) + i_N(t) = \dfrac{R_1 I_A}{R_1+R_2} + K e^{-\frac{t}{L/R_N}}$

Initial condition $i(0) = \dfrac{R_1 I_A}{R_1 + R_2} + K = I_0$

$$i(t) = \left[I_0 - \frac{R_1 I_A}{R_1 + R_2} \right] e^{-\frac{t}{L/(R_1+R_2)}} + \frac{R_1 I_A}{R_1 + R_2}$$

Example 7-6

The state variable response of a first-order RC circuit for a step function input is

$$v_c(t) = 20\,e^{-200t} - 10\text{V} \qquad t \geq 0$$

(a) what is the circuit time constant?

natural response is $e^{-\frac{t}{T_c}}$

$$T_c = \frac{1}{200} = 5\text{ ms.}$$

(b) What is the initial voltage across the capacitor?

$$v_c(0) = 20e^{0} - 10 = 20 - 10 = 10 \text{ volts}$$

(c) What is the amplitude of the forced response?

The natural response decays to zero.

The forced response is the final value at $t = \infty$

$$v_c(t \to \infty) = -10 \quad \text{volts.}$$

(d) At what time is $v_c(t) = 0$?

$$20\,e^{-200t} - 10 = 0$$

$$20e^{-200t} = 10$$

$$e^{-200t} = \frac{10}{20} = 0.5$$

$$\ln e^{-200t} = \ln(0.5)$$

$$-200t = -0.693$$

$$t = \frac{0.693}{200} = 3.466\text{ ms.}$$

Zero-state response

(7-20) $$v(t) = (V_0 - V_A)\, e^{-\frac{t}{R_T C}} + V_A$$

(7-22) $$i(t) = (I_0 - I_A)\, e^{-\frac{t}{L_N/R}} + I_A$$

Now rearrange

$$v(t) = V_0\, e^{-\frac{t}{R_T C}} + V_A\left(1 - e^{-\frac{t}{R_T C}}\right)$$

$$i(t) = \underbrace{I_0\, e^{-\frac{t}{L_N/R}}}_{\text{zero-input response}} + \underbrace{I_A\left(1 - e^{-\frac{t}{L_N/R}}\right)}_{\text{initial-state response}}$$

zero-input response	initial-state response
occurs when input = 0	occurs when initial state (V_0 or I_0) is zero
proportional to initial state	proportional to initial condition

7-3 Initial and Final Conditions

For $t \geq 0$ the state variable step responses can be written as

RC circuit $\quad v_C(t) = [v_C(0) - v_C(\infty)]\, e^{-\frac{t}{T_C}} + v_C(\infty) \qquad t \geq 0$

RL circuit $\quad i_L(t) = [i_L(0) - i_L(\infty)]\, e^{-\frac{t}{T_C}} + i_L(\infty) \qquad t \geq 0$

The general form is

$$\begin{matrix}\text{state}\\ \text{variable}\\ \text{response}\end{matrix} = \left[\begin{matrix}\text{initial value}\\ \text{of}\\ \text{state variable}\end{matrix} - \begin{matrix}\text{final value}\\ \text{of}\\ \text{state variable}\end{matrix}\right] e^{-\frac{t}{T_C}} + \begin{matrix}\text{final value}\\ \text{of}\\ \text{state variable}\end{matrix}$$

This argues that all we need are

- the initial value
- the final value
- the time constant

Use dc analysis to find final values for

- capacitor $\longrightarrow$ open
- inductor $\longrightarrow$ short

Final value must be greater than $5\,T_C$ from initial conditions

Use dc analysis to determine initial values
based upon previous final values