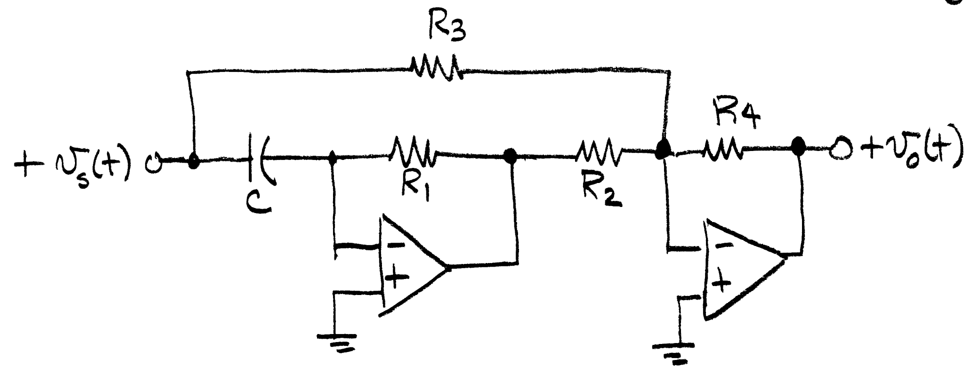
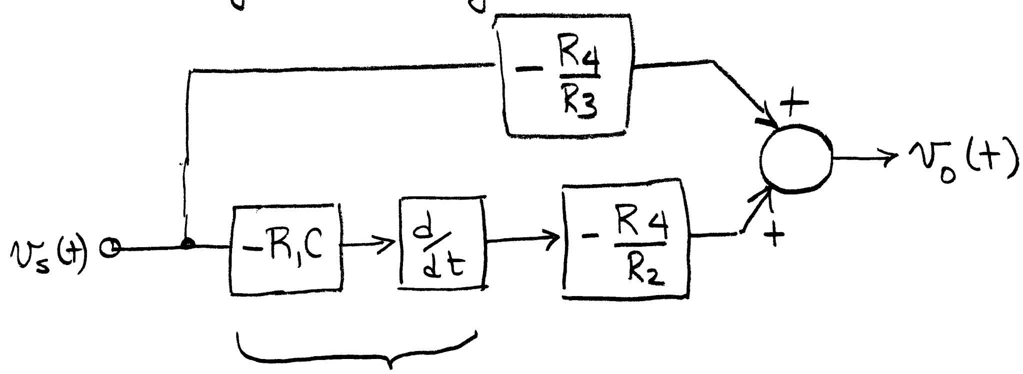
Example 6-11

Determine the input-output relationship of the given circuit.



Drawing a block diagram



this is the gain of the differentiator we separate it into a gain and a derivative.

The sum of the gains along each path gives

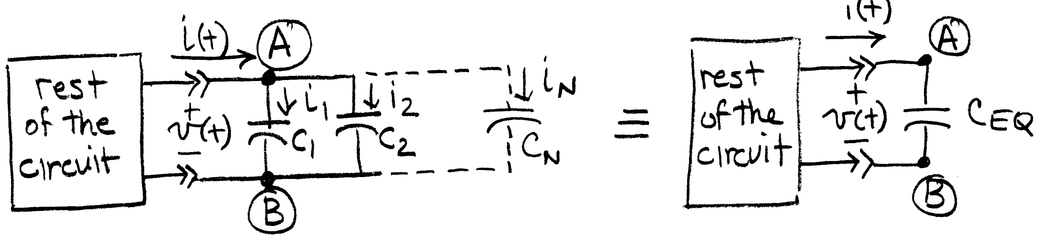
$$v_o(t) = -\frac{R_4}{R_3} v_s(t) + (-R_1C)\frac{d}{dt}\left[\left(-\frac{R_4}{R_2}\right) v_s(t)\right]$$

