Electric Circuits

SEVENTH EDITION

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CHAPTER

1 Circuit Variables

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Understand and be able to use SI units and the standard prefixes for powers of 10.
2. Know and be able to use the definitions of *voltage* and *current*.
3. Know and be able to use the definitions of *power* and *energy*.
4. Be able to use the passive sign convention to calculate the power for an ideal basic circuit element given its voltage and current.

Electrical engineering is an exciting and challenging profession for anyone who has a genuine interest in, and aptitude for, applied science and mathematics. Over the past century and a half, electrical engineers have played a dominant role in the development of systems that have changed the way people live and work. Satellite communication links, telephones, digital computers, televisions, diagnostic and surgical medical equipment, assembly-line robots, and electrical power tools are representative components of systems that define a modern technological society. As an electrical engineer, you can participate in this ongoing technological revolution by improving and refining these existing systems and by discovering and developing new systems to meet the needs of our ever-changing society.

As you embark on the study of circuit analysis, you need to gain a feel for where this study fits into the hierarchy of topics that comprise an introduction to electrical engineering. Hence we begin by presenting an overview of electrical engineering, some ideas about an engineering point of view as it relates to circuit analysis, and a review of the international system of units.

We then describe generally what circuit analysis entails. Next, we introduce the concepts of voltage and current. We follow these concepts with discussion of an ideal basic element and the need for a polarity reference system. We conclude the chapter by describing how current and voltage relate to power and energy.

HALLMARK FEATURES (OF THE SEVENTH EDITION)

Practical Perspectives

The seventh edition of *Electric Circuits* includes 13 Practical Perspectives that offer examples of real-world circuits, taken from real-world devices such as telephones, hair dryers, and automobiles. A total of 13 chapters begin with a brief description of a practical application of the material to follow. Once the chapter material is presented, the chapter concludes with a quantitative analysis of the application. Several problems pertaining to the Practical Perspective are included in the Chapter Problems and are identified with the icon ◆. The Practical Perspectives are designed to stimulate students' interest in applying circuit analysis to the design of useful circuits and devices, and to consider some of the complexities associated with making a working circuit. The following chart shows where to find the opening description, quantitative analysis, and related Chapter Problems for each Practical Perspective:

DESCRIPTION	QUANTITATIVE ANALYSIS	RELATED HOMEWORK PROBLEMS
Electric Safety, Chapter 2, Page 27	Page 54	Pages 62–63
A Rear Window Defroster, Chapter 3, Page 65	Pages 87–90	Page 105
Circuit with Realistic Resistors, Chapter 4, Page 107	Pages 155–158	Page 178
Strain Gages, Chapter 5, Page 181	Pages 201–203	Pages 214–215
Proximity Switches, Chapter 6, Page 217	Pages 247–248	Pages 260–261
A Flashing Light Circuit, Chapter 7, Page 263	Pages 304–305	Pages 326–327
An Ignition Circuit, Chapter 8, Page 329	Pages 366–369	Page 379
A Household Distribution Circuit, Chapter 9, Page 382	Page 431	Page 447
Heating Appliances, Chapter 10, Page 449	Pages 481–482	Pages 496–497
Transmission and Distribution of Electric Power, Chapter 11, Page 499	Pages 524–525	Page 534
Surge Suppressors, Chapter 13, Page 581	Page 629	Page 648
Pushbutton Telephone Circuits, Chapter 14, Page 657	Pages 687–688	Pages 694–695
Bass Volume Control, Chapter 15, Page 697	Pages 740–742	Pages 753–754

Integration of Computer Tools

Computer tools cannot replace the traditional methods for mastering the study of electric circuits. However, they can assist students in the learning process by providing a visual representation of a circuit's behavior, validating a calculated solution, reducing the computational burden of more complex circuits, and iterating toward a desired solution using parameter variation. This computational support is often invaluable in the design process.

The seventh edition includes the support of a popular computer tool, PSpice, into the main text with the addition of icons identifying Chapter Problems suited for exploration with this tool. The icon **P** identifies those problems to investigate with PSpice. Instructors are provided with computer files containing the PSpice simulation of the problems so marked.

Design Emphasis

We continue to support the emphasis on design of circuits in several ways. First, several of the new Practical Perspective discussions focus on the design aspects of the circuits. The accompanying Chapter Problems continue the discussion of the design issues in these practical examples. Second, design-oriented Chapter Problems have been labeled explicitly with the icon ❖, enabling students and instructors to identify those problems with a design focus. Third, the identification of problems suited to exploration with PSPice suggests design opportunities using this computer tool.

Chapter Problems

Users of *Electric Circuits* consistently have rated the Chapter Problems as one of the book's most attractive features. In the seventh edition, there are over 1000 problems (approximately 75% are new or revised). The problems are designed around the following objectives (in parentheses are the corresponding problem categories identified in the *Instructor's Guide*).

- To give students practice in using the analytical techniques developed in the text (Practice)
- To show students that analytical techniques are tools, not objectives (Analytical Tool)
- To give students practice in choosing the analytical method to be used in obtaining a solution (Open Method)
- To show students how the results from one solution can be used to find other information about a circuit's operation (Additional Information)
- To encourage students to challenge the solution either by using an alternate method or by testing the solution to see if it makes sense in terms of known circuit behavior (Solution Check)
- To introduce students to design oriented problems (Design)

- To give students practice in deriving and manipulating equations where quantities of interest are expressed as functions of circuit variables such as R, L, C, ω, and so forth; this type of problem also supports the design process (Derivation)
- To challenge students with problems that will stimulate their interest in both electrical and computer engineering (Practical)

Accuracy

All text and problems in the seventh edition have been triple-checked for accuracy.

PREREQUISITES

In writing the first 12 chapters of the text, we have assumed that the reader has taken a course in elementary differential and integral calculus. We have also assumed that the reader has had an introductory physics course, at either the high school or university level, that introduces the concepts of energy, power, electric charge, electric current, electric potential, and electromagnetic fields. In writing the final six chapters, we have assumed the student has had, or is enrolled in, an introductory course in differential equations.

COURSE OPTIONS

The text has been designed for use in a one-semester, two-semester, or a three-quarter sequence.

- *Single-semester course:* After covering Chapters 1–4 and Chapters 6–10 (omitting Sections 7.7 and 8.5) the instructor can choose from Chapter 5 (operational amplifiers), Chapter 11 (three-phase circuits), Chapters 13 and 14 (Laplace methods), and Chapter 18 (Two-Port Circuits) to develop the desired emphasis.
- *Two-semester sequence:* Assuming three lectures per week, the first nine chapters can be covered during the first semester, leaving Chapters 10–18 for the second semester.
- *Academic quarter schedule:* The book can be subdivided into three parts: Chapters 1–6, Chapters 7–12, and Chapters 13–18.

The introduction to operational amplifier circuits can be omitted without interference by the reader going to the subsequent chapters. For example, if Chapter 5 is omitted, the instructor can simply skip Section 7.7, Section 8.5, Chapter 15, and those problems and assessing objective problems in the chapters following Chapter 5 that pertain to operational amplifiers.

There are several appendixes at the end of the book to help readers make effective use of their mathematical background. Appendix A reviews Cramer's method of solving simultaneous linear equations and simple matrix algebra; complex numbers are reviewed in Appendix B; Appendix C contains additional material on magnetically coupled coils and ideal transformers; Appendix D contains a brief discussion of the decibel; Appendix E is dedicated to Bode diagrams; Appendix F is devoted to an abbreviated table of trigonometric identities that are useful in circuit analysis; and an abbreviated table of useful integrals is given in Appendix G. Appendix H provides answers to selected suggested problems.

PRINT SUPPLEMENTS

An innovative supplements package is available with the seventh edition. Students and professors are constantly challenged in terms of time and energy by the confines of the classroom and the importance of integrating new information and technologies into an electric circuits course. Through the following supplements, we believe we have succeeded in making some of these challenges more manageable.

PSpice for *Electric Circuits*

This supplement is published as a separate booklet to facilitate its use at a computer. It has been revised extensively from the sixth edition, most importantly to eliminate the "programming language" aspect of the original SPICE. Now, circuits are described to PSpice using a circuit schematic, and techniques for developing such schematics are presented in the supplement. This supplement continues to present topics in PSpice in the same order as those topics are presented in the text, so the content has undergone minor revision to reflect the revisions in the text.

Instructor's Solutions Manual

A printed solutions manual is made available to assist instructors in teaching their courses. The solutions manual contains solutions with supporting figures to all of the 1000-plus end-of-chapter problems in the seventh edition. This supplement is available to all adopting faculty and was checked for accuracy by several instructors. A disc containing files for PSpice solutions for all indicated problems is attached to the solutions manual. The manual is not available for sale to students.

ONEKEY

OneKey is an on-line course management solution perfect for helping manage your class and preparing student lectures, quizzes, and tests. Using OneKey, instructors can quickly create an on-line course tailored to their needs.

OneKey for the Instructor

OneKey contains complete electronic solutions files of all assessment problems and end-of-chapter problems in the textbook and makes it casy for instructors to post the selected solutions at a protected on-line site for student review. Further, OneKey provides instructors with additional presentation resources, including PowerPoint slides and the Instructor's Guide, which includes individual chapter tests.

The Instructor's Guide enables instructors to orient themselves quickly to this text and the supplement package. For easy reference, the following information is organized for each chapter:

- a chapter overview
- problem categorizations
- problem references by chapter section
- a list of examples
- chapter tests

OneKey for the Student

OneKey contains a student solutions manual with full solutions to the assessing objectives problems from the text, as well as the suggested problems. Student solutions are thorough and detailed, including all intermediate steps of the problem solution. OneKey contains a **Student Study Guide** that guides the reader through the text, poses questions throughout, and suggests problems to try. The study guide can be used in conjunction with a lecture-based course, or can be used for self-paced instruction. Additionally, a **Student Workbook** covers nine key problem-solving techniques, additional examples, and practice problems.

To learn more about OneKey, visit **http://www.prenhall.com/onekey**, or contact your local Prentice Hall rep. Instructor use and access is free. Student access codes are free with their new *Electric Circuits* textbooks, or available for sale as stand-alone items. Additional material for both instructors and students will be added.

For instructors wishing to adopt *Electric Circuits* with OneKey, please use the following ordering ISBN:

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JAMES W. NILSSON
SUSAN A. RIEDEL

A GUIDE TO USING THE SEVENTH EDITION

The seventh edition of *Electric Circuits* was designed to make it easy for the reader to categorize and locate the many elements that comprise this text. Most of these elements are color-coded, as described below.

Chapter Openers

The information in the chapter opener gives you a preview of the chapter contents. On the left-hand page you'll find several important elements. The first is the **chapter title** and the **Chapter Contents** for the chapter, including page numbers. The second element is the **Chapter Objectives**. These objectives are numbered and provide an organized method for determining what skills you are expected to master in this chapter. The third element is the **introduction** to the chapter. This provides you with an overview of the chapter contents and purpose for studying this material.

In most chapters, the right-hand page of the chapter opener contains a **Practical Perspective** (see, for example, p. 27). This is an example of a real-world circuit that is a component of a real-world device, such as a stereo, an automobile, and a hair dryer. The Practical Perspective gives you a description of a practical application of the material in the text, and frequently includes a photo of the device and a schematic of the circuit of interest. At the end of the chapter the Practical Perspective is revisited in more detail, applying the material presented in the chapter.

Fundamental Equations and Concepts

As you will discover, most chapters contain many mathematical equations. This should not come as a surprise, since mathematics in an important foundation in the study of engineering in general, and electric circuits in particular. A few of these many equations are identified as **fundamental equations** (see, for example, p. 35). You will recognize a fundamental equation because it has a yellow background and a bulleted definition in the margin that describes the equation. A fundamental equation describes an important chapter concept in mathematical form.

In a few instances, an important chapter concept cannot be described in the form of a mathematical equation. To draw your attention to **fundamental concepts**, we have set the text in blue, surrounded the blue text with a black box, and provided a bulleted definition in the margin that describes the concept (see, for example, p. 43). The marginal notes and colored background or text make it easier to locate the fundamental equations and equations concepts throughout the text.

Examples

Every chapter includes several examples that illustrate the concepts presented in the text in the form of a numerical example. There are over 130 examples in this text. On pages xiii–xv there is a comprehensive list of the examples with titles and corresponding page numbers. Examples have a title in the border to make the purpose of the example clear. Most examples fit on a single text page and are set in a yellow background. The Examples are intended to illustrate the application of a particular concept, and also to encourage good problem-solving skills, including choosing a solution technique and checking the solution using a different approach.

Assessment Problems

We begin each chapter with a list of Chapter Objectives that identify skills you can expect to master in studying the chapter material. At key points in the chapter, you are asked to stop and assess your mastery of a particular objective by solving one or more *assessment problems*. (See p. 32, under the heading "Assessing Objective 1".) These problems are set in a green background with a heading that identifies the number of the Chapter Objective being assessed and a brief description of that objective. The answer to the assessment problem immediately follows the problem statement. Solutions to assessment problems and suggested problems are available on OneKey (See OneKey for the Student on p. xxii). If you are able to solve the assessment problems for a given Objective, you have mastered that Objective.

Suggested Problems

At the conclusion of each collection of assessment problems you will find suggested Chapter Problems to solve that will further test your mastery of the Chapter Objectives. These **suggested problems** are identified in a bulleted note (see, for example, p. 32). You will also find suggested problems at other points in the text where you can stop and assess your understanding of the material just presented by solving one or more Chapter Problems. Answers to selected suggested problems are provided in Appendix H.

Practical Perspectives

Once the material in the chapter has been presented, the **Practical Perspective** that was introduced in the chapter opener is revisited. We pose a problem that relates to the Practical Perspective and present a solution to the problem, usually in the form of an example. The Practical Perspective problem enables you to understand how to apply the chapter concepts to the solution of a real-world problem. Each Practical Perspective includes suggested problems to solve that will assess your understanding of the practical application. The Practical Perspective discussion has a blue background.

Summary

Each chapter concludes with a Summary of the important concepts presented in the chapter in the form of a bulleted list. Each concept has a page number reference that points you to the discussion of the concept in the body of the text. The Summary is a good place to review the material in the chapter and assess your mastery of that material.

Chapter Problems

The final element in each chapter is a collection of **Chapter Problems**. The Chapter Problems fall into many different categories, as described elsewhere. But three categories of Chapter Problems are identified with specific icons. Some Chapter Problems lend themselves to solution using PSpice, a software circuit simulator, and are identified with a PSpice icon P. Some Chapter Problems are focused on circuit design, and are identified with a design icon ❖. The Chapter Problems that relate to the Practical Perspective are identified with a Practical Perspective icon ◆. All chapter problems are set in a green background.

ASSESSING OBJECTIVE 1

◆ **Understand and be able to use SI units and the standard prefixes for powers of 10**

1.1 How many dollars per millisecond would the federal government have to collect to retire a deficit of $100 billion in one year?

ANSWER: $3.17/ms.

NOTE ◆ *Also try Chapter Problems 1.1, 1.3, and 1.6.*

1.2 If a signal can travel in a cable at 80% of the speed of light, what length of cable, in inches, represents 1 ns?

ANSWER: 9.45".

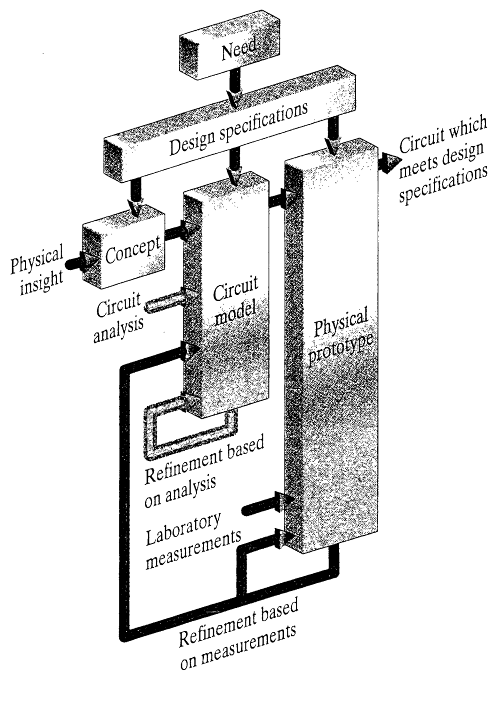


Figure 1.4 A conceptual model for electrical engineering design.

1.3 ◆ Circuit Analysis: An Overview

Before becoming involved in the details of circuit analysis, we need to take a broad look at engineering design, specifically the design of electric circuits. The purpose of this overview is to provide you with a perspective on where circuit analysis fits within the whole of circuit design. Even though this book focuses on circuit analysis, we try to provide opportunities for circuit design where appropriate.

All engineering designs begin with a need, as shown in Fig. 1.4. This need may come from the desire to improve on an existing design, or it may be something brand-new. A careful assessment of the need results in design specifications, which are measurable characteristics of a proposed design. Once a design is proposed, the design specifications allow us to assess whether or not the design actually meets the need.

A concept for the design comes next. The concept derives from a complete understanding of the design specifications coupled with an insight into the need, which comes from education and experience. The concept may be realized as a sketch, as a written description, or in some other form. Often the next step is to translate the concept into a mathematical model. A commonly used mathematical model for electrical systems is a **circuit model**.

The elements that comprise the circuit model are called **ideal circuit components**. An ideal circuit component is a mathematical model of an actual electrical component, like a battery or a light bulb. It is important for the ideal circuit component used in a circuit model to represent the behavior of the actual electrical component to an acceptable degree of accuracy. The tools of **circuit analysis**, the focus of this book, are then applied to the circuit. Circuit analysis is based on mathematical techniques and is used to predict the behavior of the circuit model and its ideal circuit components. A comparison between the desired behavior, from the design

The assignments of the reference polarity for voltage and the reference direction for current are entirely arbitrary. However, once you have assigned the references, you must write all subsequent equations to agree with the chosen references. The most widely used sign convention applied to these references is called the **passive sign convention**, which we use throughout this book. The passive sign convention can be stated as follows:

PASSIVE SIGN CONVENTION

> Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element (as in Fig. 1.5), use a positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

We apply this sign convention in all the analyses that follow. Our purpose for introducing it even before we have introduced the different types of basic circuit elements is to impress on you the fact that the selection of polarity references along with the adoption of the passive sign convention is *not* a function of the basic elements nor the type of interconnections made with the basic elements. We present the application and interpretation of the passive sign convention in power calculations in Section 1.6.

ASSESSING OBJECTIVE 2

◆ **Know and be able to use the definitions of *voltage* and *current***

1.3 The current at the terminals of the element in Fig. 1.5 is

$$i = 0, \qquad t < 0;$$

$$i = 20e^{-5000t}\,\text{A}, \quad t \geq 0.$$

Calculate the total charge (in microcoulombs) entering the element at its upper terminal.

ANSWER: 4000 μC.

NOTE ◆ *Also try Chapter Problem 1.9.*

1.4 The expression for the charge entering the upper terminal of Fig. 1.5 is

$$q = \frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\ \text{C}.$$

Find the maximum value of the current entering the terminal if $\alpha = 0.03679\ \text{s}^{-1}$.

ANSWER: 10 A.

ASSESSING OBJECTIVES 3 AND 4

◆ **Know and use the definitions of *power* and *energy*, and be able to use the passive sign convention**

1.5 Assume that a 20 V voltage drop occurs across an element from terminal 2 to terminal 1 and that a current of 4 A enters terminal 2.

a) Specify the values of v and i for the polarity references shown in Fig. 1.6(a)–(d).

b) State whether the circuit inside the box is absorbing or delivering power.

c) How much power is the circuit absorbing?

ANSWER: (a) Circuit 1.6(a): $v = -20$ V, $i = -4$A; circuit 1.6(b): $v = -20$ V, $i = 4$ A; circuit 1.6(c): $v = 20$ V, $i = -4$ A; circuit 1.6(d): $v = 20$ V, $i = 4$ A; (b) absorbing; (c) 80 W.

1.6 Assume that the voltage at the terminals of the element in Fig. 1.5 corresponding to the current in Assessment Problem 1.3 is

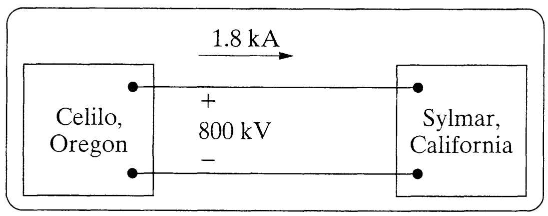
$$v = 0, \qquad t < 0;$$

$$v = 10e^{-5000t} \text{ kV}, \quad t \geq 0.$$

Calculate the total energy (in joules) delivered to the circuit element.

ANSWER: 20 J.

1.7 A high-voltage direct-current (dc) transmission line between Celilo, Oregon and Sylmar, California is operating at 800 kV and carrying 1800 A, as shown. Calculate the power (in megawatts) at the Oregon end of the line and state the direction of power flow.



ANSWER: 1440 MW, Celilo to Sylmar.

NOTE ◆ *Also try Chapter Problems 1.12, 1.17, 1.24, and 1.26.*

SUMMARY

- The International System of Units (SI) enables engineers to communicate in a meaningful way about quantitative results. Table 1.1 summarizes the base SI units; Table 1.2 presents some useful derived SI units. (See pages 10 and 11.)
- Circuit analysis is based on the variables of voltage and current. (See page 13.)
- **Voltage** is the energy per unit charge created by charge separation and has the SI unit of volt ($v = dw/dq$). (See page 14.)

- **Current** is the rate of charge flow and has the SI unit of ampere ($i = dq/dt$). (See page 14.)
- The **ideal basic circuit element** is a two-terminal component that cannot be subdivided; it can be described mathematically in terms of its terminal voltage and current. (See page 15.)
- The **passive sign convention** uses a positive sign in the expression that relates the voltage and current at the terminals of an element when the reference direction for the current through the element is in the direction of the reference voltage drop across the element. (See page 16.)
- **Power** is energy per unit of time and is equal to the product of the terminal voltage and current; it has the SI unit of watt ($p = dw/dt = vi$). (See page 17.)
- The algebraic sign of power is interpreted as follows:
  - If $p > 0$, power is being delivered to the circuit or circuit component.
  - If $p < 0$, power is being extracted from the circuit or circuit component. (See page 18.)

PROBLEMS

1.1 A high-resolution computer display monitor has 1280×1024 picture elements, or pixels. Each picture element contains 24 bits of information. If a byte is defined as 8 bits, how many megabytes (MB) are required per display?

1.2 Assume a telephone signal travels through a cable at one half the speed of light. How long does it take the signal to get across the United States if the distance is approximately 5 Mm?

1.3 Some species of bamboo can grow 250 mm/day. Assuming the individual cells in the plant are 10 μm long, how long, on average, does it take a bamboo stalk to grow a 1-cell length?

1.4 One liter (L) of paint covers approximately 10 m^2 of wall. How thick is the layer before it dries? (*Hint:* $1\text{ L} = 1 \times 10^6\text{ mm}^3$.)

1.5 A penny is approximately 1.5 mm thick. At what average velocity does a stack of pennies have to grow to accumulate 300 billion dollars in 1 year?

1.6 A double-sided $3\frac{1}{2}$" floppy disk holds 1.4 MB. The bits of data are stored on circular tracks, with 77 tracks per side. The radius of the innermost track is $\frac{1}{2}$", while the radius of the outermost tract is $1\frac{1}{2}$". The number of bits per track is the same, and there are eight bits in one byte. How much area does a bit stored on the innermost track occupy, in square micrometers?

1.7 A current of 1400 A exists in a rectangular (0.6×9 cm) copper bus bar. The current is due to free electrons moving through the bus bar at an average velocity of v meters/second. If the concentration of free electrons is 10^{29} electrons per cubic meter and if they are uniformly dispersed throughout the bus bar, then what is the average velocity of an electron?

1.8 In electronic circuits it is not unusual to encounter currents in the microampere range. Assume a 20 μA current, due to the flow of electrons.

a) What is the average number of electrons per second that flow past a fixed reference cross section that is perpendicular to the direction of flow?

b) Compare the size of this number to the number of micrometers between Miami and Seattle. You may assume the distance is 3303 mi.

1.9 The current entering the upper terminal of Fig. 1.5 is

$$i = 20 \cos 5000t \text{ A}.$$

Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for $q(t)$.

1.10 How much energy is extracted from an electron as it flows through a 9 V battery from the positive to the negative terminal? Express your answer in attojoules.

1.11 Four 1.5 V batteries supply 100 mA to a portable CD player. How much energy do the batteries supply in 3 h?

1.12 Two electric circuits, represented by boxes A and B, are connected as shown in Fig. P1.12. The reference direction for the current i in the interconnection and the reference polarity for the voltage v across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa.

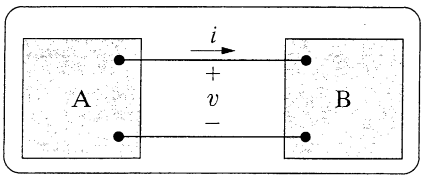
a) $i = 15$ A, $v = 20$ V

b) $i = -5$ A, $v = 100$ V

c) $i = 4$ A, $v = -50$ V

d) $i = -16$ A, $v = -25$ V

Figure P1.12



1.13 The references for the voltage and current at the terminal of a circuit element are as shown in Fig. 1.6(d). The numerical values for v and i are −20 V and 5 A.

a) Calculate the power at the terminals and state whether the power is being absorbed or delivered by the element in the box.

b) Given that the current is due to electron flow, state whether the electrons are entering or leaving terminal 2.

c) Do the electrons gain or lose energy as they pass through the element in the box?

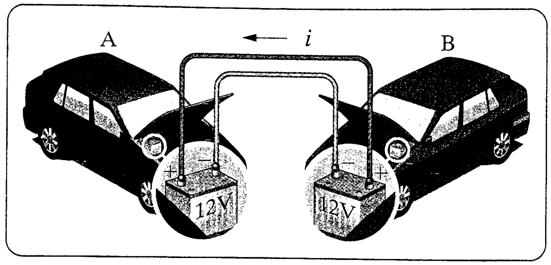
1.14 Repeat Problem 1.13 with a current of −5 A.

1.15 When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in Fig. P1.15. Assume the current i in Fig. P1.15 is measured and found to be −40 A.

a) Which car has the dead battery?

b) If this connection is maintained for 1.5 min, how much energy is transferred to the dead battery?

Figure P1.15



1.16 The manufacturer of a 6 V dry-cell flashlight battery says that the battery will deliver 15 mA for 60 continuous hours. During that time the voltage will drop from 6 V to 4 V. Assume the drop in voltage is linear with time. How much energy does the battery deliver in this 60 h interval?

1.17 P The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 50e^{-1600t} - 50e^{-400t} \text{ V},$$

$$i = 5e^{-1600t} - 5e^{-400t} \text{ mA}.$$

a) Find the power at $t = 625$ μs.

b) How much energy is delivered to the circuit element between 0 and 625 μs?

c) Find the total energy delivered to the element.

1.18 P The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 100e^{-50t} \sin 150t \text{ V},$$

$$i = 20e^{-50t} \sin 150t \text{ A}.$$

a) Find the power absorbed by the element at $t = 20$ ms.

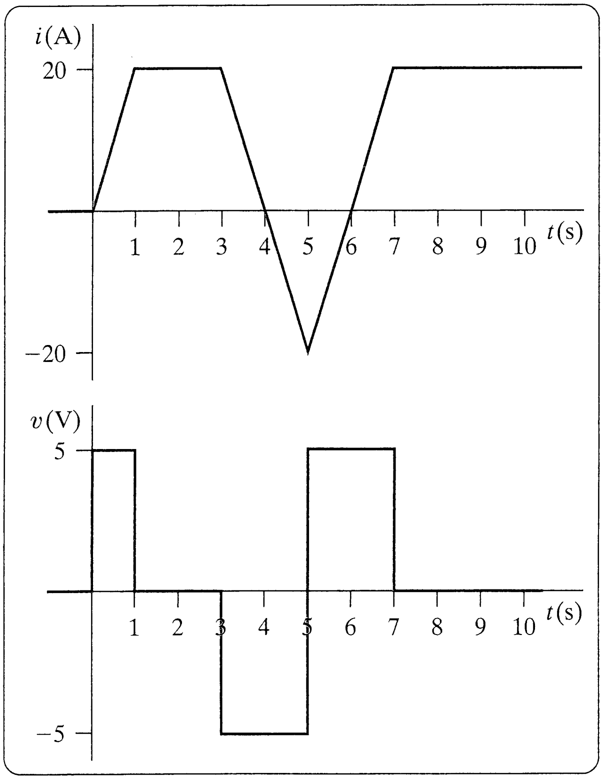
b) Find the total energy (in millijoules) absorbed by the element.

1.19 The voltage and current at the terminals of the circuit element in Fig. 1.5 are shown in Fig. P1.19.

a) Sketch the power versus t plot for $0 \le t \le 10$ s.

b) Calculate the energy delivered to the circuit element at $t = 1, 6$, and 10 s.

Figure P1.19



1.20 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \ge 0$ they are

$$v = 100e^{-500t}\text{ V},$$

$$i = 20 - 20e^{-500t}\text{ mA}.$$

a) Find the maximum value of the power delivered to the circuit.

b) Find the total energy delivered to the element.

1.21 The voltage and current at the terminals of the element in Fig. 1.5 are

$$v = 200\cos 500\pi t\text{ V},\quad i = 4.5\sin 500\pi t\text{ A}.$$

a) Find the maximum value of the power being delivered to the element.

b) Find the maximum value of the power being extracted from the element.

c) Find the average value of p in the interval $0 \le t \le 4$ ms.

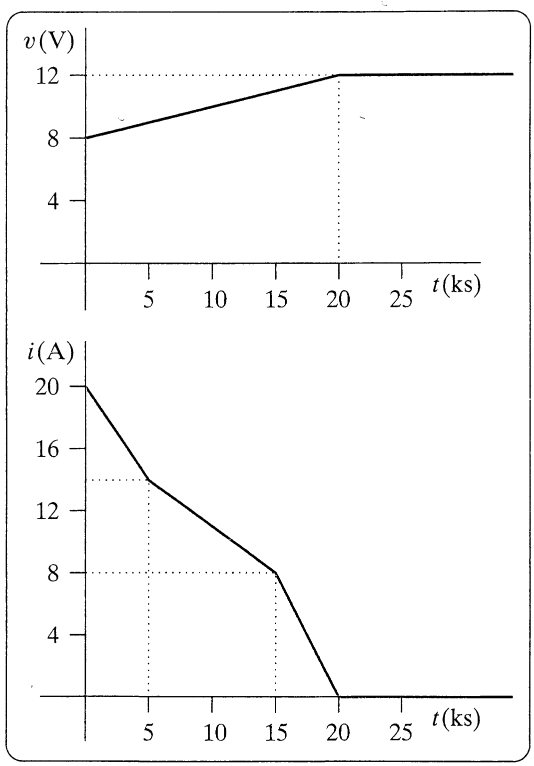
d) Find the average value of p in the interval $0 \le t \le 15$ ms.

1.22 The voltage and current at the terminals of an automobile battery during a charge cycle are shown in Fig. P1.22.

a) Calculate the total charge transferred to the battery.

b) Calculate the total energy transferred to the battery.

Figure P1.22



1.23 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = (10{,}000t + 5)e^{-400t}\ \text{V}.$$

$$i = (40t + 0.05)e^{-400t}\ \text{A}.$$

a) At what instant of time is maximum power delivered to the element?

b) Find the maximum power in watts.

c) Find the total energy delivered to the element in millijoules.

1.24 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$ and $t > 40$ s. In the interval between 0 and 40 s the expressions are

$$v = t(1 - 0.025t)\ \text{V}, \quad 0 < t < 40\ \text{s};$$

$$i = 4 - 0.2t\ \text{A}, \quad 0 < t < 40\ \text{s}.$$

a) At what instant of time is the power being delivered to the circuit element maximum?

b) What is the power at the time found in part (a)?

c) At what instant of time is the power being extracted from the circuit element maximum?

d) What is the power at the time found in part (c)?

e) Calculate the net energy delivered to the circuit at 0, 10, 20, 30, and 40 s.

1.25 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 80{,}000te^{-500t}\ \text{V}, \quad t \geq 0;$$

$$i = 15te^{-500t}\ \text{A}, \quad t \geq 0.$$

a) Find the time (in milliseconds) when the power delivered to the circuit element is maximum.

b) Find the maximum value of p in milliwatts.

c) Find the total energy delivered to the circuit element in microjoules.

1.26 The numerical values for the currents and voltages in the circuit in Fig. P1.26 are given in Table P1.26. Find the total power developed in the circuit.

Figure P1.26

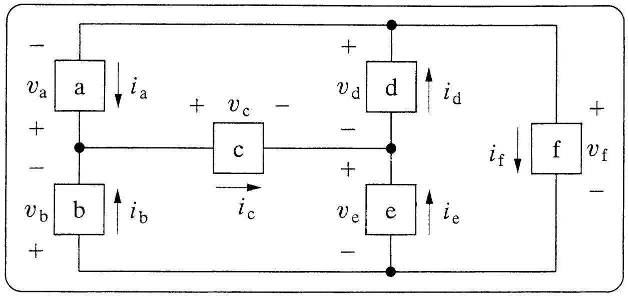


TABLE P1.26

ELEMENT	VOLTAGE (V)	CURRENT (A)
a	−18	−51
b	−18	45
c	2	−6
d	20	−20
e	16	−14
f	36	31

1.27 Assume you are an engineer in charge of a project and one of your subordinate engineers reports that the interconnection in Fig. P1.27 does not pass the power check. The data for the interconnection are given in Table P1.27.

a) Is the subordinate correct? Explain your answer.

b) If the subordinate is correct, can you find the error in the data?

Figure P1.27

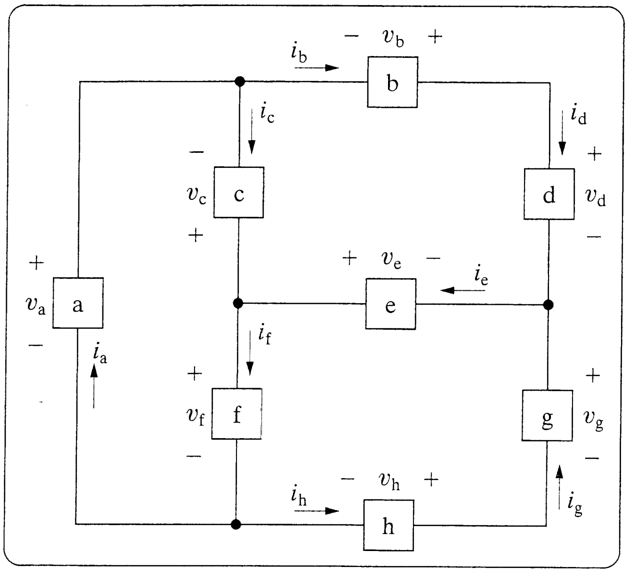


TABLE P1.27

ELEMENT	VOLTAGE (V)	CURRENT (A)
a	900	−22.5
b	105	−52.5
c	−600	−30.0
d	585	−52.5
e	−120	30.0
f	300	60.0
g	585	82.5
h	−165	82.5

1.28 The numerical values of the voltages and currents in the interconnection seen in Fig. P1.28 are given in Table P1.28. Does the interconnection satisfy the power check?

Figure P1.28

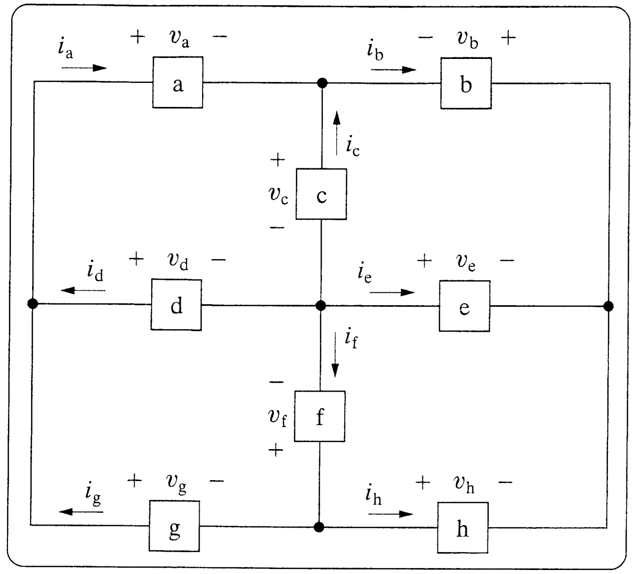


TABLE P1.28

ELEMENT	VOLTAGE (V)	CURRENT (A)
a	9	1.8
b	−15	1.5
c	45	−0.3
d	54	−2.7
e	−30	−1.0
f	−240	4.0
g	294	4.5
h	−270	−0.5

1.29 One method of checking calculations involving interconnected circuit elements is to see that the total power delivered equals the total power absorbed (conservation-of-energy principle). With this thought in mind, check the interconnection in Fig. P1.29 and state whether it satisfies this power check. The current and voltage values for each element are given in Table P1.29.

Figure P1.29

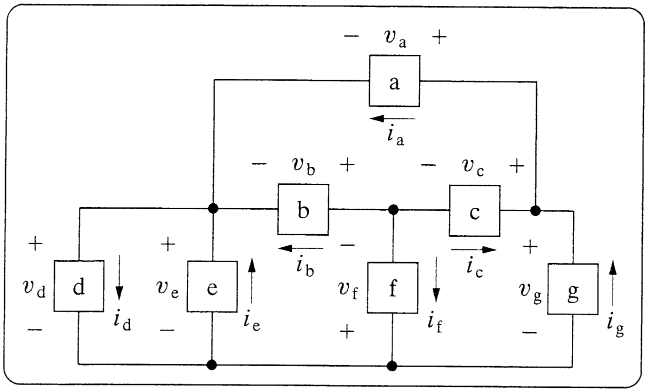


TABLE P1.29

ELEMENT	VOLTAGE (V)	CURRENT (A)
a	−160	−10
b	−100	−20
c	−60	6
d	800	−50
e	800	−20
f	−700	14
g	640	−16

1.30 a) In the circuit shown in Fig. P1.30, identify which elements have the voltage and current reference polarities defined using the passive sign convention.

b) The numerical values of the currents and voltages for each element are given in Table P1.30. How much total power is absorbed and how much is delivered in this circuit?

Figure P1.30

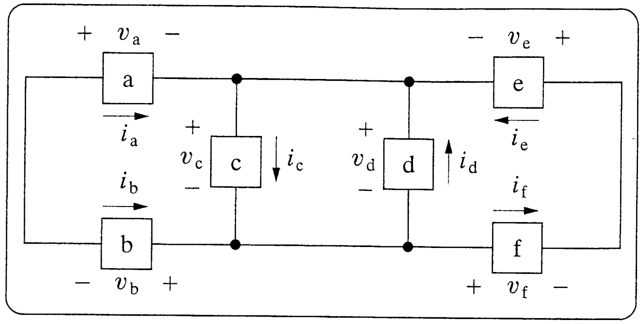


TABLE P1.30

ELEMENT	VOLTAGE (V)	CURRENT (A)
a	−8	7
b	−2	−7
c	10	15
d	10	5
e	−6	3
f	−4	3

CHAPTER

2 Circuit Elements

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Understand the symbols for and the behavior of the following ideal basic circuit elements: independent voltage and current sources, dependent voltage and current sources, and resistors.
2. Be able to state Ohm's law, Kirchhoff's current law, and Kirchhoff's voltage law, and be able to use these laws to analyze simple circuits.
3. Know how to calculate the power for each element in a simple circuit and be able to determine whether or not the power balances for the whole circuit.

There are five ideal basic circuit elements: voltage sources, current sources, resistors, inductors, and capacitors. In this chapter we discuss the characteristics of voltage sources, current sources, and resistors. Although this may seem like a small number of elements with which to begin analyzing circuits, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity; the mathematical relationships between voltage and current in sources and resistors are algebraic. Thus you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.

We will postpone introducing inductors and capacitors until Chapter 6, because their use requires that you solve integral and differential equations. However, the basic analytical techniques for solving circuits with inductors and capacitors are the same as those introduced in this chapter. So, by the time you need to begin manipulating more difficult equations, you should be very familiar with the methods of writing them.

ASSESSING OBJECTIVE 1

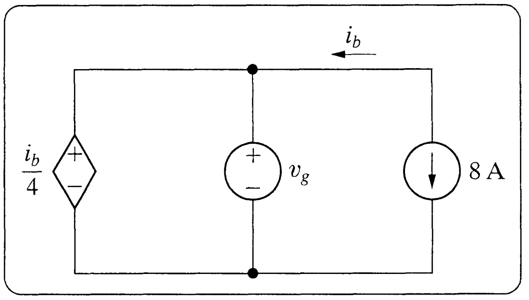
◆ **Understand ideal basic circuit elements**

2.1 For the circuit shown,

a) What value of v_g is required in order for the interconnection to be valid?

b) For this value of v_g, find the power associated with the 8 A source.

ANSWER: (a) −2 V; (b) −16 W (16 W delivered).

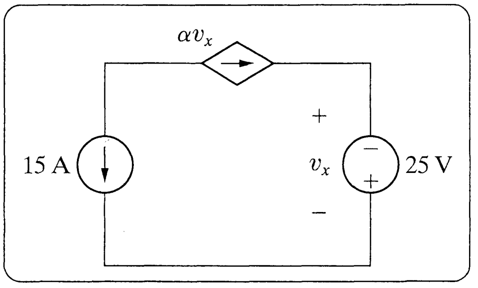


2.2 For the circuit shown,

a) What value of α is required in order for the interconnection to be valid?

b) For the value of α calculated in part (a), find the power associated with the 25 V source.

ANSWER: (a) 0.6; (b) 375 W (375 W absorbed).



NOTE ◆ *Also try Chapter Problems 2.6 and 2.7.*

2.2 ◆ Electrical Resistance (Ohm's Law)

Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element used to model this behavior is the **resistor**. Figure 2.5 shows the circuit symbol for the resistor, with R denoting the resistance value of the resistor.

Conceptually, we can understand resistance if we think about the moving electrons that make up electric current interacting with and being resisted by the atomic structure of the material through which they are moving. In the course of these interactions, some amount of electric energy is converted to thermal energy and dissipated in the form of heat. This effect may be undesirable. However, many useful electrical devices take advantage of resistance heating, including stoves, toasters, irons, and space heaters.

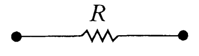


Figure 2.5 The circuit symbol for a resistor having a resistance R.

ASSESSING OBJECTIVE 2

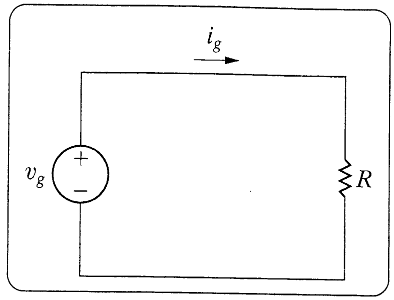
◆ **Be able to state and use Ohm's Law and Kirchhoff's current and voltage laws**

2.3 For the circuit shown,

a) If $v_g = 1$ kV and $i_g = 5$ mA, find the value of R and the power absorbed by the resistor.

b) If $i_g = 75$ mA and the power delivered by the voltage source is 3 W, find v_g, R, and the power absorbed by the resistor.

c) If $R = 300\ \Omega$ and the power absorbed by R is 480 mW, find i_g and v_g.



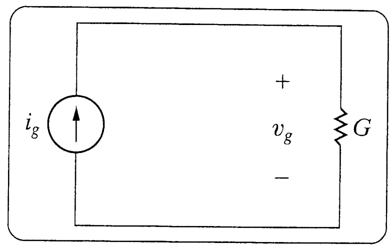
ANSWER: (a) 200 kΩ, 5 W; (b) 40 V, 533.33 Ω, 3 W; (c) 40 mA, 12 V.

2.4 For the circuit shown,

a) If $i_g = 0.5$ A and $G = 50$ mS, find v_g and the power delivered by the current source.

b) If $v_g = 15$ V and the power delivered to the conductor is 9 W, find the conductance G and the source current i_g.

c) If $G = 200\ \mu$S and the power delivered to the conductance is 8 W, find i_g and v_g.



ANSWER: (a) 10 V, 5 W; (b) 40 mS, 0.6 A; (c) 40 mA, 200 V.

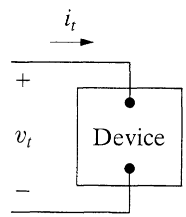
NOTE ◆ *Also try Chapter Problems 2.10 and 2.11.*

2.3 ◆ Construction of a Circuit Model

We have already stated that one reason for an interest in the basic circuit elements is that they can be used to construct circuit models of practical systems. The skill required to develop a circuit model of a device or system is as complex as the skill required to solve the derived circuit. Although this text emphasizes the skills required to solve circuits, you also will need other skills in the practice of electrical engineering, and one of the most important is modeling.

EXAMPLE ◆ 2.5 Constructing a Circuit Model Based on Terminal Measurements

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of v_t and i_t are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.



(a)

v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

(b)

Figure 2.13 The (a) device and (b) data for Example 2.5.

SOLUTION

Plotting the voltage as a function of the current yields the graph shown in Fig. 2.14(a). The equation of the line in this figure illustrates that the terminal voltage is directly proportional to the terminal current, $v_t = 4i_t$. In terms of Ohm's law, the device inside the box behaves like a 4 Ω resistor. Therefore, the circuit model for the device inside the box is a 4 Ω resistor, as seen in Fig. 2.14(b).

We come back to this technique of using terminal characteristics to construct a circuit model after introducing Kirchhoff's laws and circuit analysis.

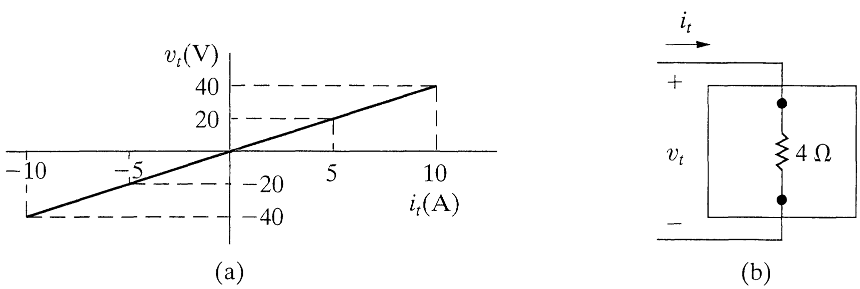


Figure 2.14 (a) The values of v_t versus i_t for the device in Fig. 2.13. (b) The circuit model for the device in Fig. 2.13.

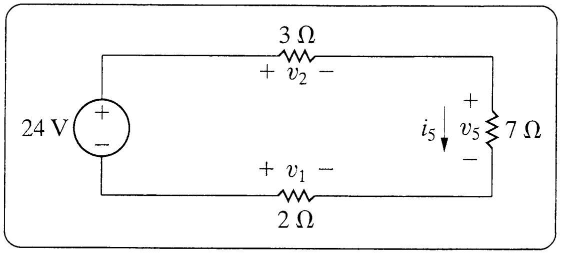
NOTE ◆ *Assess your understanding of this example by trying Chapter Problems 2.2 and 2.4.*

ASSESSING OBJECTIVE 2

◆ **Be able to state and use Ohm's law and Kirchhoff's current and voltage laws**

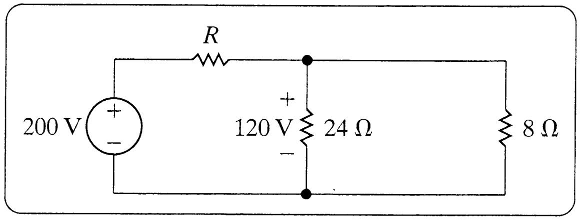
2.5 For the circuit shown, calculate (a) i_5; (b) v_1; (c) v_2; (d) v_5; and (e) the power delivered by the 24 V source.

ANSWER: (a) 2 A; (b) −4 V; (c) 6 V; (d) 14 V; (e) 48 W.



2.6 Use Ohm's law and Kirchhoff's laws to find the value of R in the circuit shown.

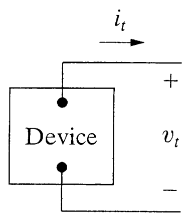
ANSWER: $R = 4\ \Omega$.



