3-3 Linearity Properties

The title of our book includes "linear" circuits
What is a linear circuit?

One that contains only linear elements and independent sources.

For a linear circuit output is a linear function of the input.

homogeneity, output ∝ input (also called proportionality)

$$f(kx) = k\,f(x)$$

Only applies when input is voltage or current.

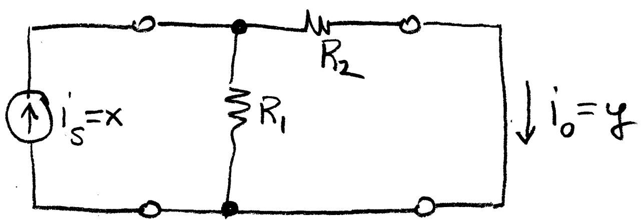
↑ increase input k times ↖ output increases k times

* additivity *

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

This is called superposition.

Example

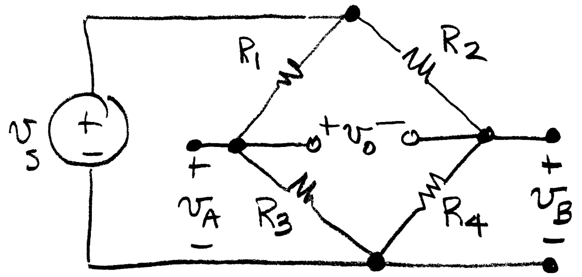


If the input current i_s is x
If the output current i_o is y
This is a current divider so $i_o = \frac{R_1}{R_1 + R_2} i_s$

can be written as $y = \frac{R_1}{R_1 + R_2} x$

this is k
the constant of proportionality
Also called gain

Example 3-10 Given the bridge circuit below



(a) Find the proportionality constant K in the input-output relationship $v_0 = K v_S$.

(b) Find the sign of K when $R_2R_3 > R_1R_4$, $R_2R_3 = R_1R_4$, and $R_2R_3 < R_1R_4$.

(a) This circuit is two side by side voltage dividers. Using the voltage division rule

$$v_A = \frac{R_3}{R_1+R_3} v_S \qquad v_B = \frac{R_4}{R_2+R_4} v_S$$

$$v_0 = v_A - v_B = \left(\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right) v_S = \frac{R_2R_3 + \cancel{R_3R_4} - R_1R_4 - \cancel{R_3R_4}}{(R_1+R_3)(R_2+R_4)} v_S$$

$$v_0 = \underbrace{\frac{R_2R_3 - R_1R_4}{(R_1+R_3)(R_2+R_4)}}_{\text{proportionality constant } K} v_S$$

(b)

$K > 0$ if $R_2R_3 > R_1R_4$

$K = 0$ if $R_2R_3 = R_1R_4$ ← this is called a balanced bridge

$K < 0$ if $R_2R_3 < R_1R_4$

Unit output method

Use proportionality property to find K.

Let output = 1 (amp or volt) and determine corresponding input.

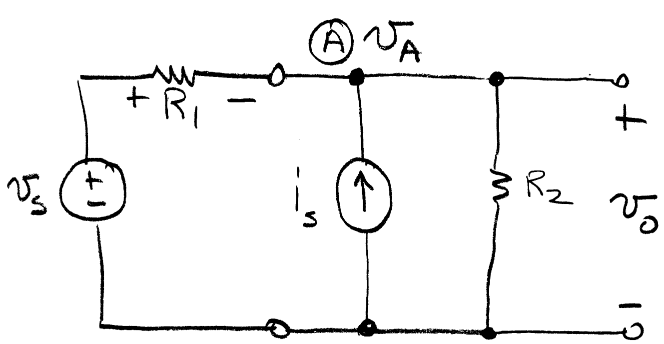
$$K = \frac{\text{Output}}{\text{Input}} = \frac{1}{K}$$

Additivity

Output of a circuit with multiple inputs can be written as a linear combination of the inputs.

