The forced sinusoid remaining after the natural component disappears is called the sinusoidal steady-state response.

8-1 Sinusoids and Phasors

The fundamental relationship between sinewaves and complex numbers comes from Euler's identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Define $\cos\theta = Re\{e^{j\theta}\}$ ⟵ we use the cosine to describe the eternal sinewave

$$\sin\theta = \mathcal{I}m\{e^{j\theta}\}$$

Expanding upon the general sinusoid

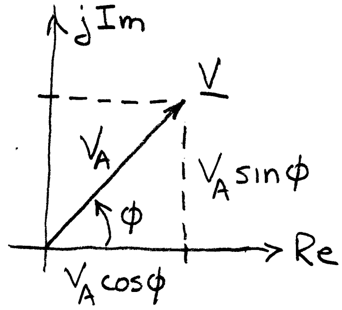
$$v(t) = V_A \cos(\omega t + \phi)$$

$$v(t) = V_A \, Re\{e^{j(\omega t+\phi)}\} = Re\{\underline{\underline{(V_A e^{j\phi})}}\, e^{j\omega t}\}$$

this is defined to be the phasor representation of the sinusoid $v(t)$

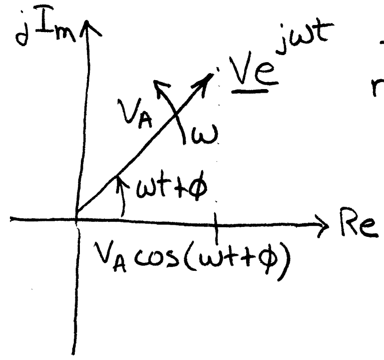
$$\underline{V} \triangleq V_A e^{j\phi} = V_A \cos\phi + j V_A \sin\phi$$

↑ the phasor $\underline{V}$ is a complex number



1. Phasors will be written with an underline ($\underline{V}$) to distinguish them from signal waveforms such as $v(t)$.

2. A phasor is determined by amplitude and phase angle and does not contain any information about the frequency.



this is a rotating phasor

Properties of phasors

Additive property

$$v(t) = v_1(t) + v_2(t) + \dots + v_N(t)$$

$$v(t) = \mathrm{Re}\left\{\underline{V}_1 e^{j\omega t}\right\} + \mathrm{Re}\left\{\underline{V}_2 e^{j\omega t}\right\} + \dots + \mathrm{Re}\left\{\underline{V}_N e^{j\omega t}\right\}$$

$$v(t) = \mathrm{Re}\left\{\underline{V}_1 e^{j\omega t} + \underline{V}_2 e^{j\omega t} + \dots + \underline{V}_N e^{j\omega t}\right\}$$

$$v(t) = \mathrm{Re}\left\{\underbrace{\left(\underline{V}_1 + \underline{V}_2 + \dots + \underline{V}_N\right)}_{\underline{V}} e^{j\omega t}\right\}$$

Derivative property

$$v(t) = \mathrm{Re}\left\{\underline{V}\, e^{j\omega t}\right\}$$

$$\frac{dv(t)}{dt} = \frac{d}{dt}\mathrm{Re}\left\{\underline{V}\, e^{j\omega t}\right\} = \mathrm{Re}\left\{\underline{V}\frac{d}{dt} e^{j\omega t}\right\}$$

$$\frac{dv(t)}{dt} = \mathrm{Re}\left\{\left(j\omega\, \underline{V}\right) e^{j\omega t}\right\}$$

the derivative of a phasor is simply $j\omega$ times the phasor. This is very useful.

Example 8-1

(a) Construct the phasors for the following signals.

$$v_1(t) = 10\cos(1000t - 45°)$$

$$\underline{V_1} = 10e^{-j45°} = 10(\cos 45° - j\sin 45°)$$

$$= 7.07 - j7.07$$

$$v_2(t) = 5\cos(1000t + 30)$$

$$\underline{V_2} = 5e^{+j30°} = 5(\cos 30° + j\sin 30°)$$

$$= 4.33 + j2.5$$

(b) Use the additive property of phasors and the phasors find in (a) to find $v(t) = v_1(t) + v_2(t)$

$$\underline{V} = \underline{V_1} + \underline{V_2} = 7.07 - j7.07 + 4.33 + j2.5 = 11.4 - j4.57$$

$$\underline{V} = 11.4 - j4.57 = 12.28\angle -21.8°$$

The corresponding waveform is

$$v(t) = \mathrm{Re}\left\{\left(12.28e^{-j21.8°}\right)e^{j1000t}\right\}$$

$$v(t) = 12.28\cos(1000t - 21.8°)$$