2.7

a) The terminal voltage and terminal current were measured on the device shown. The values of v_t and i_t are provided in the table. Using these values, create the straight line plot of v_t versus i_t. Compute the equation of the line and use the equation to construct a circuit model for the device using an ideal voltage source and a resistor.

b) Use the model constructed in (a) to predict the power that the device will deliver to a 25 Ω resistor.



(a)

v_t (V)	i_t (A)
25	0
15	0.1
5	0.2
0	0.25

(b)

ANSWER: (a) A 25 V source in series with a 100 Ω resistor; (b) 1 W.

2.8 Repeat Assessment Problem 2.7 but use the equation of the graphed line to construct a circuit model containing an ideal current source and a resistor.

ANSWER: (a) A 0.25 A current source connected between the terminals of a 100 Ω resistor; (b) 1 W.

NOTE ◆ *Also try Chapter Problems 2.14, 2.16, 2.21, and 2.23.*

- Solve the transformed Eq. (5) for i_E, and substitute this solution for i_E into Eq. (3). Use Eq. (4) to eliminate i_C in Eq. (3).
- Solve the transformed Eq. (3) for i_B, and rearrange the terms to yield

$$i_B = \frac{(V_{CC}R_2)/(R_1 + R_2) - V_0}{(R_1R_2)/(R_1 + R_2) + (1 + \beta)R_E}. \quad (2.25)$$

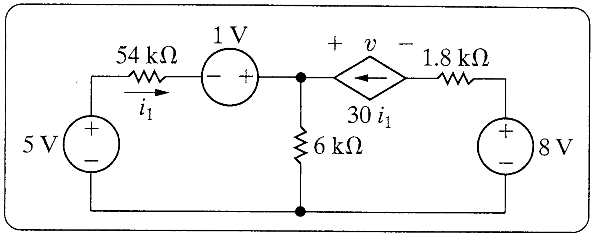
Problem 2.27 asks you to verify these steps. Note that once we know i_B, we can easily obtain the remaining currents.

ASSESSING OBJECTIVE 3

- **Know how to calculate power for each element in a simple circuit**

2.9 For the circuit shown find (a) the current i_1 in microamperes, (b) the voltage v in volts, (c) the total power generated, and (d) the total power absorbed.

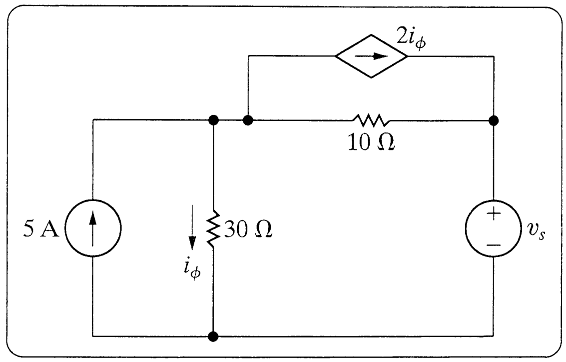
ANSWER: (a) 25 μA; (b) −2 V; (c) 6150 μW; (d) 6150 μW.



2.10 The current i_ϕ in the circuit shown is 2 A. Calculate

a) v_s,
b) the power absorbed by the independent voltage source,
c) the power delivered by the independent current source,
d) the power delivered by the controlled current source,
e) the total power dissipated in the two resistors.

ANSWER: (a) 70 V; (b) 210 W; (c) 300 W; (d) 40 W; (e) 130 W.



NOTE ◆ *Also try Chapter Problems 2.26 and 2.29.*

Practical Perspective

Electrical Safety

At the beginning of this chapter, we said that current through the body can cause injury. Let's examine this aspect of electrical safety.

You might think that electrical injury is due to burns. However, that is not the case. The most common electrical injury is to the nervous system. Nerves use electrochemical signals, and electric currents can disrupt those signals. When the current path includes only skeletal muscles, the effects can include temporary paralysis (cessation of nervous signals) or involuntary muscle contractions, which are generally not life threatening. However, when the current path includes nerves and muscles that control the supply of oxygen to the brain, the problem is much more serious. Temporary paralysis of these muscles can stop a person from breathing, and a sudden muscle contraction can disrupt the signals that regulate heartbeat. The result is a halt in the flow of oxygenated blood to the brain, causing death in a few minutes unless emergency aid is given immediately. Table 2.1 shows a range of physiological reactions to various current levels. The numbers in this table are approximate; they are obtained from an analysis of accidents because, obviously, it is not ethical to perform electrical experiments on people. Good electrical design will limit current to a few milliamperes or less under all possible conditions.

TABLE 2.1 Physiological Reactions to Current Levels in Humans

PHYSIOLOGICAL REACTION	CURRENT
Barely perceptible	3–5 mA
Extreme pain	35–50 mA
Muscle paralysis	50–70 mA
Heart stoppage	500 mA

Note: Data taken from W. F. Cooper, *Electrical Safety Engineering*, 2d ed. (London: Butterworth, 1986); and C. D. Winburn, *Practical Electrical Safety* (Monticello, N.Y.: Marcel Dekker, 1988).

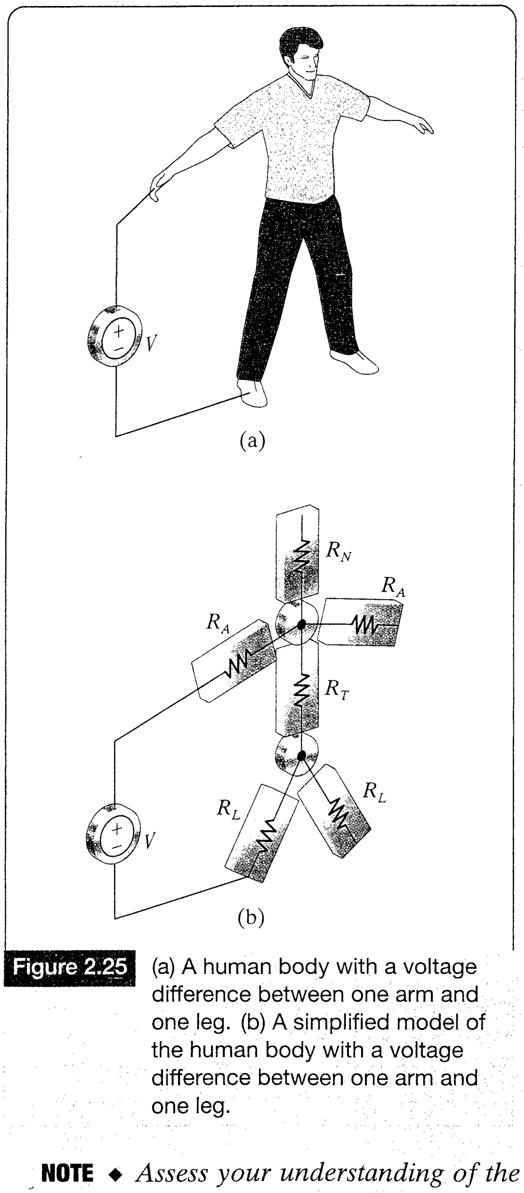


Figure 2.25 (a) A human body with a voltage difference between one arm and one leg. (b) A simplified model of the human body with a voltage difference between one arm and one leg.

Now we develop a simplified electrical model of the human body. The body acts as a conductor of current, so a reasonable starting point is to model the body using resistors. Figure 2.25 shows a potentially dangerous situation. A voltage difference exists between one arm and one leg of a human being. Figure 2.25(b) shows an electrical model of the human body in Fig. 2.25(a). The arms, legs, neck, and trunk (chest and abdomen) each have a characteristic resistance. Note that the path of the current is through the trunk, which contains the heart, a potentially deadly arrangement.

NOTE ◆ *Assess your understanding of the Practical Perspective by solving Chapter Problems 2.34–2.38.*

SUMMARY

- The circuit elements introduced in this chapter are voltage sources, current sources, and resistors:
  - An **ideal voltage source** maintains a prescribed voltage regardless of the current in the device. An **ideal current source** maintains a prescribed current regardless of the voltage across the device. Voltage and current sources are either **independent**, that is, not influenced by any other current or voltage in the circuit; or **dependent**, that is, determined by some other current or voltage in the circuit. (See pages 28 and 29.)
  - A **resistor** constrains its voltage and current to be proportional to each other. The value of the proportional constant relating voltage and current in a resistor is called its **resistance** and is measured in ohms. (See page 32.)
- **Ohm's law** establishes the proportionality of voltage and current in a resistor. Specifically,

$$v = iR \quad (2.26)$$

if the current flow in the resistor is in the direction of the voltage drop across it, or

$$v = -iR \quad (2.27)$$

if the current flow in the resistor is in the direction of the voltage rise across it. (See page 33.)

- By combining the equation for power, $p = vi$, with Ohm's law, we can determine the power absorbed by a resistor:

$$p = i^2R = v^2/R. \quad (2.28)$$

(See page 35.)

- Circuits are described by nodes and closed paths. A **node** is a point where two or more circuit elements join. When just two elements connect to form a node, they are said to be **in series**. A **closed path** is a loop traced through connecting elements, starting and ending at the same node and encountering intermediate nodes only once each. (See pages 42–44.)
- The voltages and currents of interconnected circuit elements obey Kirchhoff's laws:
  - **Kirchhoff's current law** states that the algebraic sum of all the currents at any node in a circuit equals zero. (See page 42.)
  - **Kirchhoff's voltage law** states that the algebraic sum of all the voltages around any closed path in a circuit equals zero. (See page 43.)
- A circuit is solved when the voltage across and the current in every element have been determined. By combining an understanding of independent and dependent sources, Ohm's law, and Kirchhoff's laws, we can solve many simple circuits.

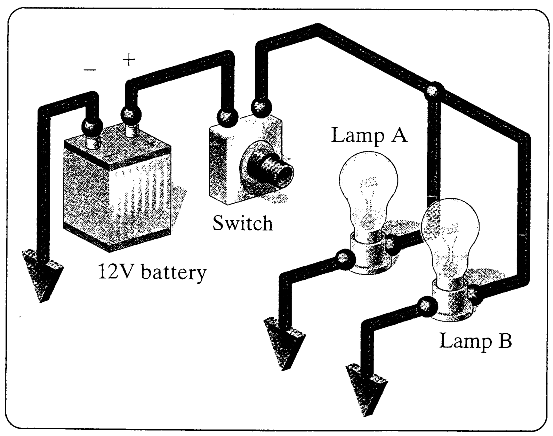
PROBLEMS

2.1 A pair of automotive headlamps is connected to a 12 V battery via the arrangements shown in Fig. P2.1. In the figure, the triangular symbol ▼ is used to indicate that the terminal is connected directly to the metal frame of the car.

a) Construct a circuit model using resistors and an independent voltage source.

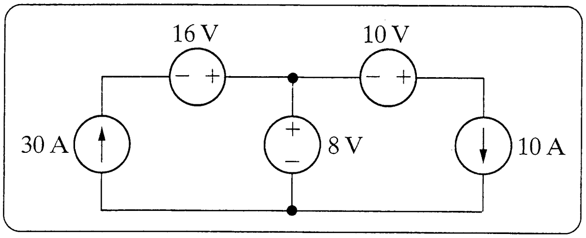
b) Identify the correspondence between the ideal circuit element and the symbol component that it represents.

Figure P2.1



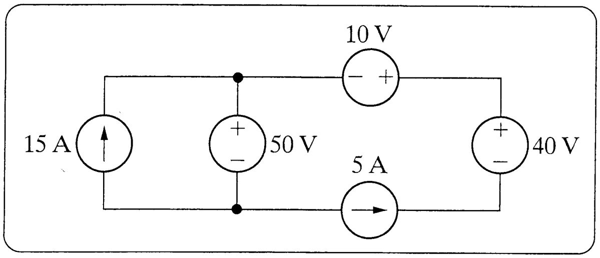
2.7 If the interconnection in Fig. P2.7 is valid, find the power developed by the current sources. If the interconnection is not valid, explain why.

Figure P2.7



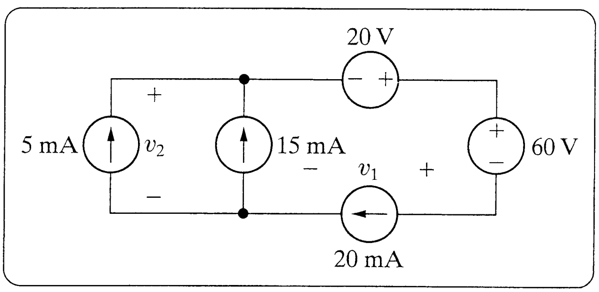
2.8 If the interconnection in Fig. P2.8 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.8



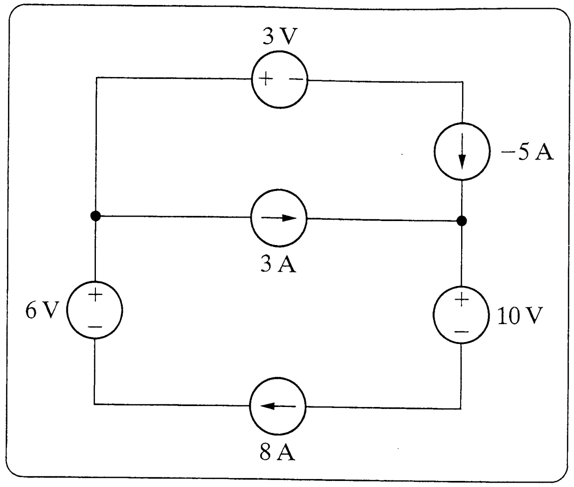
2.9 The interconnection of ideal sources can lead to an indeterminate solution. With this thought in mind, explain why the solutions for v_1 and v_2 in the circuit in Fig. P2.9 are not unique.

Figure P2.9



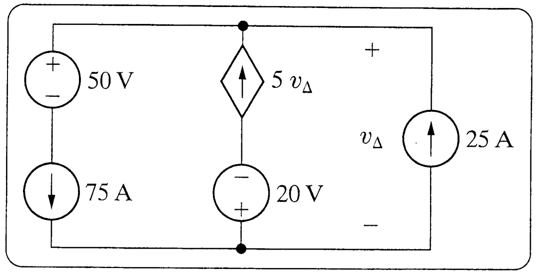
2.10 If the interconnection in Fig. P2.10 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.10



2.11 If the interconnection in Fig. P2.11 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

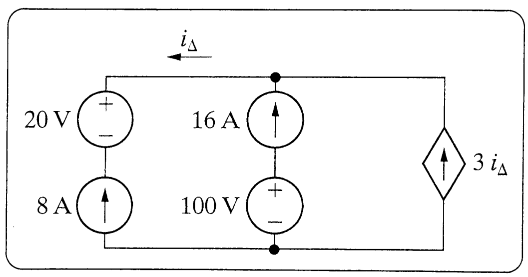
Figure P2.11



2.12 a) Is the interconnection in Fig. P2.12 valid? Explain.

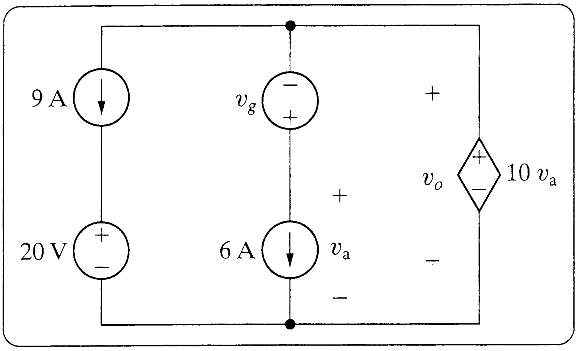
b) Can you find the total energy developed in the circuit? Explain.

Figure P2.12



2.13 Find the total power developed in the circuit in Fig. P2.13 if $v_o = 5$ V.

Figure P2.13



2.14 P Given the circuit shown in Fig. P2.14, find

a) the value of i_a,

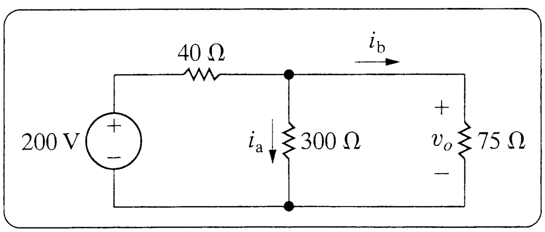
b) the value of i_b,

c) the value of v_o,

d) the power dissipated in each resistor,

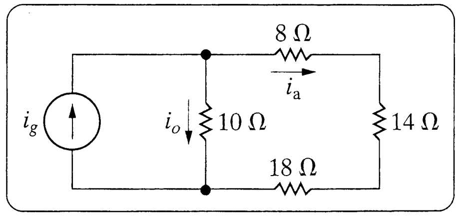
e) the power delivered by the 200 V source.

Figure P2.14



2.15 P The current i_a in the circuit shown in Fig. P2.15 is 20 A. Find (a) i_o; (b) i_g; and (c) the power delivered by the independent current source.

Figure P2.15

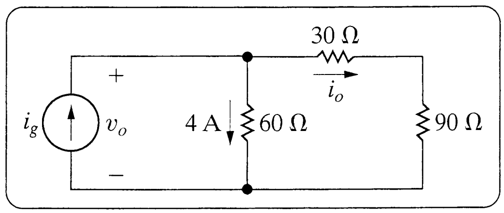


2.16 P a) Find the currents i_g and i_o in the circuit in Fig. P2.16.

b) Find the voltage v_o.

c) Verify that the total power developed equals the total power dissipated.

Figure P2.16



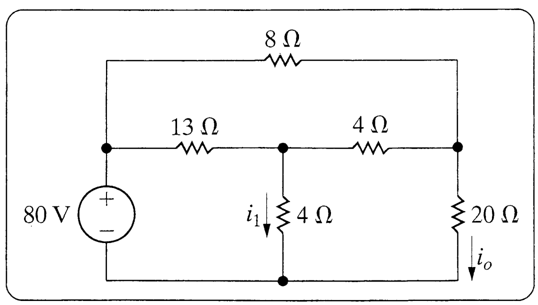
2.17 P The current i_o in the circuit in Fig. P2.17 is 2 A.

a) Find i_1.

b) Find the power dissipated in each resistor.

c) Verify that the total power dissipated in the circuit equals the power developed by the 80 V source.

Figure P2.17

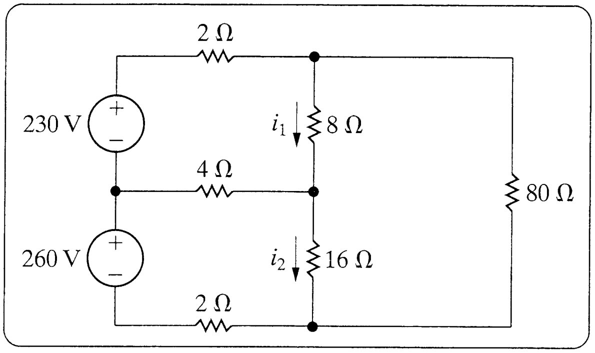


2.18 The currents i_1 and i_2 in the circuit in Fig. P2.18 are 20 A and 15 A, respectively.

a) Find the power supplied by each voltage source.

b) Show that the total power supplied equals the total power dissipated in the resistors.

Figure P2.18



2.19 The currents i_a and i_b in the circuit in Fig. P2.19 are 4 A and −2 A, respectively.

P

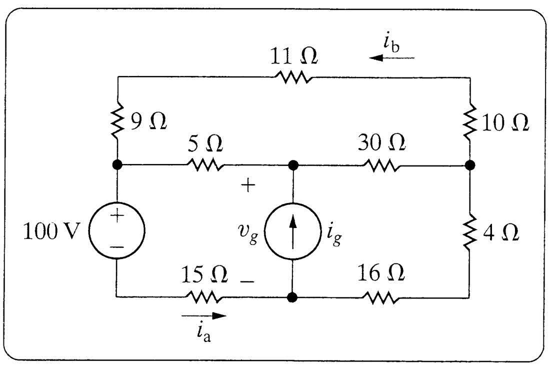
a) Find i_g.

b) Find the power dissipated in each resistor.

c) Find v_g.

d) Show that the power delivered by the current source is equal to the power absorbed by all the other elements.

Figure P2.19

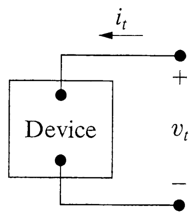


2.20 The voltage and current were measured at the terminals of the device shown in Fig. P2.20(a). The results are tabulated in Fig. P2.20(b).

a) Construct a circuit model for this device using an ideal current source and a resistor.

b) Use the model to predict the value of i_t when a 20 Ω resistor is connected across the terminals of the device.

Figure P2.20



(a)

v_t (V)	i_t (A)
50	0
65	3
80	6
95	9
110	12
125	15

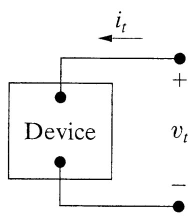
(b)

2.21 The voltage and current were measured at the terminals of the device shown in Fig. P2.21(a). The results are tabulated in Fig. P2.21(b).

a) Construct a circuit model for this device using an ideal voltage source and a resistor.

b) Use the model to predict the amount of power the device will deliver to a 40 Ω resistor.

Figure P2.21



(a)

v_t (V)	i_t (A)
−30	0
10	2
50	4
90	6
130	8

(b)

2.22 The table in Fig. P2.22(a) gives the relationship between the terminal current and voltage of the practical constant current source shown in Fig. P2.22(b).

a) Plot i_s versus v_s.

b) Construct a circuit model of this current source that is valid for $0 \leq v_s \leq 75$ V, based on the equation of the line plotted in (a).

c) Use your circuit model to predict the current delivered to a 2.5 kΩ resistor.

d) Use your circuit model to predict the open-circuit voltage of the current source.

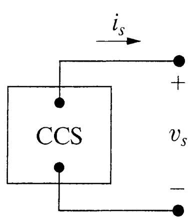
e) What is the actual open-circuit voltage?

f) Explain why the answers to (d) and (e) are not the same.

Figure P2.22

i_s (mA)	v_s (V)
20.0	0
17.5	25
15.0	50
12.5	75
9.0	100
4.0	125
0.0	140

(a)



(b)

2.23 The table in Fig. P2.23(a) gives the relationship between the terminal voltage and current of the practical constant voltage source shown in Fig. P2.23(b).

a) Plot v_s versus i_s.

b) Construct a circuit model of the practical source that is valid for $0 \leq i_s \leq 24$ A, based on the equation of the line plotted in (a). (Use an ideal voltage source in series with an ideal resistor.)

c) Use your circuit model to predict the current delivered to a 1 Ω resistor connected to the terminals of the practical source.

d) Use your circuit model to predict the current delivered to a short circuit connected to the terminals of the practical source.

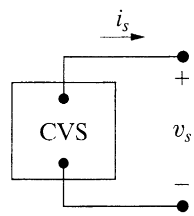
e) What is the actual short-circuit current?

f) Explain why the answers to (d) and (e) are not the same.

Figure P2.23

v_s (V)	i_s (A)
24	0
22	8
20	16
18	24
15	32
10	40
0	48

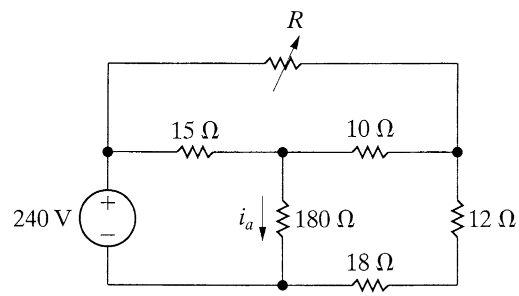
(a)



(b)

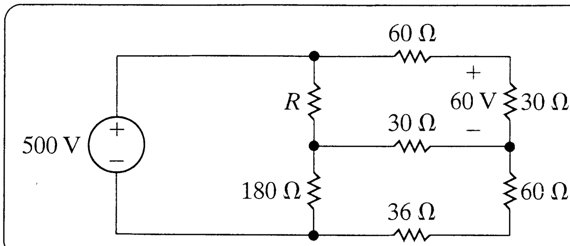
2.24 **P** The variable resistor R in the circuit in Fig. P2.24 is adjusted until i_a equals 1 A. Find the value of R.

Figure P2.24



2.25 **P** For the circuit shown in Fig. P2.25, find (a) R and (b) the power supplied by the 500 V source.

Figure P2.25



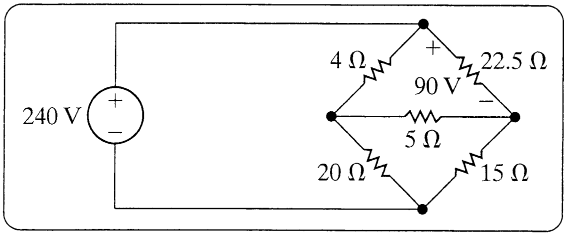
2.26 The voltage across the 22.5 Ω resistor in the circuit in Fig. P2.26 is 90 V, positive at the upper terminal.

a) Find the power dissipated in each resistor.

b) Find the power supplied by the 240 V ideal voltage source.

c) Verify that the power supplied equals the total power dissipated.

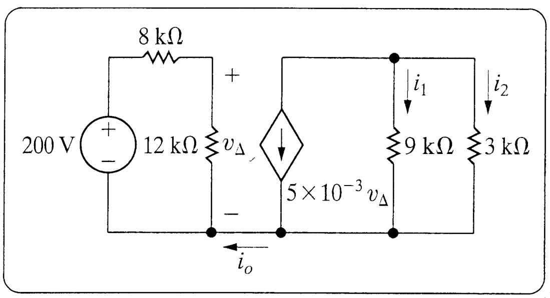
Figure P2.26



2.27 Derive Eq. 2.25. *Hint:* Use Eqs. (3) and (4) from Example 2.11 to express i_E as a function of i_B. Solve Eq. (2) for i_2 and substitute the result into both Eqs. (5) and (6). Solve the "new" Eq. (6) for i_1 and substitute this result into the "new" Eq. (5). Replace i_E in the "new" Eq. (5) and solve for i_B. Note that because i_{CC} appears only in Eq. (1), the solution for i_B involves the manipulation of only five equations.

2.28 Find (a) i_o, (b) i_1, and (c) i_2 in the circuit in Fig. P2.28.

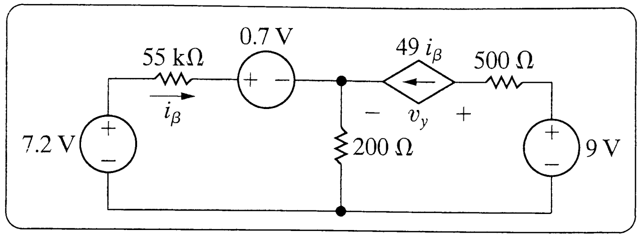
Figure P2.28



2.29 a) Find the voltage v_y in the circuit in Fig. P2.29.

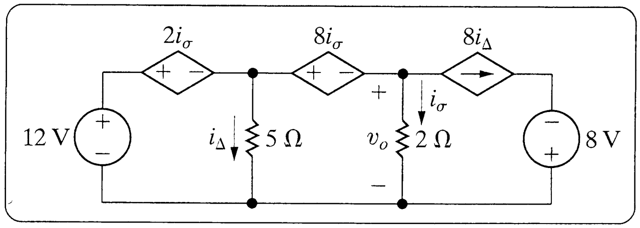
b) Show that the total power generated in the circuit equals the total power absorbed.

Figure P2.29



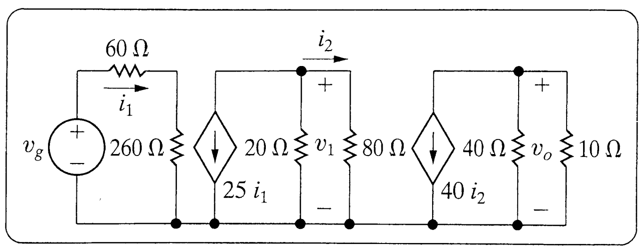
2.30 For the circuit shown in Fig. P2.30, calculate (a) i_Δ and v_o and (b) show that the power developed equals the power absorbed.

Figure P2.30



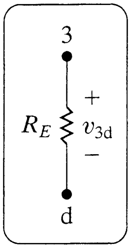
2.31 Find v_1 and v_g in the circuit shown in Fig. P2.31 when v_o equals 5 V. (*Hint:* Start at the right end of the circuit and work back toward v_g.)

Figure P2.31



2.32 For the circuit shown in Fig. 2.24, $R_1 = 40$ kΩ, $R_2 = 60$ kΩ, $R_C = 750$ Ω, $R_E = 120$ Ω, $V_{CC} = 10$ V, $V_0 = 600$ mV, and $\beta = 49$. Calculate i_B, i_C, i_E, v_{3d}, v_{bd}, i_2, i_1, v_{ab}, i_{CC}, and v_{13}. (*Note:* In the double subscript notation on voltage variables, the first subscript is positive with respect to the second subscript. See Fig. P2.32.)

Figure P2.32

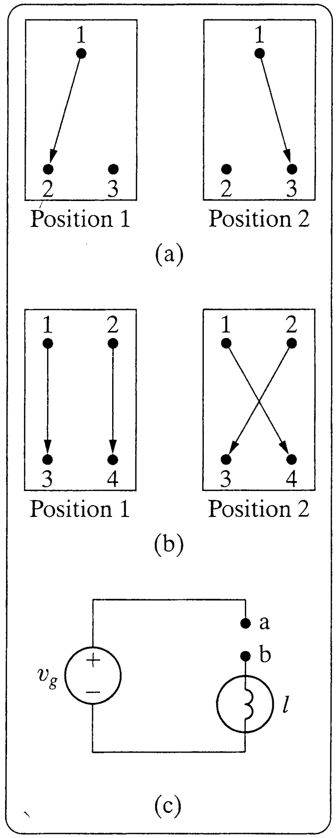


2.33 It is often desirable in designing an electric wiring system to be able to control a single appliance from two or more locations, for example, to control a lighting fixture from both the top and bottom of a stairwell. In home wiring systems, this type of control is implemented with three-way and four-way switches. A three-way switch is a three-terminal, two-position switch, and a four-way switch is a four-terminal, two-position switch. The switches are shown schematically in Fig. P2.33(a), which illustrates a three-way switch, and P2.33(b), which illustrates a four-way switch.

a) Show how two three-way switches can be connected between a and b in the circuit in Fig. P2.33(c) so that the lamp l can be turned ON or OFF from two locations.

b) If the lamp (appliance) is to be controlled from more than two locations, four-way switches are used in conjunction with two three-way switches. One four-way switch is required for each location in excess of two. Show how one four-way switch plus two three-way switches can be connected between a and b in Fig. P2.33(c) to control the lamp from three locations. (*Hint:* The four-way switch is placed between the three-way switches.)

Figure P2.33



2.34 Suppose the power company installs some equipment that could provide a 250 V shock to a human being. Is the current that results dangerous enough to warrant posting a warning sign and taking other precautions to prevent such a shock? Assume that if the source is 250 V, the resistance of the arm is 400 Ω, the resistance of the trunk is 50 Ω, and the resistance of the leg is 200 Ω. Use the model given in Fig. 2.25(b).

2.35 Based on the model and circuit shown in Fig. 2.25, draw a circuit model of the path of current through the human body for a person touching a voltage source with both hands who has both feet at the same potential as the negative terminal of the voltage source.

2.36 ◆

a) Using the values of resistance for arm, leg, and trunk provided in Problem 2.34, calculate the power dissipated in the arm, leg, and trunk.

b) The specific heat of water is 4.18×10^3 J/kg°C, so a mass of water M (in kilograms) heated by a power P (in watts) undergoes a rise in temperature at a rate given by

$$\frac{dT}{dt} = \frac{2.39 \times 10^{-4} P}{M} \,^{\circ}\text{C/s}.$$

Assuming that the mass of an arm is 4 kg, the mass of a leg is 10 kg, and the mass of a trunk is 25 kg, and that the human body is mostly water, how many seconds does it take the arm, leg, and trunk to rise the 5°C that endangers living tissue?

c) How do the values you computed in (b) compare with the few minutes it takes for oxygen starvation to injure the brain?

2.37 ◆ A person accidently grabs conductors connected to each end of a dc voltage source, one in each hand.

a) Using the resistance values for the human body provided in Problem 2.34, what is the minimum source voltage that can produce electrical shock sufficient to cause paralysis, preventing the person from letting go of the conductors?

b) Is there a significant risk of this type of accident occurring while servicing a personal computer, which typically has 5 V and 12 V sources?

2.38 ◆ To understand why the voltage level is not the sole determinant of potential injury due to electrical shock, consider the case of a static electricity shock mentioned in the Practical Perspective at the start of this chapter. When you shuffle your feet across a carpet, your body becomes charged. The effect of this charge is that your entire body represents a voltage potential. When you touch a metal doorknob, a voltage difference is created between you and the doorknob, and current flows—but the conduction material is air, not your body!

Suppose the model of the space between your hand and the doorknob is a 1 MΩ resistance. What voltage potential exists between your hand and the doorknob if the current causing the mild shock is 3 mA?

CHAPTER

3 Simple Resistive Circuits

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Be able to recognize resistors connected in series and in parallel and use the rules for combining series-connected resistors and parallel-connected resistors to yield equivalent resistance.
2. Know how to design simple voltage-divider and current-divider circuits.
3. Be able to use voltage division and current division appropriately to solve simple circuits.
4. Be able to determine the reading of an ammeter when added to a circuit to measure current; be able to determine the reading of a voltmeter when added to a circuit to measure voltage.
5. Understand how a Wheatstone bridge is used to measure resistance.
6. Know when and how to use delta-to-wye equivalent circuits to solve simple circuits.

Our analytical toolbox now contains Ohm's law and Kirchhoff's laws. In Chapter 2 we used these tools in solving simple circuits. In this chapter we continue applying these tools, but on more-complex circuits. The greater complexity lies in a greater number of elements with more complicated interconnections. This chapter focuses on reducing such circuits into simpler, equivalent circuits. We continue to focus on relatively simple circuits for two reasons: (1) It gives us a chance to acquaint ourselves thoroughly with the laws underlying more sophisticated methods, and (2) it allows us to be introduced to some circuits that have important engineering applications.

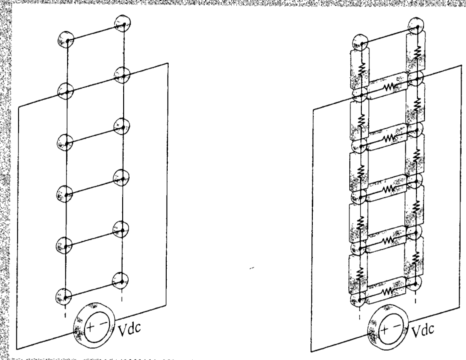
The sources in the circuits discussed in this chapter are limited to voltage and current sources that generate either constant voltages or currents; that is, voltages and currents that are invariant with time. Constant sources are often called **dc sources**. The *dc* stands for *direct current*, a description that has a historical basis but can seem misleading now. Historically, a direct current was defined as a current produced by a constant voltage. Therefore, a constant voltage became known as a direct current, or dc, voltage. The use of *dc* for *constant* stuck, and the terms *dc current* and *dc voltage* are now universally accepted in science and engineering to mean constant current and constant voltage.

Practical Perspective

A Rear Window Defroster

The rear window defroster grid on an automobile is an example of a resistive circuit that performs a useful function. One such grid structure is shown in (a) of the figure here. The grid conductors can be modeled with resistors, as shown in (b) of the figure. The number of horizontal conductors varies with the make and model of the car but typically ranges from 9 to 16.

How does this grid work to defrost the rear window? How are the properties of the grid determined? We will answer these questions in the Practical Perspective at the end of this chapter. The circuit analysis required to answer these questions arises from the goal of having uniform defrosting in both the horizontal and vertical directions.

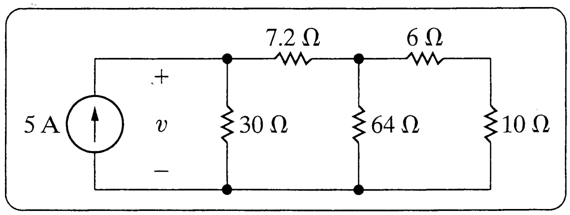


Before leaving Example 3.1, we suggest that you take the time to show that the solution satisfies Kirchhoff's current law at every node and Kirchhoff's voltage law around every closed path. (Note that there are three closed paths that can be tested.) Showing that the power delivered by the voltage source equals the total power dissipated in the resistors also is informative. (See Problems 3.3 and 3.4.)

ASSESSING OBJECTIVE 1

◆ **Be able to recognize resistors connected in series and in parallel**

3.1 For the circuit shown, find (a) the voltage v, (b) the power delivered to the circuit by the current source, and (c) the power dissipated in the 10 Ω resistor.



ANSWER: (a) 60 V; (b) 300 W; (c) 57.6 W.

NOTE ◆ *Also try Chapter Problems 3.1, 3.2, 3.5, and 3.6.*

3.3 ◆ The Voltage-Divider and Current-Divider Circuits

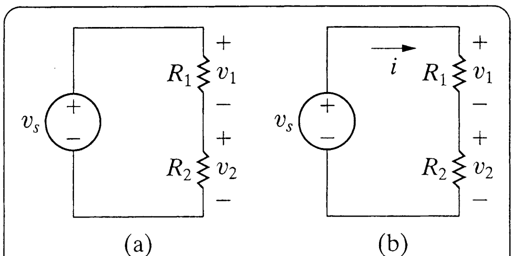


Figure 3.12 (a) A voltage-divider circuit and (b) the voltage-divider circuit with current i indicated.

At times—especially in electronic circuits—developing more than one voltage level from a single voltage supply is necessary. One way of doing this is by using a **voltage-divider circuit**, such as the one in Fig. 3.12.

We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis, we introduce the current i as shown in Fig. 3.12(b). From Kirchhoff's current law, R_1 and R_2 carry the same current. Applying Kirchhoff's voltage law around the closed loop yields

$$v_s = iR_1 + iR_2, \tag{3.19}$$

or

$$i = \frac{v_s}{R_1 + R_2}. \tag{3.20}$$

Now we can use Ohm's law to calculate v_1 and v_2:

$$v_1 = iR_1 = v_s \frac{R_1}{R_1 + R_2}, \tag{3.21}$$

$$v_2 = iR_2 = v_s \frac{R_2}{R_1 + R_2}. \tag{3.22}$$

Equations 3.21 and 3.22 show that v_1 and v_2 are fractions of v_s. Each fraction is the ratio of the resistance across which the divided voltage is

ASSESSING OBJECTIVE 2

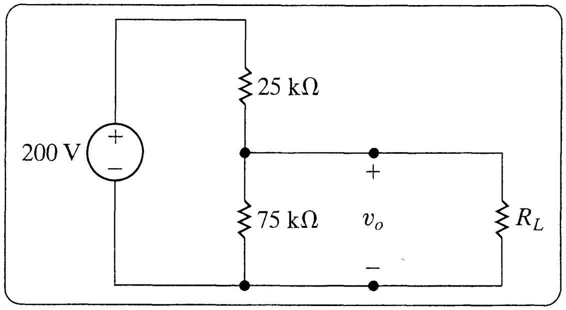
◆ **Know how to design simple voltage-divider and current-divider circuits**

3.2 a) Find the no-load value of v_o in the circuit shown.

b) Find v_o when R_L is 150 kΩ.

c) How much power is dissipated in the 25 kΩ resistor if the load terminals are accidentally short-circuited?

d) What is the maximum power dissipated in the 75 kΩ resistor?

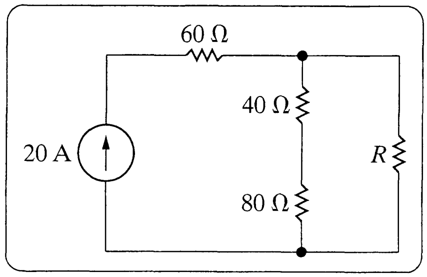


ANSWER: (a) 150 V; (b) 133.33 V; (c) 1.6 W; (d) 0.3 W.

3.3 a) Find the value of R that will cause 4 A of current to flow through the 80 Ω resistor in the circuit shown.

b) How much power will the resistor R from part (a) need to dissipate?

c) How much power will the current source generate for the value of R from part (a)?



ANSWER: (a) 30 Ω; (b) 7680 W; (c) 33,600 W

NOTE ◆ *Also try Chapter Problems 3.13, 3.14, and 3.21.*

3.4 ◆ Voltage Division and Current Division

We can now generalize the results from analyzing the voltage divider circuit in Fig. 3.12 and the current-divider circuit in Fig. 3.15. The generalizations will yield two additional and very useful circuit analysis techniques known

EXAMPLE ◆ 3.4 Using Voltage Division and Current Division to Solve a Circuit

Use current division to find the current i_o and use voltage division to find the voltage v_o for the circuit in Fig. 3.20.

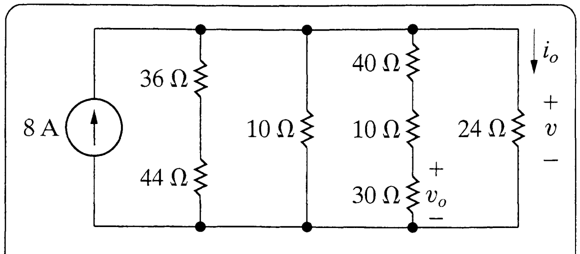


Figure 3.20 The circuit for Example 3.4.

SOLUTION

We can use Eq. 3.32 if we can find the equivalent resistance of the four parallel branches containing resistors. Symbolically,

$$R_{eq} = (36 + 44)\|10\|(40 + 10 + 30)\|24$$

$$= 80\|10\|80\|24 = \frac{1}{\frac{1}{80} + \frac{1}{10} + \frac{1}{80} + \frac{1}{24}} = 6\ \Omega.$$

Applying Eq. 3.32,

$$i_o = \frac{6}{24}(8\ \text{A}) = 2\ \text{A}.$$

We can use Ohm's law to find the voltage drop across the 24 Ω resistor:

$$v = (24)(2) = 48\ \text{V}.$$

This is also the voltage drop across the branch containing the 40 Ω, the 10 Ω, and the 3 Ω resistors in series. We can then use voltage division to determine the voltage drop v_o across the 30 Ω resistor given that we know the voltage drop across the series-connected resistors, using Eq. 3.30. To do this, we recognize that the equivalent resistance of the series-connected resistors is $40 + 10 + 30 = 80\ \Omega$:

$$v_o = \frac{30}{80}(48\ \text{V}) = 18\ \text{V}.$$

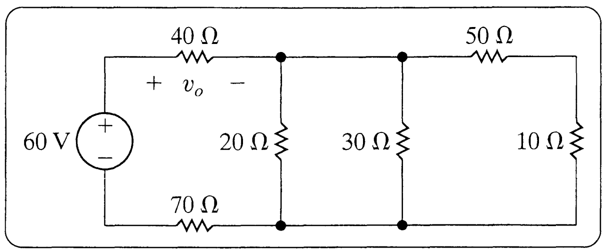
ASSESSING OBJECTIVE 3

◆ **Be able to use voltage and current division to solve simple circuits**

3.4 a) Use voltage division to determine the voltage v_o across the 40 Ω resistor in the circuit shown.

b) Use v_o from part (a) to determine the current through the 40 Ω resistor, and use this current and current division to calculate the current in the 30 Ω resistor.

c) How much power is absorbed by the 50 Ω resistor?



ANSWER: (a) 20 V; (b) 166.67 mA; (c) 347.22 mW.

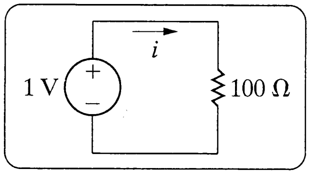
NOTE ◆ *Also try Chapter Problems 3.22 and 3.23.*

ASSESSING OBJECTIVE 4

◆ **Be able to determine the reading of ammeters and voltmeters**

3.5 a) Find the current in the circuit shown.

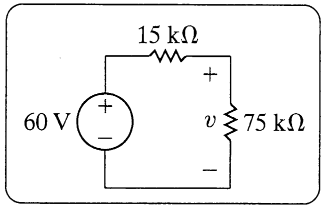
b) If the ammeter in Example 3.5(a) is used to measure the current, what will it read?



ANSWER: (a) 10 mA; (b) 9.474 mA.

3.6 a) Find the voltage v across the 75 kΩ resistor in the circuit shown.

b) If the 150 V voltmeter of Example 3.6(a) is used to measure the voltage, what will be the reading?



ANSWER: (a) 50 V; (b) 46.15 V.

NOTE ◆ *Also try Chapter Problems 3.31 and 3.34.*

3.6 ◆ Measuring Resistance — The Wheatstone Bridge

Many different circuit configurations are used to measure resistance. Here we will focus on just one, the Wheatstone bridge. The Wheatstone bridge circuit is used to precisely measure resistances of medium values, that is, in the range of 1 Ω to 1 MΩ. In commercial models of the Wheatstone bridge, accuracies on the order of ±0.1% are possible. The bridge circuit consists of four resistors, a dc voltage source, and a detector. The resistance of one of the four resistors can be varied, which is indicated in Fig. 3.26 by the arrow through R_3. The dc voltage source is usually a battery, which is indicated by the battery symbol for the voltage source v in Fig. 3.26. The detector is generally a d'Arsonval movement in the microamp range and is called a galvanometer. Figure 3.26 shows the circuit arrangement of the resistances, battery, and detector where R_1, R_2, and R_3 are known resistors and R_x is the unknown resistor.

To find the value of R_x, we adjust the variable resistor R_3 until there is no current in the galvanometer. We then calculate the unknown resistor from the simple expression

$$R_x = \frac{R_2}{R_1} R_3. \quad (3.33)$$

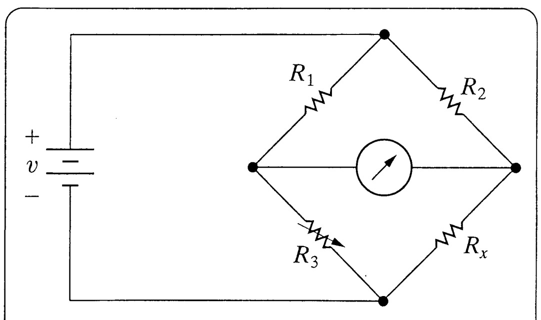


Figure 3.26 The Wheatstone bridge circuit.

because of thermal heating effects—that is, i^2R effects. Higher resistances are difficult to measure accurately because of leakage currents. In other words, if R_x is large, the current leakage in the electrical insulation may be comparable to the current in the branches of the bridge circuit.

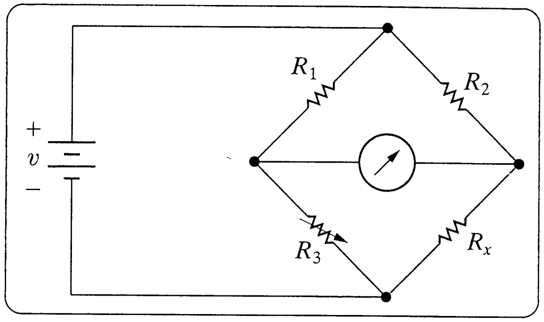
ASSESSING OBJECTIVE 5

◆ **Understand how a Wheatstone bridge is used to measure resistance**

3.7 The bridge circuit shown is balanced when $R_1 = 100\ \Omega$, $R_2 = 1000\ \Omega$, and $R_3 = 150\ \Omega$. The bridge is energized from a 5 V dc source.

a) What is the value of R_x?

b) Suppose each bridge resistor is capable of dissipating 250 mW. Can the bridge be left in the balanced state without exceeding the power-dissipating capacity of the resistors, thereby damaging the bridge?



ANSWER: (a) 1500 Ω; (b) yes.

NOTE ◆ *Also try Chapter Problem 3.49.*

3.7 ◆ Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

The bridge configuration in Fig. 3.26 introduces an interconnection of resistances that warrants further discussion. If we replace the galvanometer with its equivalent resistance R_m, we can draw the circuit shown in Fig. 3.28. We cannot reduce the interconnected resistors of this circuit to a single equivalent resistance across the terminals of the battery if restricted to the simple series or parallel equivalent circuits introduced earlier in this chapter. The interconnected resistors can be reduced to a single equivalent resistor by means of a delta-to-wye (Δ-to-Y) or pi-to-tee (π-to-T) equivalent circuit.[1]

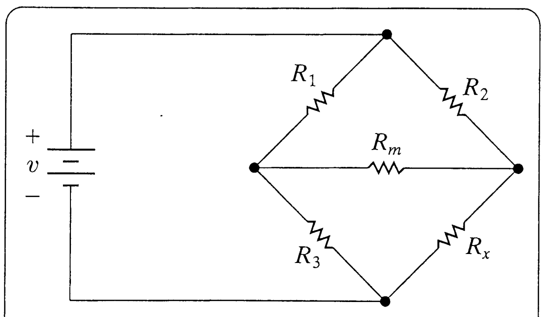


Figure 3.28 A resistive network generated by a Wheatstone bridge circuit.