$$= -\frac{R_4}{R_3} v_s(t) + R_1C\frac{R_4}{R_2}\frac{dv_s(t)}{dt}$$

Assuming OP AMPS do not saturate

6-4 Equivalent capacitance & Inductance

N capacitors in parallel



KCL @ A gives

$$i(t) = i_1(t) + i_2(t) + \dots + i_N(t)$$

since the capacitors are in parallel

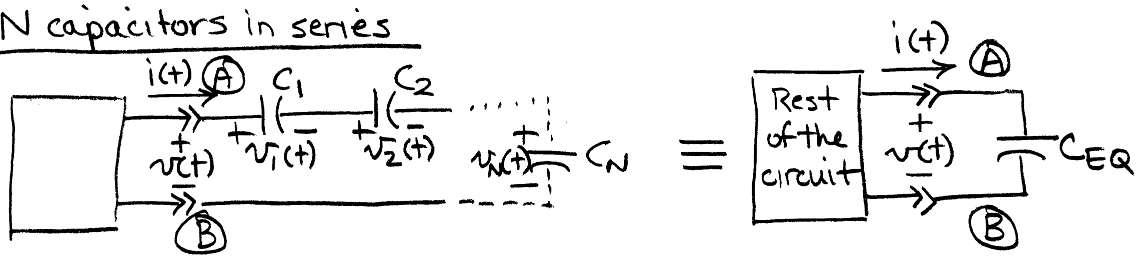
$$v_1(t) = v_2(t) = \dots = v_N(t) = v(t)$$

$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + \dots + C_N \frac{dv(t)}{dt}$$

$$i(t) = (C_1 + C_2 + \dots + C_N) \frac{dv(t)}{dt} = C_{EQ} \frac{dv(t)}{dt}$$

$$\therefore \; C_{EQ} = C_1 + C_2 + \dots + C_N$$

N capacitors in series



Do KVL around loop $v(t) = v_1(t) + v_2(t) + \dots + v_N(t)$

KCL requires that $i_1(t) = i_2(t) = \dots = i_N(t) = i(t)$

$$v(t) = v_1(0) + \frac{1}{C_1}\int_0^t i(x)\,dx + v_2(0) + \frac{1}{C_2}\int_0^t i(x)\,dx + \dots + v_N(0) + \frac{1}{C_N}\int_0^t i(x)\,dx$$

$$v(t) = \left[v_1(0) + v_2(0) + \dots + v_N(0)\right] + \underbrace{\left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}\right)}_{\frac{1}{C_{EQ}}}\int_0^t i(x)\,dx$$

$$\therefore \; \frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Inductors are the dual of capacitors.

$$L_{EQ} = L_1 + L_2 + \cdots + L_N \quad \text{(series connection)}$$

$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} \quad \text{(parallel connection)}$$

DC Equivalent Circuits

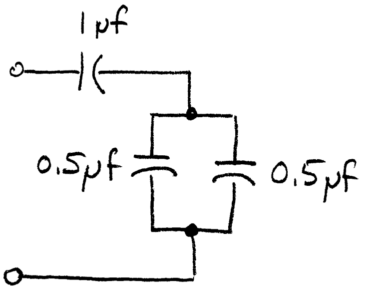
Under dc (steady state) conditions

- capacitor acts like an open circuit
- inductor acts like a short circuit

The resulting circuit typically only contains resistors.

Example 6-14 Find the equivalent capacitances and inductances for the given circuits.

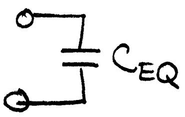
(a)

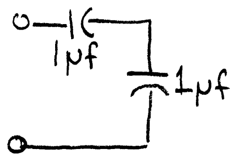


The two capacitors in parallel add.

$$0.5\,pf + 0.5\,pf = 1\,pf.$$

This gives the circuit of two capacitors in series.

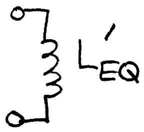


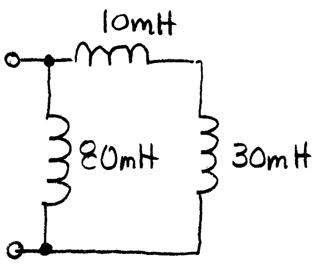


$$\frac{1}{C_{EQ}} = \frac{1}{1pf} + \frac{1}{1\mu f} = \frac{2}{1\mu f}$$

$$C_{EQ} = \frac{1}{2}\mu f$$

(b)