$$y = K_1 x_1 + K_2 x_2 + \ldots + K_n x_n$$

Figure 3-31



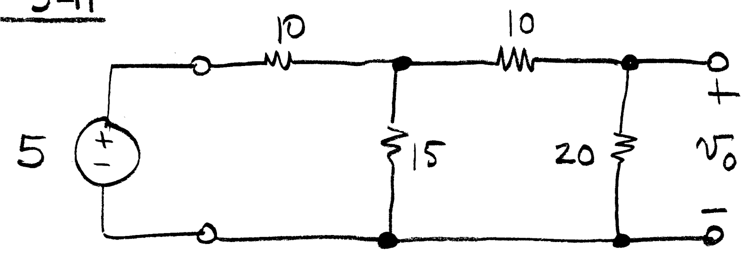
Use KCL @ (A) $\sum_{+in} i = 0$ $\quad + \frac{v_s - v_A}{R_1} + i_s - \frac{v_A}{R_2} = 0$

Rearranging $\left[\frac{1}{R_1} + \frac{1}{R_2}\right] v_A = \frac{v_s}{R_1} + i_s$

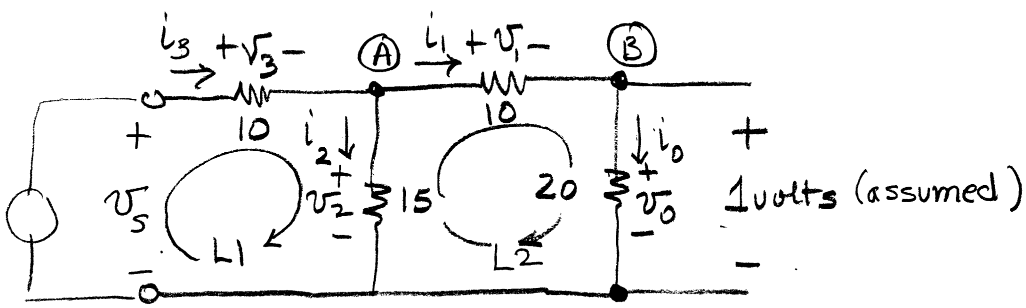
$$v_o = v_A = \left[\frac{R_2}{R_1 + R_2}\right] v_s + \left[\frac{R_1 R_2}{R_1 + R_2}\right] i_s$$

$$\therefore \quad y = K_1 x_1 + K_2 x_2$$

Example 3-11



Use the unit output method to find v_o.



By inspection v_o is the voltage across the 20Ω resistor so

$$i_o = \frac{v}{R} = \frac{1}{20} = .05\ A$$

KCL @ (B) $\sum_{+in} i = 0$ $\quad i_1 - i_o = 0 \Rightarrow i_1 = i_o = .05\,A$

Use Ohm's Law to find v_1

$$v_1 = i_1 R_1 = (.05)(10) = 0.5 \text{ volts}.$$

KVL around L_2 $\quad \sum_{\circlearrowright} v = 0 \quad -v_2 + v_1 + v_o = 0$

$$-v_2 + 0.5 + 1 = 0$$

$$v_2 = 1.5V$$

Ohm's Law $\quad i_2 = \frac{v_2}{R_2} = \frac{1.5}{15} = 0.1\ A$

KCL @ A $\quad \sum_{+in} i = 0 \quad +i_3 - i_2 - i_1 = 0$

$$i_3 - 0.1 - .05 = 0$$

$$i_3 = 0.15\ A.$$

Ohm's Law $\quad v_3 = i_3 R_3 = (0.15)(10) = 1.5\ V.$

KVL around L1 $\quad -v_s + v_3 + v_2 = 0$

$$v_s = v_3 + v_2 = 1.5 + 1.5 = 3\,V.$$

$\therefore\ K = \frac{v_o}{v_s} = \frac{1}{3}$ and output for $V_s = 5$ is $v_o = K V_s = \frac{1}{3} 5 = \frac{5}{3}$