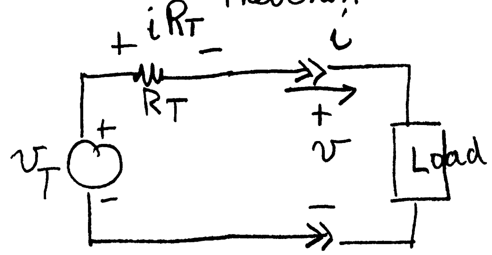
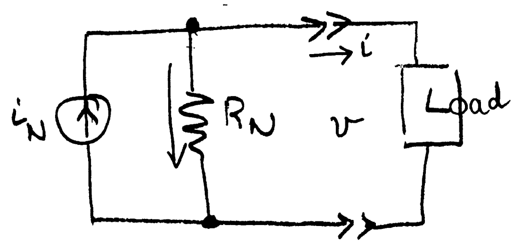
3-4 Thevenin & Norton Equivalent Circuits

Thevenin



Norton



These voltage and current source representations of the source are identical.

If each is open, i.e., remove Load

$$v_{oc} = v_T \qquad\qquad v_{oc} = i_N R_N$$

If each is shorted, i.e. replace the load by a short

$$i_{sc} = \frac{v_T}{R_T} \qquad\qquad i_{sc} = i_N$$

Everywhere in between they are equivalent. We can show this by comparing v-i equations,

by KVL $\sum v = 0$ $\qquad\qquad$ by KCL $\sum_{+in} i = 0$

$$-v_T + iR_T + v = 0 \qquad\qquad +i_N - \frac{v}{R_N} - i = 0$$

$$v = v_T - iR_T$$

rearranging

$$v = i_N R_N - iR_N$$

compare

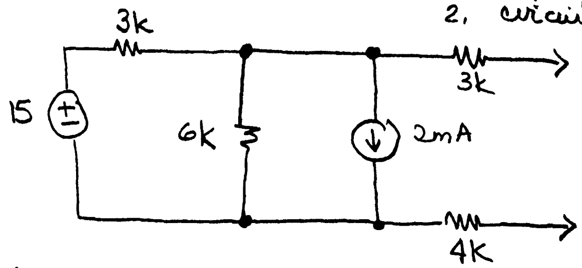
if we set $\quad R_T = R_N$

and $\quad v_T = i_N R_N$

These expressions for Norton & Thevenin are identical.

Example 3-13

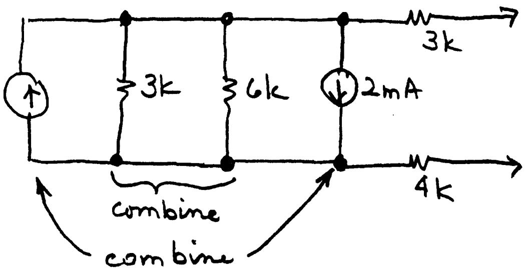
Find Thevenin equivalent
1. superposition always works.
2. circuit reductions (ladder networks)



This is a ladder network.

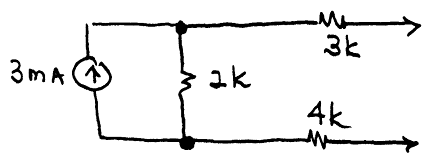
(a) source transformation

$i = \frac{15V}{3k} = 5mA$



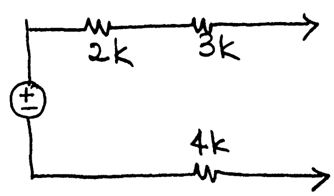
(b)

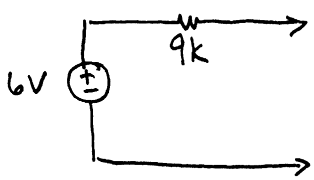
$R_{11} = \frac{3 \cdot 6}{3+6} = \frac{18}{9} = 2k.$