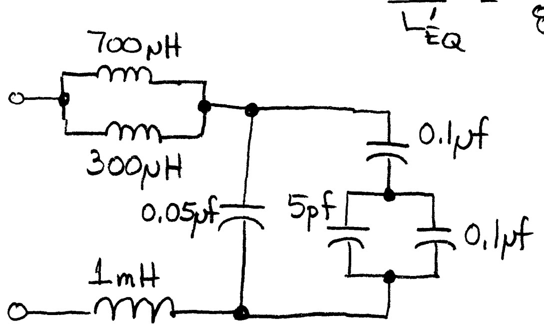
The 10mH and 30mH are in series and add.

$$L_{EQ} = 10mH + 30mH = 40\,mH$$

This result is in parallel with the 80mH inductor.

$$\frac{1}{L'_{EQ}} = \frac{1}{80} + \frac{1}{40} = \frac{3}{80} \qquad L'_{EQ} = \frac{80}{3} = 26.67\,mH$$

(c)



The inductors and capacitors must be combined separately.

For the capacitors:

5pf || 0.1μf add $\quad 0.1\mu f + .000005\,\mu f = 0.100005\,\mu f$

This capacitor is in series with the 0.1μf capacitor.

$$\frac{1}{C_{EQ}} = \frac{1}{0.1\mu f} + \frac{1}{0.100005\,\mu f} \qquad C_{EQ} \cong 0.050001$$

This is in parallel with the 0.05μf capacitor

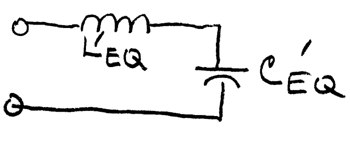
$$C'_{EQ} = 0.05\mu f + 0.0500001\,\mu f \cong 0.1\,\mu f.$$

This completes the capacitance. Proceeding to the inductors.

700μH || 300μH $\qquad \dfrac{1}{L_{EQ}} = \dfrac{1}{700\mu H} + \dfrac{1}{300\,\mu H} \qquad L_{EQ} = 210\,\mu H$

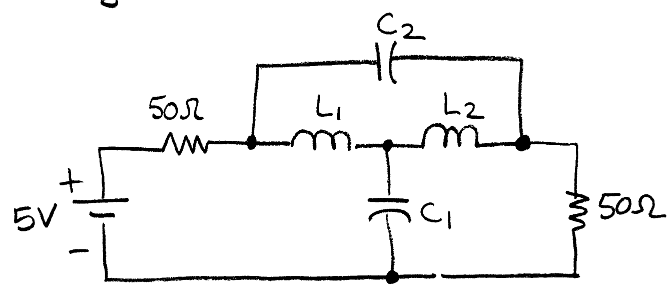
This inductance is essentially in series with the 1mH inductor

$$L'_{EQ} = 1mH + L_{EQ} = 1mH + 210\mu H = 1210\mu H$$

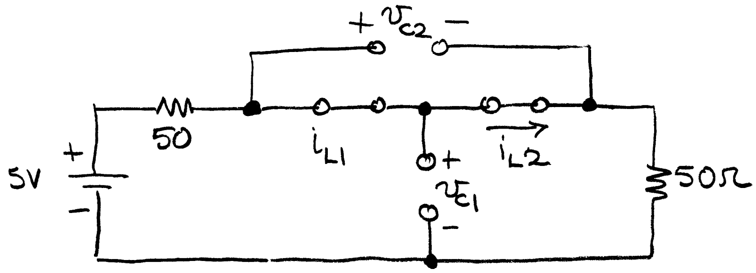


Example 6–15

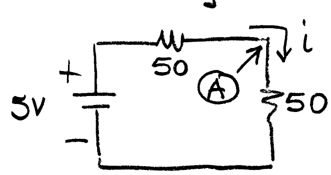
Determine the voltage across the capacitors and current through the inductors in the circuit given below.



This is a DC circuit analysis. The capacitors are opens since they are fully charged. The inductors are correspondingly shorts.



This is basically a two resistor series circuit.