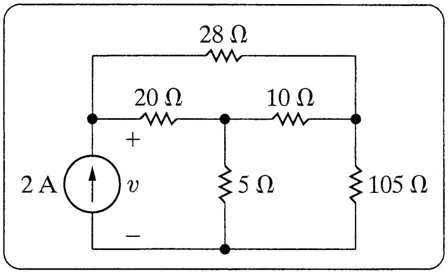
[1] Δ and Y structures are present in a variety of useful circuits, not just resistive networks. Hence the Δ-to-Y transformation is a helpful tool in circuit analysis.

ASSESSING OBJECTIVE 6

◆ **Know when and how to use delta-to-wye equivalent circuits**

3.8 Use a Y-to-Δ transformation to find the voltage v in the circuit shown.

ANSWER: 35 V.



NOTE ◆ *Also try Chapter Problems 3.53, 3.54, and 3.55.*

Practical Perspective

A Rear Window Defroster

A model of a defroster grid is shown in Fig. 3.36, where x and y denote the horizontal and vertical spacing of the grid elements. Given the dimensions of the grid, we need to find expressions for each resistor in the grid such that the power dissipated per unit length is the same in each conductor. This will ensure uniform heating of the rear window in both the x and y directions. Thus we need to find values for the grid resistors that satisfy the following relationships:

$$i_1^2\left(\frac{R_1}{x}\right) = i_2^2\left(\frac{R_2}{x}\right) = i_3^2\left(\frac{R_3}{x}\right) = i_4^2\left(\frac{R_4}{x}\right) = i_5^2\left(\frac{R_5}{x}\right), \tag{3.50}$$

$$i_1^2\left(\frac{R_a}{y}\right) = i_1^2\left(\frac{R_1}{x}\right), \tag{3.51}$$

$$i_1^2\left(\frac{R_a}{y}\right) = i_b^2\left(\frac{R_b}{y}\right) = i_c^2\left(\frac{R_c}{y}\right) = i_5^2\left(\frac{R_d}{y}\right), \tag{3.52}$$

$$i_5^2\left(\frac{R_d}{y}\right) = i_5^2\left(\frac{R_5}{x}\right). \tag{3.53}$$

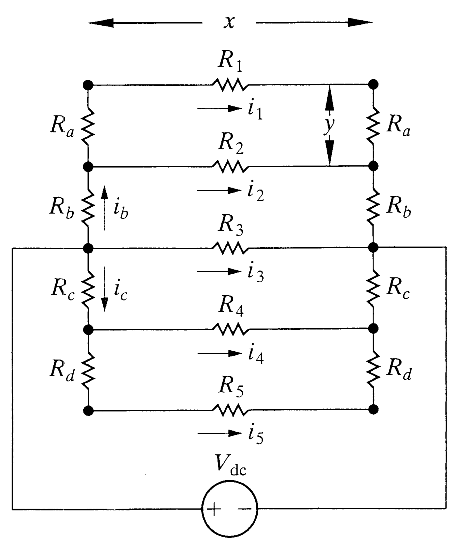


Figure 3.36 Model of a defroster grid.

PROBLEMS

3.1 For each of the circuits shown,

a) identify the resistors connected in series,

b) simplify the circuit by replacing the series-connected resistors with equivalent resistors.

3.2 For each of the circuits shown,

a) identify the resistors connected in parallel,

b) simplify the circuit by replacing the parallel-connected resistors with equivalent resistors.

3.3 P a) Find the power dissipated in each resistor in the circuit shown in Fig. 3.9.

b) Find the power delivered by the 120 V source.

c) Show that the power delivered equals the power dissipated.

3.4 P a) Show that the solution of the circuit in Fig. 3.9 (see Example 3.1) satisfies Kirchhoff's current law at junctions x and y.

b) Show that the solution of the circuit in Fig. 3.9 satisfies Kirchhoff's voltage law around every closed loop.

3.5 Find the equivalent resistance seen by the source in each of the circuits of Problem 3.1.

3.6 Find the equivalent resistance seen by the source in each of the circuits of Problem 3.2.

Figure P3.1

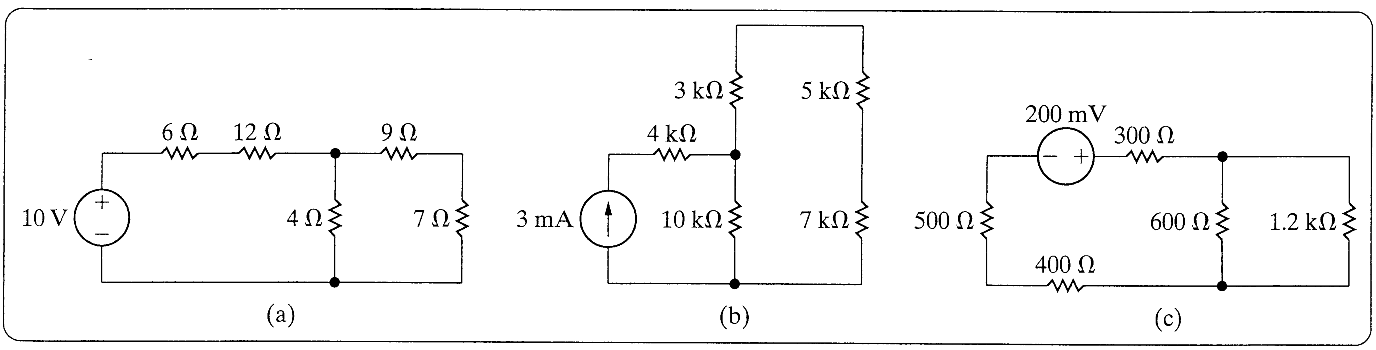
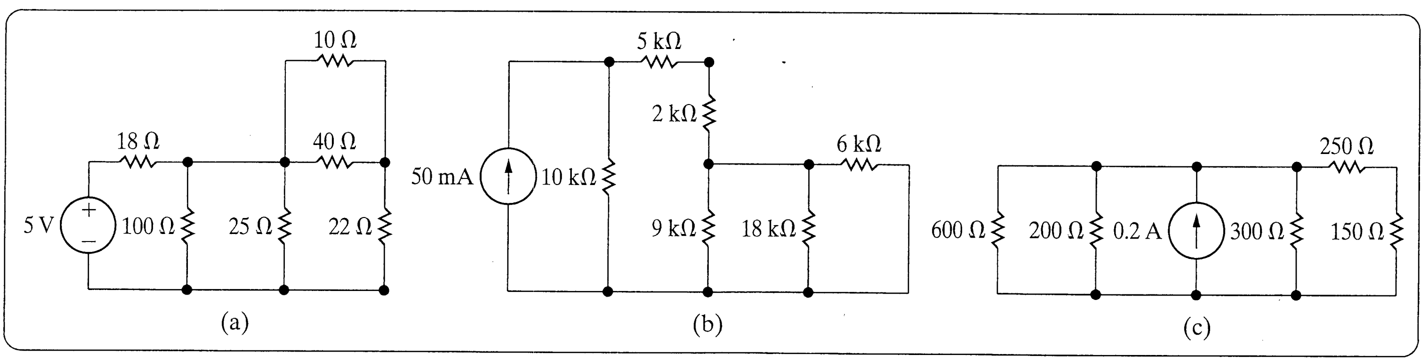


Figure P3.2



3.7 Find the equivalent resistance R_{ab} for each of the circuits in Fig. P3.7.

3.8 Find the equivalent resistance R_{ab} for each of the circuits in Fig. P3.8.

3.9 a) In the circuits in Fig. P3.9(a)–(c), find the equivalent resistance R_{ab}.

b) For each circuit find the power delivered by the source.

Figure P3.7

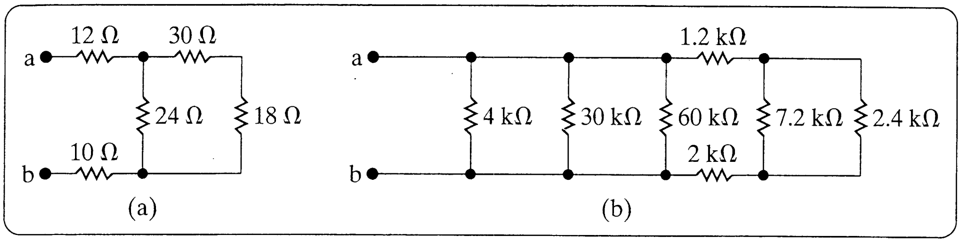


Figure P3.8

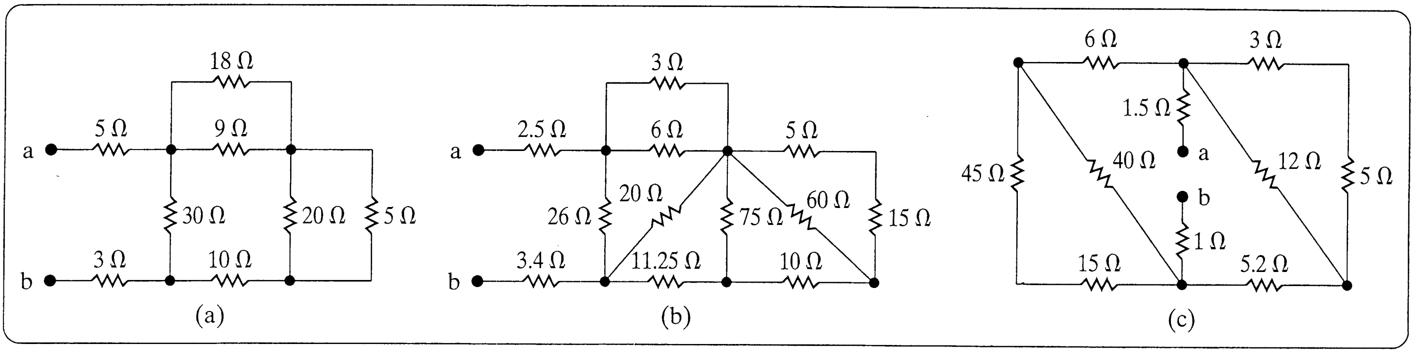
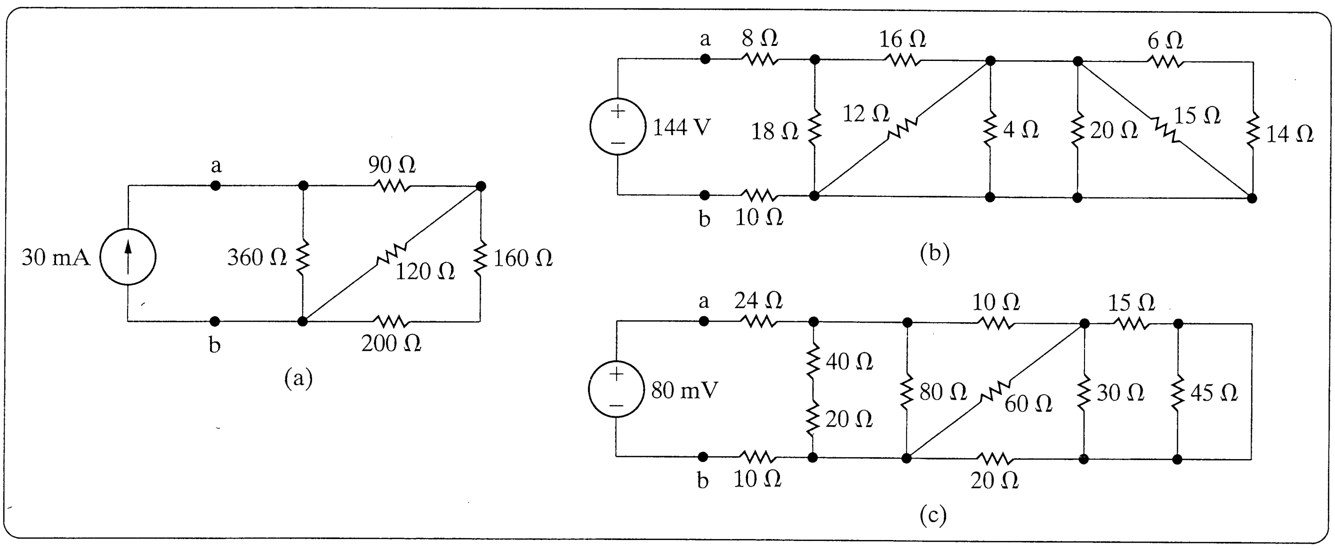
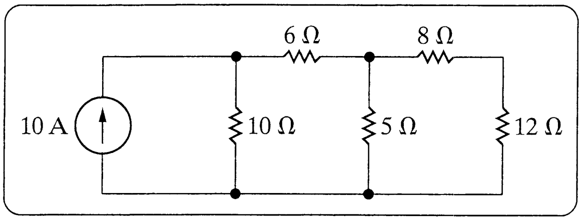


Figure P3.9



3.10 Find the power dissipated in the 5 Ω resistor in the circuit in Fig. P3.10.

Figure P3.10



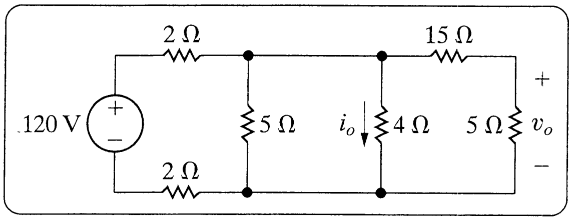
3.11 For the circuit in Fig. P3.11 calculate

a) v_o and i_o

b) the power dissipated in the 15 Ω resistor

c) the power developed by the voltage source

Figure P3.11



3.12 a) Find an expression for the equivalent resistance of two resistors of value R in parallel.

b) Find an expression for the equivalent resistance of n resistors of value R in parallel.

c) Using the results of (b), design a resistive network with an equivalent resistance of 400 Ω using 2 kΩ resistors.

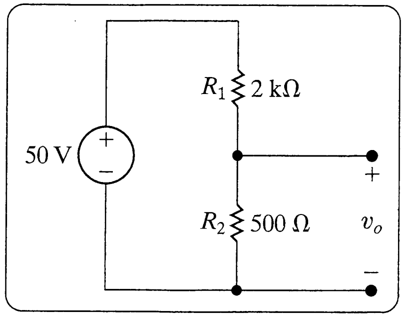
d) Using the results of (b), design a resistive network with an equivalent resistance of 12.5 kΩ using 100 kΩ resistors.

3.13 a) Calculate the no-load voltage v_o for the voltage-divider circuit shown in Fig. P3.13.

b) Calculate the power dissipated in R_1 and R_2.

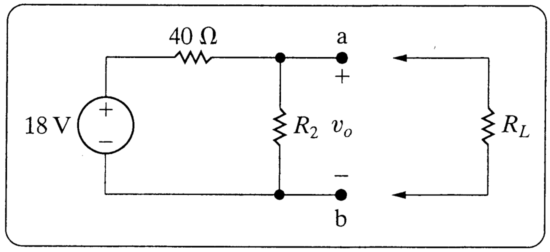
c) Assume that only 1 W resistors are available. The no-load voltage is to be the same as in (a). Specify the ohmic values of R_1 and R_2.

Figure P3.13



3.14 In the voltage-divider circuit shown in Fig. P3.14, the no-load value of v_o is 6 V. When the load resistance R_L is attached across the terminals a and b, v_o drops to 4 V. Find R_L.

Figure P3.14

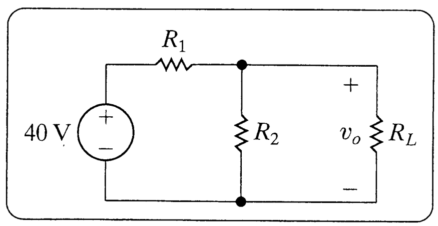


3.15 The no-load voltage in the voltage-divider circuit shown in Fig. P3.15 is 8 V. The smallest load resistor that is ever connected to the divider is 3.6 kΩ. When the divider is loaded, v_o is not to drop below 7.5 V.

a) Design the divider circuit to meet the specifications just mentioned. Specify the numerical value of R_1 and R_2.

b) Assume the power ratings of commercially available resistors are 1/16, 1/8, 1/4, 1, and 2 W. What power rating would you specify?

Figure P3.15



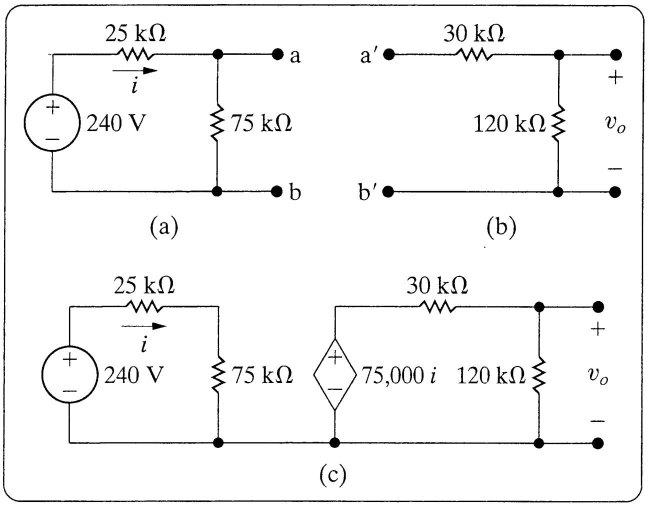
3.16 Assume the voltage divider in Fig. P3.15 has been constructed from 1 W resistors. How small can R_L be before one of the resistors in the divider is operating at its dissipation limit?

3.17 a) The voltage divider in Fig. P3.17(a) is loaded with the voltage divider shown in Fig. P3.17(b); that is, a is connected to a′, and b is connected to b′. Find v_o.

b) Now assume the voltage divider in Fig. P3.17(b) is connected to the voltage divider in Fig. P3.17(a) by means of a current-controlled voltage source as shown in Fig. P3.17(c). Find v_o.

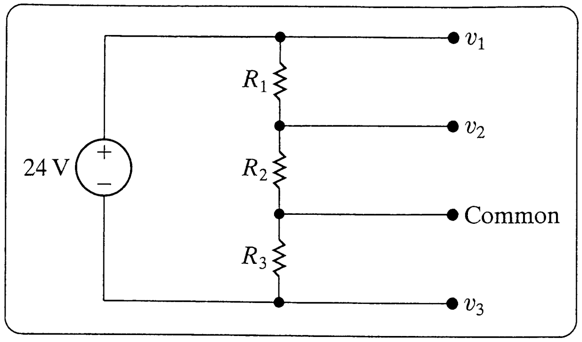
c) What effect does adding the dependent-voltage source have on the operation of the voltage divider that is connected to the 240 V source?

Figure P3.17



3.18 There is often a need to produce more than one voltage using a voltage divider. For example, the memory components of many personal computers require voltages of −12 V, 6 V, and +12 V, all with respect to a common reference terminal. Select the values of R_1, R_2, and R_3 in the circuit in Fig. P3.18 to meet the following design requirements:

Figure P3.18



a) The total power supplied to the divider circuit by the 24 V source is 36 W when the divider is unloaded.

b) The three voltages, all measured with respect to the common reference terminal, are $v_1 = 12$ V, $v_2 = 6$ V, and $v_3 = -12$ V.

3.19 A voltage divider like that in Fig. 3.13 is to be designed so that $v_o = kv_s$ at no load ($R_L = \infty$) and $v_o = \alpha v_s$ at full load ($R_L = R_o$). Note that by definition $\alpha < k < 1$.

a) Show that

$$R_L = \frac{k-\alpha}{\alpha k} R_o$$

and

$$R_2 = \frac{k-\alpha}{\alpha(1-k)} R_o.$$

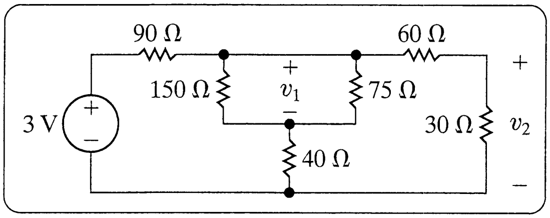
b) Specify the numerical values of R_1 and R_2 if $k = 0.85$, $\alpha = 0.80$, and $R_o = 34$ kΩ.

c) If $v_s = 60$ V, specify the maximum power that will be dissipated in R_1 and R_2.

d) Assume the load resistor is accidentally short circuited. How much power is dissipated in R_1 and R_2?

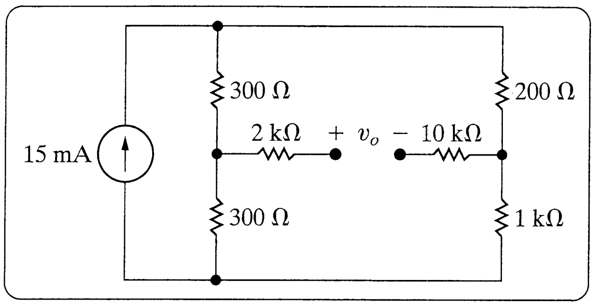
3.25 Find v_1 and v_2 in the circuit in Fig. P3.25.

Figure P3.25



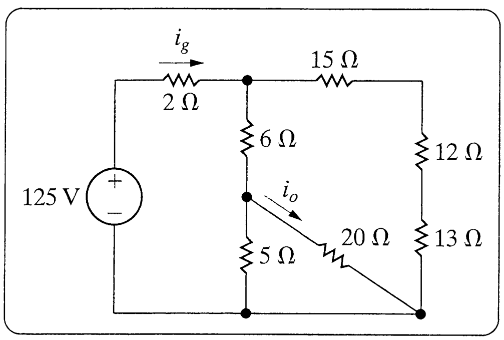
3.26 Find v_o in the circuit in Fig. P3.26.

Figure P3.26



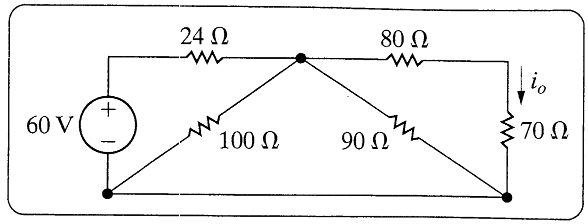
3.27 Find i_o and i_g in the circuit in Fig. P3.27.

Figure P3.27



3.28 For the circuit in Fig. P3.28, calculate (a) i_o and (b) the power dissipated in the 90 Ω resistor.

Figure P3.28

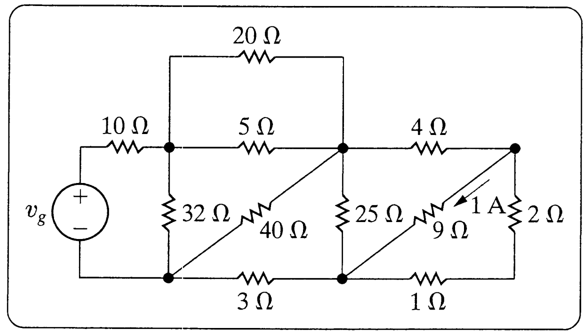


3.29 The current in the 9 Ω resistor in the circuit in Fig. P3.29 is 1 A, as shown.

a) Find v_g.

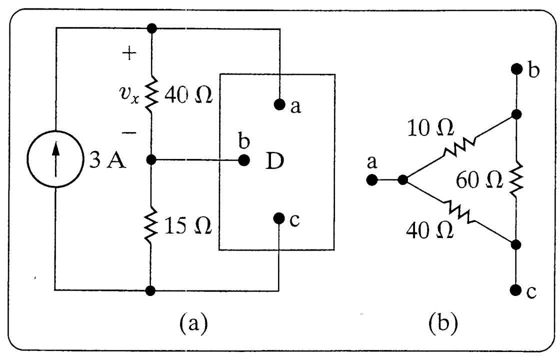
b) Find the power dissipated in the 20 Ω resistor.

Figure P3.29



3.30 In the circuit in Fig. P3.30(a) the device labeled D represents a component that has the equivalent circuit shown in Fig. P3.30(b). The labels on the terminals of D show how the device is connected to the circuit. Find v_x and the power absorbed by the device.

Figure P3.30

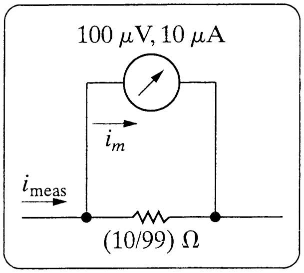


3.31 a) Show for the ammeter circuit in Fig. P3.31 that the current in the d'Arsonval movement is always 1/100th of the current being measured.

b) What would the fraction be if the 100 μV, 10 μA movement were used in a 1 A ammeter?

c) Would you expect a uniform scale on a dc d'Arsonval ammeter?

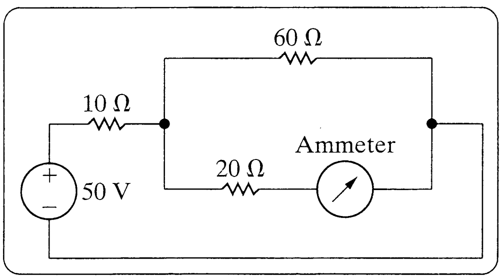
Figure P3.31



3.32 The ammeter in the circuit in Fig. P3.32 has a resistance of 0.1 Ω. What is the percentage of error in the reading of this ammeter if

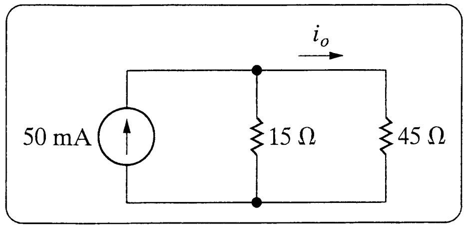
$$\% \text{ error} = \left(\frac{\text{measured value}}{\text{true value}} - 1\right) \times 100?$$

Figure P3.32



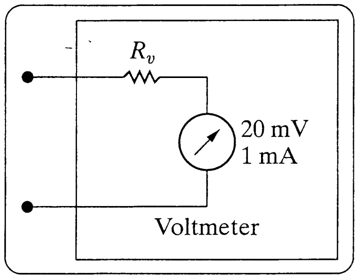
3.33 The ammeter described in Problem 3.32 is used to measure the current i_o in the circuit in Fig. P3.33. What is the percentage of error in the measured value?

Figure P3.33



3.34 A d'Arsonval voltmeter is shown in Fig. P3.34. Find the value of R_V for each of the following full-scale readings: (a) 50 V, (b) 5 V, (c) 250 mV, and (d) 25 mV.

Figure P3.34



3.35 Suppose the d'Arsonval voltmeter described in Problem 3.34(b) is used to measure the voltage across the 45 Ω resistor in Fig. P3.33.

a) What will the voltmeter read?

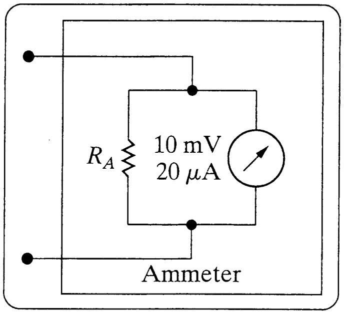
b) Using the definition of the percentage of error in a meter reading found in Problem 3.32, what is the percentage of error in the voltmeter reading?

3.36 A shunt resistor and a 50 mV, 1 mA d'Arsonval movement are used to build a 5 A ammeter. A resistance of 0.02 Ω is placed across the terminals of the ammeter. What is the new full-scale range of the ammeter?

3.37 ❖ A d'Arsonval movement is rated at 2 mA and 100 mV. Assume 0.25 W precision resistors are available to use as shunts. What is the largest full-scale-reading ammeter that can be designed? Explain.

3.38 A d'Arsonval ammeter is shown in Fig. P3.38. Design a set of d'Arsonval ammeters to read the following full-scale current readings: (a) 10 A, (b) 1 A, (c) 10 mA, and (d) 100 μA. Specify the shunt resistor R_A for each ammeter.

Figure P3.38



3.39 The elements in the circuit in Fig. 2.24 have the following values: $R_1 = 10$ kΩ, $R_2 = 50$ kΩ, $R_C = 0.5$ kΩ, $R_E = 0.3$ kΩ, $V_{CC} = 12$ V, $V_0 = 0.4$ V, and $\beta = 29$.

a) Calculate the value of i_B in microamperes.

b) Assume that a digital multimeter, when used as a dc ammeter, has a resistance of 2 kΩ. If the meter is inserted between terminals b and 2 to measure the current i_B, what will the meter read?

c) Using the calculated value of i_B in (a) as the correct value, what is the percentage of error in the measurement?

3.40 The voltage-divider circuit shown in Fig. P3.40 is designed so that the no-load output voltage is 8/10ths of the input voltage. A d'Arsonval voltmeter having a sensitivity of 100 Ω/V and a full-scale rating of 100 V is used to check the operation of the circuit.

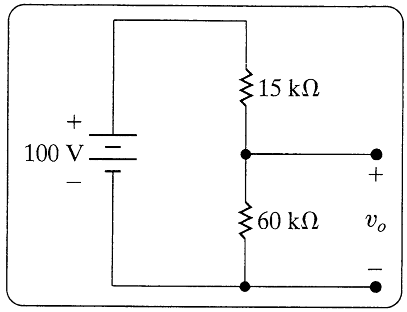
a) What will the voltmeter read if it is placed across the 100 V source?

b) What will the voltmeter read if it is placed across the 60 kΩ resistor?

c) What will the voltmeter read if it is placed across the 15 kΩ resistor?

d) Will the voltmeter readings obtained in parts (b) and (c) add to the reading recorded in part (a)? Explain why or why not.

Figure P3.40



3.41 You have been told that the dc voltage of a power supply is about 500 V. When you go to the instrument room to get a dc voltmeter to measure the power supply voltage, you find that there are only two dc voltmeters available. One voltmeter is rated 300 V full scale and has a sensitivity of 1000 Ω/V. The other voltmeter is rated 150 V full scale and has a sensitivity of 800 Ω/V.

a) How can you use the two voltmeters to check the power supply voltage?

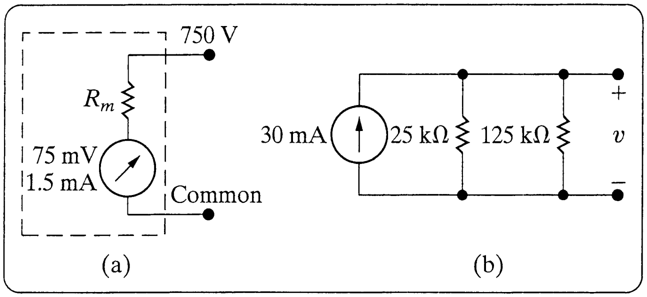
b) What is the maximum voltage that can be measured?

c) If the power supply voltage is 399 V, what will each voltmeter read?

3.42 Assume that in addition to the two voltmeters described in Problem 3.41, an 80 kΩ precision resistor is also available. The 80 kΩ resistor is connected in series with the series-connected voltmeters. This circuit is then connected across the terminals of the power supply. The reading on the 300 V voltmeter is 288 V, while the reading on the 150 V voltmeter is 115.2 V. What is the voltage of the power supply?

3.43 The voltmeter shown in Fig. P3.43(a) has a full-scale reading of 750 V. The meter movement is rated 75 mV and 1.5 mA. What is the percentage of error in the meter reading if it is used to measure the voltage v in the circuit of Fig. P3.43(b)?

Figure P3.43

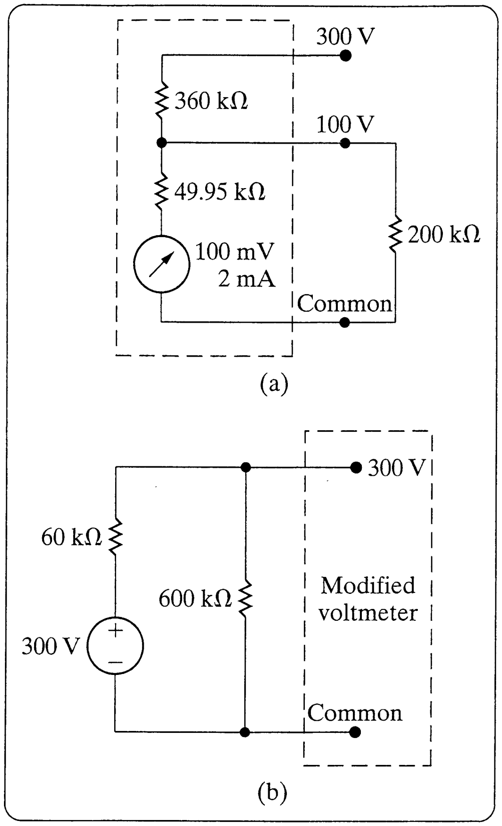


3.44 A 200 kΩ resistor is connected from the 100 V terminal to the common terminal of a dual-scale voltmeter, as shown in Fig. P3.44(a). This modified voltmeter is then used to measure the voltage across the 600 kΩ resistor in the circuit in Fig. P3.44(b).

a) What is the reading on the 300 V scale of the meter?

b) What is the percentage of error in the measured voltage?

Figure P3.44

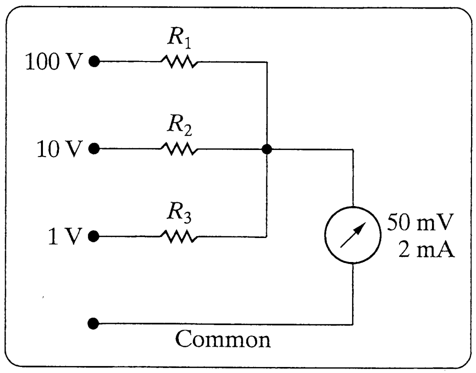


3.45 ❖ Assume in designing the multirange voltmeter shown in Fig. P3.45 that you ignore the resistance of the meter movement.

a) Specify the values of R_1, R_2, and R_3.

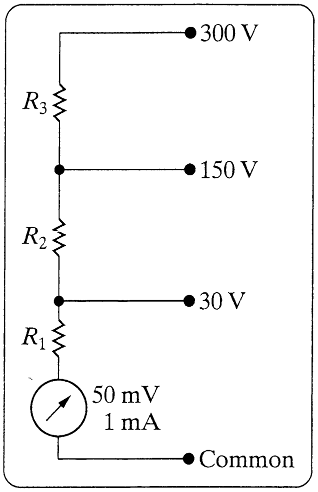
b) For each of the three ranges, calculate the percentage of error that this design strategy produces.

Figure P3.45



3.46 ❖ Design a d'Arsonval voltmeter that will have the three voltage ranges shown in Fig. P3.46.

Figure P3.46



a) Specify the values of R_1, R_2, and R_3.

b) Assume that a 750 kΩ resistor is connected between the 150 V terminal and the common terminal. The voltmeter is then connected to an unknown voltage using the common terminal and the 300 V terminal. The voltmeter reads 288 V. What is the unknown voltage?

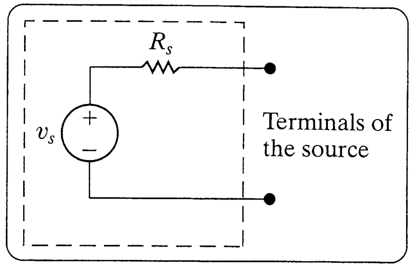
c) What is the maximum voltage the voltmeter in (b) can measure?

3.47 The circuit model of a dc voltage source is shown in Fig. P3.47. The following voltage measurements are made at the terminals of the source: (1) With the terminals of the source open, the voltage is measured at 50 mV, and (2) with a 15 MΩ resistor connected to the terminals, the voltage is measured at 48.75 mV. All measurements are made with a digital voltmeter that has a meter resistance of 10 MΩ.

a) What is the internal voltage of the source (v_s) in millivolts?

b) What is the internal resistance of the source (R_s) in kilo-ohms?

Figure P3.47



3.48 Assume the ideal voltage source in Fig. 3.26 is replaced by an ideal current source. Show that Eq. 3.33 is still valid.

3.49 P The bridge circuit shown in Fig. 3.26 is energized from a 6 V dc source. The bridge is balanced when $R_1 = 200\ \Omega$, $R_2 = 500\ \Omega$, and $R_3 = 800\ \Omega$.

a) What is the value of R_x?

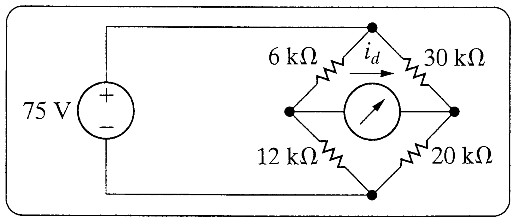
b) How much current (in milliamperes) does the dc source supply?

c) Which resistor in the circuit absorbs the most power? How much power does it absorb?

d) Which resistor absorbs the least power? How much power does it absorb?

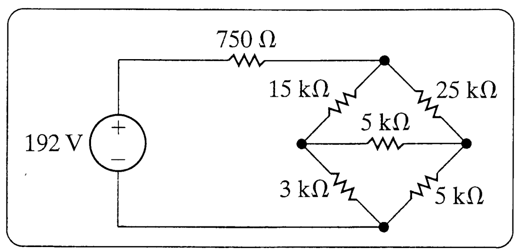
3.50 P Find the detector current i_d in the unbalanced bridge in Fig. P3.50 if the voltage drop across the detector is negligible.

Figure P3.50



3.51 P Find the power dissipated in the 3 kΩ resistor in the circuit in Fig. P3.51.

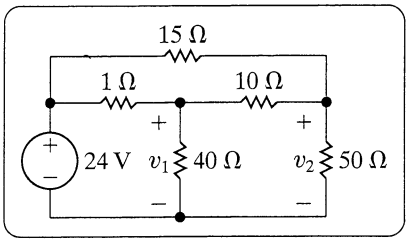
Figure P3.51



3.52 In the Wheatstone bridge circuit shown in Fig. 3.26, the ratio R_2/R_1 can be set to the following values: 0.001, 0.01, 0.1, 1, 10, 100, and 1000. The resistor R_3 can be varied from 1 to 11,110 Ω, in increments of 1 Ω. An unknown resistor is known to lie between 4 and 5 Ω. What should be the setting of the R_2/R_1 ratio so that the unknown resistor can be measured to four significant figures?

3.53 Use a Δ-to-Y transformation to find the voltages v_1 and v_2 in the circuit in Fig. P3.53.

Figure P3.53

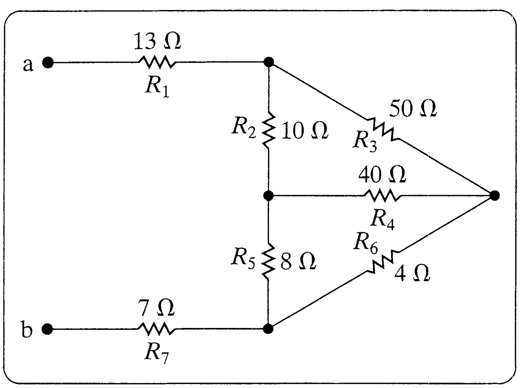


3.54
a) Find the equivalent resistance R_{ab} in the circuit in Fig. P3.54 by using a Δ-to-Y transformation involving the resistors R_2, R_3, and R_4.

b) Repeat (a) using a Y-to-Δ transformation involving resistors R_2, R_4, and R_5.

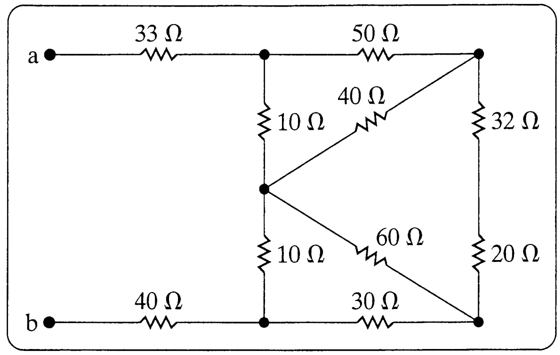
c) Give two additional Δ-to-Y or Y-to-Δ transformations that could be used to find R_{ab}.

Figure P3.54



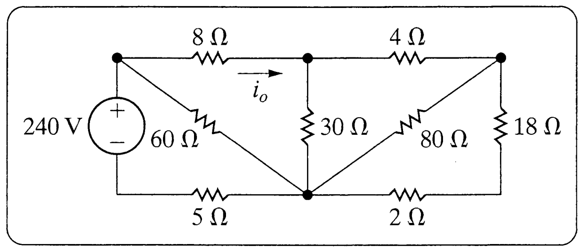
3.55 Find the equivalent resistance R_{ab} in the circuit in Fig. P3.55.

Figure P3.55



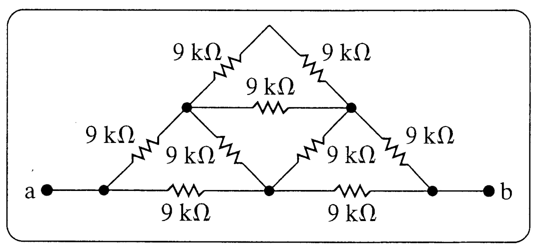
3.56 Find i_o and the power dissipated in the 30 Ω resistor in the circuit in Fig. P3.56.

Figure P3.56



3.57 Find R_{ab} in the circuit in Fig. P3.57.

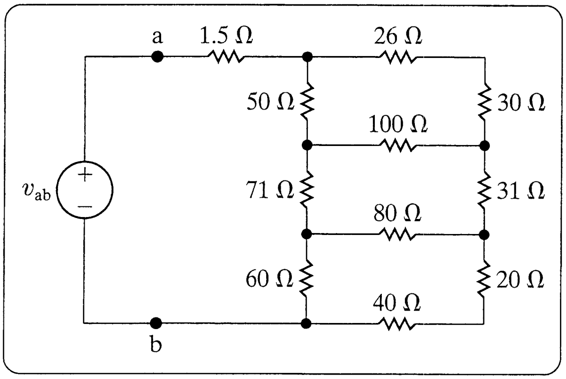
Figure P3.57



3.58 a) Find the resistance seen by the ideal voltage source in the circuit in Fig. P3.58.

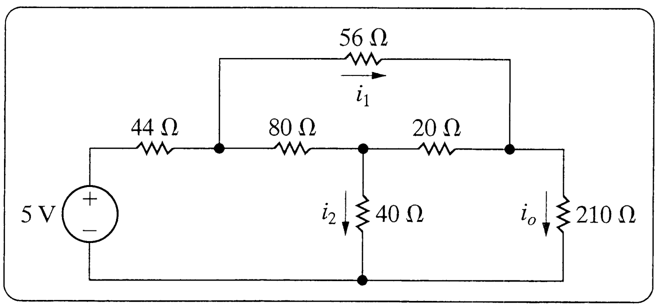
b) If v_{ab} equals 400 V, how much power is dissipated in the 31 Ω resistor?

Figure P3.58



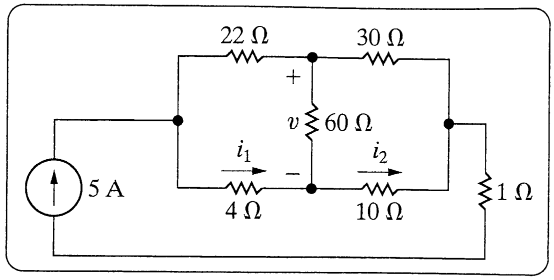
3.59 Use a Y-to-Δ transformation to find (a) i_o; (b) i_1; (c) i_2; and (d) the power delivered by the ideal voltage source in the circuit in Fig. P3.59.

Figure P3.59



3.60 For the circuit shown in Fig. P3.60, find (a) i_1, (b) v, (c) i_2, and (d) the power supplied by the current source.

Figure P3.60



3.61 Derive Eqs. 3.44–3.49 from Eqs. 3.41–3.43. The following two hints should help you get started in the right direction:

a) To find R_1 as a function of R_a, R_b, and R_c, first subtract Eq. 3.42 from Eq. 3.43 and then add this result to Eq. 3.39. Use similar manipulations to find R_2 and R_3 as functions of R_a, R_b, and R_c.

b) To find R_b as a function of R_1, R_2, and R_3, take advantage of the derivations obtained by hint (1), namely, Eqs. 3.44–3.46. Note that these equations can be divided to obtain

$$\frac{R_2}{R_3} = \frac{R_c}{R_b}, \text{ or } R_c = \frac{R_2}{R_3}R_b,$$

and

$$\frac{R_1}{R_2} = \frac{R_b}{R_a}, \text{ or } R_a = \frac{R_2}{R_1}R_b.$$

Now use these ratios in Eq. 3.43 to eliminate R_a and R_c. Use similar manipulations to find R_a and R_c as functions of R_1, R_2, and R_3.

3.62 Show that the expressions for Δ conductances as functions of the three Y conductances are

$$G_a = \frac{G_2G_3}{G_1 + G_2 + G_3},$$

$$G_b = \frac{G_1G_3}{G_1 + G_2 + G_3},$$

$$G_c = \frac{G_1G_2}{G_1 + G_2 + G_3},$$

where

$$G_a = \frac{1}{R_a}, \quad G_1 = \frac{1}{R_1}, \text{ etc.}$$

3.63 Resistor networks are sometimes used as volume-control circuits. In this application, they are referred to as *resistance attenuators* or *pads*. A typical fixed-attenuator pad is shown in Fig. P3.63. In designing an attenuation pad, the circuit designer will select the values of R_1 and R_2 so that the ratio of v_o/v_i and the resistance seen by the input voltage source R_{ab} both have a specified value.

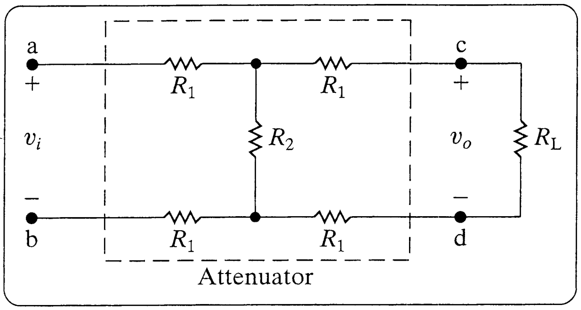
a) Show that if $R_{ab} = R_L$, then

$$R_L^2 = 4R_1(R_1 + R_2)$$

$$\frac{v_o}{v_i} = \frac{R_2}{2R_1 + R_2 + R_L}.$$

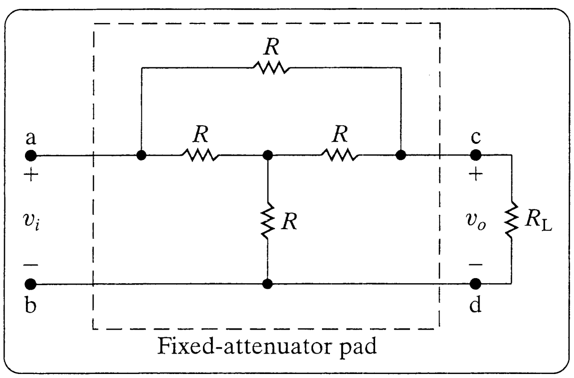
b) Select the values of R_1 and R_2 so that $R_{ab} = R_L = 600\ \Omega$ and $v_o/v_i = 0.6$.

Figure P3.63



3.64 a) The fixed-attenuator pad shown in Fig. P3.64 is called a *bridged tee*. Use a Y-to-Δ transformation to show that $R_{ab} = R_L$ if $R = R_L$.

Figure P3.64



b) Show that when $R = R_L$, the voltage ratio v_o/v_i equals 0.50.

3.65 The design equations for the bridged-tee attenuator circuit in Fig. P3.65 are

$$R_2 = \frac{2RR_L^2}{3R^2 - R_L^2}$$

$$\frac{v_o}{v_i} = \frac{3R - R_L}{3R + R_L}$$

when R_2 has the value just given.

a) Design a fixed attenuator so that $v_i = 3.5v_o$ when $R_L = 300\ \Omega$.

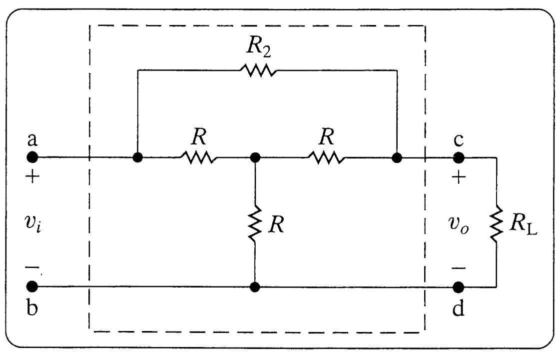
b) Assume the voltage applied to the input of the pad designed in (a) is 42 V. Which resistor in the pad dissipates the most power?

c) How much power is dissipated in the resistor in part (b)?

d) Which resistor in the pad dissipates the least power?

e) How much power is dissipated in the resistor in part (d)?