(c) source transform

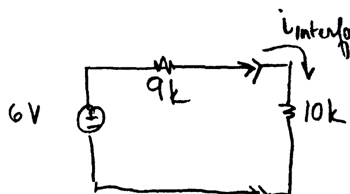
$v = iR$
$= (3mA)(2k)$
$= 6V$





---

load 1

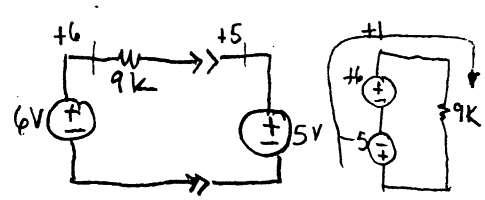


also compute v
$v = \frac{10}{10+9} \cdot 6$

$i = \frac{\mathcal{E}}{R_T} = \frac{6V}{9k+10k} = 0.3158\,mA$

$P = i^2R = (0.3158\,mA)^2(10k) = .9973\,mW$

load 2

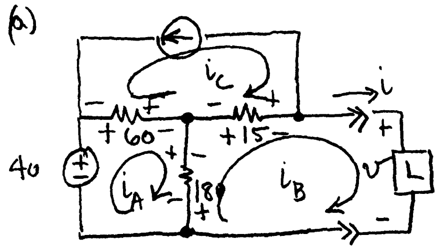


$i = \frac{1V}{9k} = 0.1111\,mA$

$p = vi = (5)(.1111) = .5555\,mW$

Example 3-14.

(a)



by inspection $i_C = -2A$, $i_B = i$

A: $-40 + 60 i_A - 60 i_C + 180 i_A - 180 i_B = 0$

B: $+180 i_B - 180 i_A + 15 i_B - 15 i_C + v = 0$

$-40 + 240 i_A + 120 - 180 i = 0$

$195 i - 180 i_A + 30 + v = 0$

must use mesh equations to find Norton in this case

$240 i_A - 180 i = -80$

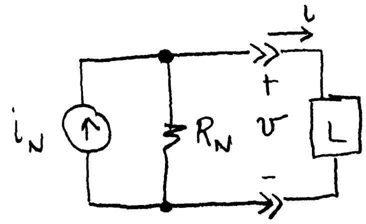
$-180 i_A + 195 i = -30 - v$

using determinants

$$i = \frac{\begin{vmatrix} 240 & -80 \\ -180 & -30 - v \end{vmatrix}}{\begin{vmatrix} 240 & -180 \\ -180 & 195 \end{vmatrix}} = \frac{-21600 - 240v}{14400}$$

$i = -1.5 - \frac{v}{60}$

compare this with equation of norton source.



apply KCL $\sum_{\text{in}} i = 0$

$+i_N - \frac{v}{R_N} - i = 0$

compare

$i = +i_N - \frac{v}{R_N}$

$\therefore$ $i_N = -1.5$ Amps.

$R_N = 60\Omega$

~~if P_{LOAD} = 5 watts find i~~

(b) $vi = 5$ watts

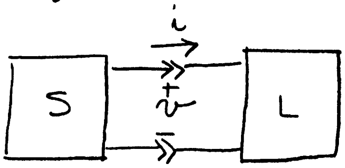
find i for $v = 5/i$

$i = -1.5 - \frac{(5/i)}{60}$

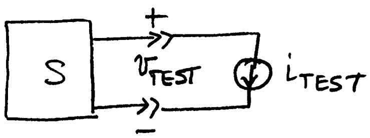
$60i^2 = -90i - 5$

$12i^2 + 18i + 1 = 0$ $\Rightarrow$ $i = -.05778$ A, $i = -1.442$ A.

