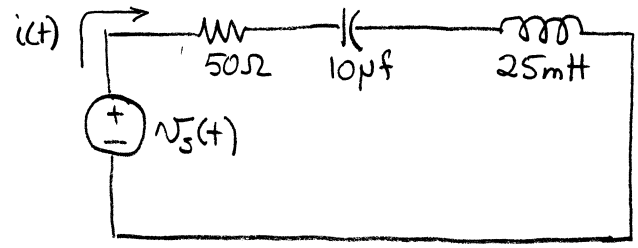
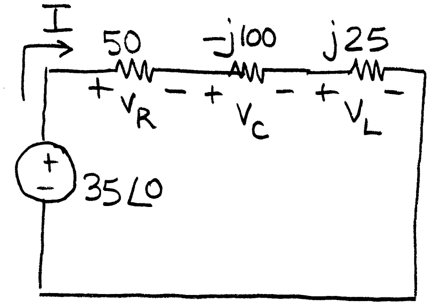
Example 8-6

The circuit shown below is operating in the sinusoidal steady state with $v_S(t) = 35 \cos 1000t$.




(a) Transform the circuit into the phasor domain.




$$Z_R = R = 50$$

$$Z_L = j\omega L = j(1000)(25\times10^{-3}) = j25$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(1000)(10\times10^{-6})} = -j100$$

(b) Solve for the phasor current $\underline{I}$.

$$Z_{EQ} = Z_R + Z_L + Z_C = 50 + j25 - j100 = 50 - j75 = 90.14\angle -56.3^\circ$$

$$\underline{I} = \frac{\underline{V}}{Z} = \frac{35\angle 0^\circ}{90.14\angle -56.3^\circ} = \frac{35}{50-j75} = 0.215 + j0.323 = 0.388\angle 56.3^\circ$$

(c) Solve for the phasor voltage across each element

$$\underline{V}_R = Z_R\underline{I} = 50(0.215 + j0.323) = 10.75 + j16.15 = 19.4\angle +56.4^\circ$$

$$\underline{V}_C = Z_C\underline{I} = (-j100)(0.215 + j0.323) = +32.3 - j21.5 = 38.8\angle +146.4^\circ$$

$$\underline{V}_L = Z_L\underline{I} = (j25)(0.215 + j0.323) = -8.075 + j5.375 = 9.70\angle -33.65^\circ$$

(d) Construct the waveforms corresponding to the phasors found in (a) and (b)

$$i(t) = 0.388 \cos(1000t + 56.3^\circ)$$

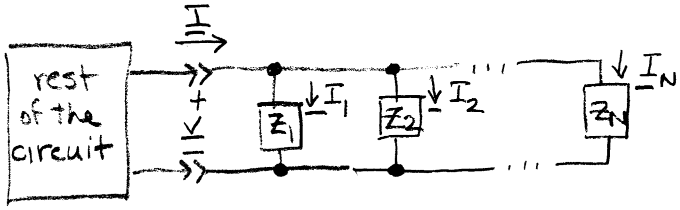
$$v_R(t) = 19.4 \cos(1000t + 56.4^\circ)$$

$$v_C(t) = 38.8 \cos(1000t + 146.4^\circ)$$

$$v_L(t) = 9.70 \cos(1000t - 33.65^\circ)$$

# Parallel equivalence and current division

When impedances are connected in parallel as shown below we may use KCL to compute an equivalent impedance




$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \cdots + \underline{I}_N$$

$$= \frac{\underline{V}}{Z_1} + \frac{\underline{V}}{Z_2} + \cdots + \frac{\underline{V}}{Z_N} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}\right)\underline{V}$$

$$\frac{1}{Z_{EQ}} = \frac{\underline{I}}{\underline{V}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_N}$$

We often use the admittance $Y$

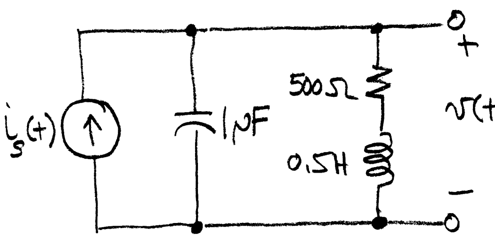
$$Y = \frac{1}{Z} = G + jB$$

susceptance ($B$)

conductance ($G$)

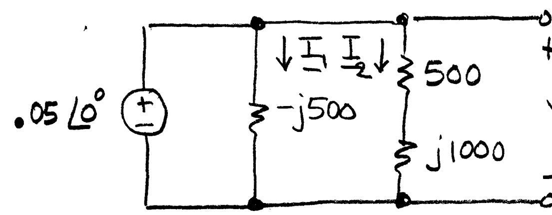
Example 8-9

The circuit shown below is operating in sinusoidal steady state with $i_s(t) = 50 \cos 2000t$ mA.




