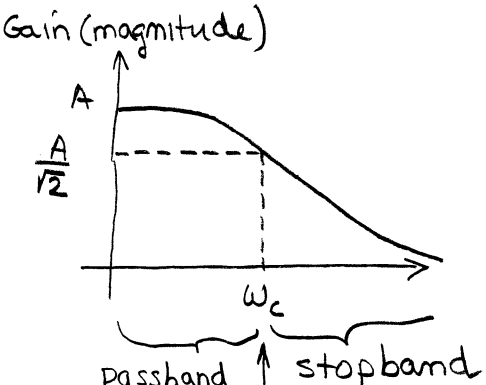
Chapter 12 – Frequency Response

frequency response – frequency dependent relationship (including both magnitude and phase) between a sinusoidal input and the resulting sinusoidal steady-state output

Bode diagram – plots of magnitude and phase versus logarithmic frequency

12-1 Frequency Response descriptors

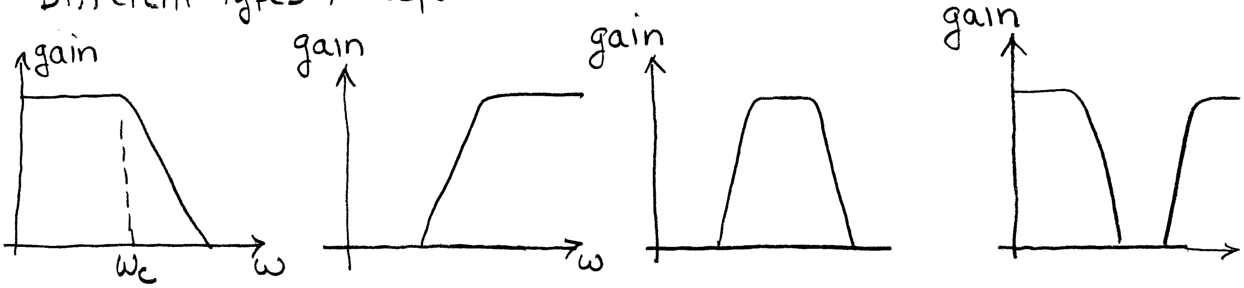


frequencies for which there is little attenuation

frequencies which are attenuated

cutoff frequency
(also called half-power point since power is $\frac{1}{2}$ of maximum power)

Different types of responses



low-pass   high-pass   band-pass   bandstop

The sinusoidal steady-state output is given by the gain function $|T(j\omega)|$ and the phase function $\theta(\omega)$.

Output amplitude $= |T(j\omega)| \times$ Input amplitude

Output phase $=$ Input phase $+ \theta(\omega)$

octave: 2:1 range of frequencies

decade: 10:1 range of frequencies

In Bode plots the gain $|T(j\omega)|$ is usually expressed in decibels

$$|T(j\omega)|_{dB} = 20\log_{10}|T(j\omega)|$$

Since the cutoff frequency occurs for the gain reduced to $\frac{1}{\sqrt{2}}$ this is a gain reduction of

$$20\log_{10}\left(\frac{1}{\sqrt{2}}|T_{MAX}|\right) = 20\log_{10}|T_{MAX}| - 20\log_{10}\sqrt{2}$$
$$= |T_{MAX}|_{dB} - 3dB$$

$\therefore$ cutoff frequency is often called the 3-dB down frequency

We will concentrate on the gain (magnitude) response.