Derivation of Thevenin's Theorem

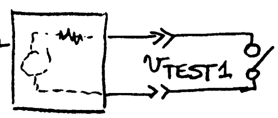


start with typical source / load



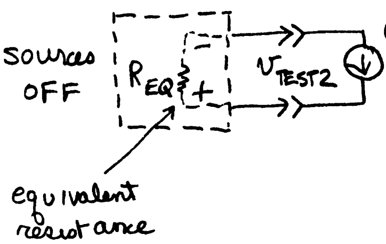
apply a current source i_{TEST}
we know i_{TEST} and let $v = v_{TEST}$

sources ON



① Then TURN i_{TEST} OFF but leave all sources on. $\Rightarrow$ $v_{TEST1} = v_{OC}$

sources OFF



② TURN i_{TEST} ON but turn all sources OFF. $\Rightarrow$ $v_{TEST2} = (-i_{TEST}) R_{EQ}$

using superposition the sum of ① and ② must be v_{TEST}

$$v_{TEST} = v_{TEST1} + v_{TEST2}$$

$$v_{TEST} = v_{OC} - i_{TEST} R_{EQ}$$

$$v = v_T - i\, R_T$$

looks exactly like KVL for Thevenin which we got on first page.

Let $\Rightarrow$ $v_{TEST} = v$

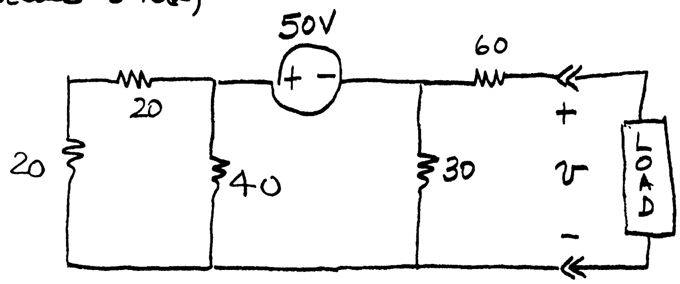
$v_{OC} = v_T$

$R_T = R_{EQ}$

$i_{TEST} = i$

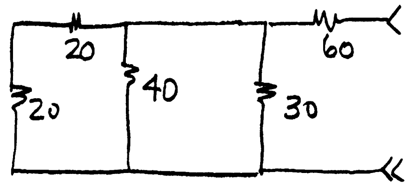
This gives us a great method to determine R_T. Simply turn off all sources and determine R_{EQ}.

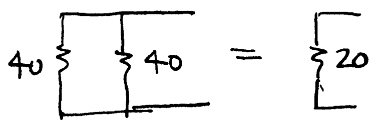
Exercise 3-16(a)

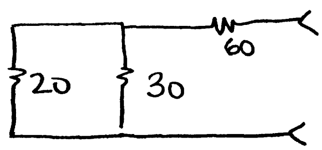


Now use what we just learned to determine Thevenin & Norton

If we eliminate all sources we can find R_T



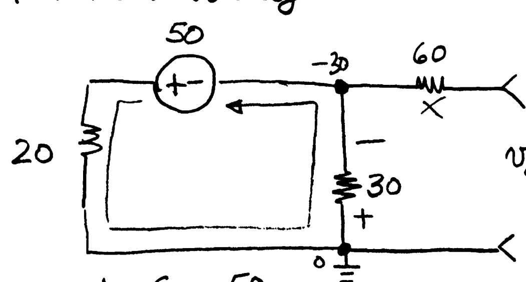




$$R = \frac{20 \cdot 30}{20+30} = \frac{60\cancel{0}}{5\cancel{0}} = 12$$

$$R_T = 12 + 60 = 72\,\Omega.$$

We then find open circuit voltage



$v_{oc} = -30$ volts.

$$i = \frac{\mathcal{E}}{R} = \frac{50}{50} = 1\,A$$

$$V = iR = (1A)(30\,\Omega) = 30 \text{ volts}$$

since they are related $\quad i_N = \dfrac{V_T}{R_T} = \dfrac{-30V}{72\,\Omega} = -417\text{ mA}$