(a) Transform the circuit into the phasor domain.

$$Z_R = R = 500\Omega$$
$$Z_L = j\omega L = j(2000)(.5) = j1000$$
$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(2000)(1\times 10^{-6})} = -j500$$




(b) Solve for the phasor voltage $\underline{V}$

$$Z_{EQ} = (-j500) \| (500 + j1000) = \frac{(-j500)(500+j1000)}{-j500+500+j1000} = 250 - j750$$

$$\underline{V} = \underline{I}\, Z_{EQ} = (.05\angle 0^\circ)(250 - j750) = 12.5 - j37.5 = 39.5\angle -71.6^\circ$$

(c) Solve for the phasor current through each branch.

$$\underline{I}_1 = \frac{\underline{V}}{-j500} = \frac{12.5 - j37.5}{-j500} = .075 + j.025 = .079\angle 18.4^\circ$$

$$\underline{I}_2 = \frac{\underline{V}}{500 + j1000} = \frac{12.5 - j37.5}{500 + j1000} = -.025 - j.025 = .035\angle -135^\circ$$

(d) Construct the wave forms corresponding to the phasors found in (b) and (c).

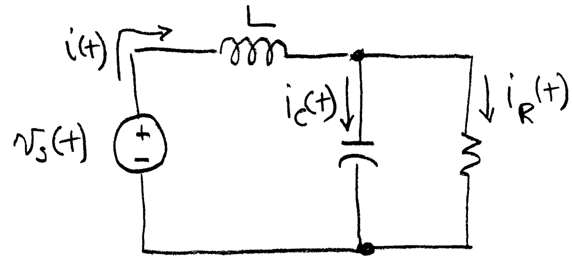
$$v(t) = Re\left[39.5 e^{-j71.6^\circ} e^{j2000t}\right] = 39.5 \cos(2000t - 71.6^\circ)$$

$$i_1(t) = Re\left[.079 e^{j18.4^\circ} e^{j2000t}\right] = .079 \cos(2000t - 18.4^\circ)$$

$$i_2(t) = Re\left[.035 e^{-j135^\circ} e^{j2000t}\right] = .035 \cos(2000t - 135^\circ)$$

Example 8-10

Find the steady-state currents $i(t)$, $i_c(t)$ and $i_R(t)$ in the circuit below for $v_s = 100 \cos 2000t$, $L = 250\,mH$, $C = 0.05\mu f$, and $R = 3k\Omega$.

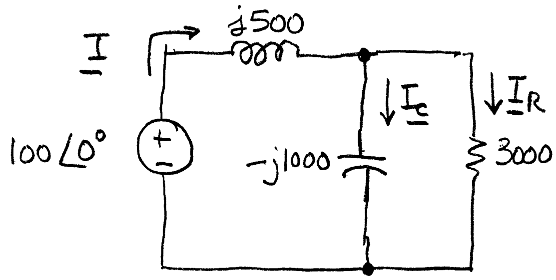



Converting to phasors.

$$Z_L = j\omega L = j(2000)(250\times10^{-3}) = j500$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(2000)(.05\times10^{-6})} = -j1000$$

$$Z_R = 3000$$




$$Z_{EQ} = j500 + (3000)\|(-j1000)$$

$$= j500 + \frac{(3000)(-j1000)}{3000-j1000}$$

$$= j500 + 300 - j900$$

$$Z_{EQ} = 300 - j400$$

$$\underline{I} = \frac{V}{Z_{EQ}} = \frac{100\angle 0°}{300-j400} = 0.12+j0.16 = 0.2\angle 53.1°$$

We can now use current division to find $\underline{I_C}$ and $\underline{I_R}$

$$\underline{I_C} = \frac{3000}{3000-j1000}(0.12+j0.16) = 0.06+j0.18 = 0.190\angle 71.56°$$

$$\underline{I_R} = \frac{-j1000}{3000-j1000}(0.12+j0.16) = 0.06-j0.02 = 0.063\angle -18.43°$$

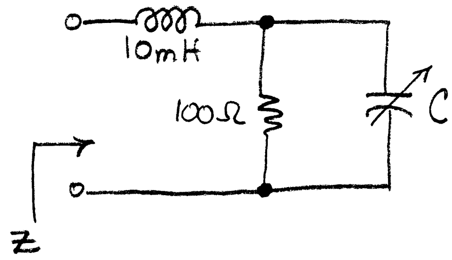
$$i(t) = Re\left\{0.2e^{j53.1°}e^{j2000t}\right\} = 0.2\cos(2000t + 53.1°)$$

$$i_c(t) = Re\left\{0.190e^{j71.56°}e^{j2000t}\right\} = 0.190\cos(2000t + 71.56°)$$

$$i_R(t) = Re\left\{0.063e^{-j18.43°}e^{j2000t}\right\} = 0.063\cos(2000t - 18.43°)$$

Example 8-12

The circuit shown below is operating in the sinusoidal steady state with $\omega = 5000$.




(a) Find the value of C that causes the input impedance Z to be purely resistive.

$$Z_R = 100\,\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(5000)C} = \frac{1}{j5000C}$$

$$Z_L = j\omega L = j(5000)(10\times10^{-3}) = j50$$

$$Z = j50 + \frac{1}{j5000C} \| 100 = j50 + \frac{\frac{1}{j5000C}\cdot 100}{\frac{1}{j5000C} + 100} = j50 + \frac{100}{1 + j5\times10^5 C}$$

$$Z = j50 + \frac{100(1 - j5\times10^5 C)}{(1+j5\times10^5C)(1-j5\times10^5C)} = j50 + \frac{100 - j5\times10^7 C}{1 + (5\times10^5 C)^2}$$

$$Z = j50 + \frac{100}{1+(5\times10^5C)^2} - j\frac{5\times10^7C}{1+(5\times10^5C)^2}$$

Set $$50 - \frac{5\times10^7C}{1+(5\times10^5C)^2} = 0$$

$$\cancel{5}0 + \cancel{5}0(5\times10^5C)^2 - \cancel{5}\times10^7C = 0$$

$$25\times10^{11}C^2 - 10^7C + 10 = 0$$

$$25\times10^{10}C^2 - 10^6C + 10 = 0$$

$$C = \frac{+10^6 \pm \sqrt{(-10^6)^2 - 4(25\times10^{10})(10)}}{2(25\times10^{10})} = 2\times10^{-6}\ \text{F}$$

(b) Find the real part of the input impedance for this value of C

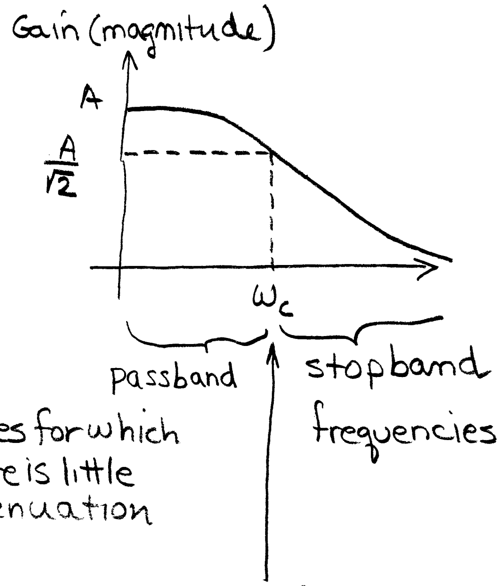
$$Z_{real} = \frac{100}{1+((5\times10^5)(2\times10^{-6}))^2} = \frac{100}{1+1} = 50\,\Omega$$

# Chapter 12 – Frequency Response

frequency response – frequency dependent relationship (including both magnitude and phase) between a sinusoidal input and the resulting sinusoidal steady-state output

Bode diagram – plots of magnitude and phase versus logarithmic frequency

## 12-1 Frequency Response descriptors

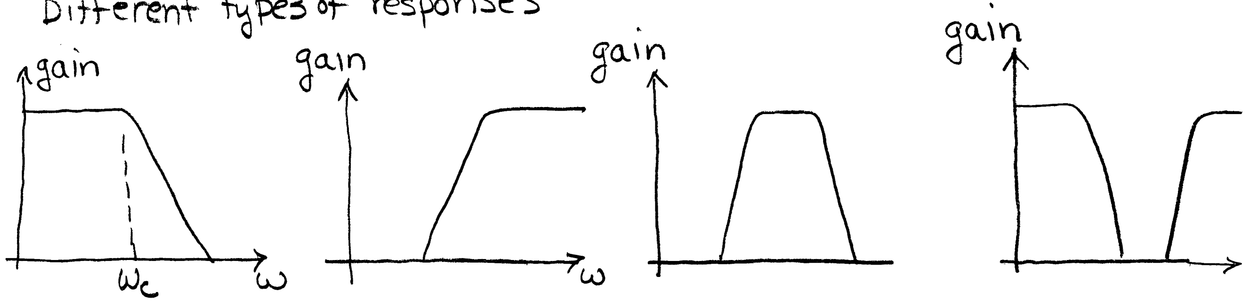



frequencies for which there is little attenuation

frequencies which are attenuated

cutoff frequency
(also called half-power point since power is $\frac{1}{2}$ of maximum power)

Different types of responses




low-pass　　high-pass　　band-pass　　bandstop