12-2 First Order Circuit Frequency Response

Consider the first-order low-pass transfer function

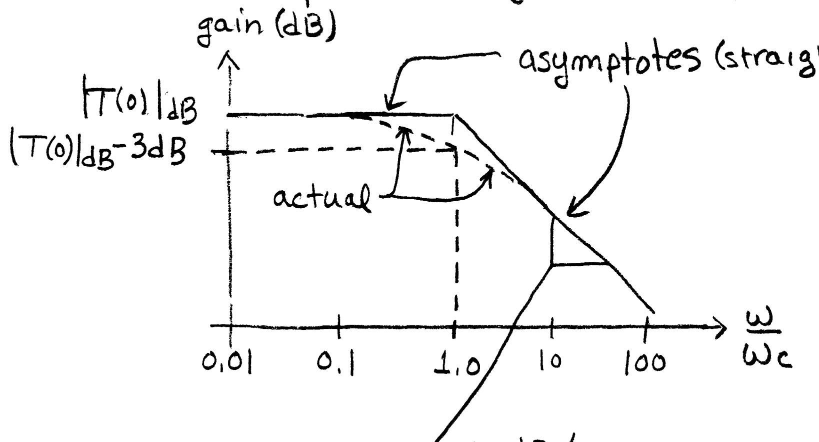
$$T(j\omega) = \frac{K}{\alpha + j\omega} = \frac{|K| e^{j\angle K}}{\sqrt{\alpha^2+\omega^2}\, e^{j\tan^{-1}(\frac{\omega}{\alpha})}}$$

$$|T(j\omega)| = \frac{|K|}{\sqrt{\alpha^2+\omega^2}} \qquad \theta(\omega) = \angle K - \tan^{-1}\left(\frac{\omega}{\alpha}\right)$$

simple magnitude division of the numerator by the denominator

this is simply the phase of the numerator minus the phase of the denominator

We can plot the magnitude response by approximating it as straight lines.



Typical response is $T(j\omega) = \dfrac{K/\alpha}{1+j\,\omega/\alpha}$

write as sum of log terms $\quad 20\log_{10}|T(j\omega)| = \overbrace{\underbrace{20\log_{10}\left|\frac{K}{\alpha}\right|}_{\text{plot as a straight line}}}^{\text{this just a constant}} - \underbrace{20\log_{10}\left|1+j\frac{\omega}{\alpha}\right|}_{\text{also plot as a straight line}}$

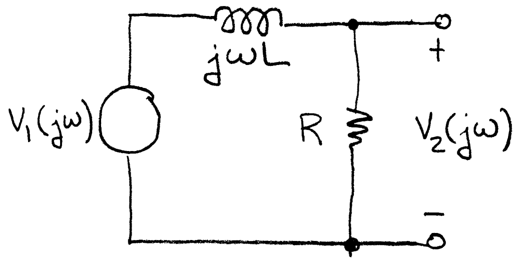
for $\omega < \alpha$ $\quad j\frac{\omega}{\alpha} \ll 1$ and we approximate $1+j\frac{\omega}{\alpha}$ as simply 1

for $\omega > \alpha$ $\quad j\frac{\omega}{\alpha} \gg 1$ and we approximate $1+j\frac{\omega}{\alpha}$ as $\frac{\omega}{\alpha}$

As ω increases by a factor of 10 (a decade)
$20\log_{10}\left|\frac{\omega}{\alpha}\right|$ increases by 20 since $\frac{\omega}{\alpha}$ increased by 10^1

Example 12-2

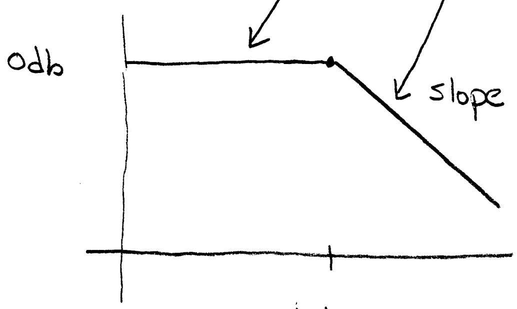
Consider the circuit shown below. Find the transfer function $T(j\omega) = \dfrac{V_2(j\omega)}{V_1(j\omega)}$ and construct the straight line approximations to the gain response.



Using voltage division $T(j\omega) = \dfrac{V_2(j\omega)}{V_1(j\omega)} = \dfrac{R}{R + j\omega L} = \dfrac{1}{1 + j\frac{\omega L}{R}}$

want all terms to be in terms of $1 + (\,)\omega$

$$20 \log |T(j\omega)| = \underbrace{20 \log (1)}_{0\,dB} - 20 \log \left| 1 + j\frac{\omega L}{R} \right|$$



slope is -20db/decade because of the minus sign due to denominator

cutoff for each term $\omega_c \frac{L}{R} = 1$

or $\omega_c = \frac{R}{L}$