Figure P3.65



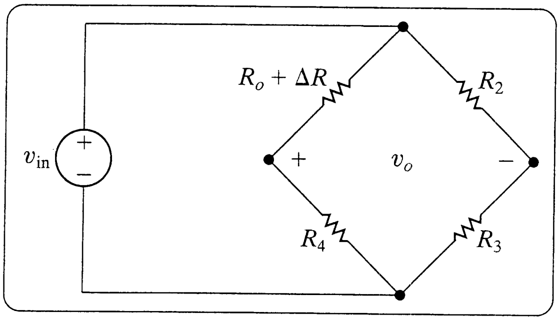
3.66 a) For the circuit shown in Fig. P3.66 the bridge is balanced when $\Delta R = 0$. Show that if $\Delta R \ll R_o$ the bridge output voltage is approximately

$$v_o \approx \frac{-\Delta R R_4}{(R_o + R_4)^2} v_{in}$$

b) Given $R_2 = 1\ \text{k}\Omega$, $R_3 = 500\ \Omega$, $R_4 = 5\ \text{k}\Omega$, and $v_{in} = 6$ V, what is the approximate bridge output voltage if ΔR is 3% of R_o?

c) Find the actual value of v_o in part (b).

Figure P3.66



3.67 a) If percent error is defined as

$$\%\ \text{error} = \left[\frac{\text{approximate value}}{\text{true value}} - 1\right] \times 100$$

show that the percent error in the approximation of v_o in Problem 3.66 is

$$\%\ \text{error} = \frac{(\Delta R)R_3}{(R_2 + R_3)R_4} \times 100.$$

b) Calculate the percent error in v_o for Problem 3.66.

3.68 Assume the error in v_o in the bridge circuit in Fig. P3.66 is not to exceed 0.5%. What is the largest percent change in R_o that can be tolerated?

3.69 a) Derive Eq. 3.65.

b) Derive Eq. 3.68.

3.70 Derive Eq. 3.70.

3.71 Suppose the grid structure in Fig. 3.36 is 1 m wide and the vertical displacement of the five horizontal grid lines is 0.025 m. Specify the numerical values of $R_1 - R_5$ and $R_a - R_d$ to achieve a uniform power dissipation of 120 W/m, using a 12 V power supply. (*Hint:* Calculate σ first, then R_3, R_1, R_a, R_b, and R_2 in that order.)

3.72 Check the solution to Problem 3.71 by showing that the total power dissipated equals the power developed by the 12 V source.

3.73 a) Design a defroster grid in Fig. 3.36 having five horizontal conductors to meet the following specifications: The grid is to be 1.5 m wide, the vertical separation between conductors is to be 0.03 m, and the power dissipation is to be 200 W/m when the supply voltage is 12 V.

b) Check your solution and make sure it meets the design specifications.

CHAPTER 4

Techniques of Circuit Analysis

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Understand and be able to use the node-voltage method to solve a circuit.
2. Understand and be able to use the mesh-current method to solve a circuit.
3. Be able to decide whether the node-voltage method or the mesh-current method is the preferred approach to solving a particular circuit.
4. Understand source transformation and be able to use it to solve a circuit.
5. Understand the concept of the Thévenin and Norton equivalent circuits and be able to construct a Thévenin or Norton equivalent for a circuit.
6. Know the condition for maximum power transfer to a resistive load and be able to calculate the value of the load resistor that satisfies this condition.

So far, we have analyzed relatively simple resistive circuits by applying Kirchhoff's laws in combination with Ohm's law. We can use this approach for all circuits, but as they become structurally more complicated and involve more and more elements, this direct method soon becomes cumbersome. In this chapter we introduce two powerful techniques of circuit analysis that aid in the analysis of complex circuit structures: the node-voltage method and the mesh-current method. These techniques give us two systematic methods of describing circuits with the minimum number of simultaneous equations.

In addition to these two general analytical methods, in this chapter we also discuss other techniques for simplifying circuits. We have already demonstrated how to use series-parallel reductions and Δ-to-Y transformations to simplify a circuit's structure. We now add source transformations and Thévenin and Norton equivalent circuits to those techniques.

We also consider two other topics that play a role in circuit analysis. One, maximum power transfer, considers the conditions necessary to ensure that the power delivered to a resistive load by a source is maximized. Thévenin equivalent circuits are used in establishing the maximum power transfer conditions. The final topic in this chapter, superposition, looks at the analysis of circuits with more than one independent source.

ASSESSING OBJECTIVE 1

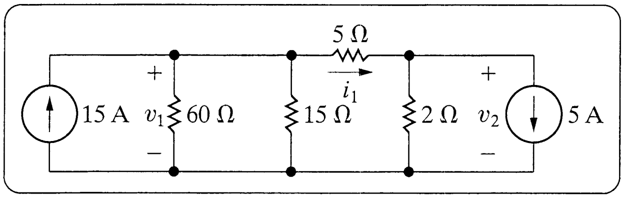
◆ **Understand and be able to use the node-voltage method**

4.1 a) For the circuit shown, use the node-voltage method to find v_1, v_2, and i_1.

b) How much power is delivered to the
circuit by the 15 A source?

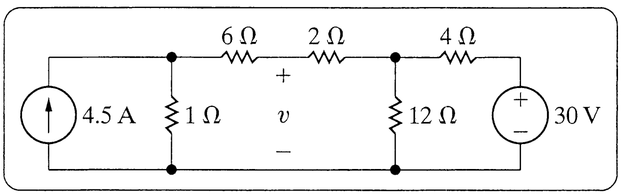
c) Repeat (b) for the 5 A source.

ANSWER: (a) 60 V, 10 V, 10 A; (b) 900 W; (c) −50 W.



4.2 Use the node-voltage method to find v in the circuit shown.

ANSWER: 15 V.



NOTE ◆ *Also try Chapter Problems 4.6, 4.9, and 4.10.*

4.3 ◆ The Node-Voltage Method and Dependent Sources

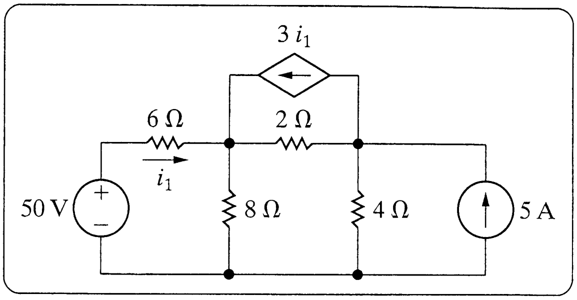
If the circuit contains dependent sources, the node-voltage equations must be supplemented with the constraint equations imposed by the presence of the dependent sources. Example 4.3 illustrates the application of the node-voltage method to a circuit containing a dependent source.

ASSESSING OBJECTIVE 1

◆ **Understand and be able to use the node-voltage method**

4.3 a) Use the node-voltage method to find the power associated with each source in the circuit shown.

b) State whether the source is delivering power to the circuit or extracting power from the circuit.



ANSWER: (a) $p_{50V} = -150$ W, $p_{3i_1} = -144$ W, $p_{5A} = -80$ W; (b) all sources are delivering power to the circuit.

NOTE ◆ *Also try Chapter Problems 4.19 and 4.20.*

4.4 ◆ The Node-Voltage Method: Some Special Cases

When a voltage source is the only element between two essential nodes, the node-voltage method is simplified. As an example, look at the circuit in Fig. 4.12. There are three essential nodes in this circuit, which means that two simultaneous equations are needed. From these three essential nodes, a reference node has been chosen and two other nodes have been labeled. But the 100 V source constrains the voltage between node 1 and the reference node to 100 V. This means that there is only one unknown node voltage (v_2). Solution of this circuit thus involves only a single node-voltage equation at node 2:

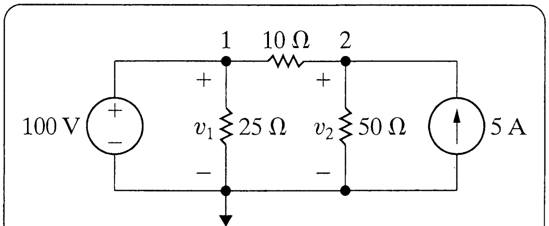


Figure 4.12 A circuit with a known node voltage.

$$\frac{v_2 - v_1}{10} + \frac{v_2}{50} - 5 = 0. \qquad (4.7)$$

But $v_1 = 100$ V, so Eq. 4.7 can be solved for v_2:

$$v_2 = 125 \text{ V}. \qquad (4.8)$$

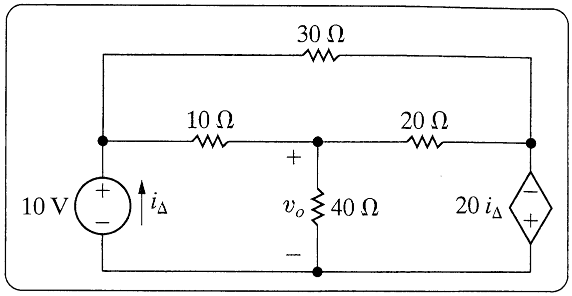
Knowing v_2, we can calculate the current in every branch. You should verify that the current into node 1 in the branch containing the independent voltage source is 1.5 A.

ASSESSING OBJECTIVE 1

◆ **Understand and be able to use the node-voltage method**

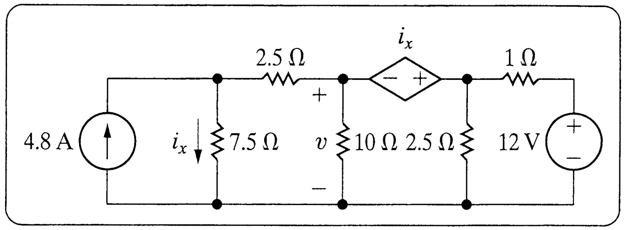
4.4 Use the node-voltage method to find v_o in the circuit shown.

ANSWER: 24 V.



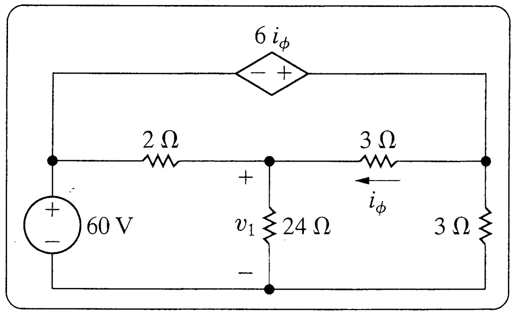
4.5 Use the node-voltage method to find v in the circuit shown.

ANSWER: 8 V.



4.6 Use the node-voltage method to find v_1 in the circuit shown.

ANSWER: 48 V.



NOTE ◆ *Also try Chapter Problems 4.21, 4.26, and 4.27.*

4.5 ◆ Introduction to the Mesh-Current Method

As stated in Section 4.1, the mesh-current method of circuit analysis enables us to describe a circuit in terms of $b_e - (n_e - 1)$ equations. Recall that a mesh is a loop with no other loops inside it. The circuit in Fig. 4.1(b) is shown again in Fig. 4.18, with current arrows inside each loop to distinguish

The mesh current i_a is identical with the branch current in the 40 V source, so the power associated with this source is

$$p_{40V} = -40i_a = -224 \text{ W}.$$

The minus sign means that this source is delivering power to the network. The current in the 20 V source is identical to the mesh current i_c; therefore

$$p_{20V} = 20i_c = -16 \text{ W}.$$

The 20 V source also is delivering power to the network.

b) The branch current in the 8 Ω resistor in the direction of the voltage drop v_o is $i_a - i_b$. Therefore

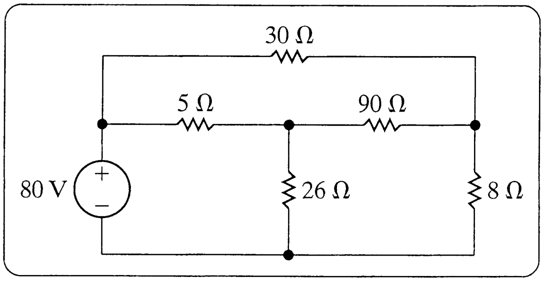
$$v_o = 8(i_a - i_b) = 8(3.6) = 28.8 \text{ V}.$$

ASSESSING OBJECTIVE 2

◆ **Understand and be able to use the mesh-current method**

4.7 Use the mesh-current method to find (a) the power delivered by the 80 V source to the circuit shown and (b) the power dissipated in the 8 Ω resistor.

ANSWER: (a) 400 W; (b) 50 W.



NOTE ◆ *Also try Chapter Problems 4.31 and 4.32.*

4.6 ◆ The Mesh-Current Method and Dependent Sources

If the circuit contains dependent sources, the mesh-current equations must be supplemented by the appropriate constraint equations. Example 4.5 illustrates the application of the mesh-current method when the circuit includes a dependent source.

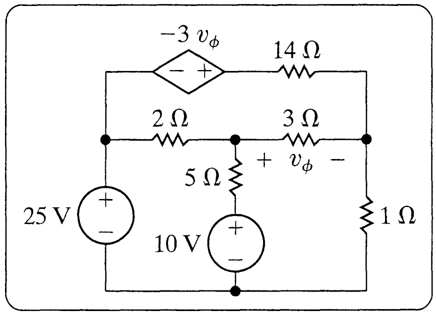
ASSESSING OBJECTIVE 2

◆ **Understand and be able to use the mesh-current method**

4.8 a) Determine the number of mesh-current equations needed to solve the circuit shown.

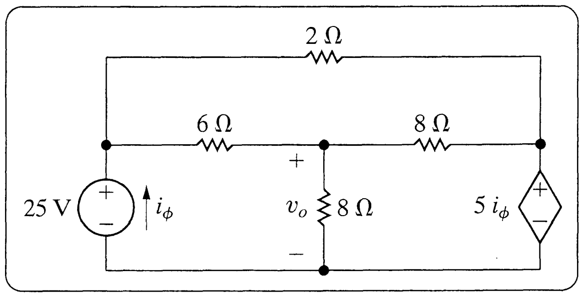
b) Use the mesh-current method to find how much power is being delivered to the dependent voltage source.

ANSWER: (a) 3; (b) −36 W.



4.9 Use the mesh-current method to find v_o in the circuit shown.

ANSWER: 16 V.



NOTE ◆ *Also try Chapter Problems 4.33 and 4.34.*

4.7 ◆ The Mesh-Current Method: Some Special Cases

When a branch includes a current source, the mesh-current method requires some additional manipulations. The circuit shown in Fig. 4.25 depicts the nature of the problem.

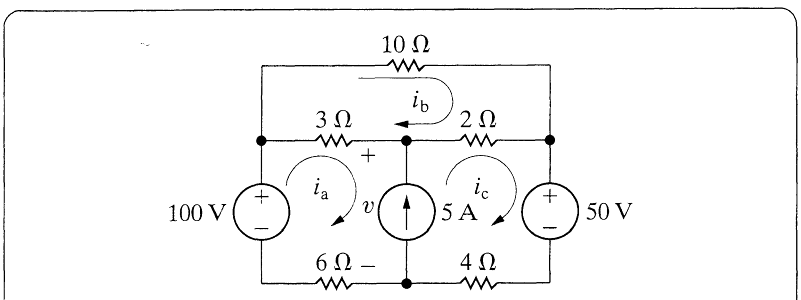


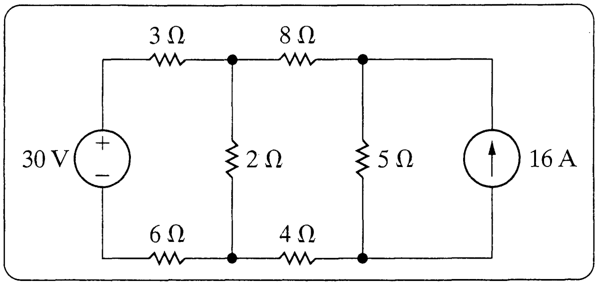
Figure 4.25 A circuit illustrating mesh analysis when a branch contains an independent current source.

ASSESSING OBJECTIVE 2

◆ **Understand and be able to use the mesh-current method**

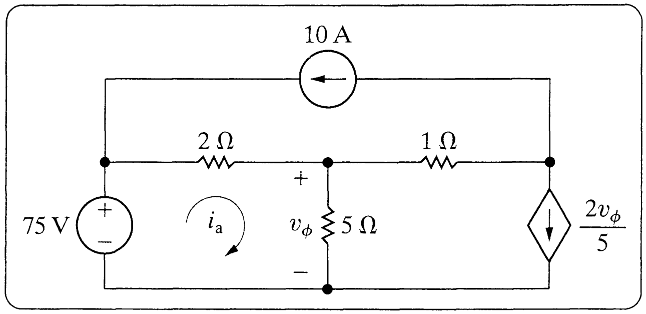
4.10 Use the mesh-current method to find the power dissipated in the 2 Ω resistor in the circuit shown.

ANSWER: 72 W.



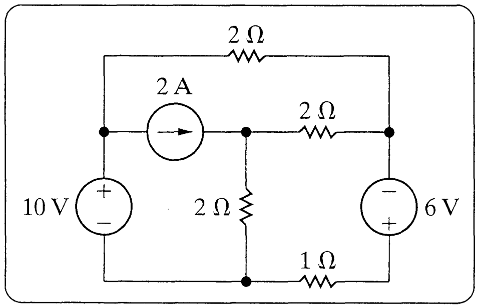
4.11 Use the mesh-current method to find the mesh current i_a in the circuit shown.

ANSWER: 15 A.



4.12 Use the mesh-current method to find the power dissipated in the 1 Ω resistor in the circuit shown.

ANSWER: 36 W.



NOTE ◆ *Also try Chapter Problems 4.37, 4.38, 4.43, and 4.46.*

4.8 ◆ The Node-Voltage Method Versus the Mesh-Current Method

The greatest advantage of both the node-voltage and mesh-current methods is that they reduce the number of simultaneous equations that must be manipulated. They also require the analyst to be quite systematic in terms of organizing and writing these equations. It is natural to ask, then, "When the node-voltage equations are based on the circuit shown in Fig. 4.35. The supermesh equation is

$$193 = 10i_a + 10i_b + 10i_c + 0.8v_\theta,$$

and the constraint equations are

$$i_b - i_a = 0.4v_\Delta = 0.8i_c; \quad v_\theta = -7.5i_b; \quad \text{and} \quad i_c - i_b = 0.5.$$

We use the constraint equations to write the supermesh equation in terms of i_a:

$$160 = 80i_a, \quad \text{or} \quad i_a = 2 \text{ A},$$

$$v_o = 193 - 20 = 173 \text{ V}.$$

The node-voltage equations are

$$\frac{v_o - 193}{10} - 0.4v_\Delta + \frac{v_o - v_a}{2.5} = 0,$$

$$\frac{v_a - v_o}{2.5} - 0.5 + \frac{v_a - (v_b + 0.8v_\theta)}{10} = 0,$$

$$\frac{v_b}{7.5} + 0.5 + \frac{v_b + 0.8v_\theta - v_a}{10} = 0.$$

The constraint equations are

$$v_\theta = -v_b, \quad v_\Delta = \left[\frac{v_a - (v_b + 0.8v_\theta)}{10}\right]2.$$

We use the constraint equations to reduce the node-voltage equations to three simultaneous equations involving v_o, v_a, and v_b. You should verify that the node-voltage approach also gives v_o = 173 V.

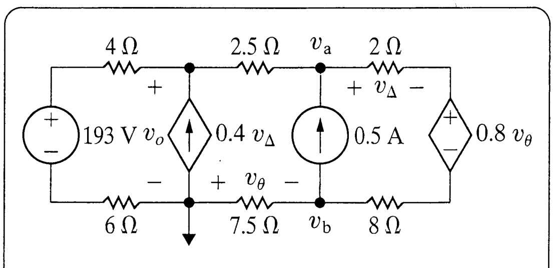


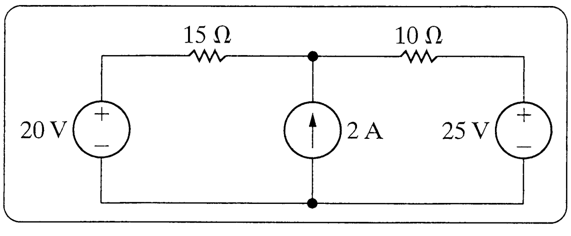
Figure 4.35 The circuit shown in Fig. 4.33 with node voltages.

ASSESSING OBJECTIVE 3

◆ **Deciding between the node-voltage and mesh-current methods**

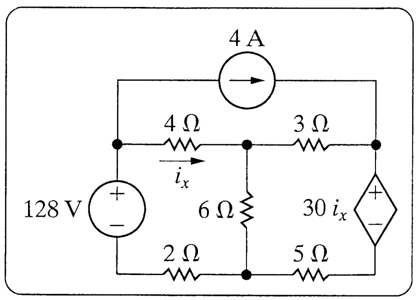
4.13 Find the power delivered by the 2 A current source in the circuit shown.

ANSWER: 70 W.



4.14 Find the power delivered by the 4 A current source in the circuit shown.

ANSWER: 40 W.



NOTE ◆ *Also try Chapter Problems 4.52 and 4.54.*

c) To find the power developed by the 8 A current source, we first find the voltage across the source. If we let v_s represent the voltage across the source, positive at the upper terminal of the source, we obtain

$$v_s + 8(10) = v_o = 20, \quad \text{or} \quad v_s = -60 \text{ V},$$

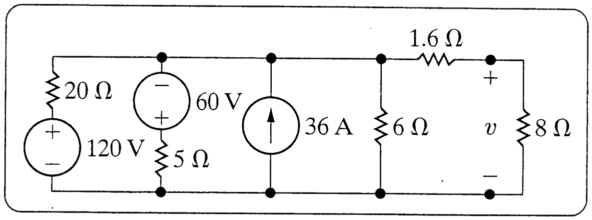
and the power developed by the 8 A source is 480 W. Note that the 125 Ω and 10 Ω resistors do not affect the value of v_o but do affect the power calculations.

ASSESSING OBJECTIVE 4

◆ **Understand source transformation**

4.15 a) Use a series of source transformations to find the voltage v in the circuit shown.

b) How much power does the 120 V source deliver to the circuit?



ANSWER: (a) 48 V; (b) 374.4 W.

NOTE ◆ *Also try Chapter Problems 4.55 and 4.58.*

4.10 ◆ Thévenin and Norton Equivalents

At times in circuit analysis, we want to concentrate on what happens at a specific pair of terminals. For example, when we plug a toaster into an outlet, we are interested primarily in the voltage and current at the terminals of the toaster. We have little or no interest in the effect that connecting the toaster has on voltages or currents elsewhere in the circuit supplying the outlet. We can expand this interest in terminal behavior to a set of appliances, each requiring a different amount of power. We then are interested in how the voltage and current delivered at the outlet change as we change appliances. In other words, we want to focus on the behavior of the circuit supplying the outlet, but only at the outlet terminals.

Thévenin and Norton equivalents are circuit simplification techniques that focus on terminal behavior and thus are extremely valuable aids in analysis. Although here we discuss them as they pertain to resistive circuits, Thévenin and Norton equivalent circuits may be used to represent any circuit made up of linear elements.

From i_{sc} and V_{Th} we get

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-5}{-50} \times 10^3 = 100\ \Omega.$$

Figure 4.51 illustrates the Thévenin equivalent for the circuit shown in Fig. 4.49. Note that the reference polarity marks on the Thévenin voltage source in Fig. 4.51 agree with the preceding equation for V_{Th}.

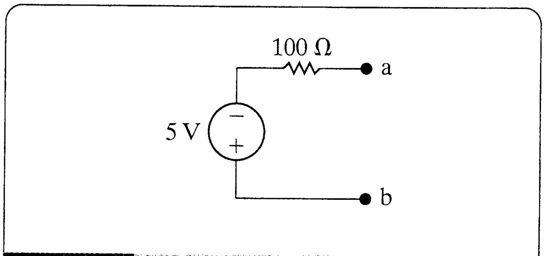


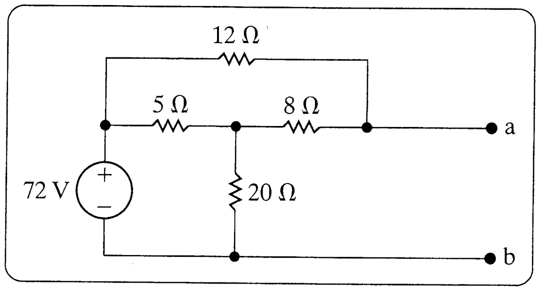
Figure 4.51 The Thévenin equivalent for the circuit shown in Fig. 4.49.

ASSESSING OBJECTIVE 5

◆ **Understand Thèvenin and Norton equivalents**

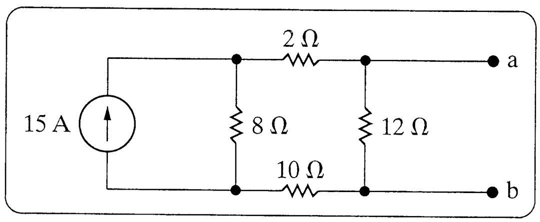
4.16 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown.

ANSWER: $V_{ab} = V_{Th} = 64.8$ V, $R_{Th} = 6\ \Omega$.



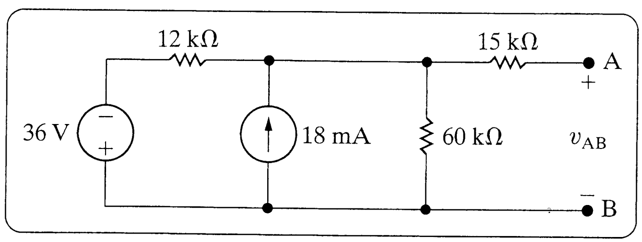
4.17 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown.

ANSWER: $I_N = 6$ A (directed toward a), $R_N = 7.5\ \Omega$.



4.18 A voltmeter with an internal resistance of 100 kΩ is used to measure the voltage v_{AB} in the circuit shown. What is the voltmeter reading?

ANSWER: 120 V.



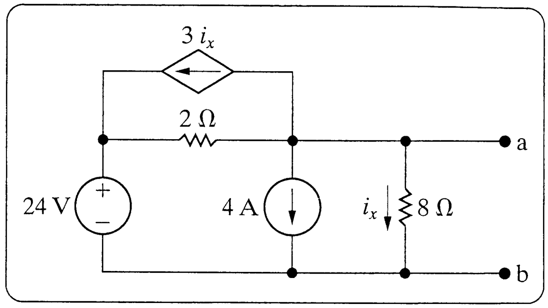
NOTE ◆ *Also try Chapter Problems 4.59, 4.62, and 4.63.*

ASSESSING OBJECTIVE 5

◆ **Understand Thèvenin and Norton equivalents**

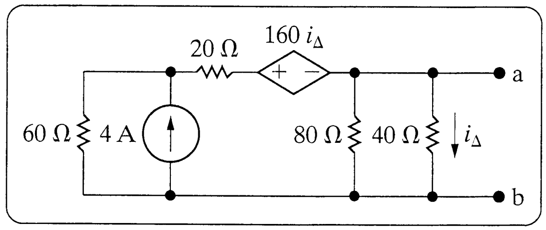
4.19 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown.

ANSWER: $V_{\text{Th}} = v_{\text{ab}} = 8$ V, $R_{\text{Th}} = 1\ \Omega$.



4.20 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown. (*Hint:* Define the voltage at the left-most node as v, and write two nodal equations with V_{Th} as the right node voltage.)

ANSWER: $V_{\text{Th}} = v_{\text{ab}} = 30$ V, $R_{\text{Th}} = 10\ \Omega$.



NOTE ◆ *Also try Chapter Problems 4.65 and 4.73.*

Using the Thévenin Equivalent in the Amplifier Circuit

At times we can use a Thévenin equivalent to reduce one portion of a circuit to greatly simplify analysis of the larger network. Let's return to the circuit first introduced in Section 2.5 and subsequently analyzed in Sections 4.4 and 4.7. To aid our discussion, we redrew the circuit and identified the branch currents of interest, as shown in Fig. 4.55.

As our previous analysis has shown, i_B is the key to finding the other branch currents. We redraw the circuit as shown in Fig. 4.56 to prepare to replace the subcircuit to the left of V_0 with its Thévenin equivalent. You should be able to determine that this modification has no effect on the branch currents i_1, i_2, i_B, and i_E.

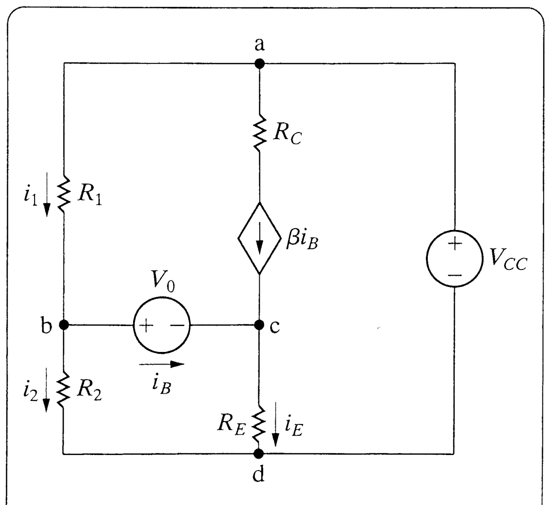


Figure 4.55 The application of a Thévenin equivalent in circuit analysis.

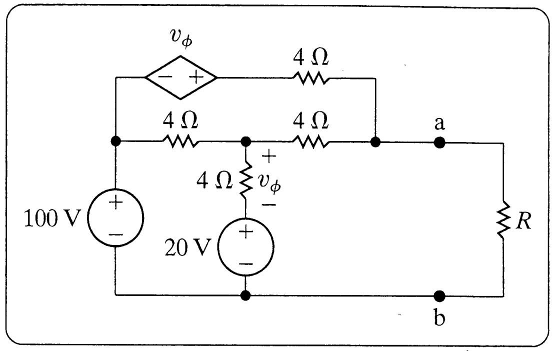
ASSESSING OBJECTIVE 6

◆ **Know the condition for and calculate maximum power transfer to resistive load**

4.21 a) Find the value of R that enables the circuit shown to deliver maximum power to the terminals a,b.

b) Find the maximum power delivered to R.

ANSWER: (a) 3 Ω; (b) 1.2 kW.



4.22 Assume that the circuit in Assessment Problem 4.21 is delivering maximum power to the load resistor R.

a) How much power is the 100 V source delivering to the network?

b) Repeat (a) for the dependent voltage source.

c) What percentage of the total power generated by these two sources is delivered to the load resistor R?

ANSWER: (a) 3000 W; (b) 800 W; (c) 31.58%.

NOTE ◆ *Also try Chapter Problems 4.75 and 4.76.*

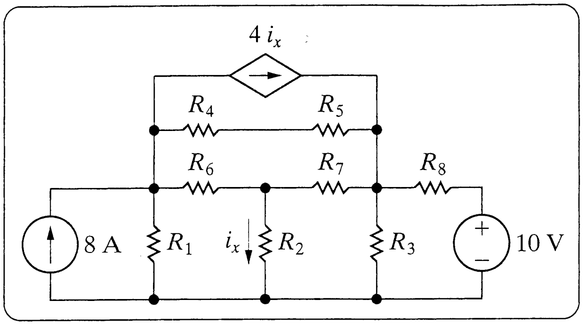
4.13 ◆ Superposition

A linear system obeys the principle of **superposition**, which states that whenever a linear system is excited, or driven, by more than one independent source of energy, the total response is the sum of the individual responses. An individual response is the result of an independent source acting alone. Because we are dealing with circuits made up of interconnected linear-circuit elements, we can apply the principle of superposition directly to the analysis of such circuits when they are driven by more than one independent energy source. At present, we restrict the discussion to simple resistive networks; however, the principle is applicable to any linear system.

PROBLEMS

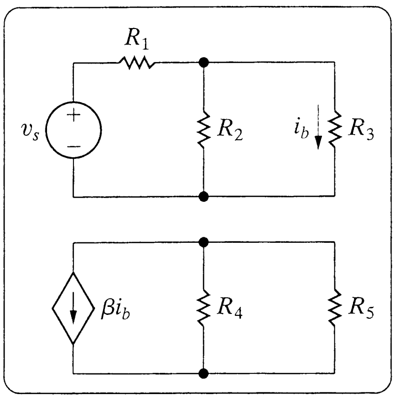
4.1 For the circuit shown in Fig. P4.1, state the numerical value of the number of (a) branches, (b) branches where the current is unknown, (c) essential branches, (d) essential branches where the current is unknown, (e) nodes, (f) essential nodes, and (g) meshes.

Figure P4.1



4.2
a) How many separate parts does the circuit in Fig. P4.2 have?
b) How many nodes?
c) How many branches are there?
d) Assume that the lower node in each part of the circuit is joined by a single conductor. Repeat the calculations in (a)–(c).

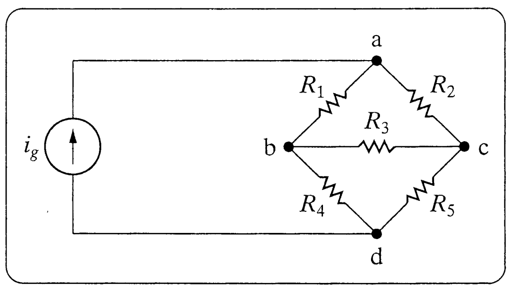
Figure P4.2



4.3
a) If only the essential nodes and branches are identified in the circuit in Fig. P4.1, how many simultaneous equations are needed to describe the circuit?
b) How many of these equations can be derived using Kirchhoff's current law?
c) How many must be derived using Kirchhoff's voltage law?
d) What two meshes should be avoided in applying the voltage law?

4.4 Assume the current i_g in the circuit in Fig. P4.4 is known. The resistors R_1–R_5 are also known.
a) How many unknown currents are there?
b) How many independent equations can be written using Kirchhoff's current law (KCL)?
c) Write an independent set of KCL equations.
d) How many independent equations can be derived from Kirchhoff's voltage law (KVL)?
e) Write a set of independent KVL equations.

Figure P4.4

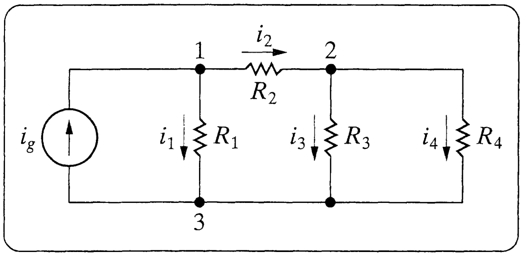


4.5 A current leaving a node is defined as positive.

a) Sum the currents at each node in the circuit shown in Fig. P4.5.

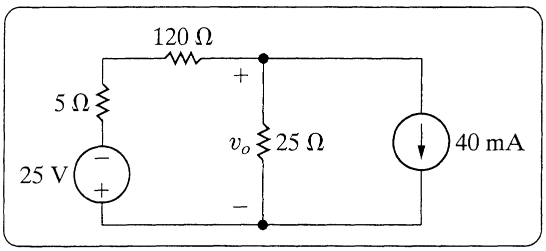
b) Show that any one of the equations in (a) can be derived from the remaining two equations.

Figure P4.5



4.6 Use the node-voltage method to find v_o in the circuit in Fig. P4.6.

Figure P4.6



4.7 a) Find the power developed by the 40 mA current source in the circuit in Fig. P4.6.

b) Find the power developed by the 25 V voltage source in the circuit in Fig. P4.6.

c) Verify that the total power developed equals the total power dissipated.

4.8 A 100 Ω resistor is connected in series with the 40 mA current source in the circuit in Fig. P4.6.

a) Find v_o.

b) Find the power developed by the 40 mA current source.

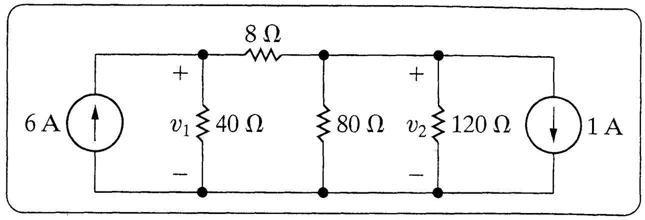
c) Find the power developed by the 25 V voltage source.

d) Verify that the total power developed equals the total power dissipated.

e) What effect will any finite resistance connected in series with the 40 mA current source have on the value of v_o?

4.9 Use the node-voltage method to find v_1 and v_2 in the circuit shown in Fig. P4.9.

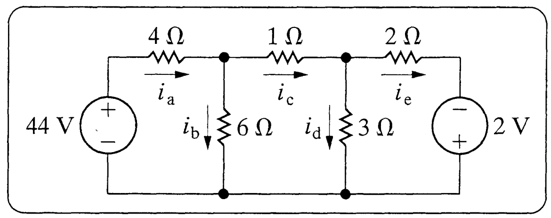
Figure P4.9



4.10 a) Use the node-voltage method to find the branch currents i_a–i_e in the circuit shown in Fig. P4.10.

b) Find the total power developed in the circuit.

Figure P4.10

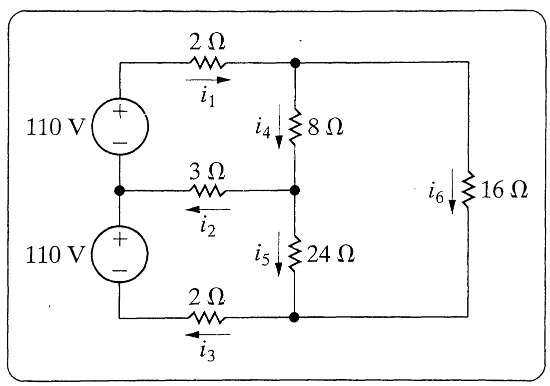


4.11 The circuit shown in Fig. P4.11 is a dc model of a residential power distribution circuit.

a) Use the node-voltage method to find the branch currents i_1–i_6.

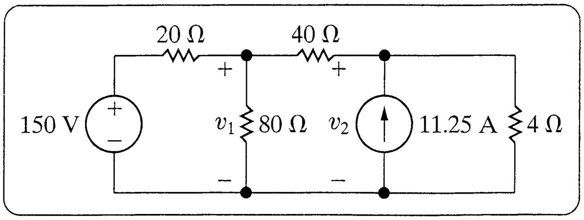
b) Test your solution for the branch currents by showing that the total power dissipated equals the total power developed.

Figure P4.11



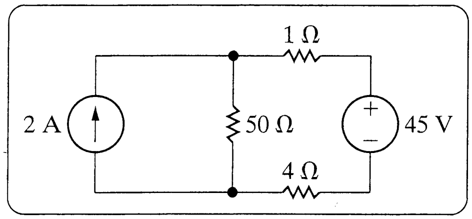
4.12 Use the node-voltage method to find v_1 and v_2 in the circuit in Fig. P4.12.

Figure P4.12



4.13 Use the node-voltage method to find how much power the 2 A source extracts from the circuit in Fig. P4.13.

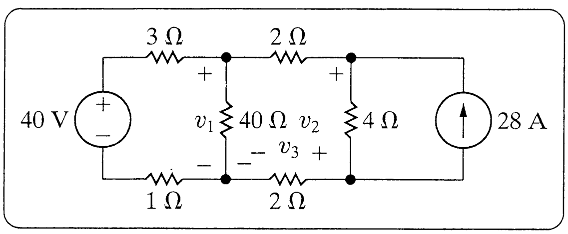
Figure P4.13



4.14
a) Use the node-voltage method to find v_1, v_2, and v_3 in the circuit in Fig. P4.14.

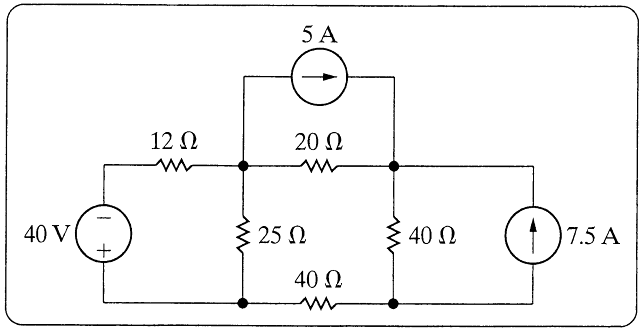
b) How much power does the 28 A current source deliver to the circuit?

Figure P4.14



4.15 Use the node-voltage method to find the total power dissipated in the circuit in Fig. P4.15.

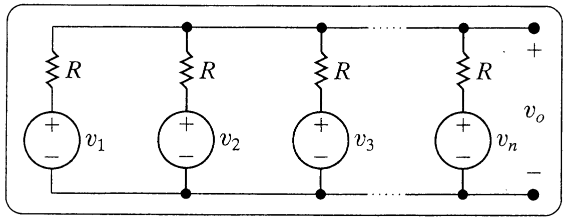
Figure P4.15



4.16
a) Use the node-voltage method to show that the output voltage v_o in the circuit in Fig. P4.16 is equal to the average value of the source voltages.

b) Find v_o if $v_1 = 120$ V, $v_2 = 60$ V, and $v_3 = -30$ V.

Figure P4.16

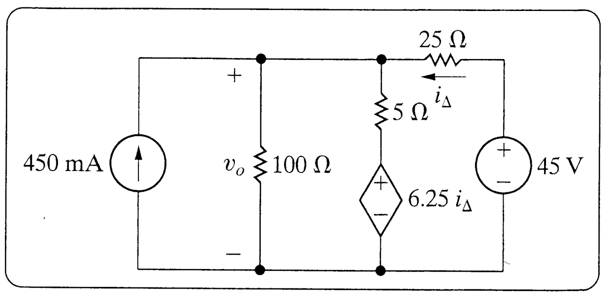


4.17
a) Use the node-voltage method to find v_o in the circuit in Fig. P4.17.

b) Find the power absorbed by the dependent source.

c) Find the total power developed by the independent sources.

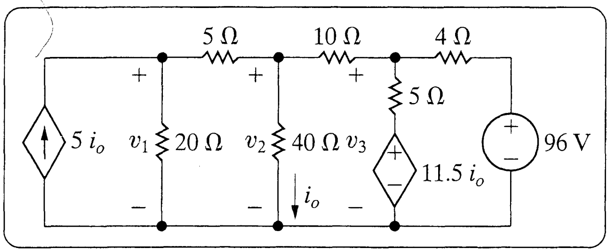
Figure P4.17



4.18 a) Find the node voltages v_1, v_2, and v_3 in the circuit in Fig. P4.18.

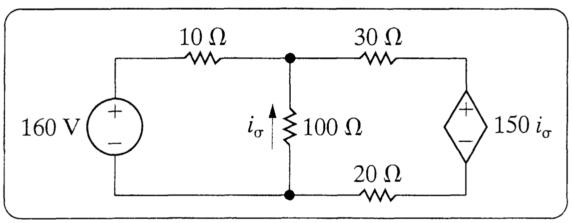
b) Find the total power dissipated in the circuit.

Figure P4.18



4.19 Use the node-voltage method to calculate the power delivered by the dependent voltage source in the circuit in Fig. P4.19.

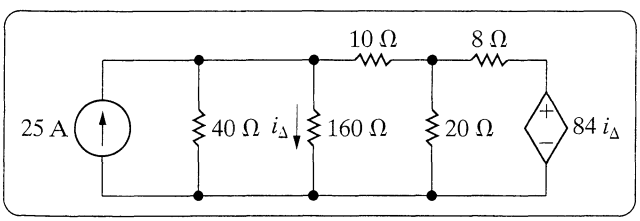
Figure P4.19



4.20 a) Use the node-voltage method to find the total power developed in the circuit in Fig. P4.20.

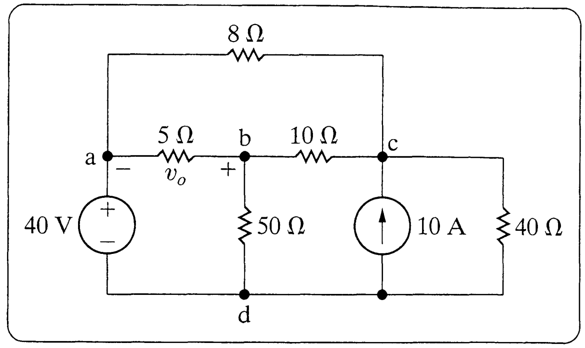
b) Check your answer by finding the total power absorbed in the circuit.

Figure P4.20



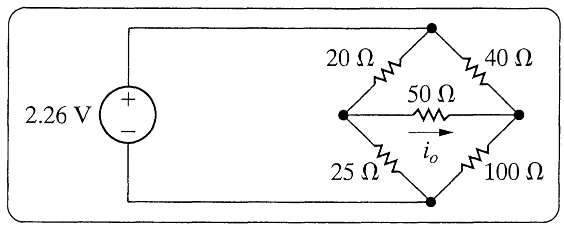
4.21 Use the node-voltage method to find the value of v_o in the circuit in Fig. P4.21.

Figure P4.21



4.22 Use the node-voltage method to find i_o in the circuit in Fig. P4.22.

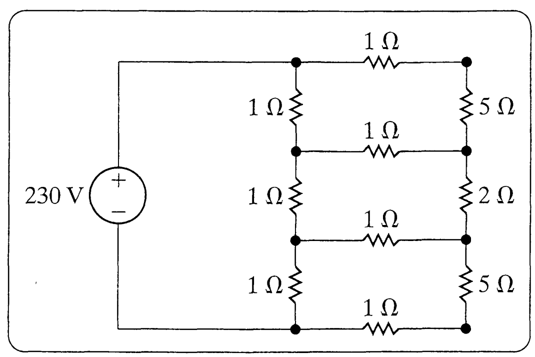
Figure P4.22



4.23 a) Use the node-voltage method to find the power dissipated in the 2 Ω resistor in the circuit in Fig. P4.23.

b) Find the power supplied by the 230 V source.

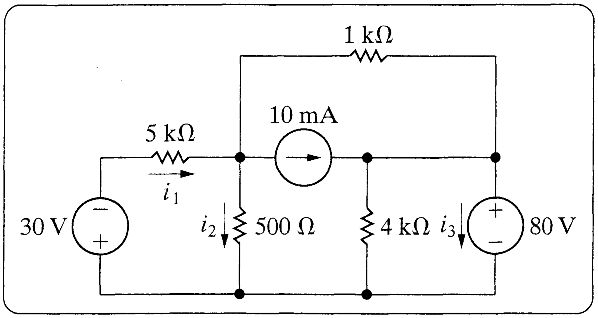
Figure P4.23



4.24 P a) Use the node-voltage method to find the branch currents i_1, i_2, and i_3 in the circuit in Fig. P4.24.

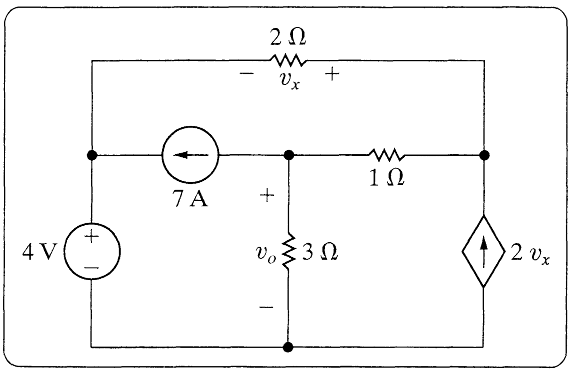
b) Check your solution for i_1, i_2, and i_3 by showing that the power dissipated in the circuit equals the power developed.

Figure P4.24



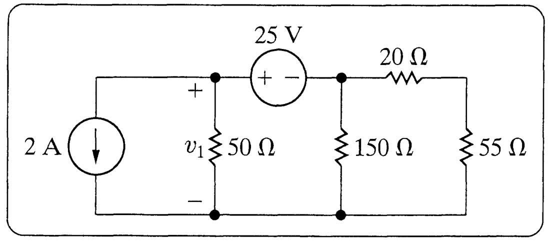
4.25 P Use the node-voltage method to find the value of v_o in the circuit in Fig. P4.25.