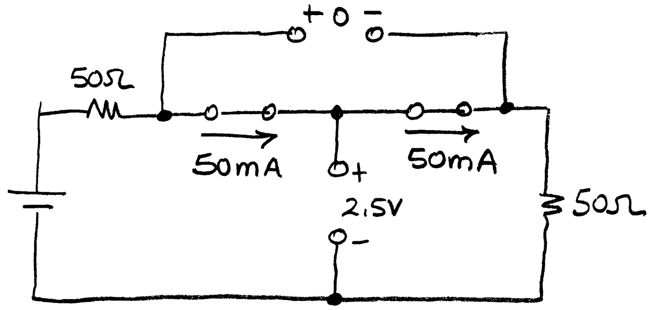


$$i = \frac{5V}{50+50} = 0.050\ A$$

The voltage at Ⓐ is $v_A = iR$

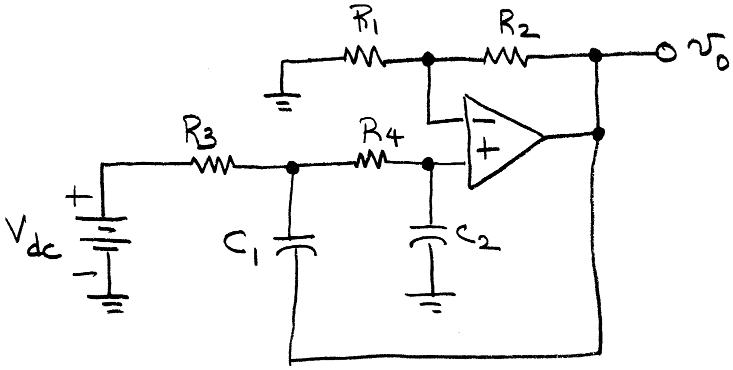
$$v_A = (.05)(50) = 2.5V$$

The circuit values are then

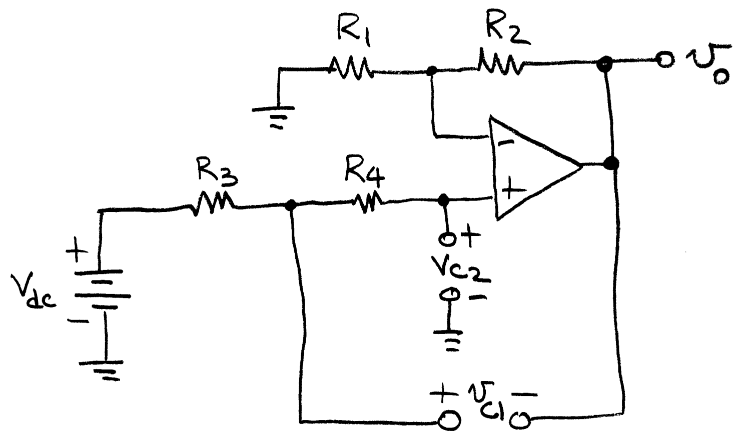


Exercise 6-11

Find the OP AMP output voltage.

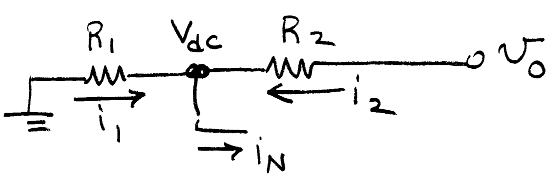


Since this is a dc circuit we replace the capacitors by open circuits and solve.



There is no current i_P into the OP AMP so there are no voltage drops across either R_3 or R_4. $\Rightarrow V_P = V_{dc}$

Since $V_P = V_{dc} = V_N$ we can analyze the inverting side of the OP AMP.



Using KCL $\sum_{in} i = 0$

$$i_1 - \cancel{i_N}^{\,0} + i_2 = 0$$

$$\frac{0 - V_{dc}}{R_1} + \frac{v_o - V_{dc}}{R_2} = 0$$

$$\therefore \frac{v_o}{R_2} = V_{dc}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_o = \frac{R_1 + R_2}{\cancel{R_1} R_2} \cancel{R_1}\, V_{dc}$$

Note that: $V_{c2} = V_{dc} - 0 = V_{dc}$

$$V_{c1} = V_{dc} - \frac{R_1 + R_2}{R_2} V_{dc} = \frac{2R_2 - R_1}{R_2} V_{dc}$$