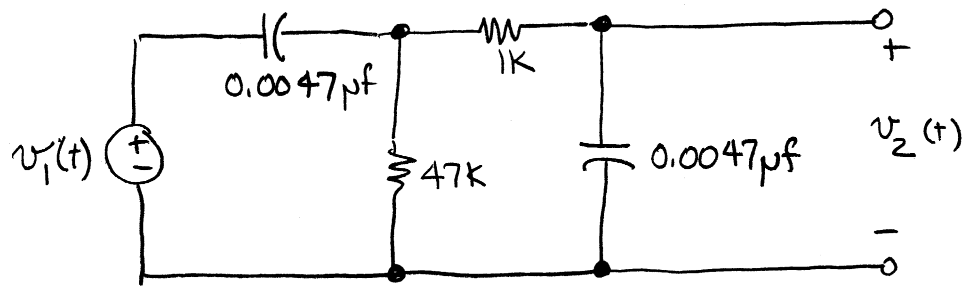
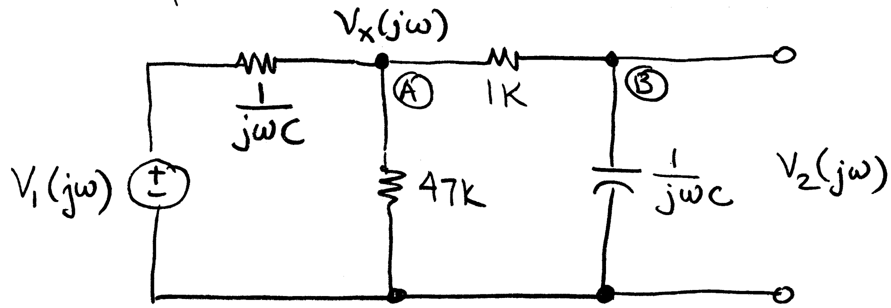
Analysis of directly coupled high-pass / low-pass filter cascade.



Convert to phasors.



Do KCL at nodes A and B. Assume the voltage at node A is $V_x(j\omega)$.

$$\frac{V_1 - V_x}{\frac{1}{j\omega C}} - \frac{V_x - 0}{47k} - \frac{V_x - V_2}{1k} = 0 \qquad (1)$$

$$\frac{V_x - V_2}{1k} - \frac{V_2 - 0}{\frac{1}{j\omega C}} = 0$$

$$\frac{V_x}{1K} - \frac{V_2}{1K} - j\omega C V_2 = 0$$

$$V_x = 1K\left(\frac{1}{1K} + j\omega C\right) V_2 = \quad 1 + j\omega(1k)C \qquad (2)$$

Expanding (1) gives

$$j\omega C V_1 - j\omega C V_x - \frac{1}{47k} V_x - \frac{V_x}{1K} + \frac{V_2}{1K} = 0$$

Multiply through by 1k

$$j\omega(1k)C V_1 - j\omega(1k)C V_x - \frac{1}{47}V_x - V_x + V_2 = 0$$

Collect terms

$$j\omega(1k)C V_1 - \underset{\substack{\uparrow \\ \text{substituting from(2)}}}{V_x}\left[j\omega(1k)C + \frac{1}{47} + 1\right] + V_2 = 0$$

$$j\omega(1k)C V_1 - \left[1 + j\omega(1k)C\right]\left[1 + \frac{1}{47} + j\omega(1k)C\right]V_2 + V_2 = 0$$

Rearranging

$$T(j\omega) = \frac{V_2}{V_1} = \frac{j\omega(1k)C}{\left[1 + j\omega(1k)C\right]\left[1 + \frac{1}{47} + j\omega(1k)C\right] + 1}$$