Figure P4.25



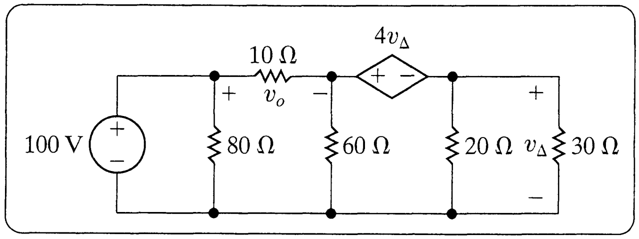
4.26 P Use the node-voltage method to find v_1 and the power delivered by the 25 V voltage source in the circuit in Fig. P4.26.

Figure P4.26



4.27 P Use the node-voltage method to find v_o in the circuit in Fig. P4.27.

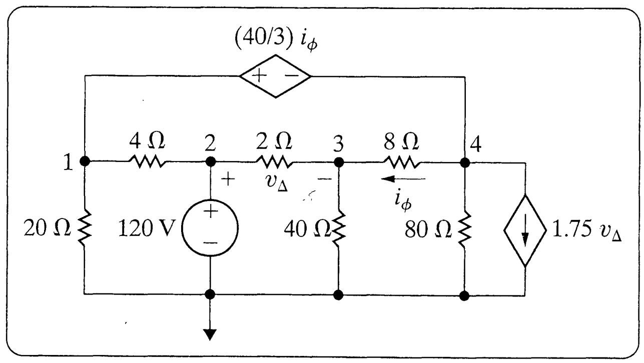
Figure P4.27



4.28 P Assume you are a project engineer and one of your staff is assigned to analyze the circuit shown in Fig. P4.28. The reference node and node numbers given on the figure were assigned by the analyst. Her solution gives the values of v_3 and v_4 as 108 V and 81.6 V, respectively.

Test these values by checking the total power developed in the circuit against the total power dissipated. Do you agree with the solution submitted by the analyst?

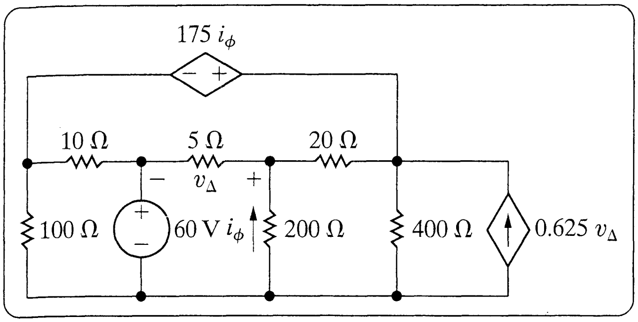
Figure P4.28



4.29 Use the node-voltage method to find the power developed by the 60 V source in the circuit in Fig. P4.29.

P

Figure P4.29



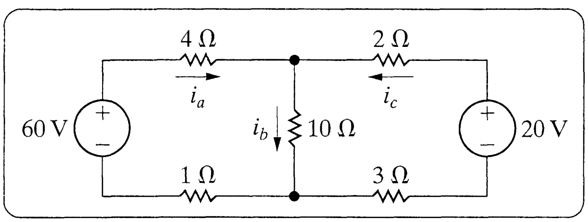
4.30 Show that when Eqs. 4.16, 4.17, and 4.19 are solved for i_B, the result is identical to Eq. 2.25.

4.31 a) Use the mesh-current method to find the branch currents i_a, i_b, and i_c in the circuit in Fig. P4.31.

P

b) Repeat (a) if the polarity of the 60 V source is reversed.

Figure P4.31

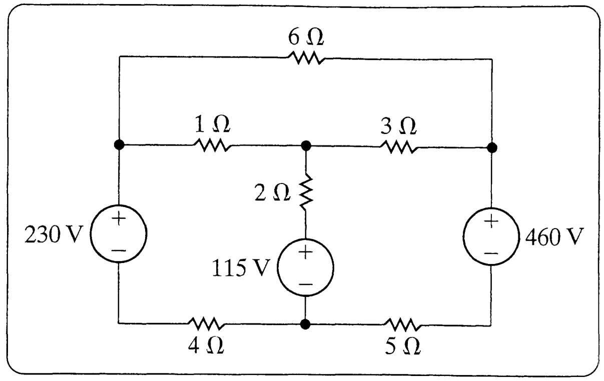


4.32 a) Use the mesh-current method to find the total power developed in the circuit in Fig. P4.32.

P

b) Check your answer by showing that the total power developed equals the total power dissipated.

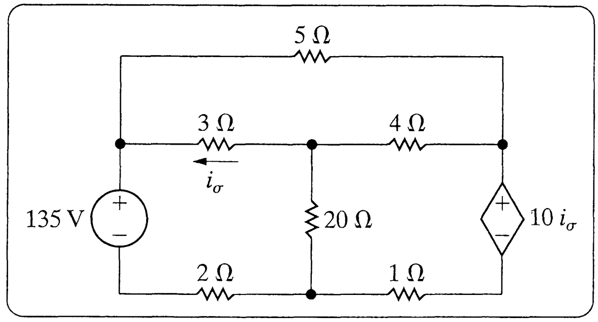
Figure P4.32



4.33 Use the mesh-current method to find the power dissipated in the 20 Ω resistor in the circuit in Fig. P4.33.

P

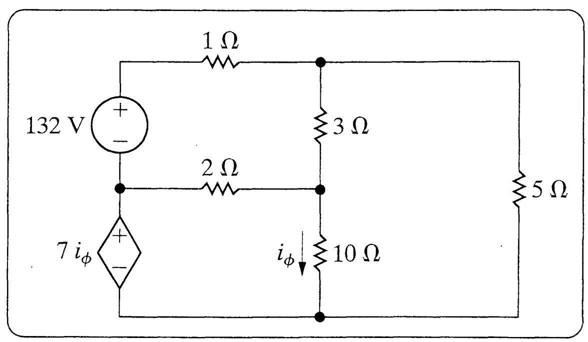
Figure P4.33



4.34 Use the mesh-current method to find the power delivered by the dependent voltage source in the circuit seen in Fig. P4.34.

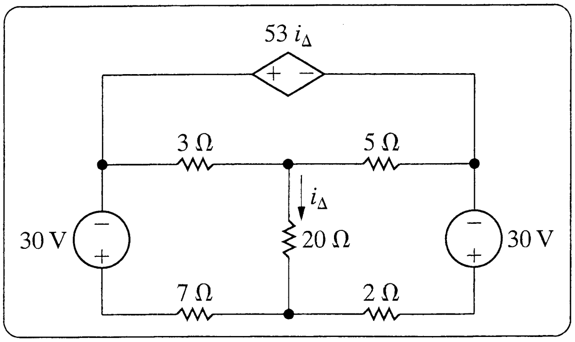
P

Figure P4.34



4.35 Use the mesh-current method to find the power developed in the dependent voltage source in the circuit in Fig. P4.35.

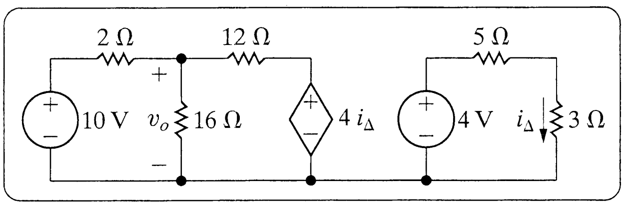
Figure P4.35



4.36 a) Use the mesh-current method to find v_o in the circuit in Fig. P4.36.

b) Find the power delivered by the dependent source.

Figure P4.36

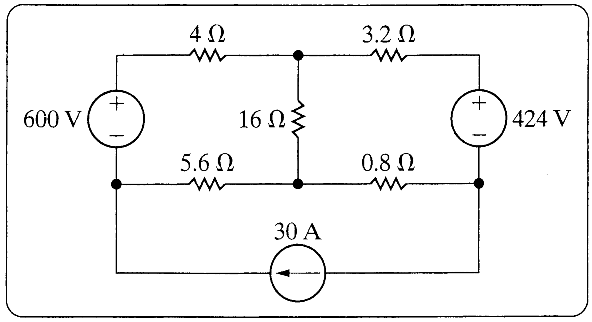


4.37 a) Use the mesh-current method to find how much power the 30 A current source delivers to the circuit in Fig. P4.37.

b) Find the total power delivered to the circuit.

c) Check your calculations by showing that the total power developed in the circuit equals the total power dissipated.

Figure P4.37

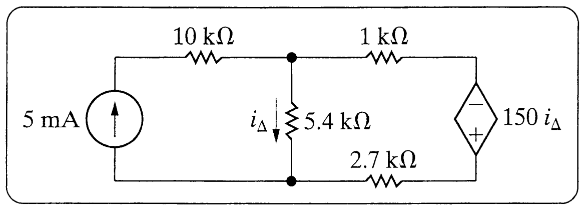


4.38 a) Use the mesh-current method to solve for i_Δ in the circuit in Fig. P4.38.

b) Find the power delivered by the independent current source.

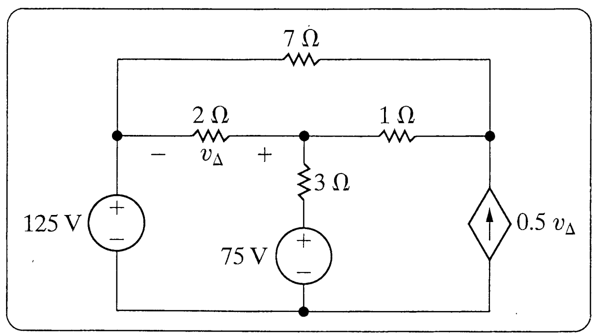
c) Find the power delivered by the dependent voltage source.

Figure P4.38



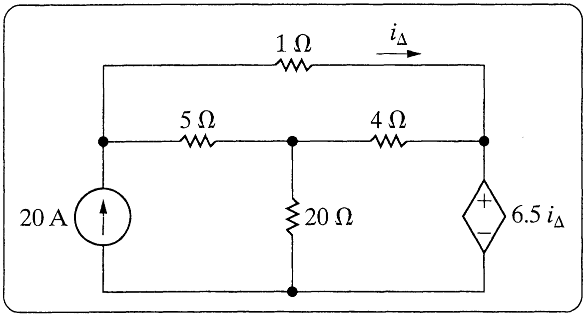
4.39 Use the mesh-current method to find the total power developed in the circuit in Fig. P4.39.

Figure P4.39



4.40 Use the mesh-current method to find the power developed by the 20 A source in the circuit in Fig. P4.40.

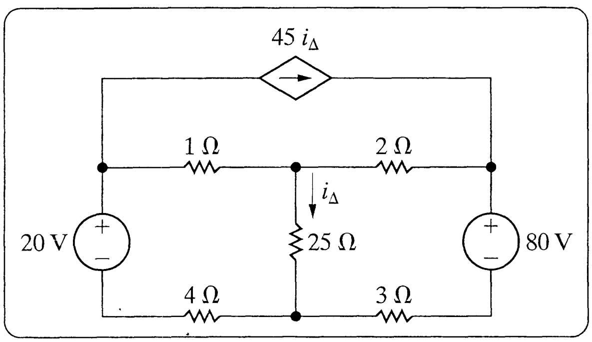
Figure P4.40



4.41 a) Use the mesh-current method to find the power delivered to the 2 Ω resistor in the circuit in Fig. P4.41.

b) What percentage of the total power developed in the circuit is delivered to the 2 Ω resistor?

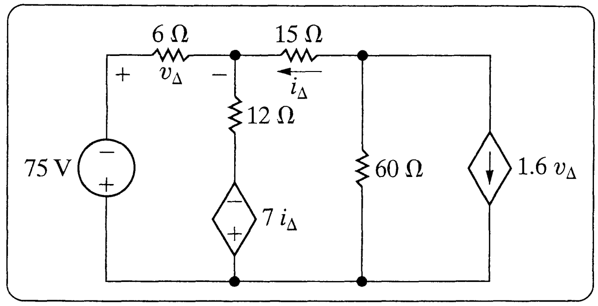
Figure P4.41



4.42 a) Use the mesh-current method to determine which sources in the circuit in Fig. P4.42 are generating power.

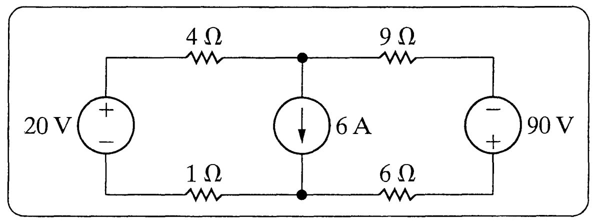
b) Find the total power dissipated in the circuit.

Figure P4.42



4.43 Use the mesh-current method to find the total power dissipated in the circuit in Fig. P4.43.

Figure P4.43



4.44 Assume the 20 V source in the circuit in Fig. P4.43 is increased to 120 V. Find the total power dissipated in the circuit.

4.45 a) Assume the 20 V source in the circuit in Fig. P4.43 is changed to 60 V. Find the total power dissipated in the circuit.

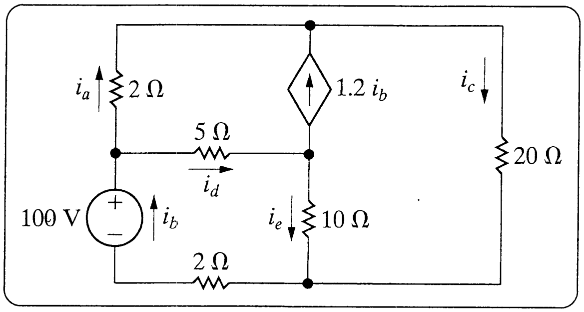
b) Repeat (a) if the 6 A current source is replaced by a short circuit.

c) Explain why the answers to (a) and (b) are the same.

4.46 a) Use the mesh-current method to find the branch currents in i_a–i_e in the circuit in Fig. P4.46.

b) Check your solution by showing that the total power developed in the circuit equals the total power dissipated.

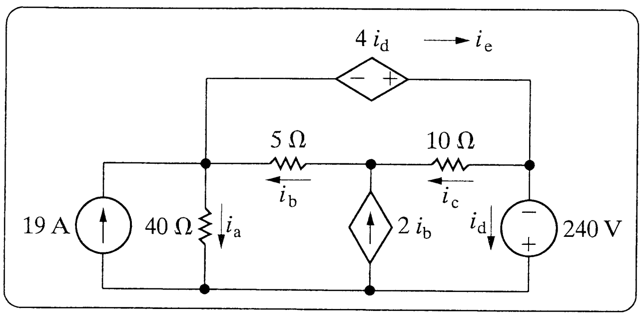
Figure P4.46



4.47 a) Find the branch currents i_a–i_e for the circuit shown in Fig. P4.47.

b) Check your answers by showing that the total power generated equals the total power dissipated.

Figure P4.47



4.48 The circuit in Fig. P4.48 is a direct-current version of a typical three-wire distribution system. The resistors R_a, R_b, and R_c represent the resistances of the three conductors that connect the three loads R_1, R_2, and R_3 to the 125/250 V voltage supply. The resistors R_1 and R_2 represent loads connected to the 125 V circuits, and R_3 represents a load connected to the 250 V circuit.

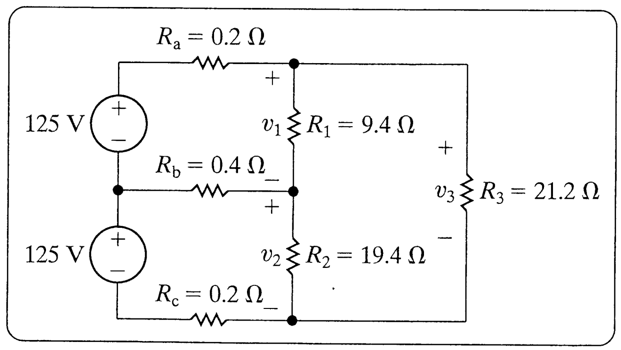
a) Calculate v_1, v_2, and v_3.

b) Calculate the power delivered to R_1, R_2, and R_3.

c) What percentage of the total power developed by the sources is delivered to the loads?

d) The R_b branch represents the neutral conductor in the distribution circuit. What adverse effect occurs if the neutral conductor is opened? (*Hint:* Calculate v_1 and v_2 and note that appliances or loads designed for use in this circuit would have a nominal voltage rating of 125 V.)

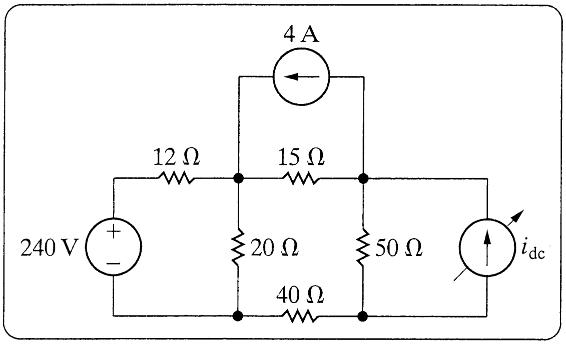
Figure P4.48



4.49 Show that whenever $R_1 = R_2$ in the circuit in Fig. P4.48, the current in the neutral conductor is zero. (*Hint:* Solve for the neutral conductor current as a function of R_1 and R_2).

4.50 The variable dc current source in the circuit in Fig. P4.50 is adjusted so that the power developed by the 4 A current source is zero. Find the value of i_{dc}.

Figure P4.50

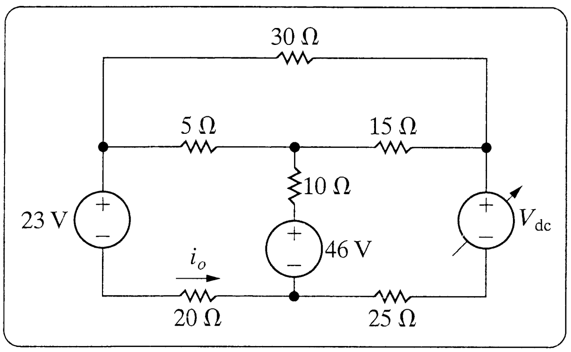


4.51 The variable dc voltage source in the circuit in Fig. P4.51 is adjusted so that i_o is zero.

a) Find the value of V_{dc}.

b) Check your solution by showing the power developed equals the power dissipated.

Figure P4.51



4.52 Assume you have been asked to find the power dissipated in the 10 Ω resistor in the circuit in Fig. P4.52.

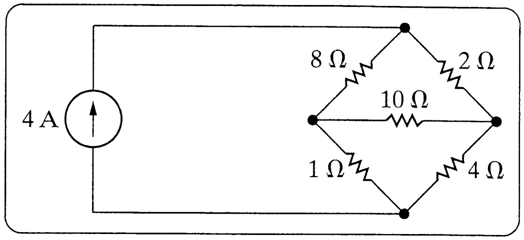
a) Which method of circuit analysis would you recommend? Explain why.

b) Use your recommended method of analysis to find the power dissipated in the 10 Ω resistor.

c) Would you change your recommendation if the problem had been to find the power developed by the 4 A current source? Explain.

d) Find the power delivered by the 4 A current source.

Figure P4.52



4.53 A 20 Ω resistor is placed in parallel with the 4 A current source in the circuit in Fig. P4.52. Assume you have been asked to calculate the power developed by the current source.

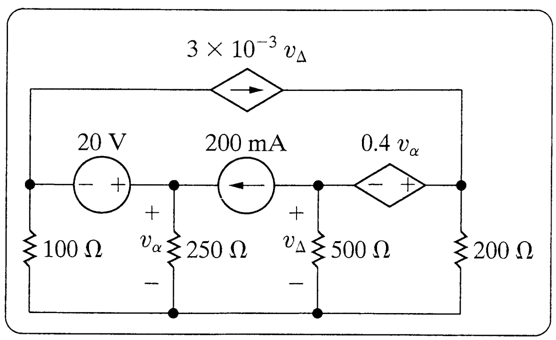
a) Which method of circuit analysis would you recommend? Explain why.

b) Find the power developed by the current source.

4.54 a) Would you use the node-voltage or mesh-current method to find the power absorbed by the 20 V source in the circuit in Fig. P4.54? Explain your choice.

b) Use the method you selected in (a) to find the power.

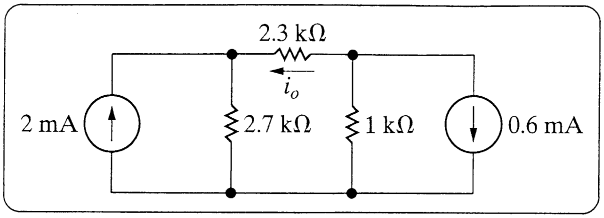
Figure P4.54



4.55 a) Use a series of source transformations to find the current i_o in the circuit in Fig. P4.55.

b) Verify your solution by using the node-voltage method to find i_o.

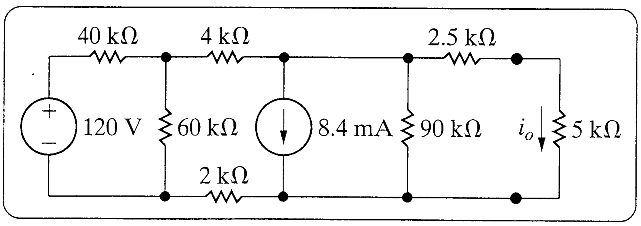
Figure P4.55



4.56 a) Find the current in the 5 kΩ resistor in the circuit in Fig. P4.56 by making a succession of appropriate source transformations.

b) Using the result obtained in (a), work back through the circuit to find the power developed by the 120 V source.

Figure P4.56



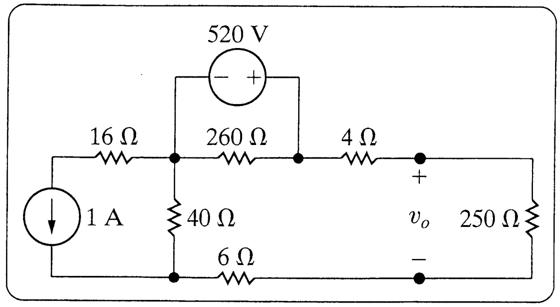
4.57 a) Use source transformations to find v_o in the circuit in Fig. P4.57.

b) Find the power developed by the 520 V source.

c) Find the power developed by the 1 A current source.

d) Verify that the total power developed equals the total power dissipated.

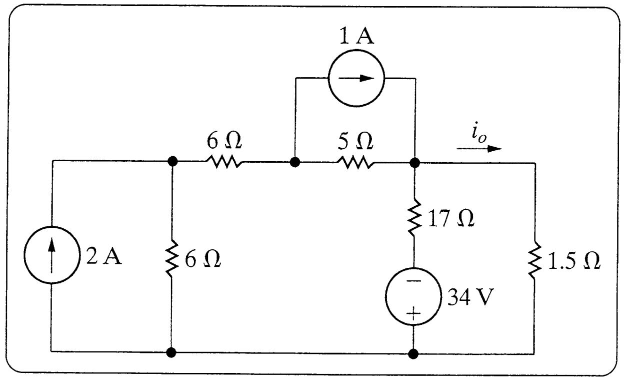
Figure P4.57



4.58 a) Use a series of source transformations to find i_o in the circuit in Fig. P4.58.

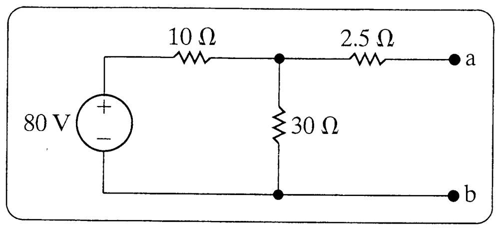
b) Verify your solution by using the mesh-current method to find i_o.

Figure P4.58



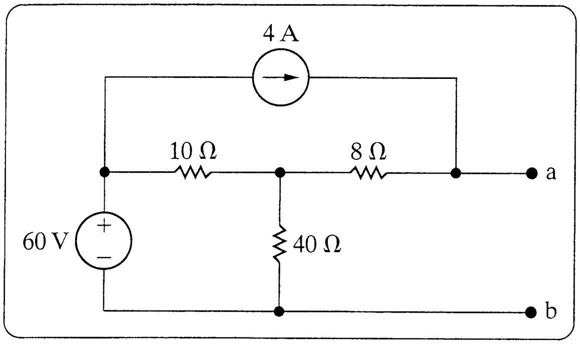
4.59 Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.59.

Figure P4.59



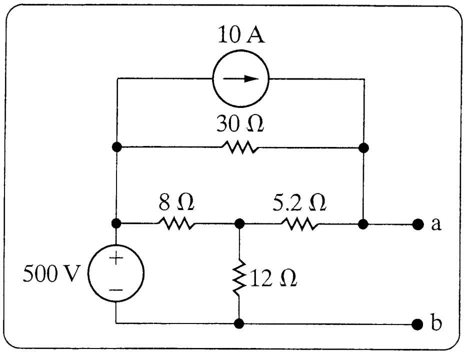
4.60 Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.60.

Figure P4.60



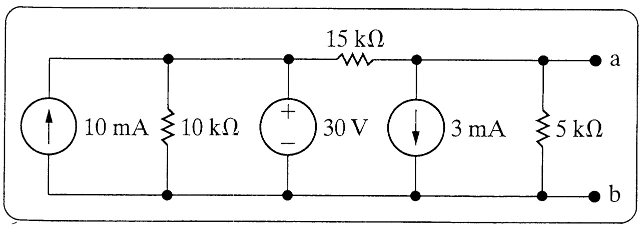
4.61 Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.61.

Figure P4.61



4.62 Find the Norton equivalent with respect to the terminals a,b in the circuit in Fig. P4.62.

Figure P4.62

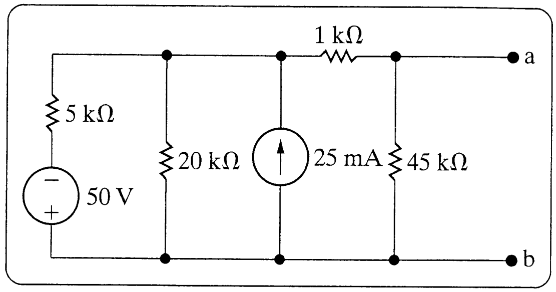


4.63 A voltmeter with a resistance of 85.5 kΩ is used to measure the voltage v_{ab} in the circuit in Fig. P4.63.

a) What is the voltmeter reading?

b) What is the percentage of error in the voltmeter reading if the percentage of error is defined as [(measured − actual)/actual] × 100?

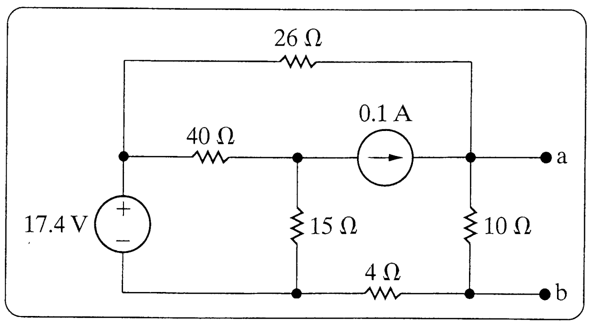
Figure P4.63



4.64 a) Find the Thévenin equivalent with respect to the terminals a,b for the circuit in Fig. P4.64 by finding the open-circuit voltage and the short-circuit current.

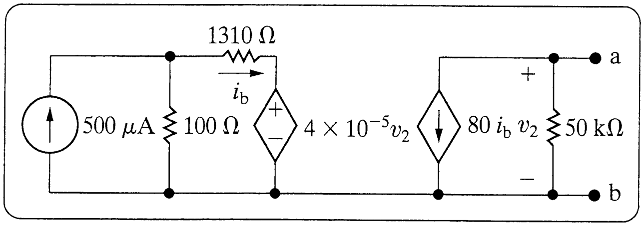
b) Solve for the Thévenin resistance by removing the independent sources. Compare your result to the Thévenin resistance found in (a).

Figure P4.64



4.65 Determine the Thévenin equivalent with respect to the terminals a,b for the circuit shown in Fig. P4.65.

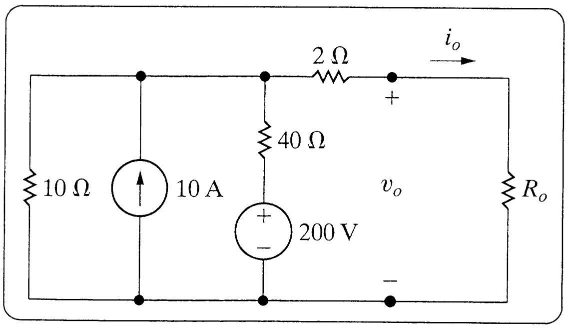
Figure P4.65



4.66 An automobile battery, when connected to a car radio, provides 12.72 V to the radio. When connected to a set of headlights, it provides 12 V to the headlights. Assume the radio can be modeled as a 6.36 Ω resistor and the headlights can be modeled as a 0.60 Ω resistor. What are the Thévenin and Norton equivalents for the battery?

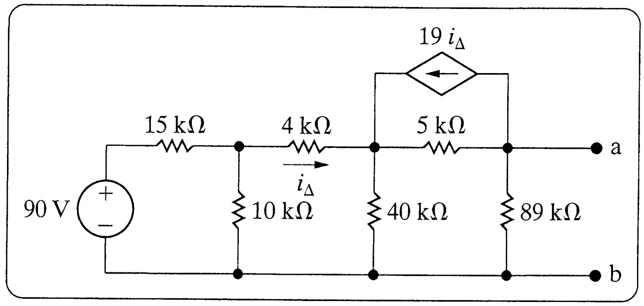
4.67 Determine i_o and v_o in the circuit shown in Fig. P4.67 when R_o is 0, 2, 6, 10, 15, 20, 30, 40, 50, and 70 Ω.

Figure P4.67



4.68 Find the Thévenin equivalent with respect to the terminals a,b for the circuit seen in Fig. P4.68.

Figure P4.68

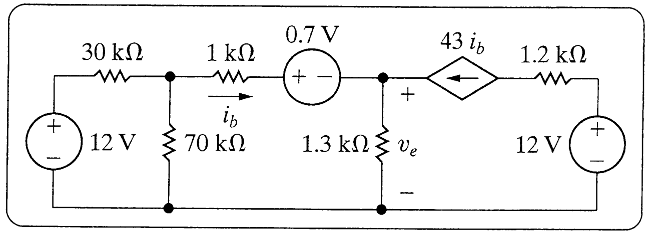


4.69 When a voltmeter is used to measure the voltage v_e in Fig. P4.69, it reads 5.5 V.

a) What is the resistance of the voltmeter?

b) What is the percentage of error in the voltage measurement?

Figure P4.69

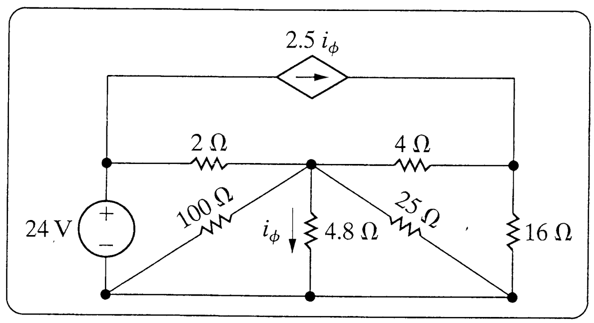


4.70 When an ammeter is used to measure the current i_ϕ in the circuit shown in Fig. P4.70, it reads 6 A.

a) What is the resistance of the ammeter?

b) What is the percentage of error in the current measurement?

Figure P4.70



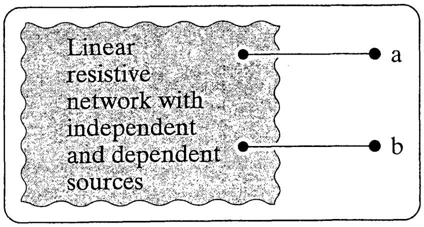
4.71 A Thévenin equivalent can also be determined from measurements made at the pair of terminals of interest. Assume the following measurements were made at the terminals a,b in the circuit in Fig. P4.71.

When a 20 kΩ resistor is connected to the terminals a,b, the voltage v_{ab} is measured and found to be 100 V.

When a 50 kΩ resistor is connected to the terminals a,b, the voltage is measured and found to be 200 V.

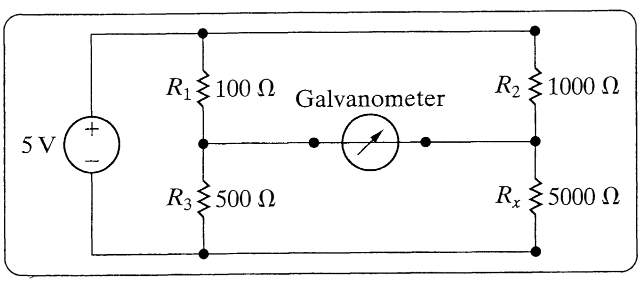
Find the Thévenin equivalent of the network with respect to the terminals a,b.

Figure P4.71



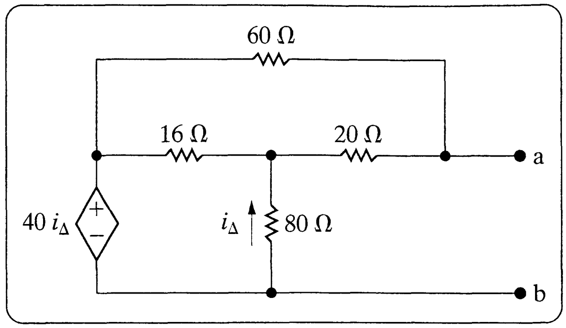
4.72 The Wheatstone bridge in the circuit shown in Fig. P4.72 is balanced when R_3 equals 500 Ω. If the galvanometer has a resistance of 50 Ω, how much current will the galvanometer detect when the bridge is unbalanced by setting R_3 to 501 Ω? (*Hint:* Find the Thévenin equivalent with respect to the galvanometer terminals when $R_3 = 501$ Ω. Note that once we have found this Thévenin equivalent, it is easy to find the amount of unbalanced current in the galvanometer branch for different galvanometer movements.)

Figure P4.72



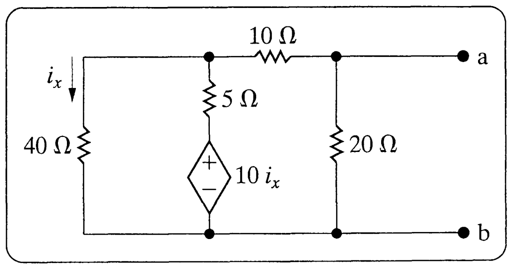
4.73 Find the Thévenin equivalent with respect to the terminals a,b in the circuit in Fig. P4.73.

Figure P4.73



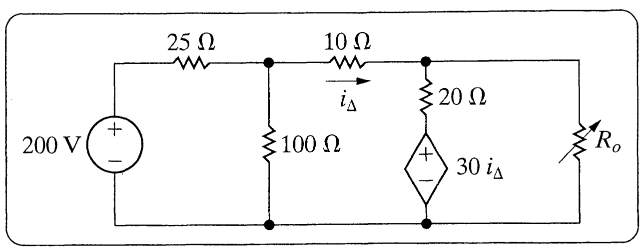
4.74 Find the Thévenin equivalent with respect to the terminals a,b for the circuit seen in Fig. P4.74.

Figure P4.74



4.75 The variable resistor (R_o) in the circuit in Fig. P4.75 is adjusted until the power dissipated in the resistor is 250 W. Find the values of R_o that satisfy this condition.

Figure P4.75

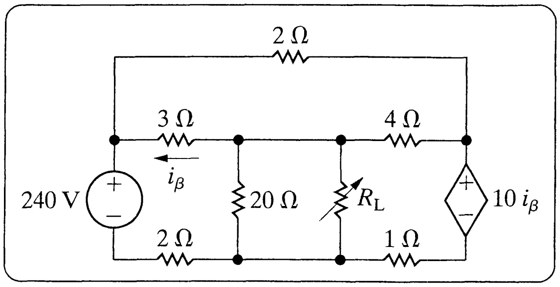


4.76 The variable resistor (R_L) in the circuit in Fig. P4.76 is adjusted for maximum power transfer to R_L.

a) Find the numerical value of R_L.

b) Find the maximum power transferred to R_L.

Figure P4.76

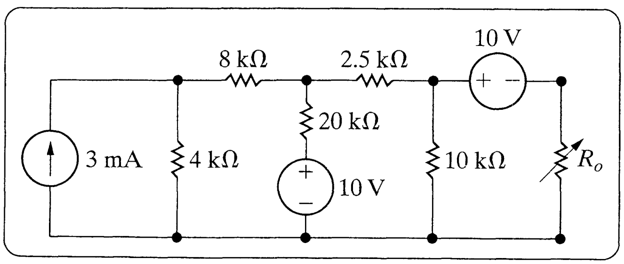


4.77 The variable resistor in the circuit in Fig. P4.77 is adjusted for maximum power transfer to R_o.

a) Find the value of R_o.

b) Find the maximum power that can be delivered to R_o.

Figure P4.77



4.78 What percentage of the total power developed in the circuit in Fig. P4.77 is delivered to R_o when R_o is set for maximum power transfer?

4.79 A variable resistor R_o is connected across the terminals a,b in the circuit in Fig. P4.68. The variable resistor is adjusted until maximum power is transferred to R_o.

a) Find the value of R_o.

b) Find the maximum power delivered to R_o.

c) Find the percentage of the total power developed in the circuit that is delivered to R_o.

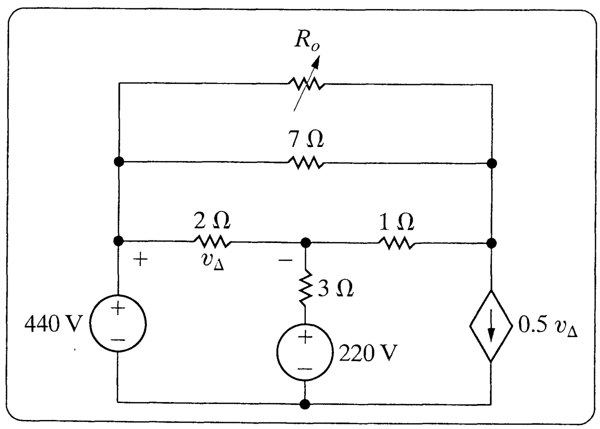
4.80 a) Calculate the power delivered for each value of R_o used in Problem 4.67.

b) Plot the power delivered to R_o versus the resistance R_o.

c) At what value of R_o is the power delivered to R_o a maximum?

4.81 The variable resistor (R_o) in the circuit in Fig. P4.81 is adjusted for maximum power transfer to R_o. What percentage of the total power developed in the circuit is delivered to R_o?

Figure P4.81



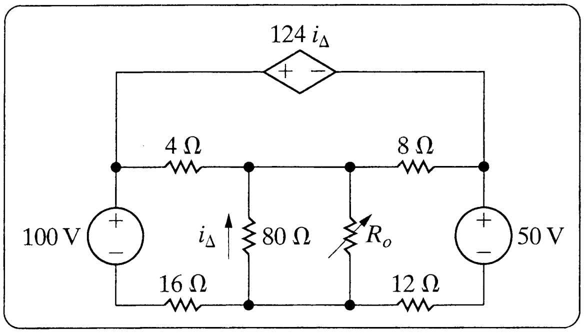
4.82 The variable resistor (R_o) in the circuit in Fig. P4.82 is adjusted for maximum power transfer to R_o.

P

a) Find the value of R_o.

b) Find the maximum power that can be delivered to R_o.

Figure P4.82



4.83 What percentage of the total power developed in the circuit in Fig. P4.82 is delivered to R_o?

P

4.84 The variable resistor (R_o) in the circuit in Fig. P4.84 is adjusted until it absorbs maximum power from the circuit.

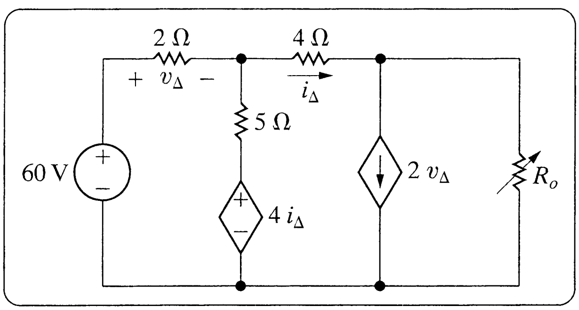
P

a) Find the value of R_o.

b) Find the maximum power.

c) Find the percentage of the total power developed in the circuit that is delivered to R_o.

Figure P4.84



4.85 The variable resistor in the circuit in Fig. P4.85 is adjusted for maximum power transfer to R_o.

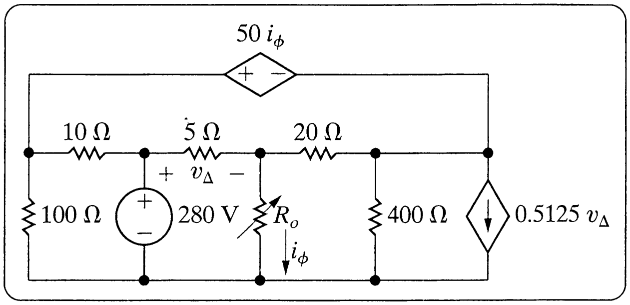
P

a) Find the numerical value of R_o.

b) Find the maximum power delivered to R_o.

c) How much power does the 280 V source deliver to the circuit when R_o is adjusted to the value found in (a)?

Figure P4.85

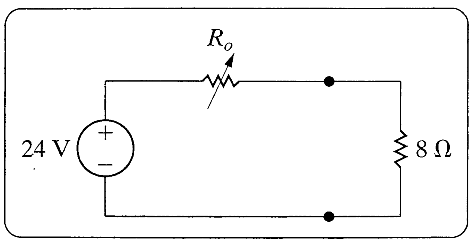


4.86 a) Find the value of the variable resistor R_o in the circuit in Fig. P4.86 that will result in maximum power dissipation in the 8 Ω resistor. (*Hint:* Hasty conclusions could be hazardous to your career.)

P

b) What is the maximum power that can be delivered to the 8 Ω resistor?

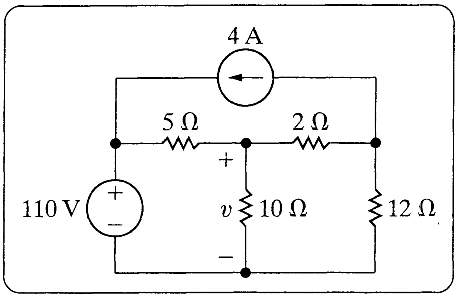
Figure P4.86



4.87 a) Use the principle of superposition to find the voltage v in the circuit of Fig. P4.87.

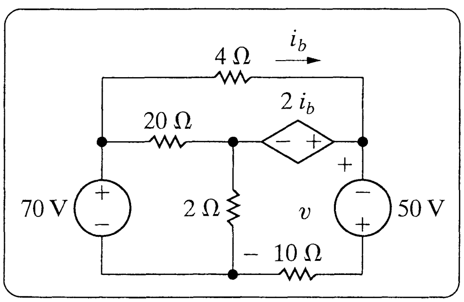
b) Find the power dissipated in the 10 Ω resistor.

Figure P4.87



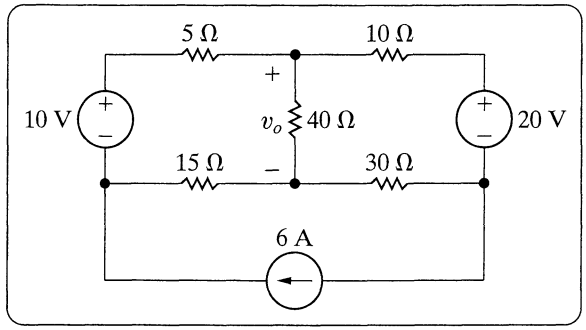
4.88 Use the principle of superposition to find the voltage v in the circuit of Fig. P4.88.

Figure P4.88



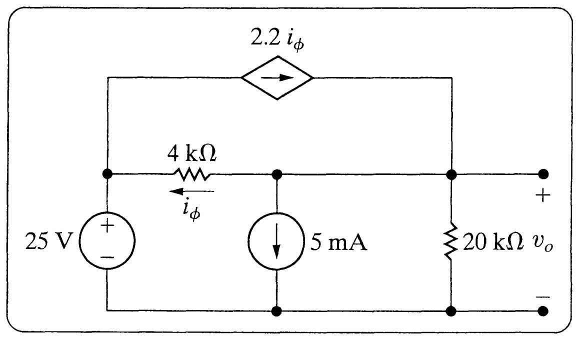
4.89 Use the principle of superposition to find the voltage v_o in the circuit in Fig. P4.89.

Figure P4.89



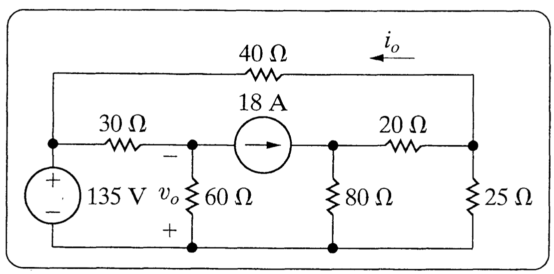
4.90 Use the principle of superposition to find v_o in the circuit in Fig. P4.90.

Figure P4.90



4.91 Use superposition to solve for i_o and v_o in the circuit in Fig. P4.91.

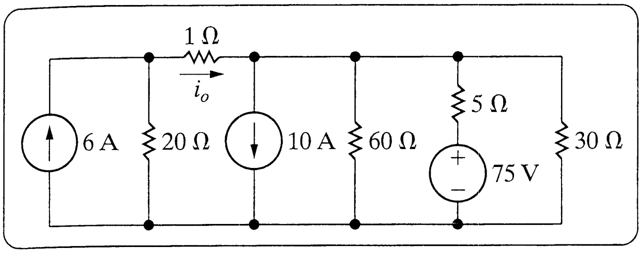
Figure P4.91



4.92 Use the principle of superposition to find the current i_o in the circuit shown in Fig. P4.92.

P

Figure P4.92

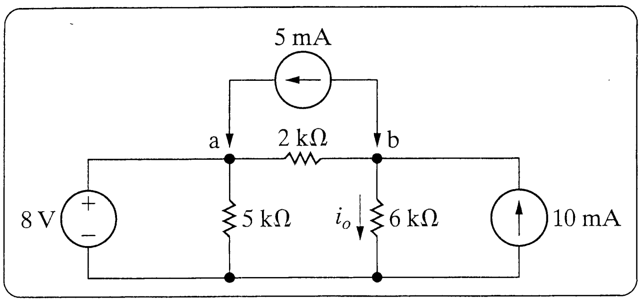


4.93 a) In the circuit in Fig. P4.93, before the 5 mA current source is attached to the terminals a,b, the current i_o is calculated and found to be 3.5 mA. Use superposition to find the value of i_o after the current source is attached.

P

b) Verify your solution by finding i_o when all three sources are acting simultaneously.

Figure P4.93



4.94 Laboratory measurements on a dc voltage source yield a terminal voltage of 75 V with no load connected to the source and 60 V when loaded with a 20 Ω resistor.

a) What is the Thévenin equivalent with respect to the terminals of the dc voltage source?

b) Show that the Thévenin resistance of the source is given by the expression

$$R_{\text{Th}} = \left(\frac{v_{\text{Th}}}{v_o} - 1\right) R_{\text{L}},$$

where

v_{Th} = the Thévenin voltage

v_o = the terminal voltage corresponding to the load resistance R_{L}.

4.95 Two ideal dc voltage sources are connected by electrical conductors that have a resistance of r Ω/m, as shown in Fig. P4.95. A load having a resistance of R Ω moves between the two voltage sources. Let x equal the distance between the load and the source v_1, and let L equal the distance between the sources.

a) Show that

$$v = \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}.$$

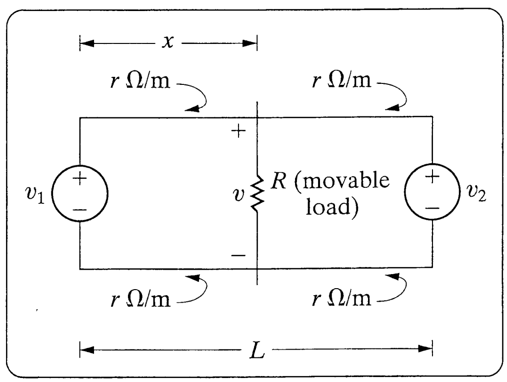
b) Show that the voltage v will be minimum when

$$x = \frac{L}{v_2 - v_1}\left[-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_1 - v_2)^2}\right].$$

c) Find x when $L = 16$ km, $v_1 = 1000$ V, $v_2 = 1200$ V, $R = 3.9$ Ω, and $r = 5 \times 10^{-5}$ Ω/m.

d) What is the minimum value of v for the circuit of part (c)?

