We can substitute the element equations into the node equations and solve the problem.

@ B $\quad i_1 - i_2 - i_4 = 0$

$$\frac{v_A - v_B}{2R} - \frac{v_B - v_C}{2R} - \frac{v_B - 0}{R/2} = 0$$

$$\frac{1}{2R} v_A - \left(\frac{1}{2R} + \frac{1}{2R} + \frac{1}{R/2}\right) v_B + \left(\frac{1}{2R}\right) v_C = 0$$

@ C $\quad i_3 + i_2 - i_5 = 0$

$$\frac{v_A - v_C}{R/2} + \frac{v_B - v_C}{2R} - \left(\frac{v_C - 0}{R}\right) = 0$$

$$\left(\frac{1}{R/2}\right) v_A + \left(\frac{1}{2R}\right) v_B + \left(-\frac{1}{R/2} - \frac{1}{2R} - \frac{1}{R}\right) v_C = 0$$

Substituting $v_A = v_S$ and putting into standard form.

@ C $\quad \left(\frac{1}{2R}\right) v_B + \left(-\frac{1}{R/2} - \frac{1}{2R} - \frac{1}{R}\right) v_C = -\frac{1}{R/2} v_S$

@ B $\quad \left(-\frac{1}{2R} - \frac{1}{2R} - \frac{1}{R/2}\right) v_B + \left(\frac{1}{2R}\right) v_C = -\frac{1}{2R} v_S$

@ C $\quad \frac{1}{2R} v_B - \frac{7}{2R} v_C = -\frac{2}{R} v_S$

@ B $\quad -\frac{3}{R} v_B + \frac{1}{2R} v_C = -\frac{1}{2R} v_S$

$\frac{1}{2} v_B - \frac{7}{2} v_C = -2 v_S$ $\qquad$ divided out all the R's

$-3 v_B + \frac{1}{2} v_C = -\frac{1}{2} v_S$

This can be solved by Cramer's Rule

$$v_B = \frac{\Delta_B}{\Delta} = \frac{\begin{bmatrix} -2v_s & -\frac{7}{2} \\ -\frac{1}{2}v_s & +\frac{1}{2} \end{bmatrix}}{\begin{bmatrix} \frac{1}{2} & -\frac{7}{2} \\ -3 & +\frac{1}{2} \end{bmatrix}} = \frac{(-v_s)-(\frac{7}{4}v_s)}{(\frac{1}{4})-(\frac{21}{2})} = \frac{-4v_s-7v_s}{1-42}$$

$$v_B = \frac{-11v_s}{-41} = 0.2683\,v_s$$

$$v_c = \frac{\Delta_c}{\Delta} = \frac{\begin{bmatrix} \frac{1}{2} & -2v_s \\ -3 & -\frac{1}{2}v_s \end{bmatrix}}{\begin{bmatrix} \frac{1}{2} & -\frac{7}{2} \\ -3 & +\frac{1}{2} \end{bmatrix}} = \frac{(-\frac{1}{4}v_s)-(6v_s)}{(\frac{1}{4})-(\frac{21}{2})} = \frac{-v_s-24v_s}{1-42}$$

$$v_c = \frac{-25v_s}{-41} = 0.60976\,v_s$$

The input current is then given by

$$i_{in} = i_1 + i_3$$

$$= \frac{v_A - v_B}{2R} + \frac{v_A - v_c}{R/2} = \frac{1}{R}\left[\frac{v_s}{2} - \frac{0.2683v_s}{2} + \frac{v_s}{1/2} - \frac{.60976v_s}{1/2}\right]$$

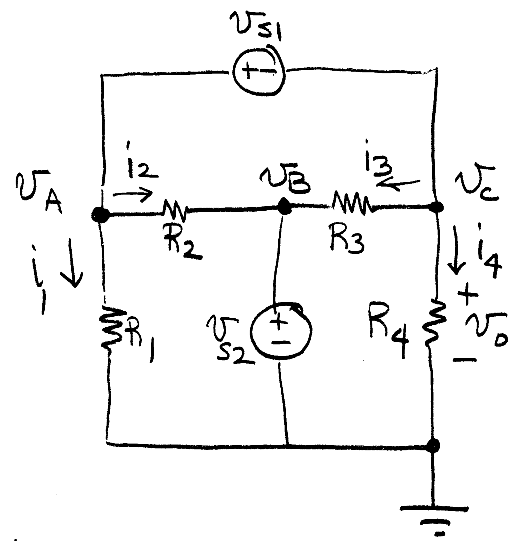
$$= \frac{1}{R}\left[0.5v_s - 0.13415v_s + 2v_s - 1.2195v_s\right]$$

$$i_{in} = \frac{1.1463}{R}v_s$$

$$R_{in} \equiv \frac{v_s}{i_{in}} = \frac{v_s}{\frac{1.1463v_s}{R}} = 0.8723\,R$$

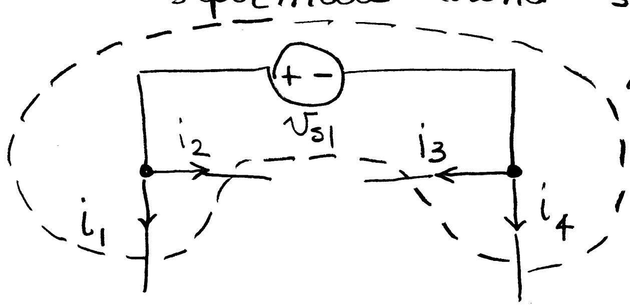
Example 3-6

For the circuit given below
(a) Formulate the node-voltage equations
(b) Solve for v_o using $R_1 = R_4 = 2k\Omega$ and $R_2 = R_3 = 4k\Omega$,



This circuit cannot be solved since we cannot write a current for v_A or v_C through v_{S1}. We cannot use a source transformation on v_{S1} since it does not have a series resistance. We also cannot ground either node A or C since that would change the circuit. Therefore, we must use a "supernode".

Construct a super node around v_{S1}.



"super node"

For a supernode $\sum_{out} i = 0$ $\qquad$ $i_1 + i_2 + i_3 + i_4 = 0$

$$i_1 = \frac{v_A - 0}{R_1}$$

$$i_2 = \frac{v_A - v_B}{R_2} \qquad v_B = v_{S2}$$

$$i_3 = \frac{v_C - v_B}{R_3}$$

$$i_4 = \frac{v_C - 0}{R_4} \qquad v_C = v_o$$

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$\left(\frac{v_A}{R_1}\right) + \left(\frac{v_A - v_{S2}}{R_2}\right) + \left(\frac{v_C - v_{S2}}{R_3}\right) + \left(\frac{v_C}{R_4}\right) = 0$$

We have one other equation

$$v_A - v_C = v_{S1}$$

Rearranging
$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_A + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) v_C = \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_{S2}$$
$$v_A - v_C = v_{S1}$$

Putting in numerical values

$$\frac{1}{R_1} = .00050$$

$$\frac{1}{R_2} = .00025$$

$$\frac{1}{R_3} = .00025$$

$$\frac{1}{R_4} = .00050$$

$$(75 \times 10^{-5})\, v_A + (75 \times 10^{-5})\, v_C = (50 \times 10^{-5})\, v_{S2}$$
$$v_A - v_C = v_{S1}$$

$$3v_A + 3v_C = 2v_{S2}$$

multiplying by 3 and subtracting $\rightarrow$ $3v_A - 3v_C = 3v_{S1}$

$$6v_C = 2v_{S2} - 3v_{S1}$$
$$v_C = \frac{1}{3}v_{S2} - \frac{1}{2}v_{S1}$$

$$v_o = v_C = \frac{1}{3}v_{S2} - \frac{1}{2}v_{S1}$$