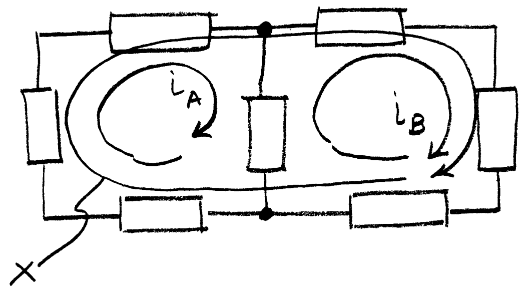
3-2 Mesh Current

A loop is a closed path formed by passing through an ordered sequence of nodes without passing through any node more than once.

A mesh is a special type of loop that does not enclose any elements.

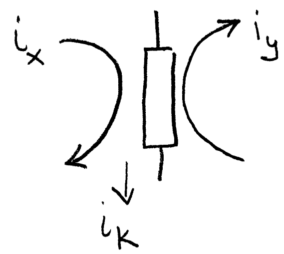


in this drawing i_A, i_B form meshes; but x is only a loop because it encloses an element

If the K-th two-terminal element is contained in meshes X and Y, then the element current can be expressed in terms of the two mesh currents as

$$i_K = i_X - i_Y$$

where X is the mesh whose reference direction agrees with the reference direction of i_K



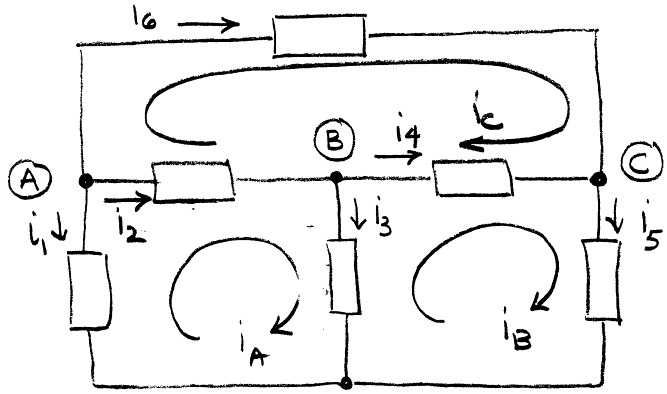
$$i_K = i_x - i_y$$

Mesh current analysis:

1. Identify a mesh current with every mesh and a voltage across every circuit element.
2. Write KVL connection equations in terms of the element voltages around every mesh.
3. Use KCL and the element i-v relationships (usually Ohm's Law) to express the element voltages in terms of the mesh currents.
4. Substitute the element constraints into the connection equations from 2 and arrange into standard form.

Example 3-7

The mesh currents are $i_A = 10A$, $i_B = 5A$, and $i_C = -3A$. Find the element currents i_1 through i_6



By inspection

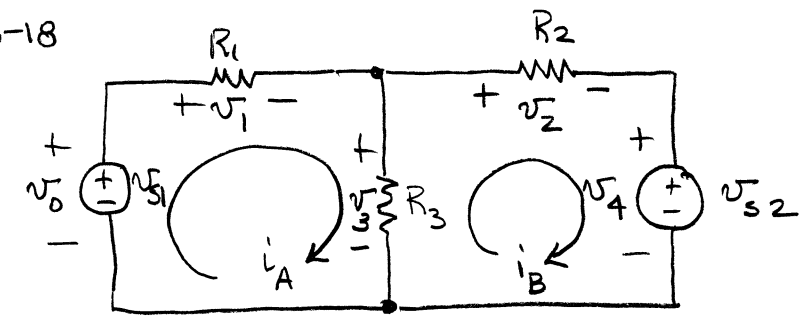
$$i_6 = i_C = -3\,A$$
$$i_5 = i_B = 5\,A$$
$$i_1 = -i_A = -10\,A$$

outside elements have only one current

$$i_2 = i_A - i_C = 10 - (-3) = 13\,A.$$
$$i_3 = i_A - i_B = 10 - (5) = 5\,A$$
$$i_4 = i_B - i_C = 5 - (-3) = 8\,A$$

inside elements have two currents

Figure 3-18



mesh A: $\sum_{\circlearrowright} v \quad -v_0 + v_1 + v_3 = 0$

mesh B: $\sum_{\circlearrowright} v \quad -v_3 + v_2 + v_4 = 0$

element equations.

$$v_1 = i_A R_1 \qquad v_0 = v_{S1}$$

$$v_2 = i_B R_2 \qquad v_4 = v_{S2}$$

$$v_3 = (i_A - i_B) R_3$$

Substitute
$$-v_{S1} + i_A R_1 + (i_A - i_B) R_3 = 0$$
$$-(i_A - i_B) R_3 + i_B R_2 + v_{S2} = 0$$

Put into standard form

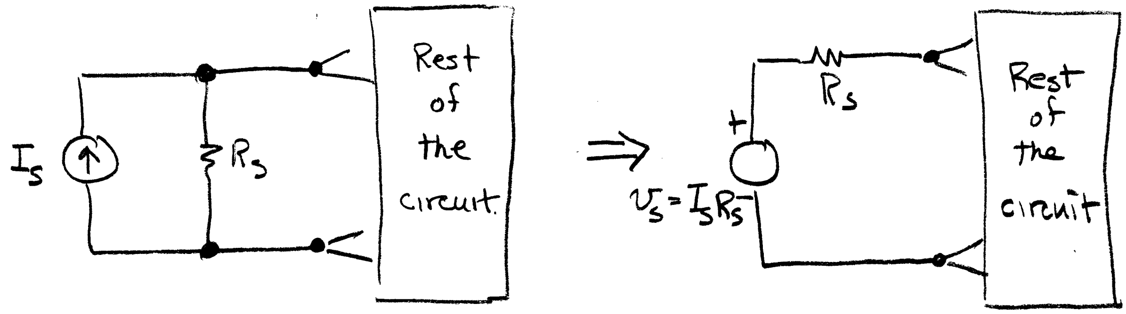
$$(R_1 + R_3)\, i_A + (-R_3)\, i_B = v_{S1}$$
$$(-R_3)\, i_A + (R_2 + R_3)\, i_B = -v_{S2}$$

Solve using Cramer's Rule

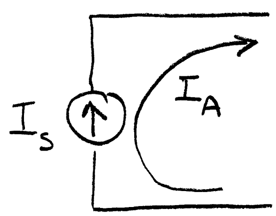
$$i_A = \frac{\Delta_A}{\Delta} = \frac{\begin{vmatrix} v_{S1} & -R_3 \\ -v_{S2} & R_2+R_3 \end{vmatrix}}{\begin{vmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{vmatrix}} = \frac{(R_2+R_3)\, v_{S1} - R_3 v_{S2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$i_B = \frac{\Delta_B}{\Delta} = \frac{\begin{vmatrix} R_1+R_3 & v_{S1} \\ -R_3 & -v_{S2} \end{vmatrix}}{\begin{vmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3 \end{vmatrix}} = \frac{R_3 v_{S1} - (R_1+R_3)\, v_{S2}}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

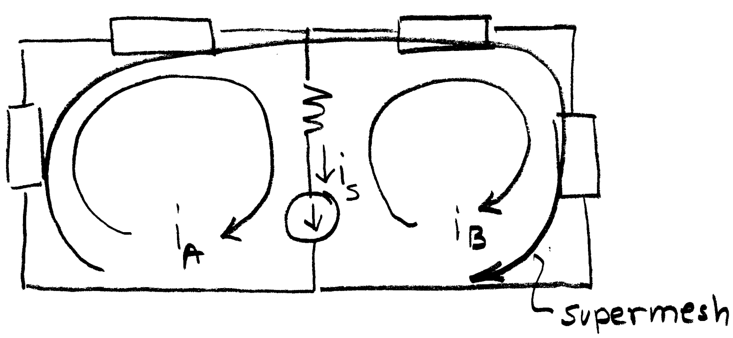
Three methods of dealing with current sources in mesh analysis.



I. Whenever there is a parallel R with the current source transform to an equivalent current source.



II. If a current source I_s is contained in only one mesh then that mesh current is determined by I_s.



III. If a current source is contained in two meshes or is not connected in parallel with a resistance we can create a super mesh which excludes the current source and any series elements.

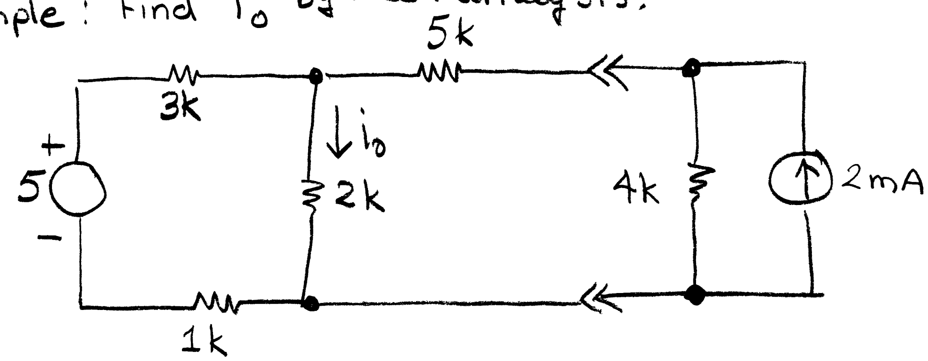
Write KVL for supermesh using i_A and i_B.

Write one additional equation

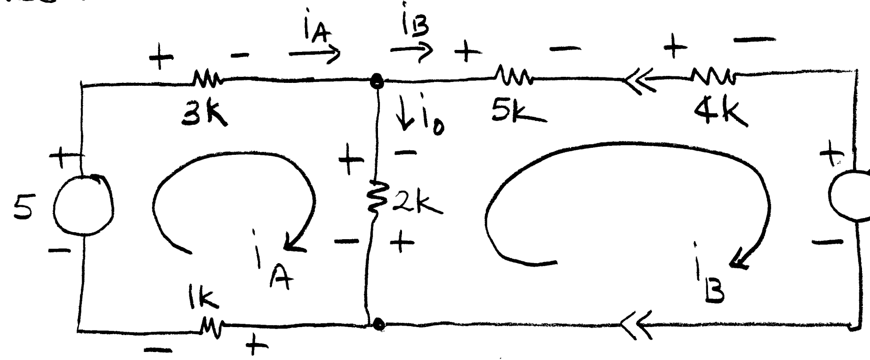
$$i_s = i_A - i_B$$

Solve.

Example: Find i_o by mesh analysis.



Use a source transformation to get rid of the current source.



$v = iR = (2mA)(4k)$
$v = 8$ volts

KVL for mesh A: $-5 + 3000\, i_A + 2000\, i_A - 2000\, i_B + 1000\, i_A = 0$

KVL for mesh B: $+2000\, i_B - 2000\, i_A + 5000\, i_B + 4000\, i_B + 8 = 0$

standard form

$$6000\, i_A - 2000\, i_B = 5$$
$$-2000\, i_A + 11\,000\, i_B = -8$$

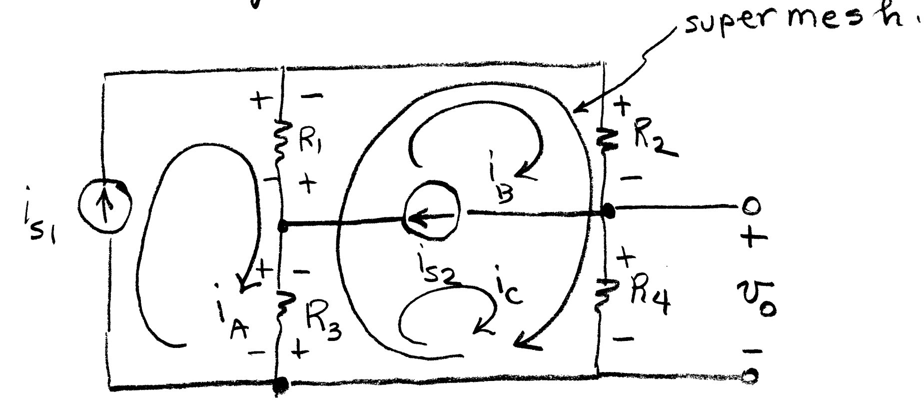
Solving gives
$$i_A = 0.6290 \text{ mA}$$
$$i_B = -0.6129 \text{ mA}$$

Now do KCL @ node to determine i_o

$$\sum_{in} i \qquad +i_A - i_o - i_B = 0$$
$$(0.6290) - i_o - (-0.6129) = 0$$
$$i_o = 0.6290 + 0.6129 = 1.2419 \text{ mA}$$

Example 3-9

Use mesh-current equations to find v_o.



Since we have a current source i_{S1} which is isolated the mesh current $i_A = i_{S1}$.

However, we have a current source i_{S2} which is contained in two meshes. Write a super mesh around it.

For the super mesh write KVL

$$(+i_c - i_A)R_3 + (i_B - i_A)R_1 + i_B R_2 + i_c R_4 = 0$$

and use the constraint that $i_{S2} = i_B - i_c$

Then substitute and solve

$$i_c(R_3+R_4) - i_A(R_1+R_3) + i_B(R_1+R_2) = 0$$

rearranging $(R_1+R_2)\,i_B + (R_3+R_4)\,i_c = (R_1+R_3)\,i_{S1}$

$$i_c = i_B - i_{S2}$$

Solving gives $(R_1+R_2)\,i_B + (R_3+R_4)\,i_B - (R_3+R_4)\,i_{S2} = (R_1+R_3)\,i_{S1}$

$$i_c = \frac{(R_1+R_3)\,i_{S1} + (R_3+R_4)\,i_{S2}}{R_1+R_2+R_3+R_4}$$

$$v_o = i_c R_4 = R_4\,\frac{(R_1+R_3)\,i_{S1} + (R_3+R_4)\,i_{S2}}{R_1+R_2+R_3+R_4}$$