Figure P4.95

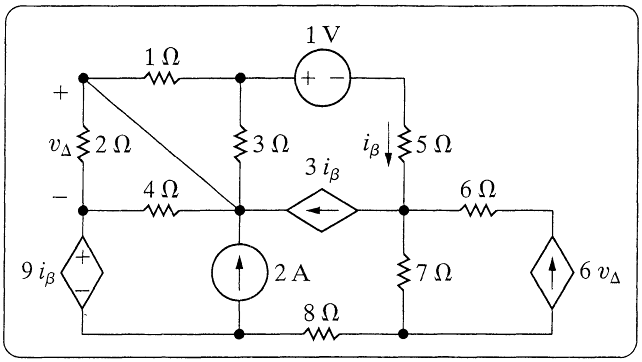


4.96 Assume your supervisor has asked you to determine the power developed by the 1 V source in the circuit in Fig. P4.96. Before calculating the power developed by the 1 V source, the supervisor asks you to submit a proposal describing how you plan to attack the problem. Furthermore, he asks you to explain why you have chosen your proposed method of solution.

a) Describe your plan of attack, explaining your reasoning.

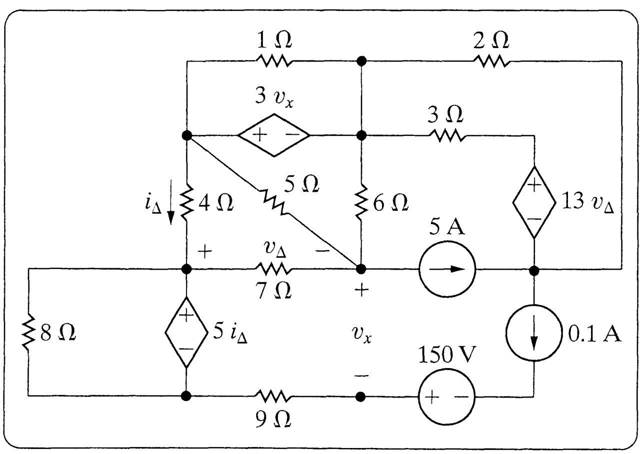
b) Use the method you have outlined in (a) to find the power developed by the 1 V source.

Figure P4.96



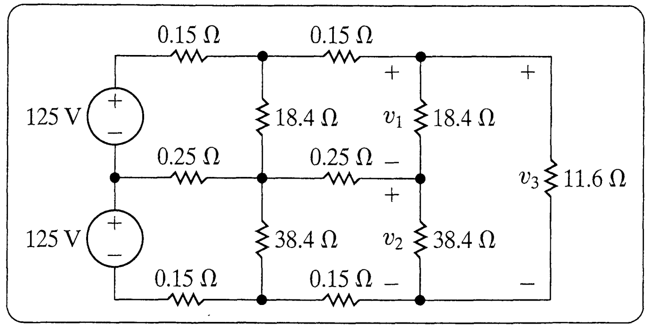
4.97 Find the power absorbed by the 5 A current source in the circuit in Fig. P4.97.

Figure P4.97



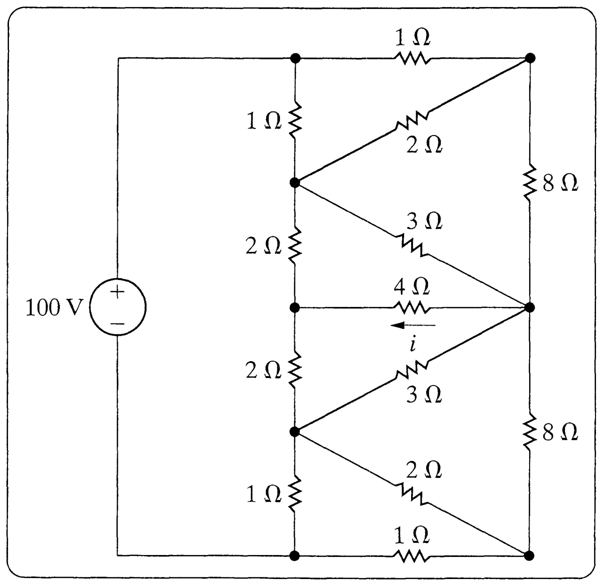
4.98 Find v_1, v_2, and v_3 in the circuit in Fig. P4.98.

Figure P4.98



4.99 Find i_1 in the circuit in Fig. P4.99.

Figure P4.99



4.100 For the circuit in Fig. 4.69 derive the expressions for the sensitivity of v_1 and v_2 to changes in the source currents I_{g1} and I_{g2}.

4.101 ◆ P Assume the nominal values for the components in the circuit in Fig. 4.69 are: $R_1 = 25\ \Omega$; $R_2 = 5\ \Omega$; $R_3 = 50\ \Omega$; $R_4 = 75\ \Omega$; $I_{g1} = 12$ A; and $I_{g2} = 16$ A. Predict the values of v_1 and v_2 if I_{g1} decreases to 11 A and all other components stay at their nominal values. Check your predictions using a tool like PSpice or MATLAB.

4.102 ◆ P Repeat Problem 4.101 if I_{g2} increases to 17 A, and all other components stay at their nominal values. Check your predictions using a tool like PSpice or MATLAB.

4.103 ◆ P Repeat Problem 4.101 if I_{g1} decreases to 11 A and I_{g2} increases to 17 A. Check your predictions using a tool like PSpice or MATLAB.

4.104 ◆ Use the results given in Table 4.2 to predict the values of v_1 and v_2 if R_1 and R_3 increase to 10% above their nominal values and R_2 and R_4 decrease to 10% below their nominal values. I_{g1} and I_{g2} remain at their nominal values. Compare your predicted values of v_1 and v_2 with their actual values.

CHAPTER

5

The Operational Amplifier

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Be able to name the five op amp terminals and describe and use the voltage and current constraints and the resulting simplifications they lead to in an ideal op amp.
2. Be able to analyze simple circuits containing ideal op amps, and recognize the following op amp circuits: inverting amplifier, summing amplifier, noninverting amplifier, and difference amplifier.
3. Understand the more realistic model for an op amp and be able to use this model to analyze simple circuits containing op amps.

The electronic circuit known as an operational amplifier has become increasingly important. However, a detailed analysis of this circuit requires an understanding of electronic devices such as diodes and transistors. You may wonder, then, why we are introducing the circuit before discussing the circuit's electronic components. There are several reasons. First, you can develop an appreciation for how the operational amplifier can be used as a circuit building block by focusing on its terminal behavior. At an introductory level, you need not fully understand the operation of the electronic components that govern terminal behavior. Second, the circuit model of the operational amplifier requires the use of a dependent source. Thus you have a chance to use this type of source in a practical circuit rather than as an abstract circuit component. Third, you can combine the operational amplifier with resistors to perform some very useful functions, such as scaling, summing, sign changing, and subtracting. Finally, after introducing inductors and capacitors in Chapter 6, we can show you how to use the operational amplifier to design integrating and differentiating circuits.

Our focus on the terminal behavior of the operational amplifier implies taking a black box approach to its operation; that is, we are not interested in the internal structure of the amplifier nor in the currents and voltages that exist in this structure. The important thing to remember is that the internal behavior of the amplifier accounts for the voltage and current constraints imposed at the terminals. (For now, we ask that you accept these constraints on faith.)

Practical Perspective

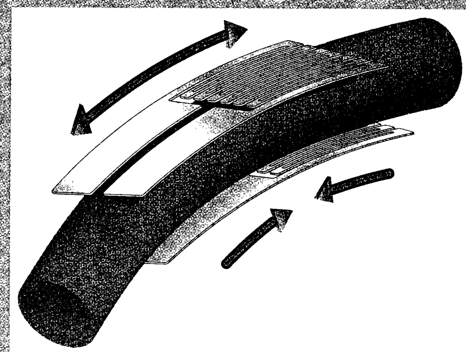
Strain Gages

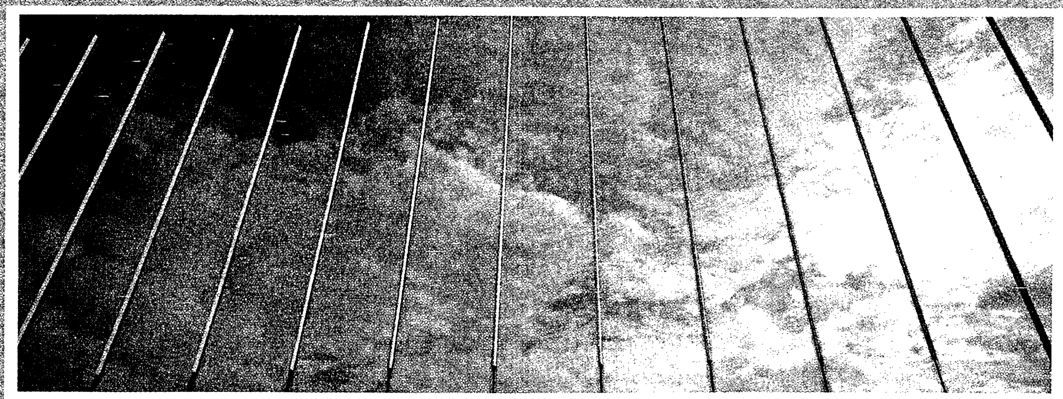
How could you measure the amount of bending in a metal bar such as the one shown in the figure without physically contacting the bar? One method would be to use a strain gage. A strain gage is a type of **transducer**. A transducer is a device that measures a quantity by converting it into a more convenient form. The quantity we wish to measure in the metal bar is the bending angle, but measuring the angle directly is quite difficult and could even be dangerous. Instead, we attach a strain gage (shown in the line drawing here) to the metal bar. A strain gage is a grid of thin wires whose resistance changes when the wires are lengthened or shortened:

$$\Delta R = 2R\frac{\Delta L}{L}$$

where R is the resistance of the gage at rest, $\Delta L/L$ is the fractional lengthening of the gage (which is the definition of "strain"), the constant 2 is typical of the manufacturer's gage factor, and ΔR is the change in resistance due to the bending of the bar. Typically, pairs of strain gages are attached to opposite sides of a bar. When the bar is bent, the wires in one pair of gages get longer and thinner, increasing the resistance, while the wires in the other pair of gages get shorter and thicker, decreasing the resistance.

But how can the change in resistance be measured? One way would be to use an ohmmeter. However, the change in resistance experienced by the strain gage is typically much smaller than could be accurately measured by an ohmmeter. Usually the pairs of strain gages are connected to form a Wheatstone bridge, and the voltage difference between two legs of the bridge is measured. In order to make an accurate measurement of the voltage difference, we use an operational amplifier circuit to amplify, or increase, the voltage difference. After we introduce the operational amplifier and some of the important circuits that employ these devices, we will present the circuit used together with the strain gages for measuring the amount of bending in a metal bar.





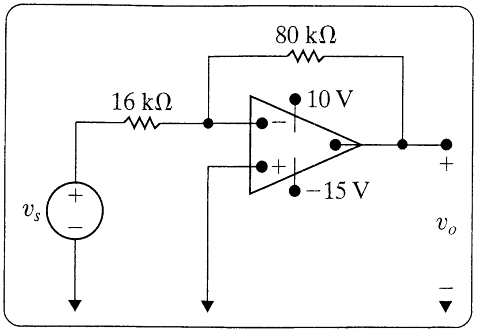
ASSESSING OBJECTIVE 1

◆ **Use voltage and current constraints in an ideal op amp**

5.1 Assume that the op amp in the circuit shown is ideal.

a) Calculate v_o for the following values of v_s: 0.4, 2.0, 3.5, −0.6, −1.6, and −2.4 V.

b) Specify the range of v_s required to avoid amplifier saturation.



ANSWER: (a) −2, −10, −15, 3, 8, and 10 V;
(b) $-2\ \text{V} \leq v_s \leq 3\ \text{V}$.

NOTE ◆ *Also try Chapter Problems 5.1–5.3.*

5.3 ◆ The Inverting-Amplifier Circuit

We are now ready to discuss the operation of some important op amp circuits, using Eqs. 5.2 and 5.3 to model the behavior of the device itself. Figure 5.9 shows an inverting-amplifier circuit. We assume that the op amp is operating in its linear region. Note that, in addition to the op amp, the circuit consists of two resistors (R_f and R_s), a voltage signal source (v_s), and a short circuit connected between the noninverting input terminal and the common node.

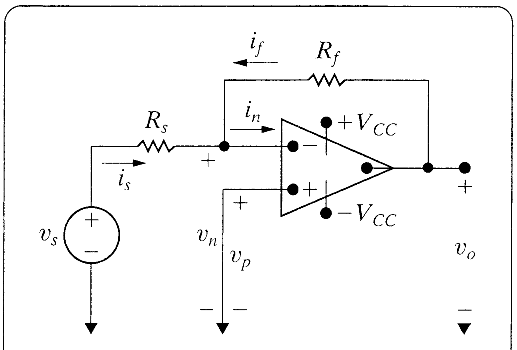


Figure 5.9 An inverting-amplifier circuit.

We now analyze this circuit, assuming an ideal op amp. The goal is to obtain an expression for the output voltage, v_o, as a function of the source voltage, v_s. We employ a single node-voltage equation at the inverting terminal of the op amp, given as

$$i_s + i_f = i_n. \tag{5.6}$$

The voltage constraint of Eq. 5.2 sets the voltage at $v_n = 0$, because the voltage at $v_p = 0$. Therefore,

$$i_s = \frac{v_s}{R_s}, \tag{5.7}$$

$$i_f = \frac{v_o}{R_f}. \tag{5.8}$$

Now we invoke the constraint stated in Eq. 5.3, namely,

$$i_n = 0. \tag{5.9}$$

ASSESSING OBJECTIVE 2

◆ **Be able to analyze simple circuits containing ideal op amps**

5.2 The source voltage v_s in the circuit in Assessment Problem 5.1 is −640 mV. The 80 kΩ feedback resistor is replaced by a variable resistor R_x. What range of R_x allows the inverting amplifier to operate in its linear region?

ANSWER: $0 \leq R_x \leq 250\ \text{k}\Omega$.

NOTE ◆ *Also try Chapter Problems 5.6 and 5.7.*

5.4 ◆ The Summing-Amplifier Circuit

The output voltage of a summing amplifier is an inverted, scaled sum of the voltages applied to the input of the amplifier. Figure 5.11 shows a summing amplifier with three input voltages.

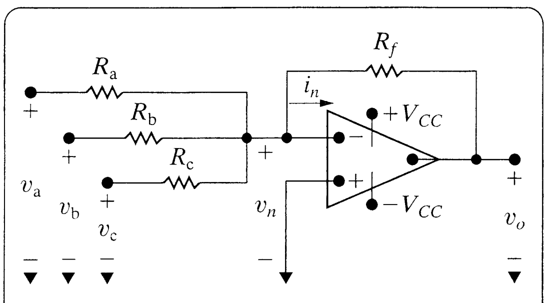


Figure 5.11 A summing amplifier.

We obtain the relationship between the output voltage v_o and the three input voltages, v_a, v_b, and v_c, by summing the currents away from the inverting input terminal:

$$\frac{v_n - v_a}{R_a} + \frac{v_n - v_b}{R_b} + \frac{v_n - v_c}{R_c} + \frac{v_n - v_o}{R_f} + i_n = 0. \tag{5.13}$$

Assuming an ideal op amp, we can use the voltage and current constraints together with the ground imposed at v_p by the circuit to see that $v_n = v_p = 0$ and $i_n = 0$. This reduces Eq. 5.13 to

INVERTING-SUMMING AMPLIFIER EQUATION

$$v_o = -\left(\frac{R_f}{R_a}v_a + \frac{R_f}{R_b}v_b + \frac{R_f}{R_c}v_c\right). \tag{5.14}$$

Equation 5.14 states that the output voltage is an inverted, scaled sum of the three input voltages.

If $R_a = R_b = R_c = R_s$, then Eq. 5.14 reduces to

$$v_o = -\frac{R_f}{R_s}(v_a + v_b + v_c). \tag{5.15}$$

Finally, if we make $R_f = R_s$, the output voltage is just the inverted sum of the input voltages. That is,

$$v_o = -(v_a + v_b + v_c). \tag{5.16}$$

Although we illustrated the summing amplifier with just three input signals, the number of input voltages can be increased as needed. For example, you might wish to sum 16 individually recorded audio signals to form a single audio signal. The summing amplifier configuration in Fig. 5.11 could include 16 different input resistor values so that each of the input audio tracks appears in the output signal with a different amplification factor. The summing amplifier thus plays the role of an audio mixer. As with inverting-amplifier circuits, the scaling factors in summing-amplifier circuits are determined by the external resistors $R_f, R_a, R_b, R_c, \ldots, R_n$.

ASSESSING OBJECTIVE 2

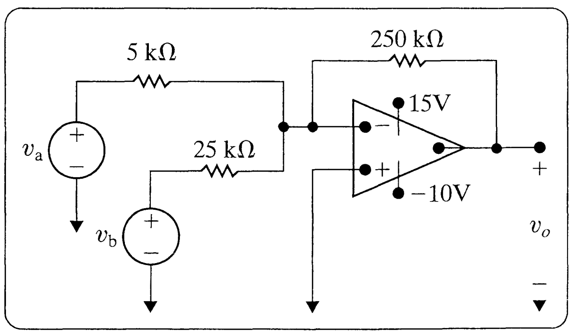
◆ **Be able to analyze simple circuits containing ideal op amps**

5.3 a) Find v_o in the circuit shown if $v_a = 0.1$ V and $v_b = 0.25$ V.

b) If $v_b = 0.25$ V, how large can v_a be before the op amp saturates?

c) If $v_a = 0.10$ V, how large can v_b be before the op amp saturates?

d) Repeat (a), (b), and (c) with the polarity of v_b reversed.



ANSWER: (a) −7.5 V; (b) 0.15 V; (c) 0.5 V; (d) −2.5, 0.25, and 2 V.

NOTE ◆ *Also try Chapter Problems 5.16, 5.17, and 5.19.*

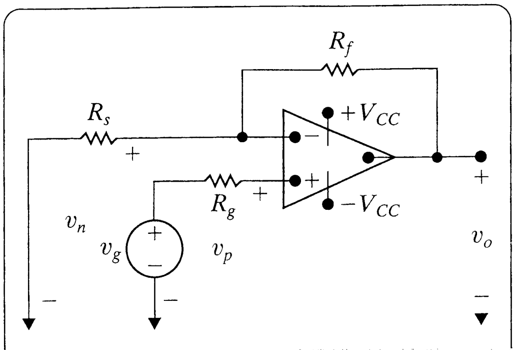


Figure 5.12 A noninverting amplifier.

5.5 ◆ The Noninverting-Amplifier Circuit

Figure 5.12 depicts a noninverting-amplifier circuit. The signal source is represented by v_g in series with the resistor R_g. In deriving the expression for the output voltage as a function of the source voltage, we assume an ideal op amp operating within its linear region. Thus, as before, we use Eqs. 5.2 and 5.3 as the basis for the derivation. Because the op amp input current is zero, we can write $v_p = v_g$ and, from Eq. 5.2, $v_n = v_g$ as well. Now, because the input current is zero ($i_n = i_p = 0$), the resistors R_f and R_s form an unloaded voltage divider across v_o. Therefore,

$$v_n = v_g = \frac{v_o R_s}{R_s + R_f}. \quad (5.17)$$

Solving Eq. 5.17 for v_o gives us the sought-after expression:

NONINVERTING-AMPLIFIER EQUATION

$$v_o = \frac{R_s + R_f}{R_s} v_g. \quad (5.18)$$

Operation in the linear region requires that

$$\frac{R_s + R_f}{R_s} < \left| \frac{V_{CC}}{v_g} \right|.$$

Note again that, because of the ideal op amp assumption, we can express the output voltage as a function of the input voltage and the external resistors—in this case, R_s and R_f.

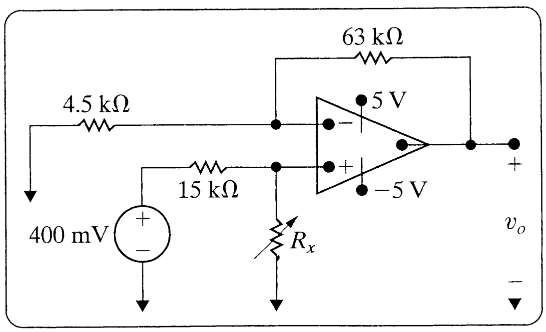
ASSESSING OBJECTIVE 2

◆ **Be able to analyze simple circuits containing ideal op amps**

5.4 Assume that the op amp in the circuit shown is ideal.

a) Find the output voltage when the variable resistor is set to 60 kΩ.

b) How large can R_x be before the amplifier saturates?



ANSWER: (a) 4.8 V; (b) 75 kΩ.

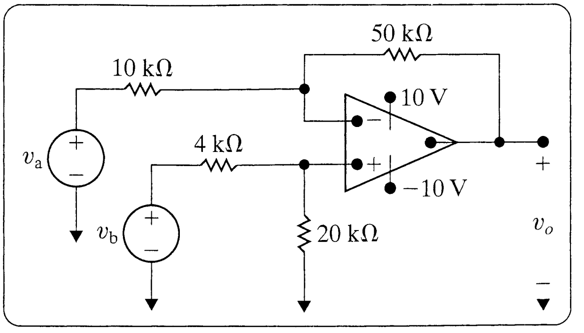
NOTE ◆ *Also try Chapter Problems 5.22 and 5.23.*

ASSESSING OBJECTIVE 2

◆ **Be able to analyze simple circuits containing ideal op amps**

5.5 a) In the difference amplifier shown, $v_b = 4.0$ V. What range of values for v_a will result in linear operation?

b) Repeat (a) with the 20 kΩ resistor decreased to 8 kΩ.



ANSWER: (a) $2 \text{ V} \leq v_a \leq 6$ V;
(b) $1.2 \text{ V} \leq v_a \leq 5.2$ V.

NOTE ◆ *Also try Chapter Problems 5.29–5.31.*

The Difference Amplifier—Another Perspective

We can examine the behavior of a difference amplifier more closely if we redefine its inputs in terms of two other voltages. The first is the **differential mode** input, which is the difference between the two input voltages in Fig. 5.13:

$$v_{dm} = v_b - v_a. \tag{5.25}$$

The second is the **common mode** input, which is the average of the two input voltages in Fig. 5.13:

$$v_{cm} = (v_a + v_b)/2. \tag{5.26}$$

Using Eqs. 5.25 and 5.26, we can now represent the original input voltages, v_a and v_b, in terms of the differential mode and common mode voltages, v_{dm} and v_{cm}:

$$v_a = v_{cm} - \frac{1}{2}v_{dm} \tag{5.27}$$

$$v_b = v_{cm} + \frac{1}{2}v_{dm}. \tag{5.28}$$

Substituting Eqs. 5.27 and 5.28 into Eq. 5.22 gives the output of the difference amplifier in terms of the differential mode and common mode

ASSESSING OBJECTIVE 3

◆ **Understand the more realistic model for an op amp**

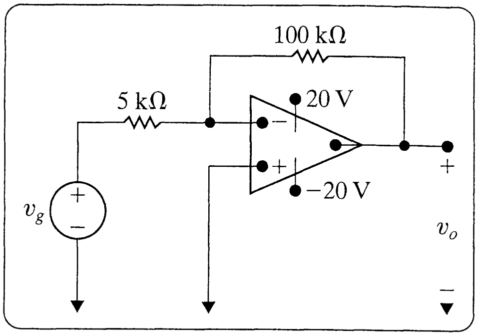
5.6 The inverting amplifier in the circuit shown has an input resistance of 500 kΩ, an output resistance of 5 kΩ, and an open-loop gain of 300,000. Assume that the amplifier is operating in its linear region.

a) Calculate the voltage gain (v_o/v_g) of the amplifier.

b) Calculate the value of v_n in microvolts when $v_g = 1$ V.

c) Calculate the resistance seen by the signal source (v_g).

d) Repeat (a)–(c) using the ideal model for the op amp.



ANSWER: (a) −19.9985; (b) 69.995 μV; (c) 5000.35 Ω; (d) −20, 0 μV, 5 kΩ.

NOTE ◆ *Also try Chapter Problems 5.39 and 5.40.*

Practical Perspective

Strain Gages

Changes in the shape of elastic solids are of great importance to engineers who design structures that twist, stretch, or bend when subjected to external forces. An aircraft frame is a prime example of a structure in which engineers must take into consideration elastic strain. The intelligent application of strain gages requires information about the physical structure of the gage, methods of bonding the gage to the surface of the structure, and the orientation of the gage relative to the forces exerted on the structure. Our purpose here is to point out that strain gage measurements are important in engineering applications, and a knowledge of electric circuits is germane to their proper use.

- A summing amplifier is an op amp circuit producing an output voltage that is a scaled sum of the input voltages. (See page 190.)
- A noninverting amplifier is an op amp circuit producing an output voltage that is a scaled replica of the input voltage. (See page 192.)
- A difference amplifier is an op amp circuit producing an output voltage that is a scaled replica of the input voltage difference. (See page 193.)
- The two voltage inputs to a difference amplifier can be used to calculate the common mode and difference mode voltage inputs, v_{cm} and v_{dm}. The output from the difference amplifier can be written in the form

$$v_o = A_{cm} v_{cm} + A_{dm} v_{dm},$$

where A_{cm} is the common mode gain, and A_{dm} is the differential mode gain. (See page 195.)

- In an ideal difference amplifier, $A_{cm} = 0$. To measure how nearly ideal a difference amplifier is, we use the common mode rejection ratio:

$$\text{CMRR} = \left| \frac{A_{dm}}{A_{cm}} \right|.$$

An ideal difference amplifier has an infinite CMRR. (See page 197.)

PROBLEMS

5.1 The op amp in the circuit in Fig. P5.1 is ideal.

P

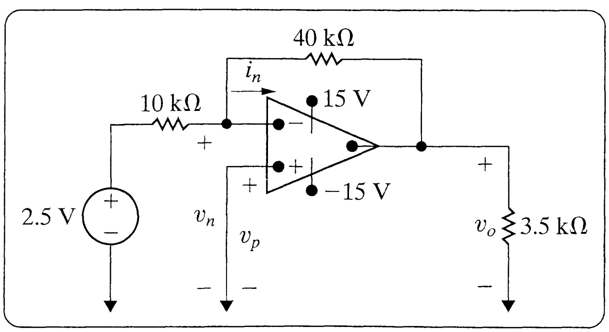
a) Label the five op amp terminals with their names.

b) What ideal op amp constraint determines the value of i_n? What is this value?

c) What ideal op amp constraint determines the value of $(v_p - v_n)$? What is this value?

d) Calculate v_o.

Figure P5.1



5.2 The op amp in the circuit in Fig. P5.2 is ideal.

P

a) Calculate v_o if $v_a = 4$ V and $v_b = 0$ V.

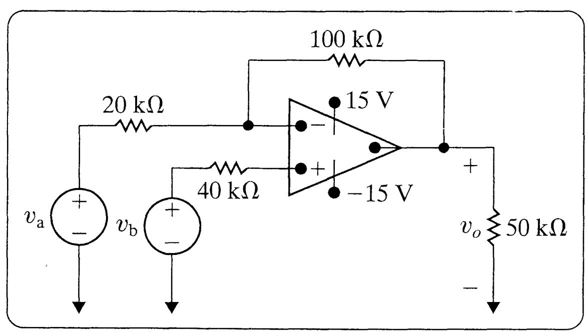
b) Calculate v_o if $v_a = 2$ V and $v_b = 0$ V.

c) Calculate v_o if $v_a = 2$ V and $v_b = 1$ V.

d) Calculate v_o if $v_a = 1$ V and $v_b = 2$ V.

e) If $v_b = 1.6$ V, specify the range of v_a such that the amplifier does not saturate.

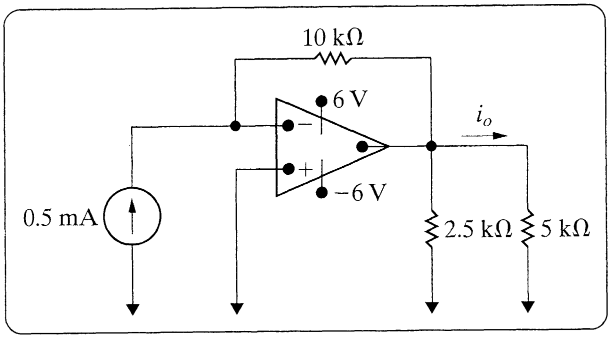
Figure P5.2



5.3 Find i_o in the circuit in Fig. P5.3 if the op amp is ideal.

P

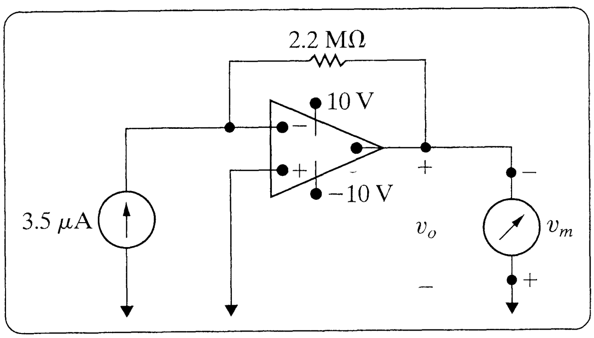
Figure P5.3



5.4 A voltmeter with a full-scale reading of 10 V is used to measure the output voltage in the circuit in Fig. P5.4. What is the reading of the voltmeter? Assume the op amp is ideal.

P

Figure P5.4



5.5 The op amp in the circuit in Fig. P5.5 is ideal. Calculate the following:

P

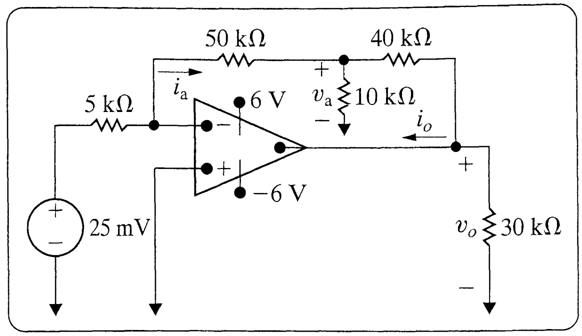
a) v_a

b) v_o

c) i_a

d) i_o

Figure P5.5



5.6 ❖

a) Design an inverting amplifier using an ideal op amp that has a gain of 6. Use only 20 kΩ resistors.

b) If you wish to amplify a 3 V input signal using the circuit you designed in part (a), what are the smallest power supply signals you can use?

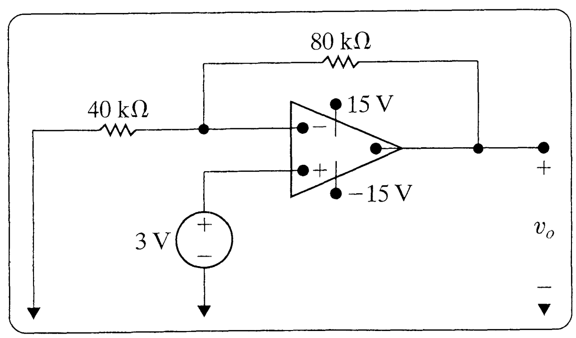
5.7 The op amp in the circuit of Fig. P5.7 is ideal.

P

a) What op amp circuit configuration is this?

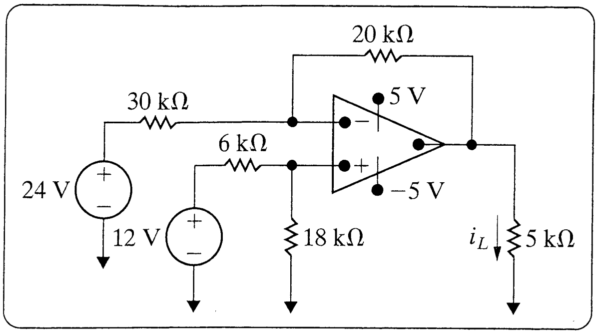
b) Calculate v_o.

Figure P5.7



5.8 Find i_L (in microamperes) in the circuit in Fig. P5.8.

Figure P5.8

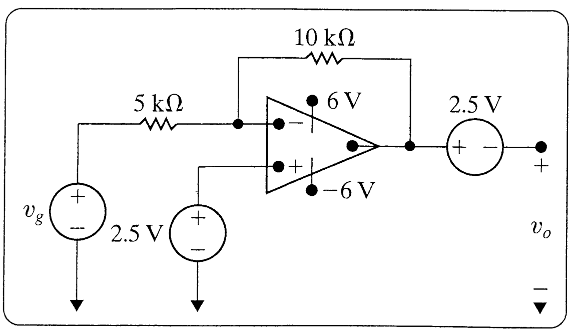


5.9 A circuit designer claims the circuit in Fig. P5.9 will produce an output voltage that will vary between ±5 as v_g varies between 0 and 5 V. Assume the op amp is ideal.

a) Draw a graph of the output voltage v_o as a function of the input voltage v_g for $0 \le v_g \le$ 5 V.

b) Do you agree with the designer's claim?

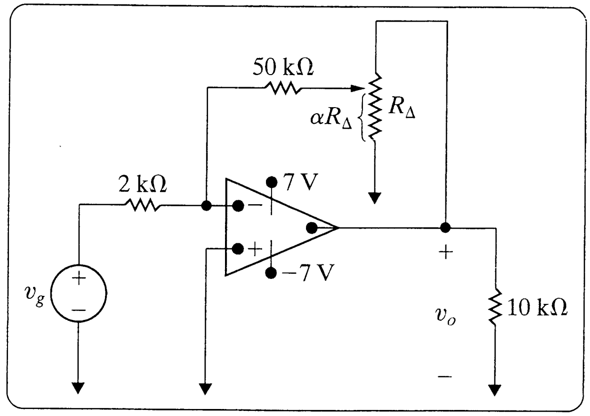
Figure P5.9



5.10 a) The op amp in the circuit shown in Fig. P5.10 is ideal. The adjustable resistor R_Δ has a maximum value of 100 kΩ, and α is restricted to the range of $0.2 \le \alpha \le 1.0$. Calculate the range of v_o if $v_g = 40$ mV.

b) If α is not restricted, at what value of α will the op amp saturate?

Figure P5.10

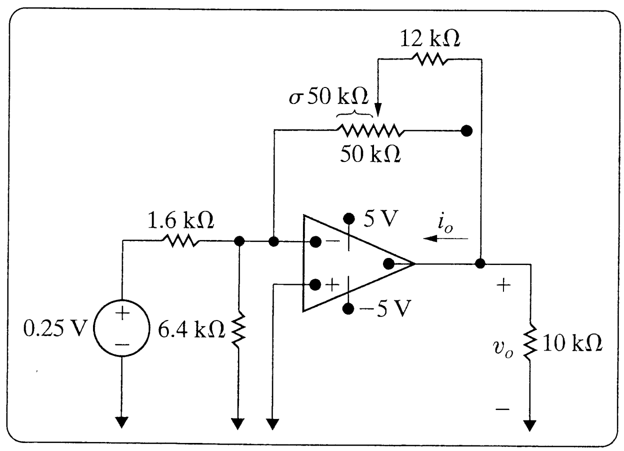


5.11 The op amp in the circuit in Fig. P5.11 is ideal.

a) Find the range of values for σ in which the op amp does not saturate.

b) Find i_o (in microamperes) when $\sigma = 0.272$.

Figure P5.11



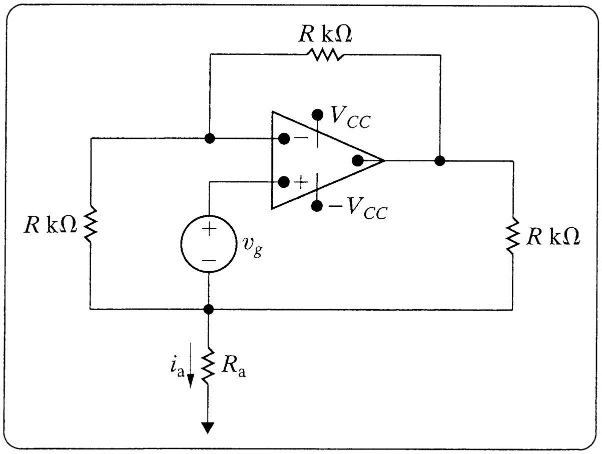
5.12 a) Show that when the ideal op amp in Fig. P5.12 is operating in its linear region,

$$i_a = \frac{3v_g}{R}.$$

b) Show that the ideal op amp will saturate when

$$R_a = \frac{R(\pm V_{CC} - 2v_g)}{3v_g}.$$

Figure P5.12



5.13 P The circuit inside the shaded area in Fig. P5.13 is a constant current source for a limited range of values of R_L.

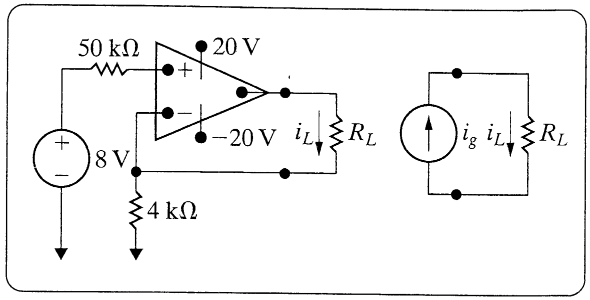
a) Find the value of i_L for $R_L = 4$ kΩ.

b) Find the maximum value for R_L for which i_L will have the value in (a).

c) Assume that $R_L = 16$ kΩ. Explain the operation of the circuit. You can assume that $i_n = i_p \approx 0$ under all operating conditions.

d) Sketch i_L versus R_L for $0 \leq R_L \leq 16$ kΩ.

Figure P5.13



5.14 P The op amps in the circuit in Fig. P5.14 are ideal.

a) Find i_a.

b) Find the value of the left source voltage for which $i_a = 0$.

5.15 P Assume that the ideal op amp in the circuit in Fig. P5.15 is operating in its linear region.

a) Calculate the power delivered to the 600 Ω resistor.

b) Repeat (a) with the op amp removed from the circuit, that is, with the 600 Ω resistor connected in the series with the voltage source and the 29.4 kΩ resistor.

c) Find the ratio of the power found in (a) to that found in (b).

d) Does the insertion of the op amp between the source and the load serve a useful purpose? Explain.

Figure P5.14

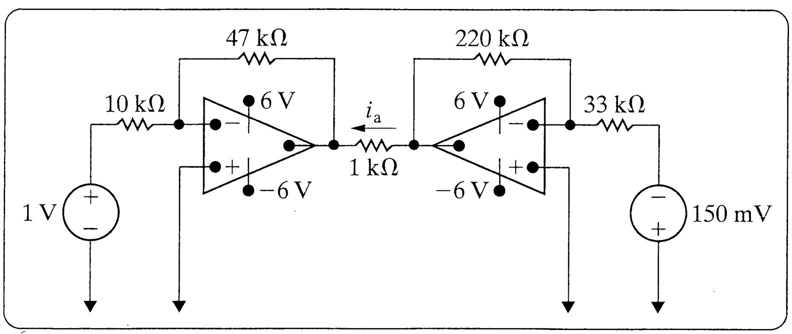
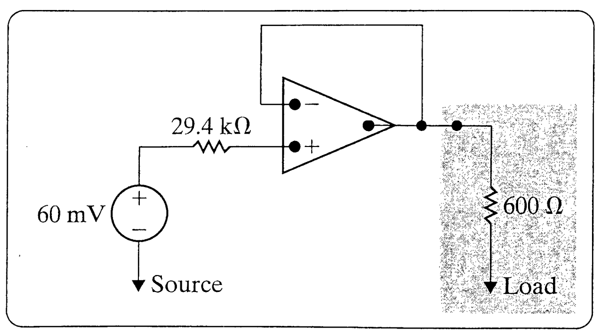


Figure P5.15



5.16 The op amp in Fig. P5.16 is ideal.

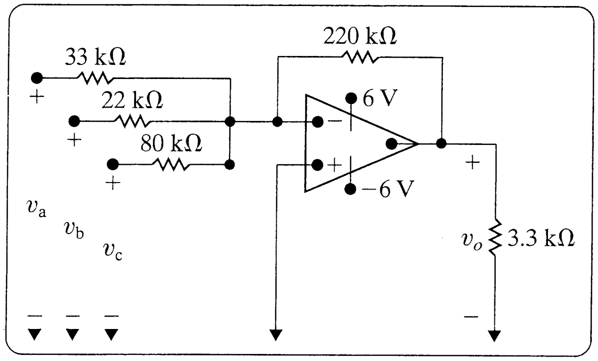
P

a) What circuit configuration is shown in this figure?

b) Find v_o if $v_a = 1.2$ V, $v_b = -1.5$ V, and $v_c = 4$ V.

c) The voltages v_a and v_c remain at 1.2 V and 4 V, respectively. What are the limits on v_b if the op amp operates within its linear region?

Figure P5.16

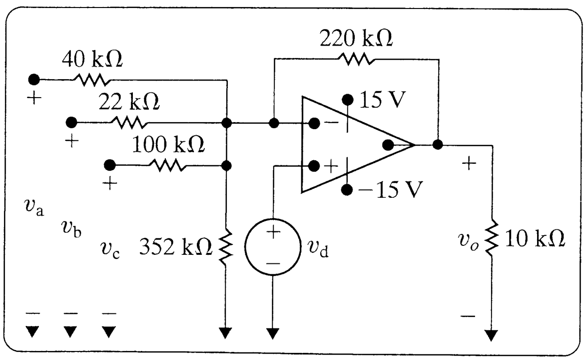


5.17 a) The op amp in Fig. P5.17 is ideal. Find v_o if $v_a = 4$ V, $v_b = 9$ V, $v_c = 13$ V, and $v_d = 8$ V.

P

b) Assume v_b, v_c, and v_d retain their values as given in (a). Specify the range of v_a such that the op amp operates within its linear region.

Figure P5.17



5.18 The 220 kΩ feedback resistor in the circuit in Fig. P5.17 is replaced by a variable resistor R_f. The voltages v_a–v_d have the same values as given in Problem 5.17(a).

P

a) What value of R_f will cause the op amp to saturate? Note that $0 \le R_f \le \infty$.

b) When R_f has the value found in (a), what is the current (in microamperes) into the output terminal of the op amp?

5.19 Design an inverting summing amplifier so that

❖ P

$$v_o = -(2v_a + 4v_b + 6v_c + 8v_d).$$

If the feedback resistor (R_f) is chosen to be 48 kΩ, draw a circuit diagram of the amplifier and specify the values of R_a, R_b, R_c, and R_d.

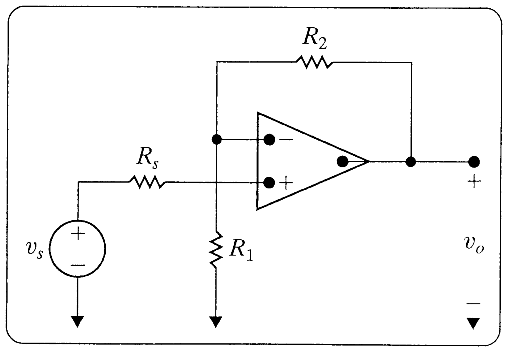
5.20 Assume that the ideal op amp in the circuit seen in Fig. P5.20 is operating in its linear region.

a) Show that $v_o = [(R_1 + R_2)/R_1]v_s$.

b) What happens if $R_1 \to \infty$ and $R_2 \to 0$?

c) Explain why this circuit is referred to as a voltage follower when $R_1 = \infty$ and $R_2 = 0$.

Figure P5.20



5.21 Refer to the circuit in Fig. 5.11, where the op amp is assumed to be ideal. Given that $R_a = 3$ kΩ, $R_b = 5$ kΩ, $R_c = 25$ kΩ, $v_a = 150$ mV, $v_b = 100$ mV, $v_c = 250$ mV, and $V_{CC} = \pm 6$ V, specify the range of R_f for which the op amp operates within its linear region.

P

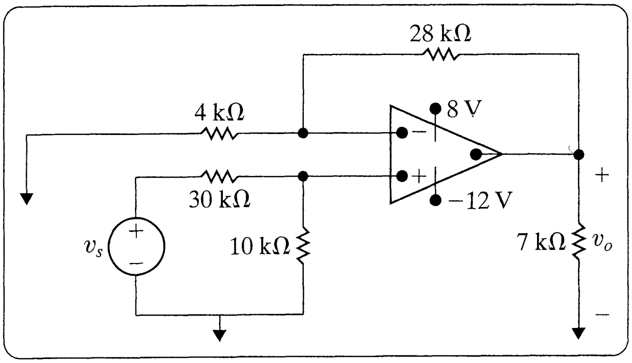
5.22 The op amp in the circuit of Fig. P5.22 is ideal.

a) What op amp circuit configuration is this?

b) Find v_o in terms of v_s.

c) Find the range of values for v_s such that v_o does not saturate and the op amp remains in its linear region of operation.

Figure P5.22



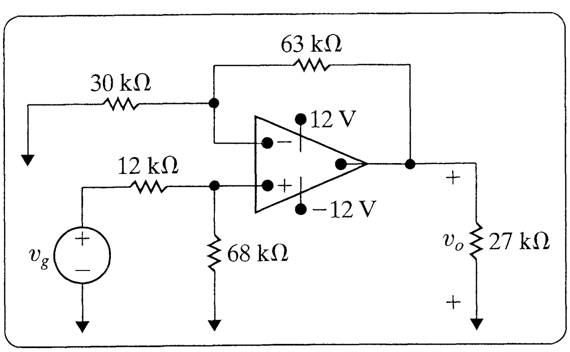
5.23 The op amp in the circuit shown in Fig. P5.23 is ideal.

a) Calculate v_o when v_g equals 4 V.

b) Specify the range of values of v_g so that the op amp operates in a linear mode.

c) Assume that v_g equals 2 V and that the 63 kΩ resistor is replaced with a variable resistor. What value of the variable resistor will cause the op amp to saturate?

Figure P5.23



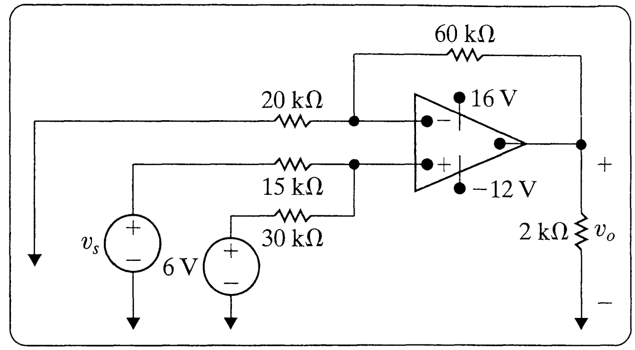
5.24 The op amp in the circuit of Fig. P5.24 is ideal.

a) What op amp circuit configuration is this?

b) Find v_o in terms of v_s.

c) Find the range of values for v_s such that v_o does not saturate and the op amp remains in its linear region of operation.

Figure P5.24



5.25 The op amp in the circuit shown in Fig. P5.25 is ideal. The signal voltages v_a and v_b are 400 mV and 1200 mV, respectively.

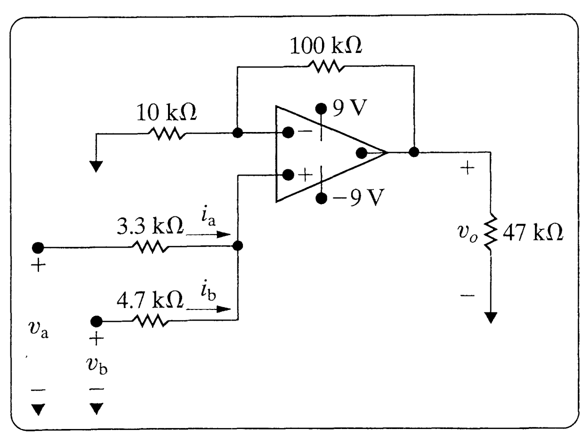
a) What circuit configuration is shown in the figure?

b) Calculate v_o in volts.

c) Find i_a and i_b in microamperes.

d) What are the weighting factors associated with v_a and v_b?

Figure P5.25



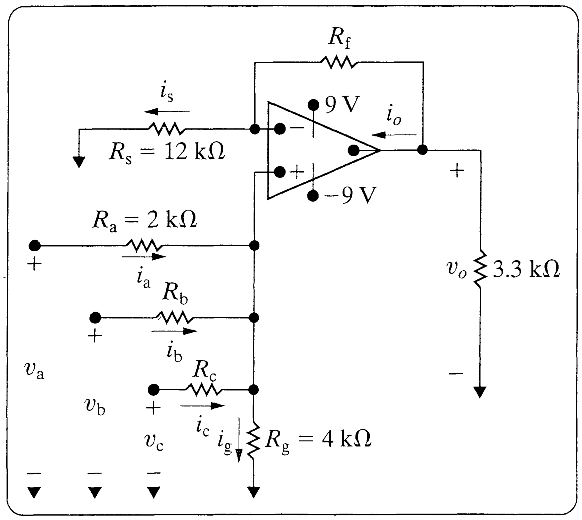
5.26 The op amp in the noninverting summing amplifier of Fig. P5.26 is ideal.

a) Specify the values of R_f, R_b, and R_c so that

$$v_o = 3v_a + 2v_b + v_c.$$

b) Find (in microamperes) i_a, i_b, i_c, i_g, and i_s when $v_a = 0.80$ V, $v_b = 1.5$ V, and $v_c = 2.1$ V.

Figure P5.26



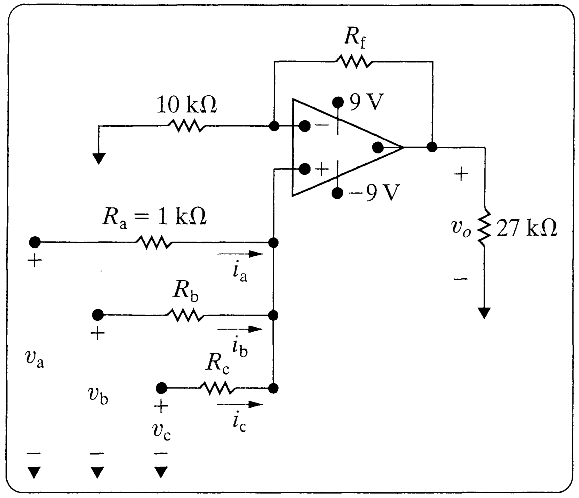
5.27 The circuit in Fig. P5.27 is a noninverting summing amplifier. Assume the op amp is ideal. Design the circuit so that

$$v_o = 5v_a + 4v_b + v_c.$$

a) Specify the numerical values of R_b, R_c, and R_f.

b) Calculate (in microamperes) i_a, i_b, and i_c when $v_a = 0.5$ V, $v_b = 1.0$ V, and $v_c = 1.5$ V.

Figure P5.27



5.28 a) Use the principle of superposition to derive Eq. 5.22.

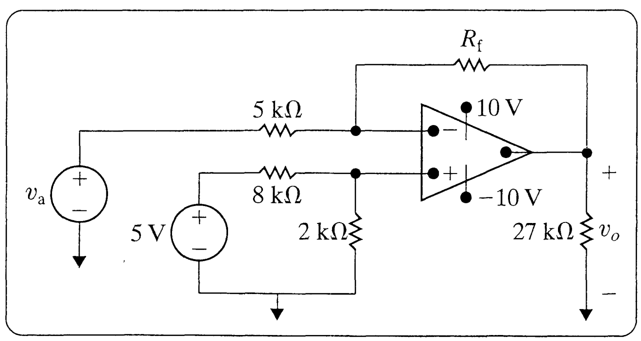
b) Derive Eqs. 5.23 and 5.24.

5.29 The op amp in the circuit of Fig. P5.29 is ideal. What value of R_f will give the equation

$$v_o = 5 - 4v_a$$

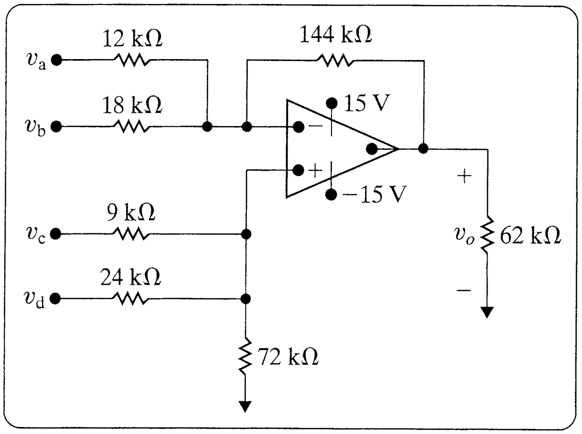
for this circuit.

Figure P5.29



5.30 The op amp in the adder-subtracter circuit shown in Fig. P5.30 is ideal.

Figure P5.30



a) Find v_o when $v_a = 0.5$ V, $v_b = 0.3$ V, $v_c = 0.6$ V, and $v_d = 0.8$ V.

b) If v_a, v_b, and v_d are held constant, what values of v_c will not saturate the op amp?

5.31 The resistors in the difference amplifier shown in Fig. 5.13 are $R_a = 10\,\text{k}\Omega$, $R_b = 100\,\text{k}\Omega$, $R_c = 33\,\text{k}\Omega$, and $R_d = 47\,\text{k}\Omega$. The signal voltages v_a and v_b are 0.67 and 0.8 V, respectively, and $V_{CC} = \pm 5$ V.

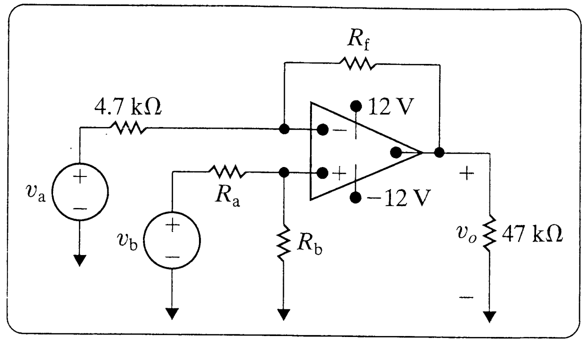
a) Find v_o.

b) What is the resistance seen by the signal source v_a?

c) What is the resistance seen by the signal source v_b?

5.32 Design the difference-amplifier circuit in Fig. P5.32 so that $v_o = 10(v_b - v_a)$, and the voltage source v_b sees an input resistance of $220\,\text{k}\Omega$. Specify the values of R_a, R_b, and R_f. Use the ideal model for the op amp.

Figure P5.32

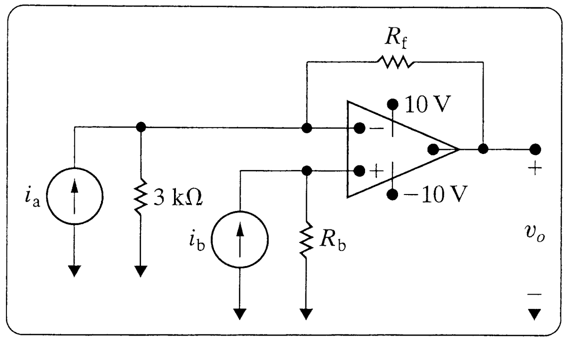


5.33 Select the values of R_b and R_f in the circuit in Fig. P5.33 so that

$$v_o = 2000(i_b - i_a).$$

The op amp is ideal.

Figure P5.33



5.34 Design a difference amplifier (Fig. 5.13) to meet the following criteria: $v_o = 3v_b - 4v_a$. The resistance seen by the signal source v_b is 470 kΩ, and the resistance seen by the signal source v_a is 22 kΩ when the output voltage v_o is zero. Specify the values of R_a, R_b, R_c, and R_d.

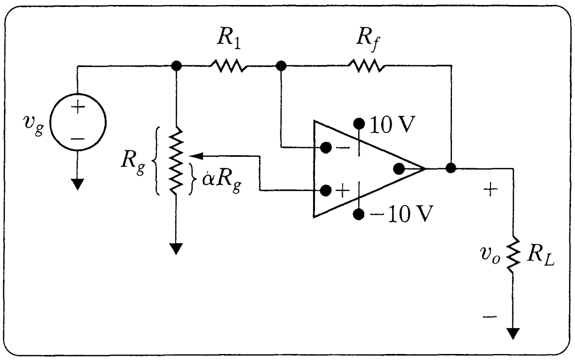
5.35 The op amp in the circuit of Fig. P5.35 is ideal.

a) Plot v_o versus α when $R_f = 4R_1$ and $v_g \doteq 2$ V. Use increments of 0.1 and note by hypothesis that $0 \leq \alpha \leq 1.0$.

b) Write an equation for the straight line you plotted in (a). How are the slope and intercept of the line related to v_g and the ratio R_f/R_1?

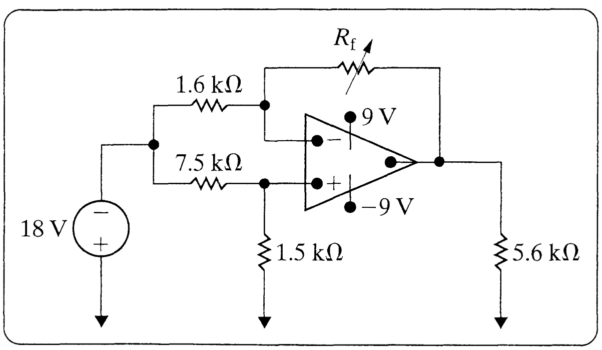
c) Using the results from (b), choose values for v_g and the ratio R_f/R_1 such that $v_o = -6\alpha + 4$.

Figure P5.35



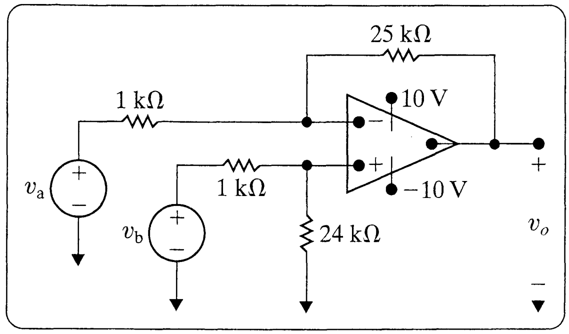
5.36 P The resistor R_f in the circuit in Fig. P5.36 is adjusted until the ideal op amp saturates. Specify R_f in kilohms.

Figure P5.36



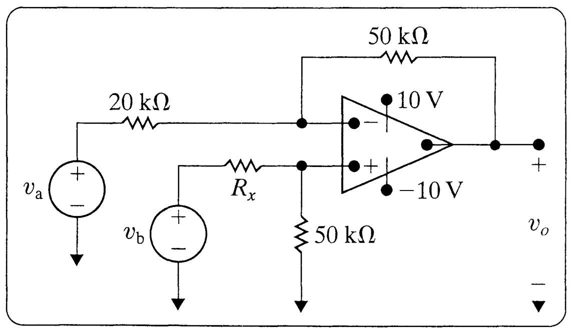
5.37 In the difference amplifier shown in Fig. P5.37, compute (a) the differential mode gain, (b) the common mode gain, and (c) the CMRR.

Figure P5.37



5.38 In the difference amplifier shown in Fig. P5.38, what value of R_x yields a CMRR ≥ 1000?

Figure P5.38



5.39 P Repeat Assessment Problem 5.6, given that the inverting amplifier is loaded with a 1 kΩ resistor.

5.40 The op amp in the noninverting amplifier circuit of Fig. P5.40 has an input resistance of 400 kΩ, an output resistance of 5 kΩ, and an open-loop gain of 20,000. Assume that the op amp is operating in its linear region.

P

a) Calculate the voltage gain (v_o/v_g).

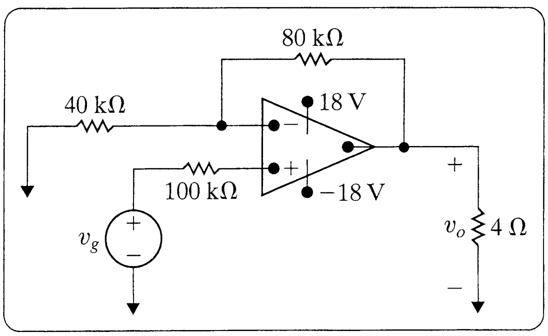
b) Find the inverting and noninverting input voltages v_n and v_p (in millivolts) if $v_g = 1$ V.

c) Calculate the difference $(v_p - v_n)$ in microvolts when $v_g = 1$ V.

d) Find the current drain in picoamperes on the signal source v_g when $v_g = 1$ V.

e) Repeat (a)–(d) assuming an ideal op amp.

Figure P5.40



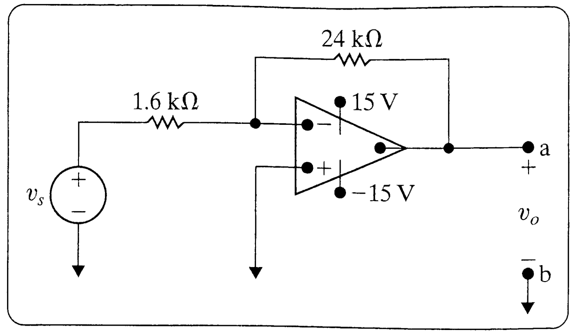
5.41 a) Find the Thévenin equivalent circuit with respect to the output terminals a,b for the inverting amplifier of Fig. P5.41. The dc signal source has a value of 880 mV. The op amp has an input resistance of 500 kΩ, an output resistance of 2 kΩ, and an open-loop gain of 100,000.

P

b) What is the output resistance of the inverting amplifier?

c) What is the resistance (in ohms) seen by the signal source v_s when the load at the terminals a,b is 330 Ω?

Figure P5.41



5.42 Repeat Problem 5.41 assuming an ideal op amp.

P

5.43 Assume the input resistance of the op amp in Fig. P5.43 is infinite and its output resistance is zero.

P

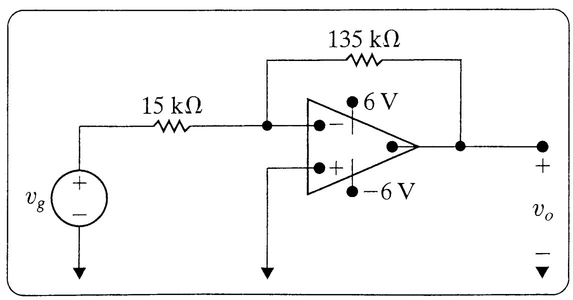
a) Find v_o as a function of v_g and the open-loop gain A.

b) What is the value of v_o if $v_g = 0.4$ V and $A = 90$?

c) What is the value of v_o if $v_g = 0.4$ V and $A = \infty$?

d) How large does A have to be so that v_o is 95% of its value in (c)?

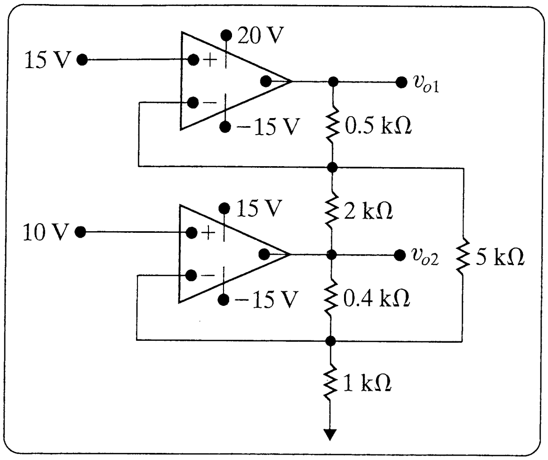
Figure P5.43



5.44 Derive Eq. 5.60.

5.45 The two op amps in the circuit in Fig. P5.45 are ideal. Calculate v_{o1} and v_{o2}.

Figure P5.45



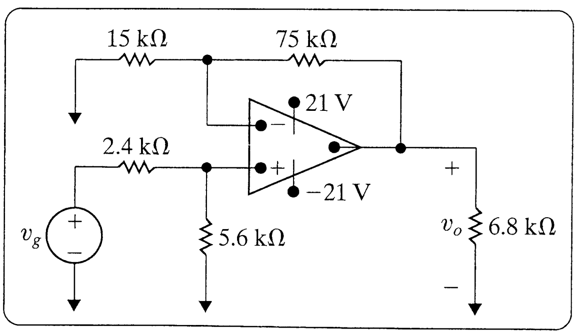
5.46 The signal voltage v_g in the circuit shown in Fig. P5.46 is described by the following equations:

$$v_g = 0, \qquad t \leq 0,$$

$$v_g = 10\sin(\pi/3)t \text{ V}, \qquad 0 \leq t \leq \infty.$$

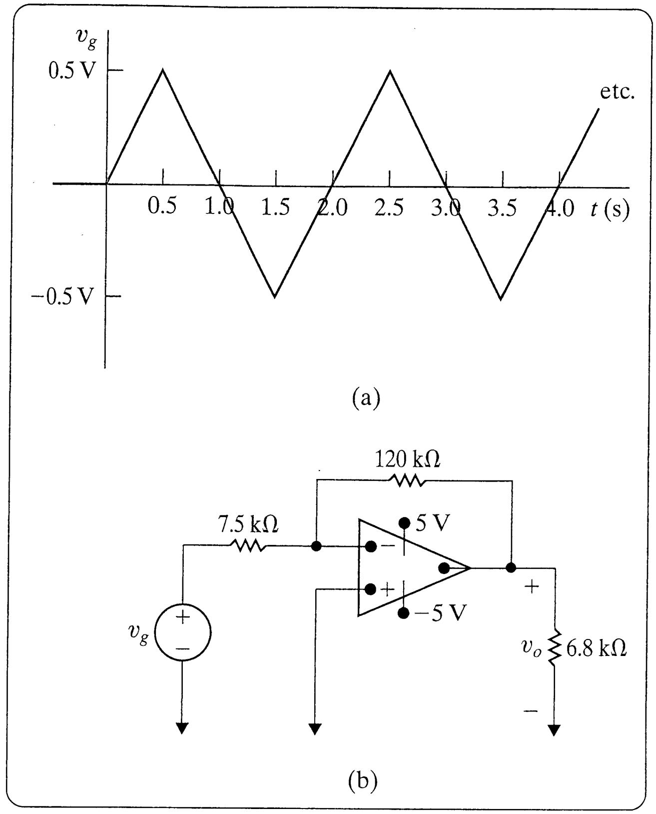
Sketch v_o versus t, assuming the op amp is ideal.

Figure P5.46



5.47 The voltage v_g shown in Fig. P5.47(a) is applied to the inverting amplifier shown in Fig. P5.47(b). Sketch v_o versus t, assuming the op amp is ideal.

Figure P5.47



5.48 ◆ Suppose the strain gages in the bridge in Fig. 5.18 have the value 120 Ω ± 1%. The power supplies to the op amp are ±15 V, and the reference voltage, v_{ref}, is taken from the positive power supply.

a) Calculate the value of R_f so that when the strain gage that is lengthening reaches its maximum length, the output voltage is 5 V.

b) Suppose that we can accurately measure 50 mV changes in the output voltage. What change in strain gage resistance can be detected in milliohms?

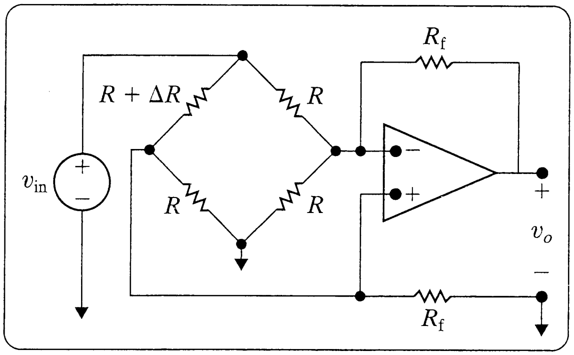
5.49 a) For the circuit shown in Fig. P5.49, show that if $\Delta R \ll R$, the output voltage of the op amp is approximately

$$v_o \approx \frac{R_f}{R^2}\frac{(R+R_f)}{(R+2R_f)}(-\Delta R)v_{in}.$$

b) Find v_o if $R_f = 470$ kΩ, $R = 10$ kΩ, $\Delta R = 95$ Ω, and $v_{in} = 15$ V.

c) Find the actual value of v_o in (b).

Figure P5.49



5.50 a) If percent error is defined as

$$\% \text{ error} = \left[\frac{\text{approximate value}}{\text{true value}} - 1\right] \times 100,$$

show that the percent error in the approximation of v_o in Problem 5.49 is

$$\% \text{ error} = \frac{\Delta R}{R}\frac{(R+R_f)}{(R+2R_f)} \times 100.$$

b) Calculate the percent error in v_o for Problem 5.49.

5.51 Assume the percent error in the approximation of v_o in the circuit in Fig. P5.49 is not to exceed 1%. What is the largest percent change in R that can be tolerated?

5.52 Assume the resistor in the variable branch of the bridge circuit in Fig. P5.49 is $R - \Delta R$.

a) What is the expression for v_o if $\Delta R \ll R$?

b) What is the expression for the percent error in v_o as a function of R, R_f, and ΔR?

c) Assume the resistance in the variable arm of the bridge circuit in Fig. P5.49 is 9810 Ω and the values of R, R_f, and v_{in} are the same as in Problem 5.49(b). What is the approximate value of v_o?

d) What is the percent error in the approximation of v_o when the variable arm resistance is 9810 Ω?

CHAPTER 6

Inductance, Capacitance, and Mutual Inductance

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Know and be able to use the equations for voltage, current, power, and energy in an inductor; understand how an inductor behaves in the presence of constant current, and the requirement that the current be continuous in an inductor.
2. Know and be able to use the equations for voltage, current, power, and energy in a capacitor; understand how a capacitor behaves in the presence of constant voltage, and the requirement that the voltage be continuous in a capacitor.
3. Be able to combine inductors with initial conditions in series and in parallel to form a single equivalent inductor with an initial condition; be able to combine capacitors with initial conditions in series and in parallel to form a single equivalent capacitor with an initial condition.
4. Understand the basic concept of mutual inductance and be able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.

We begin this chapter by introducing the last two ideal circuit elements mentioned in Chapter 2, namely, inductors and capacitors. Be assured that the circuit analysis techniques introduced in Chapters 3 and 4 apply to circuits containing inductors and capacitors. Therefore, once you understand the terminal behavior of these elements in terms of current and voltage, you can use Kirchhoff's laws to describe any interconnections with the other basic elements. Like other components, inductors and capacitors are easier to describe in terms of circuit variables rather than electromagnetic field variables. However, before we focus on the circuit descriptions, a brief review of the field concepts underlying these basic elements is in order.

An inductor is an electrical component that opposes any change in electrical current. It is composed of a coil of wire wound around a supporting core whose material may be magnetic or nonmagnetic. The behavior of inductors is based on phenomena associated with magnetic fields. The source of the magnetic field is charge in motion, or current. If the current is varying with time, the magnetic field is varying with time. A time-varying magnetic field induces a voltage in any conductor linked by the field. The circuit parameter of **inductance** relates the induced voltage to the current. We discuss this quantitative relationship in Section 6.1.

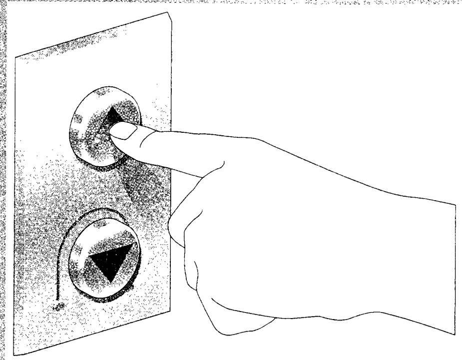
A capacitor is an electrical component that consists of two conductors separated by an insulator or dielectric material. The capacitor is the only device other than a battery that

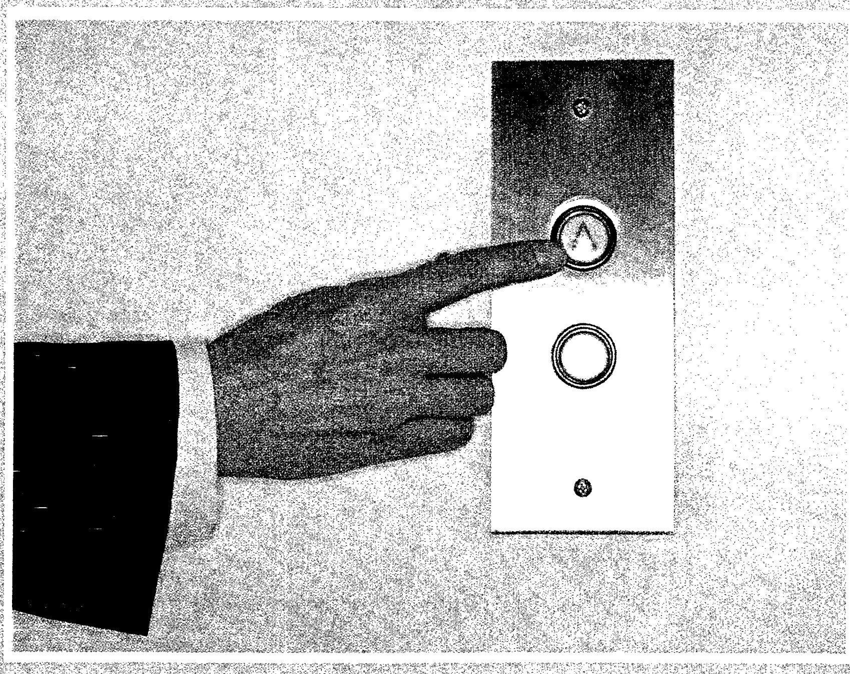
Practical Perspective

Proximity Switches

The electrical devices we use in our daily lives contain many switches. Most switches are mechanical, such as the one used in the flashlight introduced in Chapter 2. Mechanical switches use an actuator that is pushed, pulled, slid, or rotated, causing two pieces of conducting metal to touch and create a short circuit. Sometimes designers prefer to use switches without moving parts, to increase the safety, reliability, convenience, or novelty of their products. Such switches are called proximity switches. Proximity switches can employ a variety of sensor technologies. For example, some elevator doors stay open whenever a light beam is obstructed.

Another sensor technology used in proximity switches detects people by responding to the disruption they cause in electric fields. This type of proximity switch is used in some desk lamps that turn on and off when touched and in elevator buttons with no moving parts (as shown in the figure). The switch is based on a capacitor. As you are about to discover in this chapter, a capacitor is a circuit element whose terminal characteristics are determined by electric fields. When you touch a capacitive proximity switch, you produce a change in the value of a capacitor, causing a voltage change, which activates the switch. The design of a capacitive touch-sensitive switch is the topic of the Practical Perspective example at the end of this chapter.





ASSESSING OBJECTIVE 1

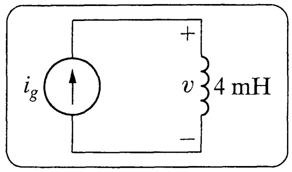
◆ **Know and be able to use the equations for voltage, current, power, and energy in an inductor**

6.1 The current source in the circuit shown generates the current pulse

$$i_g(t) = 0, \qquad t < 0,$$

$$i_g(t) = 8e^{-300t} - 8e^{-1200t} \text{ A}, \qquad t \geq 0.$$

Find (a) $v(0)$; (b) the instant of time, greater than zero, when the voltage v passes through zero; (c) the expression for the power delivered to the inductor; (d) the instant when the power delivered to the inductor is maximum; (e) the maximum power; (f) the maximum energy stored in the inductor; and (g) the instant of time when the stored energy is maximum.



ANSWER: (a) 28.8 V; (b) 1.54 ms; (c) $-76.8e^{-600t}+384e^{-1500t}-307.2e^{-2400t}$; (d) 411.05 μs; (e) 32.72 W; (f) 28.57 mJ; (g) 1.54 ms.

NOTE ◆ *Also try Chapter Problems 6.1 and 6.3.*

6.2 ◆ The Capacitor

The circuit parameter of capacitance is represented by the letter C, is measured in farads (F), and is symbolized graphically by two short parallel conductive plates, as shown in Fig. 6.10(a). Because the farad is an extremely large quantity of capacitance, practical capacitor values usually lie in the picofarad (pF) to microfarad (μF) range.

The graphic symbol for a capacitor is a reminder that capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating, material. This condition implies that electric charge is not transported through the capacitor. Although applying a voltage to the terminals of the capacitor cannot move a charge through the dielectric, it can displace a charge within the dielectric. As the voltage varies with time, the displacement of charge also varies with time, causing what is known as the **displacement current**.

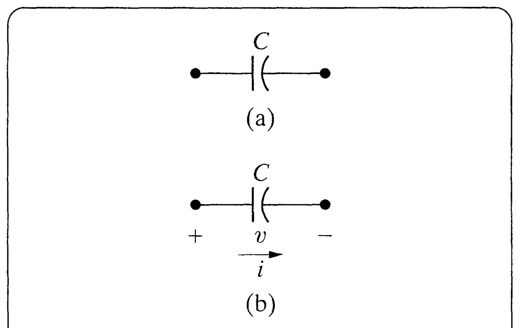


Figure 6.10 (a) The circuit symbol for a capacitor. (b) Assigning reference voltage and current to the capacitor, following the passive sign convention.

At the terminals, the displacement current is indistinguishable from a conduction current. The current is proportional to the rate at which the voltage across the capacitor varies with time, or, mathematically,

CAPACITOR i-v EQUATION

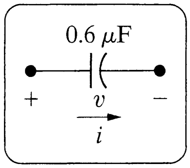
$$i = C\frac{dv}{dt}, \tag{6.13}$$

where i is measured in amperes, C in farads, v in volts, and t in seconds.

ASSESSING OBJECTIVE 2

◆ **Know and be able to use the equations for voltage, current, power, and energy in a capacitor**

6.2 The voltage at the terminals of the 0.6 μF capacitor shown in the figure is 0 for $t < 0$ and $40e^{-15,000t} \sin 30,000t$ V for $t \geq 0$. Find (a) $i(0)$; (b) the power delivered to the capacitor at $t = \pi/80$ ms; and (c) the energy stored in the capacitor at $t = \pi/80$ ms.



ANSWER: (a) 0.72 A; (b) −649.2 mW; (c) 126.13 μJ.

6.3 The current in the capacitor of Assessment Problem 6.2 is 0 for $t < 0$ and $3 \cos 50,000t$ A for $t \geq 0$. Find (a) $v(t)$; (b) the maximum power delivered to the capacitor at any one instant of time; and (c) the maximum energy stored in the capacitor at any one instant of time.

ANSWER: (a) 100 sin 50,000t V; (b) 150 W; (c) 3 mJ.

NOTE ◆ *Also try Chapter Problems 6.14 and 6.15.*

6.3 ◆ Series-Parallel Combinations of Inductance and Capacitance

Just as series-parallel combinations of resistors can be reduced to a single equivalent resistor, series-parallel combinations of inductors or capacitors can be reduced to a single inductor or capacitor. Figure 6.13 shows inductors in series. Here, the inductors are forced to carry the same current; thus we define only one current for the series combination. The voltage drops across the individual inductors are

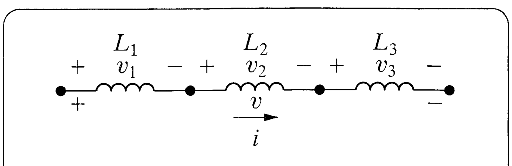


Figure 6.13 Inductors in series.

$$v_1 = L_1\frac{di}{dt}, \qquad v_2 = L_2\frac{di}{dt}, \qquad \text{and} \qquad v_3 = L_3\frac{di}{dt}.$$

Capacitors connected in parallel must carry the same voltage. Therefore, if there is an initial voltage across the original parallel capacitors, this same initial voltage appears across the equivalent capacitance C_{eq}. The derivation of the equivalent circuit for parallel capacitors is left as an exercise. (See Problem 6.31.)

We say more about series-parallel equivalent circuits of inductors and capacitors in Chapter 7, where we interpret results based on their use.

ASSESSING OBJECTIVE 3

◆ **Be able to combine inductors or capacitors in series and in parallel to form a single equivalent inductor**

6.4 The initial values of i_1 and i_2 in the circuit shown are +3 A and −5 A, respectively. The voltage at the terminals of the parallel inductors for $t \geq 0$ is $-30e^{-5t}$ mV.

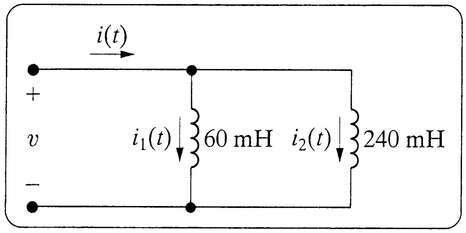
a) If the parallel inductors are replaced by a single inductor, what is its inductance?

b) What is the initial current and its reference direction in the equivalent inductor?

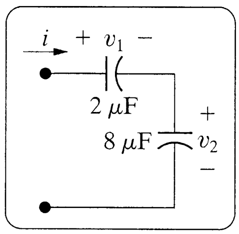
c) Use the equivalent inductor to find $i(t)$.

d) Find $i_1(t)$ and $i_2(t)$. Verify that the solutions for $i_1(t)$, $i_2(t)$, and $i(t)$ satisfy Kirchhoff's current law.

ANSWER: (a) 48 mH; (b) 2 A, up; (c) $0.125e^{-5t} - 2.125$ A; (d) $i_1(t) = 0.1e^{-5t} + 2.9$ A, $i_2(t) = 0.025e^{-5t} - 5.025$ A.



6.5 The current at the terminals of the two capacitors shown is $240e^{-10t}$ μA for $t \geq 0$. The initial values of v_1 and v_2 are −10 V and −5 V, respectively. Calculate the total energy trapped in the capacitors as $t \to \infty$. (*Hint:* Don't combine the capacitors in series—find the energy trapped in each, and then add.)



ANSWER: 20 μJ.

NOTE ◆ *Also try Chapter Problems 6.21, 6.22, 6.26, and 6.27.*

$$i_2 R_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0. \tag{6.32}$$

The Procedure for Determining Dot Markings

We shift now to two methods of determining dot markings. The first assumes that we know the physical arrangement of the two coils and the mode of each winding in a magnetically coupled circuit. The following six steps, applied here to Fig. 6.23, determine a set of dot markings:

a) Arbitrarily select one terminal—say, the D terminal—of one coil and mark it with a dot.

b) Assign a current into the dotted terminal and label it i_D.

c) Use the right-hand rule[1] to determine the direction of the magnetic field established by i_D *inside* the coupled coils and label this field ϕ_D.

d) Arbitrarily pick one terminal of the second coil—say, terminal A—and assign a current into this terminal, showing the current as i_A.

e) Use the right-hand rule to determine the direction of the flux established by i_A *inside* the coupled coils and label this flux ϕ_A.

f) Compare the directions of the two fluxes ϕ_D and ϕ_A. If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current (i_A) enters. (In Fig. 6.23, the fluxes ϕ_D and ϕ_A have the same reference direction, and therefore a dot goes on terminal A.) If the fluxes have different reference directions, place a dot on the terminal of the second coil where the test current leaves.

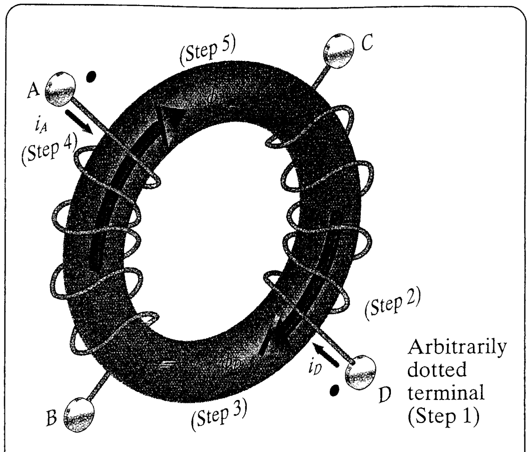


Figure 6.23 A set of coils showing a method for determining a set of dot markings.

The relative polarities of magnetically coupled coils can also be determined experimentally. This capability is important because in some situations, determining how the coils are wound on the core is impossible. One experimental method is to connect a dc voltage source, a resistor, a switch, and a dc voltmeter to the pair of coils, as shown in Fig. 6.24. The shaded box covering the coils implies that physical inspection of the coils is not possible. The resistor R limits the magnitude of the current supplied by the dc voltage source.

The coil terminal connected to the positive terminal of the dc source via the switch and limiting resistor receives a polarity mark, as shown in Fig. 6.24. When the switch is closed, the voltmeter deflection is observed. If the momentary deflection is upscale, the coil terminal connected to the positive terminal of the voltmeter receives the polarity mark. If the deflection is downscale, the coil terminal connected to the negative terminal of the voltmeter receives the polarity mark.

Example 6.6 shows how to use the dot markings to formulate a set of circuit equations in a circuit containing magnetically coupled coils.

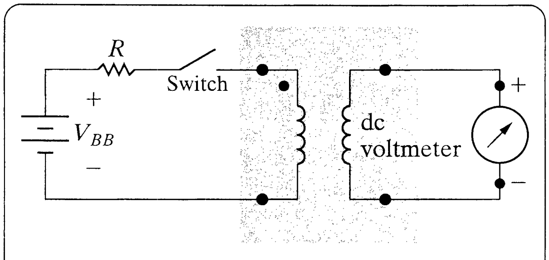


Figure 6.24 An experimental setup for determining polarity marks.

[1] See discussion of Faraday's law on page 240.

ASSESSING OBJECTIVE 4

◆ **Use the dot convention to write mesh-current equations for mutually coupled coils**

6.6 a) Write a set of mesh-current equations for the circuit in Example 6.6 if the dot on the 4 H inductor is at the right-hand terminal, the reference direction of i_g is reversed, and the 60 Ω resistor is increased to 780 Ω.

b) Verify that if there is no energy stored in the circuit at $t = 0$, and if $i_g = 1.96 - 1.96e^{-4t}$ A, the solutions to the differential equations derived in (a) of this Assessment Problem are

$$i_1 = -0.4 - 11.6e^{-4t} + 12e^{-5t} \text{ A},$$

$$i_2 = -0.01 - 0.99e^{-4t} + e^{-5t} \text{ A}.$$

ANSWER: (a) $4(di_1/dt) + 25i_1 + 8(di_2/dt) - 20i_2 = -5i_g - 8(di_g/dt)$ and $8(di_1/dt) - 20i_1 + 16(di_2/dt) + 800i_2 = -16(di_g/dt)$; (b) verification.

NOTE ◆ *Also try Chapter Problem 6.34.*

6.5 ◆ A Closer Look at Mutual Inductance

In order to fully explain the circuit parameter mutual inductance, and to examine the limitations and assumptions made in the qualitative discussion presented in Section 6.4, we begin with a more quantitative description of self-inductance than was previously provided.

A Review of Self-Inductance

The concept of inductance can be traced to Michael Faraday, who did pioneering work in this area in the early 1800s. Faraday postulated that a magnetic field consists of lines of force surrounding the current-carrying conductor. Visualize these lines of force as energy-storing elastic bands that close on themselves. As the current increases and decreases, the elastic bands (that is, the lines of force) spread and collapse about the conductor. The voltage induced in the conductor is proportional to the number of lines

Practical Perspective

Proximity Switches

At the beginning of this chapter we introduced the capacitive proximity switch. There are two forms of this switch. The one used in table lamps is based on a single-electrode switch. It is left to your investigation in Problem 6.50. In the example here, we consider the two-electrode switch used in elevator call buttons.

EXAMPLE

The elevator call button is a small cup into which the finger is inserted, as shown in Fig. 6.31. The cup is made of a metal ring electrode and a circular plate electrode that are insulated from each other. Sometimes two concentric rings embedded in insulating plastic are used instead. The electrodes are covered with an insulating layer to prevent direct contact with the metal. The resulting device can be modeled as a capacitor, as shown in Fig. 6.32.

Unlike most capacitors, the capacitive proximity switch permits you to insert an object, such as a finger, between the electrodes. Because your finger is much more conductive than the insulating covering surrounding the electrodes, the circuit responds as though another electrode, connected to ground, has been added. The result is a three-terminal circuit containing three capacitors, as shown in Fig. 6.33.

The actual values of the capacitors in Figs. 6.32 and 6.33 are in the range of 10 to 50 pF, depending on the exact geometry of the switch, how the finger is inserted, whether the person is wearing gloves, and so forth. For the following problems, assume that all capacitors have the same value of 25 pF. Also assume the elevator call button is placed in the capacitive equivalent of a voltage-divider circuit, as shown in Fig. 6.34.

a) Calculate the output voltage with no finger present.
b) Calculate the output voltage when a finger touches the button.

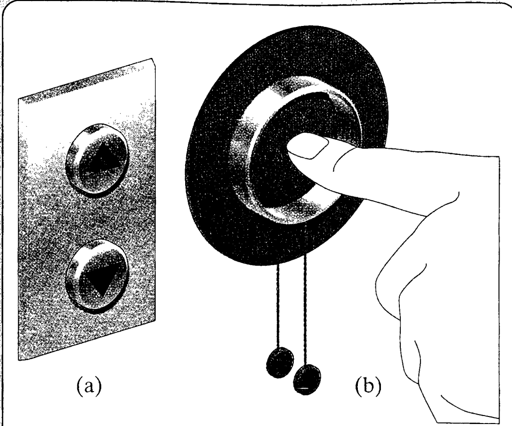


Figure 6.31 An elevator call button. (a) Front view. (b) Side view.

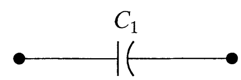


Figure 6.32 A capacitor model of the two-electrode proximity switch used in elevator call buttons.

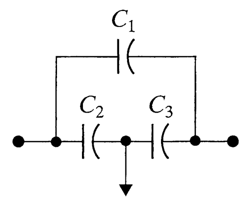


Figure 6.33 A circuit model of a capacitive proximity switch activated by finger touch.

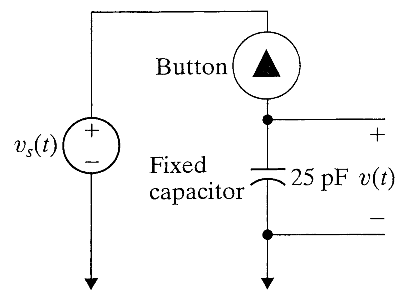


Figure 6.34 An elevator call button circuit.

Solution

a) Begin by redrawing the circuit in Fig. 6.34 with the call button replaced by its capacitive model from Fig. 6.32. The resulting circuit is shown in Fig. 6.35. Write the current equation at the single node:

$$C_1 \frac{d(v - v_s)}{dt} + C_2 \frac{dv}{dt} = 0. \tag{6.70}$$

Rearrange this equation to produce a differential equation for the output voltage $v(t)$:

$$\frac{dv}{dt} = \frac{C_1}{C_1 + C_2} \frac{dv_s}{dt}. \tag{6.71}$$

SUMMARY

- **Inductance** is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field. (See page 218.)
- **Capacitance** is a linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field. (See page 225.)
- Inductors and capacitors are passive elements; they can store and release energy, but they cannot generate or dissipate energy. (See page 218.)
- The instantaneous power at the terminals of an inductor or capacitor can be positive or negative, depending on whether energy is being delivered to or extracted from the element.
- An inductor:
  - does not permit an instantaneous change in its terminal current
  - does permit an instantaneous change in its teminal voltage
  - behaves as a short circuit in the presence of a constant terminal current (See page 219.)
- A capacitor:
  - does not permit an instantaneous change in its terminal voltage
  - does permit an instantaneous change in its terminal current
  - behaves as an open circuit in the presence of a constant terminal voltage (See page 226.)
- Equations for voltage, current, power, and energy in ideal inductors and capacitors are given in Table 6.1.
- Inductors in series or in parallel can be replaced by an equivalent inductor. Capacitors in series or in parallel can be replaced by an equivalent capacitor. The equations are summarized in Table 6.2. See Section 6.3 for a discussion on how to handle the initial conditions for series and parallel equivalent circuits involving inductors and capacitors.

TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors

INDUCTORS	
$v = L\frac{di}{dt}$	(V)
$i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$	(A)
$p = vi = Li\frac{di}{dt}$	(W)
$w = \frac{1}{2}Li^2$	(J)
CAPACITORS	
$v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$	(V)
$i = C\frac{dv}{dt}$	(A)
$p = vi = Cv\frac{dv}{dt}$	(W)
$w = \frac{1}{2}Cv^2$	(J)

TABLE 6.2 Equations for Series- and Parallel-Connected Inductors and Capacitors

SERIES-CONNECTED
$L_{eq} = L_1 + L_2 + \cdots + L_n$
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$
PARALLEL-CONNECTED
$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
$C_{eq} = C_1 + C_2 + \cdots + C_n$

- **Mutual inductance**, M, is the circuit parameter relating the voltage induced in one circuit to a time-varying current in another circuit. Specifically,

$$v_1 = L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt}$$

$$v_2 = M_{21}\frac{di_1}{dt} + L_2\frac{di_2}{dt},$$

where v_1 and i_1 are the voltage and current in circuit 1, and v_2 and i_2 are the voltage and current in circuit 2. For coils wound on nonmagnetic cores, $M_{12} = M_{21} = M$. (See page 243.)

- The **dot convention** establishes the polarity of mutually induced voltages:

> When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

Or, alternatively,

> When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

(See page 236.)

- The relationship between the self-inductance of each winding and the mutual inductance between windings is

$$M = k\sqrt{L_1 L_2}.$$

The **coefficient of coupling**, k, is a measure of the degree of magnetic coupling. By definition, $0 \leq k \leq 1$. (See page 244.)

- The energy stored in magnetically coupled coils in a linear medium is related to the coil currents and inductances by the relationship

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2.$$

(See page 246.)

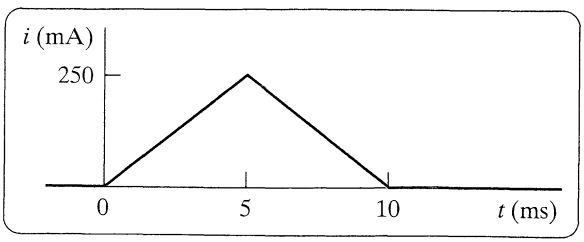
PROBLEMS

6.1 The triangular current pulse shown in Fig. P6.1 is applied to a 20 mH inductor.

a) Write the expressions that describe $i(t)$ in the four intervals $t < 0$, $0 \leq t \leq 5$ ms, $5 \text{ ms} \leq t \leq 10$ ms, and $t > 10$ ms.

b) Derive the expressions for the inductor voltage, power, and energy. Use the passive sign convention.

Figure P6.1

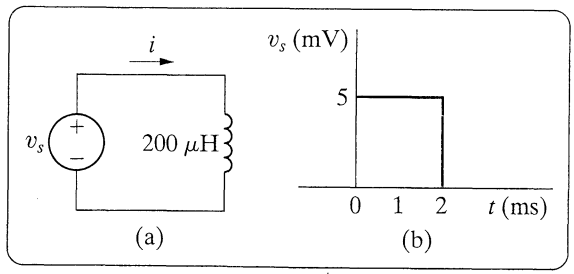


6.2 The voltage at the terminals of the 200 μH inductor in Fig. P6.2(a) is shown in Fig. P6.2(b). The inductor current i is known to be zero for $t \leq 0$.

a) Derive the expressions for i for $t \geq 0$.

b) Sketch i versus t for $0 \leq t \leq \infty$.

Figure P6.2

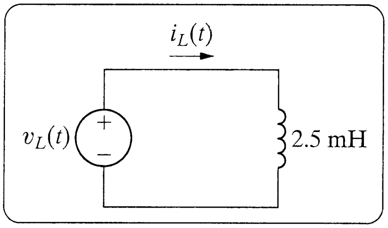


6.3 The current in the 2.5 mH inductor in Fig. P6.3 is known to be 0 A for $t < 0$. The inductor voltage for $t \geq 0$ is given by the expression

$$v_L(t) = \begin{cases} 3e^{-4t}\ \text{mV}, & 0 \leq t \leq 2\ \text{s} \\ -3e^{-4(t-2)}\ \text{mV}, & 2\ \text{s} < t \leq \infty \end{cases}$$

Sketch $v_L(t)$ and $i_L(t)$ for $0 \leq t \leq \infty$.

Figure P6.3



6.4 The current in a 50 μH inductor is known to be

$$i_L = 18te^{-10t}\ \text{A} \quad \text{for } t \geq 0.$$

a) Find the voltage across the inductor for $t > 0$. (Assume the passive sign convention.)

b) Find the power (in microwatts) at the terminals of the inductor when $t = 200$ ms.

c) Is the inductor absorbing or delivering power at 200 ms?

d) Find the energy (in microjoules) stored in the inductor at 200 ms.

e) Find the maximum energy (in microjoules) stored in the inductor and the time (in microseconds) when it occurs.

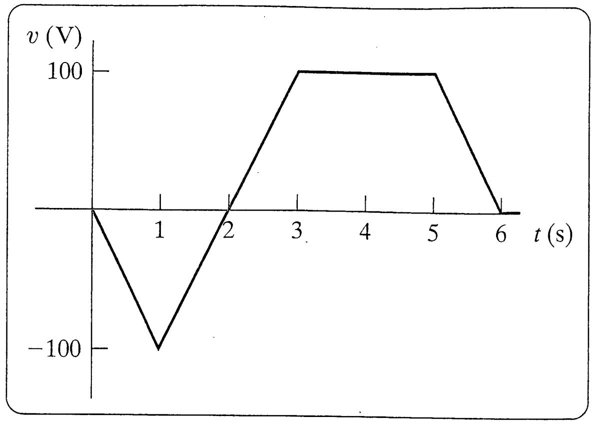
6.5 The current in and the voltage across a 5 H inductor are known to be zero for $t \leq 0$. The voltage across the inductor is given by the graph in Fig. P6.5 for $t \geq 0$.

a) Derive the expression for the current as a function of time in the intervals $0 \leq t \leq 1$ s, $1\ \text{s} \leq t \leq 3$ s, $3\ \text{s} \leq t \leq 5$ s, $5\ \text{s} \leq t \leq 6$ s, and $6\ \text{s} \leq t \leq \infty$.

b) For $t > 0$, what is the current in the inductor when the voltage is zero?

c) Sketch i versus t for $0 \leq t \leq \infty$.

Figure P6.5



6.6 The current in a 25 mH inductor is known to be −10 A for $t \leq 0$ and $-[10\cos 400t + 5\sin 400t]e^{-200t}$ A for $t \geq 0$. Assume the passive sign convention.

a) At what instant of time is the voltage across the inductor maximum?

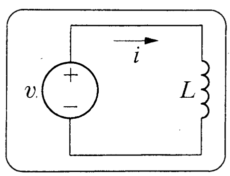
b) What is the maximum voltage?

6.7 a) Find the inductor current in the circuit in Fig. P6.7 if $v = 30\sin 500t$ V, $L = 15$ mH, and $i(0) = -4$ A.

b) Sketch $v, i, p,$ and w versus t. In making these sketches, use the format used in Fig. 6.8. Plot over one complete cycle of the voltage waveform.

c) Describe the subintervals in the time interval between 0 and 4π ms when power is being absorbed by the inductor. Repeat for the subintervals when power is being delivered by the inductor.

Figure P6.7



6.8 The current in a 20 mH inductor is known to be

$$i = 40 \text{ mA}, \quad t \leq 0;$$

$$i = A_1e^{-10{,}000t} + A_2e^{-40{,}000t} \text{ A}, \quad t \geq 0.$$

The voltage across the inductor (passive sign convention) is 28 V at $t = 0$.

a) Find the expression for the voltage across the inductor for $t > 0$.

b) Find the time, greater than zero, when the power at the terminals of the inductor is zero.

6.9 Assume in Problem 6.8 that the value of the voltage across the inductor at $t = 0$ is -68 V instead of 28 V.

a) Find the numerical expressions for i and v for $t \geq 0$.

b) Specify the time intervals when the inductor is storing energy and the time intervals when the inductor is delivering energy.

c) Show that the total energy extracted from the inductor is equal to the total energy stored.

6.10 The current in a 2 H inductor is

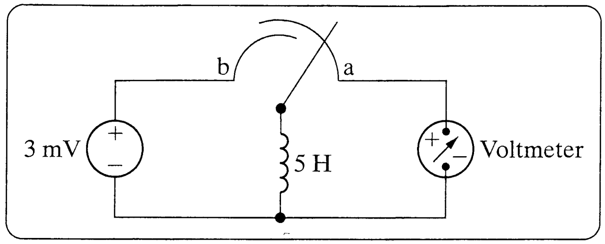
$$i = 5 \text{ A}, \quad t \leq 0;$$

$$i = (B_1 \cos 1.6t + B_2 \sin 1.6t)e^{-0.4t} \text{ A}, \quad t \geq 0.$$

The voltage across the inductor (passive sign convention) is 28 V at $t = 0$. Calculate the power at the terminals of the inductor at $t = 5$ s. State whether the inductor is absorbing or delivering power.

6.11 Initially there was no energy stored in the 5 H inductor in the circuit in Fig. P6.11 when it was placed across the terminals of the voltmeter. At $t = 0$ the inductor was switched instantaneously to position b where it remained for 1.6 s before returning instantaneously to position a. The d'Arsonval voltmeter has a full-scale reading of 20 V and a sensitivity of 1000 Ω/V. What will the reading of the voltmeter be at the instant the switch returns to position a if the inertia of the d'Arsonval movement is negligible?

Figure P6.11



6.12 Evaluate the integral

$$\int_0^{\infty} p\,dt$$

for Example 6.2. Comment on the significance of the result.

6.13 The expressions for voltage, power, and energy derived in Example 6.5 involved both integration and manipulation of algebraic expressions. As an engineer, you cannot accept such results on faith alone. That is, you should develop the habit of asking yourself, "Do these results make sense in terms of the known behavior of the circuit they purport to describe?" With these thoughts in mind, test the expressions of Example 6.5 by performing the following checks:

a) Check the expressions to see whether the voltage is continuous in passing from one time interval to the next.

b) Check the power expression in each interval by selecting a time within the interval and seeing whether it gives the same result as the corresponding product of v and i. For example, test at 10 and 30 μs.

c) Check the energy expression within each interval by selecting a time within the interval and seeing whether the energy equation gives the same result as $\frac{1}{2}Cv^2$. Use 10 and 30 μs as test points.

6.14 A 20 μF capacitor is subjected to a voltage pulse having a duration of 1 s. The pulse is described by the following equations:

$$v_c(t) = \begin{cases} 30t^2 \text{ V}, & 0 \le t \le 0.5 \text{ s}; \\ 30(t-1)^2 \text{ V}, & 0.5 \text{ s} \le t \le 1.0 \text{ s}; \\ 0 & \text{elsewhere.} \end{cases}$$

Sketch the current pulse that exists in the capacitor during the 1 s interval.

6.15 The rectangular-shaped current pulse shown in Fig. P6.15 is applied to a 5 μF capacitor. The initial voltage on the capacitor is a 12 V drop in the reference direction of the current. Assume the passive sign convention. Derive the expression for the capacitor voltage for the time intervals in (a)–(e).

a) $0 \le t \le 5\,\mu$s;

b) $5\,\mu\text{s} \le t \le 20\,\mu$s;

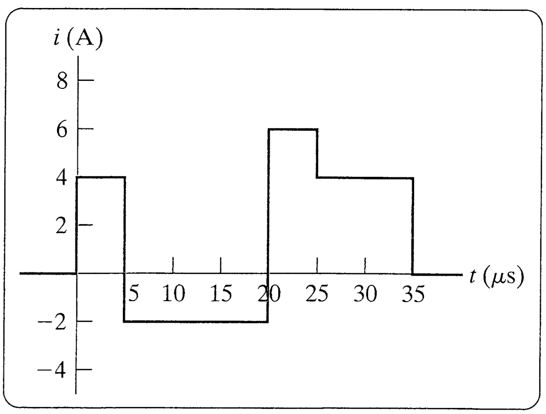
c) $20\,\mu\text{s} \le t \le 25\,\mu$s;

d) $25\,\mu\text{s} \le t \le 35\,\mu$s;

e) $35\,\mu\text{s} \le t \le \infty$;

f) Sketch $v(t)$ over the interval $-50\,\mu\text{s} \le t \le 300\,\mu$s.

Figure P6.15



6.16 The voltage at the terminals of the capacitor in Fig. 6.10 is known to be

$$v = \begin{cases} -10 \text{ V}, & t \le 0; \\ 40 - 1e^{-1000t}(50\cos 500t + 20\sin 500t) \text{ V} & t \ge 0. \end{cases}$$

Assume $C = 0.8\,\mu$F.

a) Find the current in the capacitor for $t < 0$.

b) Find the current in the capacitor for $t > 0$.

c) Is there an instantaneous change in the voltage across the capacitor at $t = 0$?

d) Is there an instantaneous change in the current in the capacitor at $t = 0$?

e) How much energy (in microjoules) is stored in the capacitor at $t = \infty$?

6.17 The current pulse shown in Fig. P6.17 is applied to a 0.25 μF capacitor. The initial voltage on the capacitor is zero.

a) Find the charge on the capacitor at $t = 30\,\mu$s.

b) Find the voltage on the capacitor at $t = 50\,\mu$s.

c) How much energy is stored in the capacitor by the current pulse?

Figure P6.17

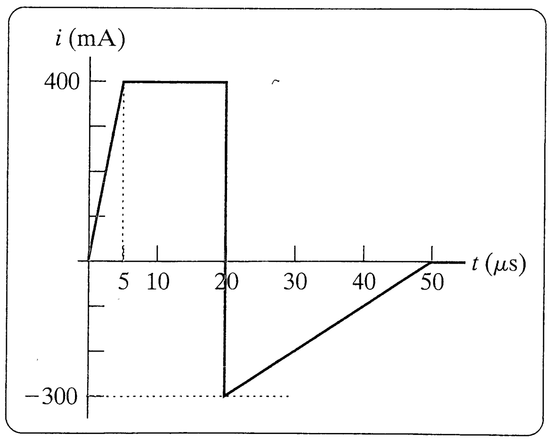
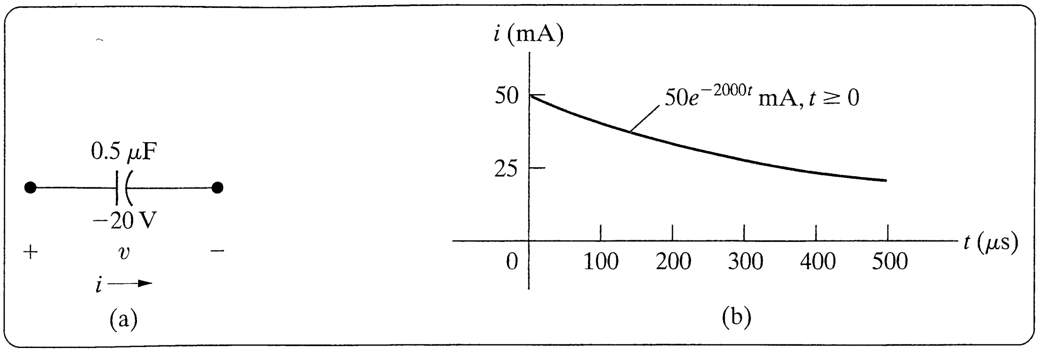


Figure P6.18



6.18 The initial voltage on the 0.5 μF capacitor shown in Fig. P6.18(a) is −20 V. The capacitor current has the waveform shown in Fig. P6.18(b).

a) How much energy, in microjoules, is stored in the capacitor at $t = 500\ \mu$s?

b) Repeat (a) for $t = \infty$.

6.19 The voltage across the terminals of a 0.25 μF capacitor is

$$v = \begin{cases} 50\ \text{V}, & t \leq 0; \\ A_1te^{-4000t} + A_2e^{-4000t}\ \text{V}, & t \geq 0. \end{cases}$$

The initial current in the capacitor is 400 mA. Assume the passive sign convention.

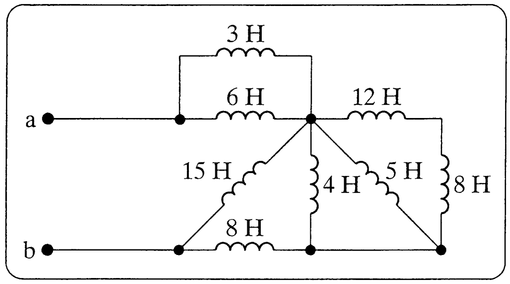
a) What is the initial energy stored in the capacitor?

b) Evaluate the coefficients A_1 and A_2.

c) What is the expression for the capacitor current?

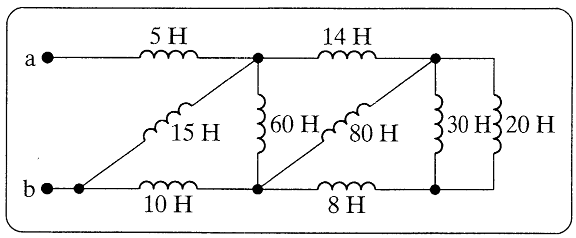
6.20 Assume that the initial energy stored in the inductors of Fig. P6.20 is zero. Find the equivalent inductance with respect to the terminals a,b.

Figure P6.20



6.21 Assume that the initial energy stored in the inductors of Fig. P6.21 is zero. Find the equivalent inductance with respect to the terminals a,b.

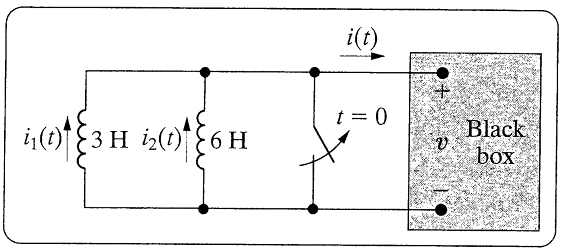
Figure P6.21



6.22 The two parallel inductors in Fig. P6.22 are connected across the terminals of a black box at $t = 0$. The resulting voltage v for $t \geq 0$ is known to be $12e^{-t}$ V. It is also known that $i_1(0) = 2$ A and $i_2(0) = 4$ A.

a) Replace the original inductors with an equivalent inductor and find $i(t)$ for $t \geq 0$.
b) Find $i_1(t)$ for $t \geq 0$.
c) Find $i_2(t)$ for $t \geq 0$.
d) How much energy is delivered to the black box in the time interval $0 \leq t \leq \infty$?
e) How much energy was initially stored in the parallel inductors?
f) How much energy is trapped in the ideal inductors?
g) Do your solutions for i_1 and i_2 agree with the answer obtained in (f)?

Figure P6.22



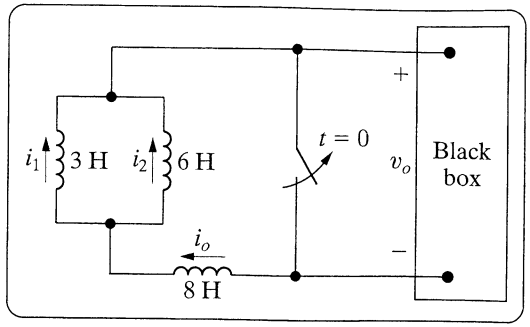
6.23 The three inductors in the circuit in Fig. P6.23 are connected across the terminals of a black box at $t = 0$. The resulting voltage for $t \geq 0$ is known to be

$$v_b = 160e^{-4t} \text{ V}.$$

If $i_1(0) = 1$ A and $i_2(0) = 3$ A, find

a) $i_o(0)$;
b) $i_o(t), t \geq 0$;
c) $i_1(t), t \geq 0$;
d) $i_2(t), t \geq 0$;
e) the initial energy stored in the three inductors;
f) the total energy delivered to the black box; and
g) the energy trapped in the ideal inductors.

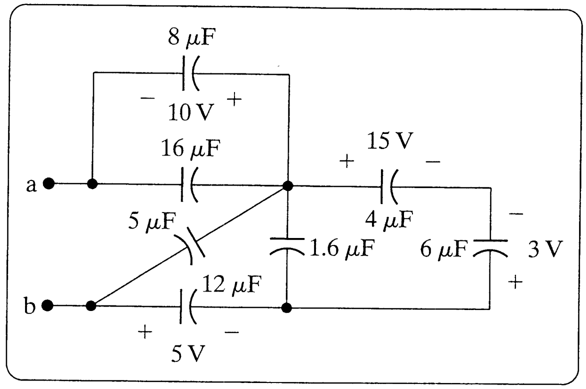
Figure P6.23



6.24 For the circuit shown in Fig. P6.23, what percentage of the total energy delivered to the black box has been delivered when $t = 200$ ms?

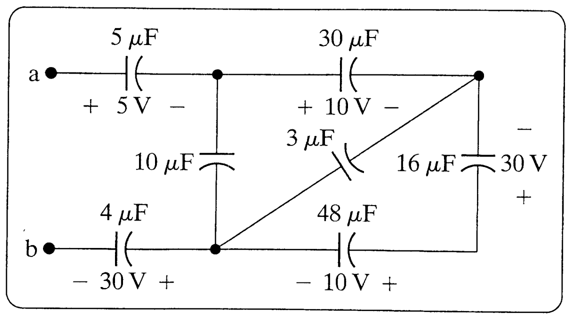
6.25 Find the equivalent capacitance with respect to the terminals a,b for the circuit shown in Fig. P6.25.

Figure P6.25



6.26 Find the equivalent capacitance with respect to the terminals a,b for the circuit shown in Fig. P6.26.

Figure P6.26



6.27 The two series-connected capacitors in Fig. P6.27 are connected to the terminals of a black box at $t = 0$. The resulting current $i(t)$ for $t \geq 0$ is known to be $20e^{-t}$ μA.

a) Replace the original capacitors with an equivalent capacitor and find $v_o(t)$ for $t \geq 0$.

b) Find $v_1(t)$ for $t \geq 0$.

c) Find $v_2(t)$ for $t \geq 0$.

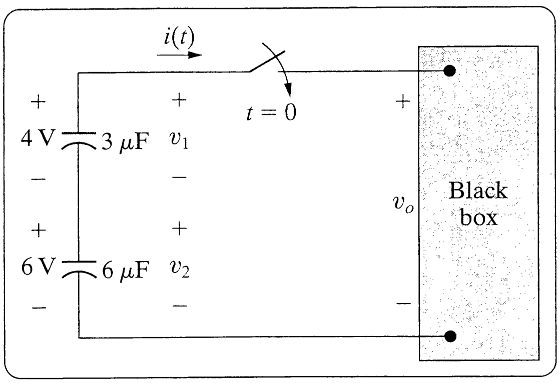
d) How much energy is delivered to the black box in the time interval $0 \leq t \leq \infty$?

e) How much energy was initially stored in the series capacitors?

f) How much energy is trapped in the ideal capacitors?

g) Do the solutions for v_1 and v_2 agree with the answer obtained in (f)?

Figure P6.27

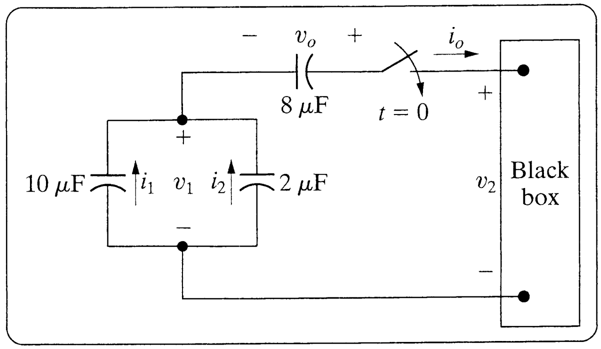


6.28 The four capacitors in the circuit in Fig. P6.28 are connected across the terminals of a black box at $t = 0$. The resulting current i_o for $t \geq 0$ is known to be

$$i_o = 1.92e^{-20t} \text{ mA}.$$

If $v_o(0) = -5$ V and $v_1(0) = 25$ V, find the following for $t \geq 0$: (a) $v_2(t)$, (b) $v_o(t)$, (c) $v_1(t)$, (d) $i_1(t)$, and (e) $i_2(t)$.

Figure P6.28



6.29 For the circuit in Fig. P6.28, calculate

a) the initial energy stored in the capacitors;

b) the final energy stored in the capacitors;

c) the total energy delivered to the black box;

d) the percentage of the initial energy stored that is delivered to the black box; and

e) the percentage of the total energy delivered that is delivered in the first 40 ms.

6.30 Derive the equivalent circuit for a series connection of ideal capacitors. Assume that each capacitor has its own initial voltage. Denote these initial voltages as $v_1(t_0)$, $v_2(t_0)$, and so on. (*Hint:* Sum the voltages across the string of capacitors, recognizing that the series connection forces the current in each capacitor to be the same.)

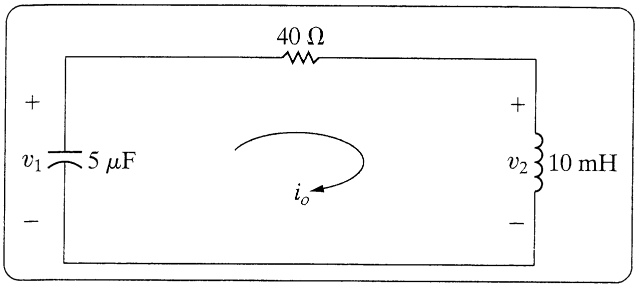
6.31 Derive the equivalent circuit for a parallel connection of ideal capacitors. Assume that the initial voltage across the paralleled capacitors is $v(t_0)$. (*Hint:* Sum the currents into the string of capacitors, recognizing that the parallel connection forces the voltage across each capacitor to be the same.)

6.32 The current in the circuit in Fig. P6.32 is known to be

$$i_o = 5e^{-2000t}(2\cos 4000t + \sin 4000t)\ \text{A}$$

for $t \geq 0^+$. Find $v_1(0^+)$ and $v_2(0^+)$.

Figure P6.32

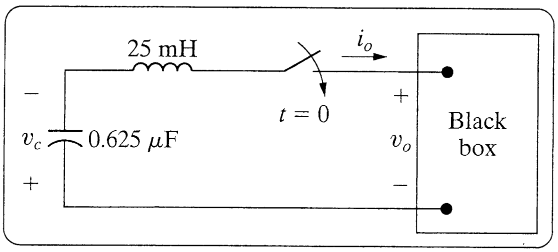


6.33 At $t = 0$, a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Fig. P6.33. For $t \geq 0$, it is known that

$$i_o = 1.5e^{-16{,}000t} - 0.5e^{-4000t}\ \text{A}.$$

If $v_c(0) = -50$ V find v_o for $t \geq 0$.

Figure P6.33



6.34 There is no energy stored in the circuit in Fig. P6.34 at the time the switch is opened.

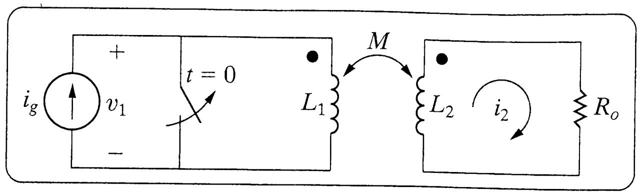
a) Derive the differential equation that governs the behavior of i_2 if $L_1 = 4$ H, $L_2 = 16$ H, $M = 2$ H, and $R_o = 32\ \Omega$.

b) Show that when $i_g = 8 - 8e^{-t}$ A, $t \geq 0$, the differential equation derived in (a) is satisfied when $i_2 = e^{-t} - e^{-2t}$ A, $t \geq 0$.

c) Find the expression for the voltage v_1 across the current source.

d) What is the initial value of v_1? Does this make sense in terms of known circuit behavior?

Figure P6.34



6.35 Let v_o represent the voltage across the 16 H inductor in the circuit in Fig. 6.25. Assume v_o is positive at the dot. As in Example 6.6, $i_g = 16 - 16e^{-5t}$ A.

a) Can you find v_o without having to differentiate the expressions for the currents? Explain.

b) Derive the expression for v_o.

c) Check your answer in (b) using the appropriate current derivatives and inductances.

6.36 Let v_g represent the voltage across the current source in the circuit in Fig. 6.25. The reference for v_g is positive at the upper terminal of the current source.

a) Find v_g as a function of time when $i_g = 16 - 16e^{-5t}$ A.

b) What is the initial value of v_g?

c) Find the expression for the power developed by the current source.

d) How much power is the current source developing when t is infinite?

e) Calculate the power dissipated in each resistor when t is infinite.

6.37 a) Show that the differential equations derived in (a) of Example 6.6 can be rearranged as follows:

$$4\frac{di_1}{dt} + 25i_1 - 8\frac{di_2}{dt} - 20i_2 = 5i_g - 8\frac{di_g}{dt};$$

$$-8\frac{di_1}{dt} - 20i_1 + 16\frac{di_2}{dt} + 80i_2 = 16\frac{di_g}{dt}.$$

b) Show that the solutions given in (a) of Example 6.6 satisfy the differential equations given in part (a) of this problem.

6.38 Two magnetically coupled coils are wound on a nonmagnetic core. The self-inductance of coil 1 is 288 mH, the mutual inductance is 90 mH, the coefficient of coupling is 0.75, and the physical structure of the coils is such that $\mathcal{P}_{11} = \mathcal{P}_{22}$.

a) Find L_2 and the turns ratio N_1/N_2.

b) If $N_1 = 1200$, what is the value of $\mathcal{P}_1$ and $\mathcal{P}_2$?

6.39 The self-inductances of two magnetically coupled coils are $L_1 = 180\ \mu$H and $L_2 = 500\ \mu$H. The coupling medium is nonmagnetic. If coil 1 has 300 turns and coil 2 has 500 turns, find $\mathcal{P}_{11}$ and $\mathcal{P}_{21}$ (in nanowebers per ampere) when the coefficient of coupling is 0.6.

6.40 Two magnetically coupled coils have self-inductances of 27 mH and 3 mH, respectively. The mutual inductance between the coils is 7.2 mH.

a) What is the coefficient of coupling?

b) For these two coils, what is the largest value that M can have?

c) Assume that the physical structure of these coupled coils is such that $\mathcal{P}_1 = \mathcal{P}_2$. What is the turns ratio N_1/N_2 if N_1 is the number of turns on the 27 mH coil?

6.41 The self-inductances of two magnetically coupled coils are 36 mH and 9 mH, respectively. The 36 mH coil has 200 turns, and the coefficient of coupling between the coils is 0.8. The coupling medium is nonmagnetic. When coil 1 is excited with coil 2 open, the flux linking only coil 1 is 0.1 as large as the flux linking coil 2. When coil 2 is excited, with coil 1 open, the flux linking coil 2 is only 0.125 as large as the flux linking coil 1.

a) How many turns does coil 2 have?

b) What is the value of $\mathcal{P}_2$ in nanowebers per ampere?

c) What is the value of $\mathcal{P}_{11}$ in nanowebers per ampere?

d) What is the ratio (ϕ_{22}/ϕ_{12})?

6.42 The physical construction of four pairs of magnetically coupled coils is shown in Fig. P6.42. (See page 259.) Assume that the magnetic flux is confined to the core material in each structure. Show two possible locations for the dot markings on each pair of coils.

6.43 a) Starting with Eq. 6.59, show that the coefficient of coupling can also be expressed as

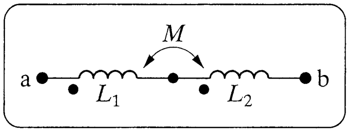
$$k = \sqrt{\left(\frac{\phi_{21}}{\phi_1}\right)\left(\frac{\phi_{12}}{\phi_2}\right)}.$$

b) On the basis of the fractions ϕ_{21}/ϕ_1 and ϕ_{12}/ϕ_2, explain why k is less than 1.0.

6.44 a) Show that the two coupled coils in Fig. P6.44 can be replaced by a single coil having an inductance of $L_{ab} = L_1 + L_2 + 2M$. (*Hint:* Express v_{ab} as a function of i_{ab}.)

b) Show that if the connections to the terminals of the coil labeled L_2 are reversed, $L_{ab} = L_1 + L_2 - 2M$.

Figure P6.44



6.45 The polarity markings on two coils are to be determined experimentally. The experimental setup is shown in Fig. P6.45. Assume that the terminal connected to the positive terminal of the battery has been given a polarity mark as shown. When the switch is *opened,* the dc voltmeter kicks upscale. Where should the polarity mark be placed on the coil connected to the voltmeter?

Figure P6.45

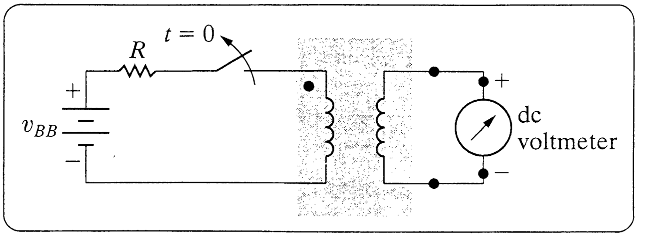
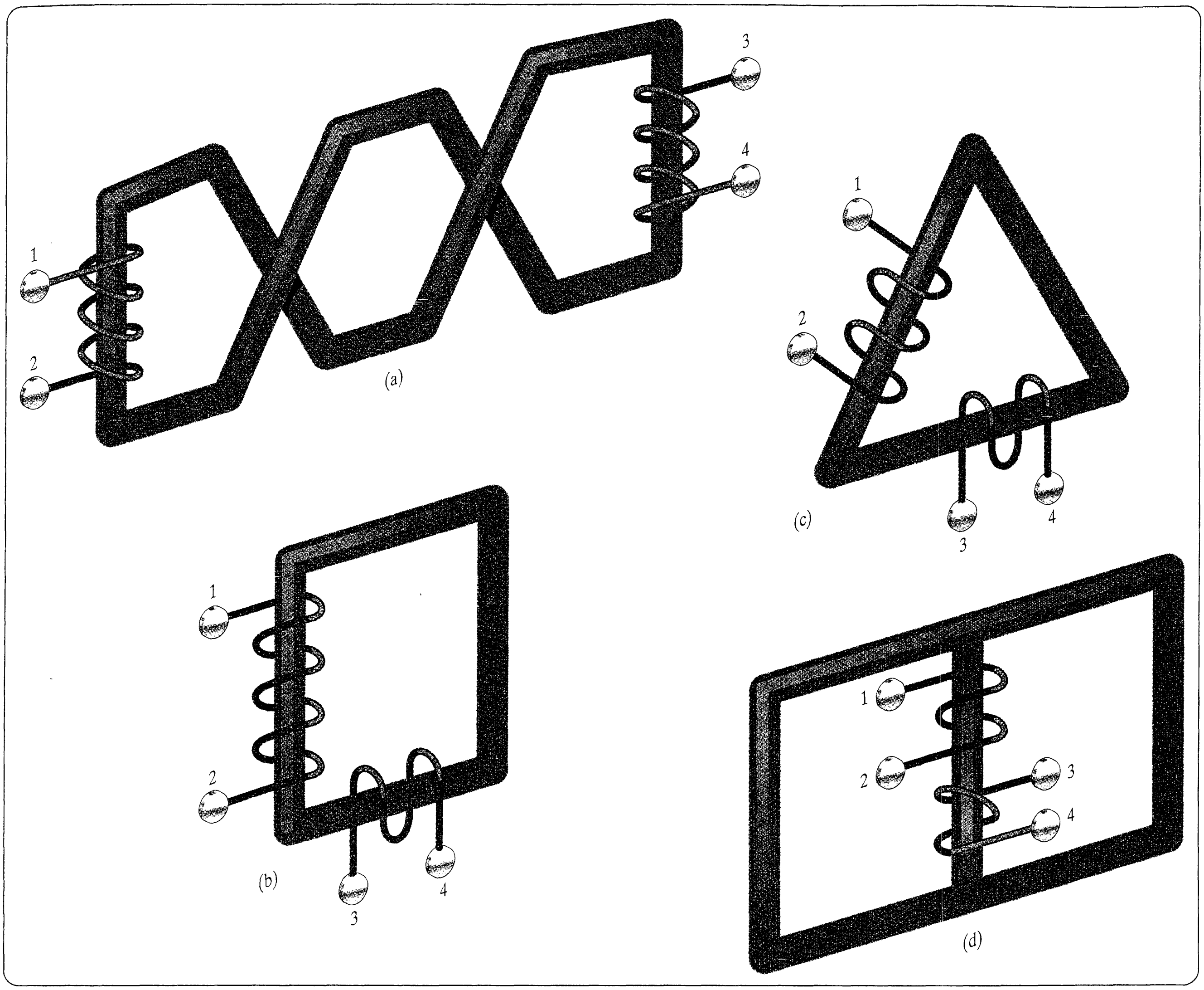


Figure P6.42



6.46 a) Show that the two magnetically coupled coils in Fig. P6.46 can be replaced by a single coil having an inductance of

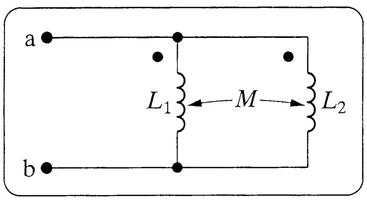
$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}.$$

(*Hint:* Let i_1 and i_2 be clockwise mesh currents in the left and right "windows" of Fig. P6.46, respectively. Sum the voltages around the two meshes. In mesh 1 let v_{ab} be the unspecified applied voltage. Solve for di_1/dt as a function of v_{ab}.)

b) Show that if the magnetic polarity of coil 2 is reversed, then

$$L_{ab} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}.$$

Figure P6.46



6.47 The self-inductances of the coils in Fig. 6.30 are $L_1 = 18$ mH and $L_2 = 32$ mH. If the coefficient of coupling is 0.85, calculate the energy stored in the system in millijoules when (a) $i_1 = 6$ A, $i_2 = 9$ A; (b) $i_1 = -6$ A, $i_2 = -9$ A; (c) $i_1 = -6$ A, $i_2 = 9$ A; and (d) $i_1 = 6$ A, $i_2 = -9$ A.

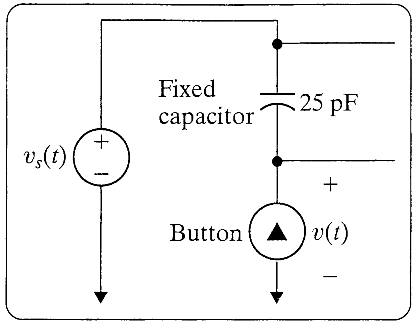
6.48 The coefficient of coupling in Problem 6.47 is increased to 1.0.

a) If i_1 equals 6 A, what value of i_2 results in zero stored energy?

b) Is there any physically realizable value of i_2 that can make the stored energy negative?

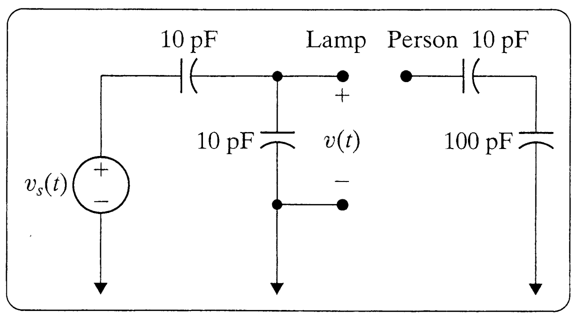
6.49 ◆ Rework the Practical Perspective example, except that this time, put the button on the bottom of the divider circuit, as shown in Fig. P6.49. Calculate the output voltage $v(t)$ when a finger is present.

Figure P6.49



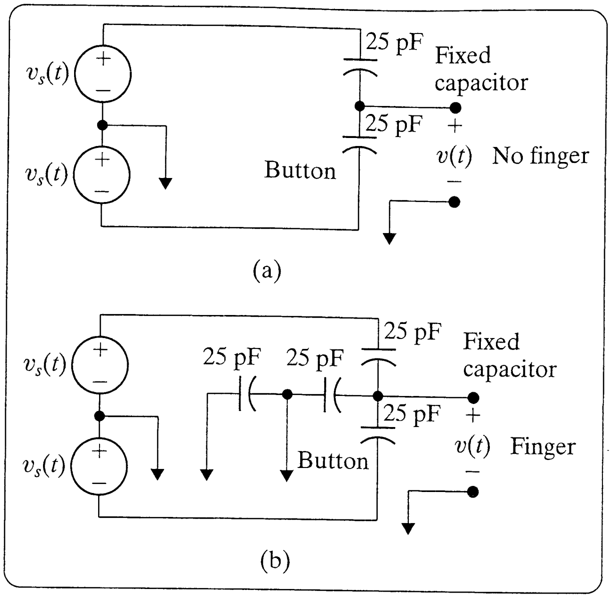
6.50 ◆ Some lamps are made to turn on or off when the base is touched. These use a one-terminal variation of the capacitive switch circuit discussed in the Practical Perspective. Figure P6.50 shows a circuit model of such a lamp. Calculate the change in the voltage $v(t)$ when a person touches the lamp. Assume all capacitors are initially discharged.

Figure P6.50



6.51 ◆ In the Practical Perspective example, we calculated the output voltage when the elevator button is the upper capacitor in a voltage divider. In Problem 6.49, we calculated the voltage when the button is the bottom capacitor in the divider, and we got the same result! You may wonder if this will be true for all such voltage dividers. Calculate the voltage difference (finger versus no finger) for the circuits in Figs. P6.51(a) and (b), which use two identical voltage sources.

Figure P6.51



CHAPTER

7

Response of First-Order *RL* and *RC* Circuits

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1 Be able to determine the natural response of both *RL* and *RC* circuits.
2 Be able to determine the step response of both *RL* and *RC* circuits.
3 Know how to analyze circuits with sequential switching.
4 Be able to analyze op amp circuits containing resistors and a single capacitor.

In Chapter 6, we noted that an important attribute of inductors and capacitors is their ability to store energy. We are now in a position to determine the currents and voltages that arise when energy is either released or acquired by an inductor or capacitor in response to an abrupt change in a dc voltage or current source. In this chapter, we will focus on circuits that consist only of sources, resistors, and either (but not both) inductors or capacitors. For brevity, such configurations are called ***RL*** (resistor-inductor) and ***RC*** (resistor-capacitor) **circuits**.

Our analysis of *RL* and *RC* circuits will be divided into three phases. In the first phase, we consider the currents and voltages that arise when stored energy in an inductor or capacitor is suddenly released to a resistive network. This happens when the inductor or capacitor is abruptly disconnected from its dc source. Thus we can reduce the circuit to one of the two equivalent forms shown in Fig. 7.1 on page 264. The currents and voltages that arise in this configuration are referred to as the **natural response** of the circuit, to emphasize that the nature of the circuit itself, not external sources of excitation, determines its behavior.

In the second phase of our analysis, we consider the currents and voltages that arise when energy is being acquired by an inductor or capacitor due to the sudden application of a dc voltage or current source. This response is referred to as the **step response**. The process for finding both the natural and step responses is the same; thus, in the third phase of our analysis, we develop

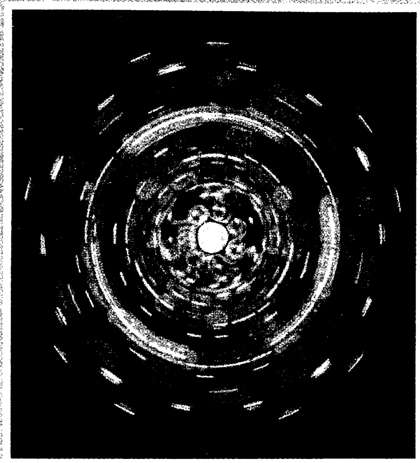
Practical Perspective

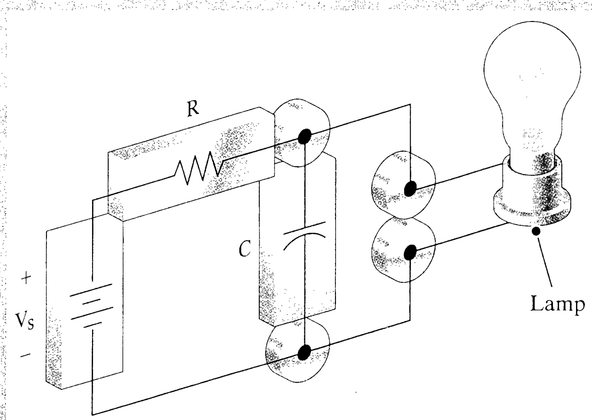
A Flashing Light Circuit

You can probably think of many different applications that require a flashing light. A still camera used to take pictures in low light conditions employs a bright flash of light to illuminate the scene for just long enough to record the image on film. Generally, the camera cannot take another picture until the circuit that creates the flash of light has "re-charged."

Other applications use flashing lights as warning for hazards, such as tall antenna towers, construction sites, and secure areas. In designing circuits to produce a flash of light the engineer must know the requirements of the application. For example, the design engineer has to know whether the flash is controlled manually by operating a switch (as in the case of a camera) or if the flash is to repeat itself automatically at a predetermined rate. The engineer also has to know if the flashing light is a permanent fixture (as on an antenna) or a temporary installation (as at a construction site). Another question that has to be answered is whether a power source is readily available.

Many of the circuits that are used today to control flashing lights are based on electronic circuits that are beyond the scope of this text. Nevertheless we can get a feel for the thought process involved in designing a flashing light circuit by analyzing a circuit consisting of a dc voltage source, a resistor, a capacitor, and a lamp that is designed to discharge a flash of light at a critical voltage. Such a circuit is shown in the figure. We shall discuss this circuit at the end of the chapter.





been opened, the energy stored in the two inductors is

$$w = \frac{1}{2}(5)(1.6)^2 + \frac{1}{2}(20)(-1.6)^2 = 32 \text{ J.}$$

d) We obtain the total energy delivered to the resistive network by integrating the expression for the instantaneous power from zero to infinity:

$$w = \int_0^\infty p\,dt = \int_0^\infty 1152e^{-4t}\,dt$$

$$= 1152 \left.\frac{e^{-4t}}{-4}\right|_0^\infty = 288 \text{ J.}$$

This result is the difference between the initially stored energy (320 J) and the energy trapped in the parallel inductors (32 J). The equivalent inductor for the parallel inductors (which predicts the terminal behavior of the parallel combination) has an initial energy of 288 J; that is, the energy stored in the equivalent inductor represents the amount of energy that will be delivered to the resistive network at the terminals of the original inductors.

ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural response of both *RL* and *RC* circuits**

7.1 The switch in the circuit shown has been closed for a long time and is opened at $t = 0$.

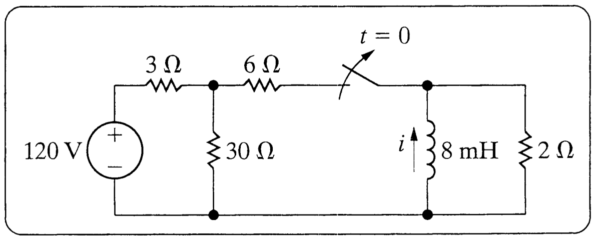
a) Calculate the initial value of i.

b) Calculate the initial energy stored in the inductor.

c) What is the time constant of the circuit for $t > 0$?

d) What is the numerical expression for $i(t)$ for $t \geq 0$?

e) What percentage of the initial energy stored has been dissipated in the 2 Ω resistor 5 ms after the switch has been opened?

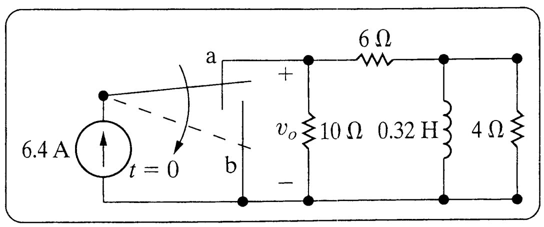


ANSWER: (a) −12.5 A; (b) 625 mJ; (c) 4 ms; (d) $-12.5e^{-250t}$ A; (e) 91.8%.

7.2 At $t = 0$, the switch in the circuit shown moves instantaneously from position a to position b.

a) Calculate v_o for $t \geq 0^+$.

b) What percentage of the initial energy stored in the inductor is eventually dissipated in the 4 Ω resistor?



ANSWER: (a) $-8e^{-10t}$ V; (b) 80%.

NOTE ◆ *Also try Chapter Problems 7.1–7.3.*

d) The total energy delivered to the 250 kΩ resistor is

$$w = \int_0^\infty p\,dt = \int_0^\infty \frac{400e^{-2t}}{250{,}000}\,dt = 800\ \mu\text{J}.$$

Comparing the results obtained in (b) and (c) shows that

$$800\ \mu\text{J} = (5800 - 5000)\ \mu\text{J}.$$

The energy stored in the equivalent capacitor in Fig. 7.15 is $\frac{1}{2}(4 \times 10^{-6})(400)$, or 800 μJ. Because this capacitor predicts the terminal behavior of the original series-connected capacitors, the energy stored in the equivalent capacitor is the energy delivered to the 250 kΩ resistor.

ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural response of both *RL* and *RC* circuits**

7.3 The switch in the circuit shown has been closed for a long time and is opened at $t = 0$. Find

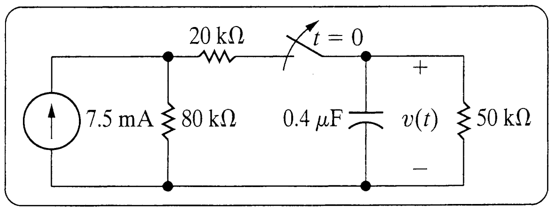
a) the initial value of $v(t)$

b) the time constant for $t > 0$

c) the numerical expression for $v(t)$ after the switch has been opened

d) the initial energy stored in the capacitor

e) the length of time required to dissipate 75% of the initially stored energy.

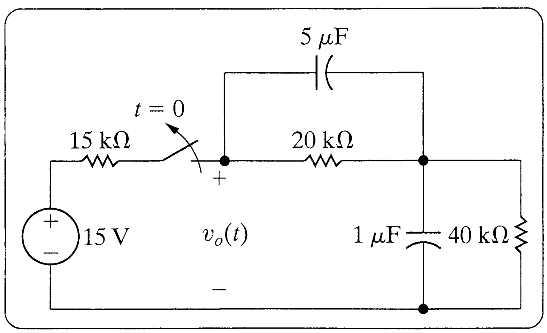


ANSWER: (a) 200 V; (b) 20 ms; (c) $200e^{-50t}$ V; (d) 8 mJ; (e) 13.86 ms.

7.4 The switch in the circuit shown has been closed for a long time before being opened at $t = 0$.

a) Find $v_o(t)$ for $t \geq 0$.

b) What percentage of the initial energy stored in the circuit has been dissipated after the switch has been open for 60 ms?



ANSWER: (a) $8e^{-25t} + 4e^{-10t}$ V; (b) 81.05%.

NOTE ◆ *Also try Chapter Problems 7.21 and 7.22.*

e) Figure 7.20 shows the graphs of $i(t)$ and $v(t)$ versus t. Note that the instant of time when the current equals zero corresponds to the instant of time when the inductor voltage equals the source voltage of 24 V, as predicted by Kirchhoff's voltage law.

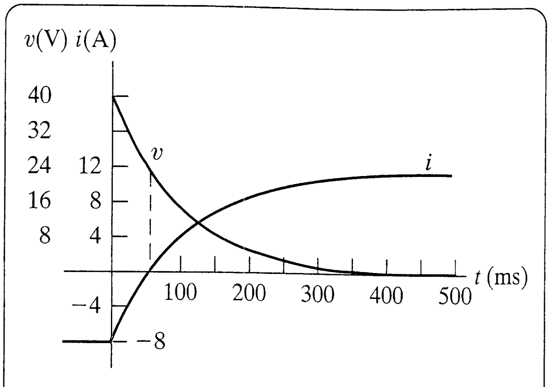


Figure 7.20 The current and voltage waveforms for Example 7.5.

ASSESSING OBJECTIVE 2

◆ **Be able to determine the step response of both *RL* and *RC* circuits**

7.5 Assume that the switch in the circuit shown in Fig. 7.19 has been in position b for a long time, and at $t = 0$ it moves to position a. Find (a) $i(0^+)$; (b) $v(0^+)$; (c) τ, $t > 0$; (d) $i(t)$, $t \geq 0$; and (e) $v(t)$, $t \geq 0^+$.

ANSWER: (a) 12 A; (b) −200 V; (c) 20 ms; (d) $-8 + 20e^{-50t}$ A, $t \geq 0$; (e) $-200e^{-50t}$ V, $t \geq 0^+$.

NOTE ◆ *Also try Chapter Problems 7.33–7.35.*

We can also describe the voltage $v(t)$ across the inductor in Fig. 7.16 directly, not just in terms of the circuit current. We begin by noting that the voltage across the resistor is the difference between the source voltage and the inductor voltage. We write

$$i(t) = \frac{V_s}{R} - \frac{v(t)}{R}, \tag{7.44}$$

where V_s is a constant. Differentiating both sides with respect to time yields

$$\frac{di}{dt} = -\frac{1}{R}\frac{dv}{dt}. \tag{7.45}$$

ASSESSING OBJECTIVE 2

◆ **Be able to determine the step response of both *RL* and *RC* circuits**

7.6 a) