Example 8-2

(a) Construct the phasors representing the following signals

$$i_A(t) = 5\cos(377t + 50°)$$
$$i_B(t) = 5\cos(377t + 170°)$$
$$i_C(t) = 5\cos(377t - 70°)$$

$$\underline{I_A} = 5e^{j50°} = 5(\cos 50° + j\sin 50°) = 3.214 + j\,3.830$$
$$\underline{I_B} = 5e^{j170°} = 5(\cos 170° + j\sin 170°) = -4.924 + j\,0.868$$
$$\underline{I_C} = 5e^{-j70°} = 5(\cos(-70°) + j\sin(-70°)) = 1.710 - j\,4.698$$

(b) Use the additive property of phasors and the phasors found in (a) to find the sum of these waveforms.

$$\underline{I} = \underline{I_A} + \underline{I_B} + \underline{I_C} = (3.214 - 4.924 + 1.710) + j(3.830 + 0.868 - 4.698)$$
$$\underline{I} = 0 + j0$$

Example 8-3

Use the derivative property of phasors to find the time derivative of $v(t) = 15\cos(200t - 30°)$

$$v(t) = 15\cos(200t - 30°)$$
$$\underline{V} = 15e^{-j30°}$$
$$\underline{V}' = (j200)15e^{-j30} = j300\,e^{-j30} = 300\,e^{-j30}e^{j90°}$$

$j\omega$

$$\underline{V}' = 300\,e^{j60°}$$

$$\frac{dv}{dt} = 300\cos(200t + 60°)$$

Example 8-4

(a) Convert the following phasors into sinusoidal waveforms.

$$\underline{V_1} = 20 + j20 \qquad \omega = 500$$

convert to polar $\underline{V_1} = 28.28 \angle 45°$

$$v_1(t) = 28.28 \cos(500t + 45°)$$

$$\underline{V_2} = 10\sqrt{2}\, e^{-j45°} \qquad \omega = 500$$

$$v_2(t) = 14.14 \cos(500t - 45°)$$

(b) Use phasor addition to find the sinusoidal waveform

$$v_3(t) = v_1(t) + v_2(t)$$

$$\underline{V_3} = (20 + j20) + (10.0 - j\,10.0)$$

$$\underline{V_3} = 30 + j\,10 = 31.62 \angle 18.44°$$

$$v_3(t) = 31.62 \cos(500t + 18.4°)$$

8-2 Phasor circuit analysis

Connection constraints in phasor form

KVL: The algebraic sum of phasor voltages around a loop is zero.

KCL: The algebraic sum of phasor currents at a node is zero.

Device constraints in phasor form

Resistor: $v_R(t) = R\, i_R(t)$ $\qquad \underline{V_R} = R\, \underline{I_R}$

Inductor: $v_L(t) = L \dfrac{d i_L(t)}{dt}$ $\qquad \underline{V_L} = L j\omega\, \underline{I_L}$

used derivative property

$\underline{V_L} = j\omega L\, \underline{I_L}$

Capacitor $i_C(t) = C \dfrac{d v_C(t)}{dt}$ $\qquad \underline{I_C} = C j\omega\, \underline{V_C}$

used derivative property

$\underline{V_C} = \dfrac{1}{j\omega C}\, \underline{I_C}$

The Impedance Concept

All of the above relationships are of the form $\underline{V} = Z\, \underline{I}$

This is analagous to Ohm's Law where Z is defined to be the impedance.

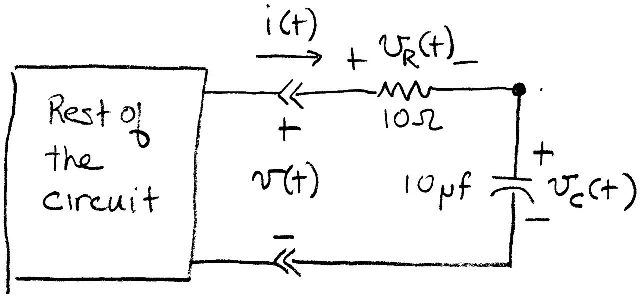
For resistors $\quad Z_R = R$

For inductors: $\quad Z_L = j\omega L$

For capacitors: $\quad Z_C = \dfrac{1}{j\omega C}$

Example 8-5

The circuit shown below is operating in the sinusoidal steady-state with $i(t) = 4\cos(5000t)$. Find the steady-state voltage.



You start the problem by finding the impedances of the individual components.

$$Z_R = R = 10\Omega$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{j(5000)(10\times 10^{-6})} = -j20$$

this comes from cos (5000t)

Since R and C are in series the same current flows through each.

$$\underline{I} = 4\angle 0^\circ = 4 + j0$$

$$\underline{V_R} = Z_R \underline{I} = (10)(4+j0) = 40 + j0$$

$$\underline{V_c} = Z_c \underline{I} = (-j20)(4+j0) = 0 - j80$$

Applying KVL gives

$$-\underline{V} + \underline{V_R} + \underline{V_c} = 0$$

$$\underline{V} = \underline{V_R} + \underline{V_c} = (40+j0) + (0-j80) = 40 - j80 = 89.44\angle -63.4^\circ$$

$$\therefore\ v(t) = 89.4\cos(5000t - 63.4^\circ)$$

8-3 Basic Circuit Analysis with Phasors

STEP 1: Transform the circuit into the phasor domain by representing input and response sinusoids as phasors and the passive circuit elements by their impedances.

STEP 2: Use standard algebraic circuit analysis techniques to the phason domain circuit to solve for the desired phasor responses.

STEP 3: Inverse transform the phasor responses back into the time domain.

For N elements connected in series

$$Z_{EQ} = Z_1 + Z_2 + \ldots + Z_N$$

$$Z_{EQ} = \underbrace{R}_{\text{resistance}} + j\underbrace{X}_{\text{reactance}}$$

reactance: + for inductor, − for capacitor

Since the current is the same for each element connected in a series connection

$$\underline{V}_k = Z_k \underline{I} = \underbrace{\frac{Z_k}{Z_{EQ}}}_{\text{voltage divider}} \underline{I}$$