To plot this I will make two approximations

(1) $1 + \frac{1}{47} \approx 1$

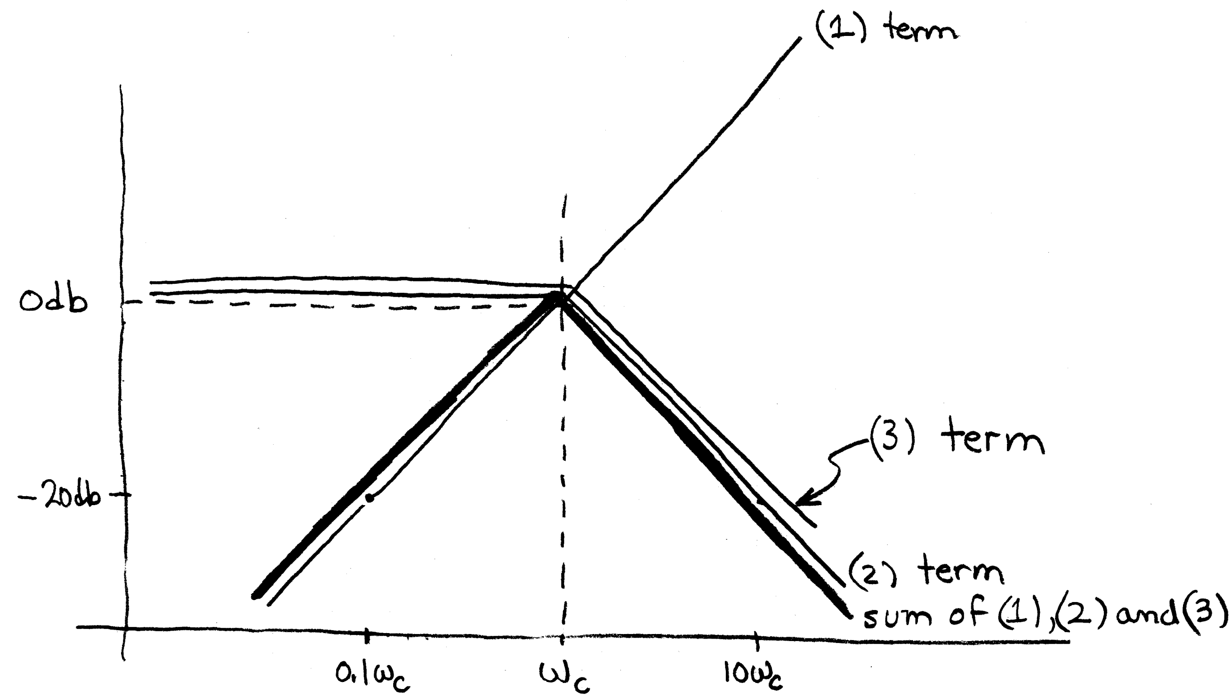
(2) I can neglect the +1 term in the denominator. As we will see this will only change the shape a little for $\omega \to 0$

$$T(j\omega) \cong \frac{j\omega(1k)C}{[1 + j\omega(1k)C][1 + j\omega(1k)C]}$$

Taking logs

$$20\log_{10}|T(j\omega)| \cong 20\log_{10}|\omega(1k)C| - 20\log_{10}|1 + j\omega(1k)C|$$
$$- 20\log_{10}|1 + j\omega(1k)C|$$

Note: I could have combined the last two terms but do not since they are easier to plot this way.



(1) Start with the $20 \log_{10} |\omega (1k) C|$ term

This term is 0db when $\omega (1k) C = 1$

or $\omega_c = \frac{1}{(1000)(.0047 \times 10^{-6})} = 2.13 \times 10^{5} \frac{rad}{sec}$

[in terms of laboratory $f_c \cong \frac{\omega}{2\pi} = 33862$ Hz

(2) Plot the second term $-20 \log_{10} |1 + j\omega (1k) C|$

substituting $C = 0.0047 \mu f$

$$-20 \log_{10} |1 + j\omega (4.7 \times 10^{-6})|$$

This term has $\omega_c = \frac{1}{4.7 \times 10^{-6}} = 2.13 \times 10^{5} \text{ rad/sec}$

It is for $\omega \ll \omega_c$ $\quad -20 \log_{10}(1) \rightarrow 0$

$\omega \gg \omega_c$ $\quad -20 \log_{10} |\omega (4.7 \times 10^{5})|$

So it does nothing (0db) for $\omega < \omega_c$ and decreases at -20db/decade for $\omega > \omega_c$.

This cancels out the (1) term for $\omega > \omega_c$

(3) The third term repeats the second term

(4) Adding together gives a band-pass characteristic centered at $\omega = 2.13 \times 10^{5}$ rad/sec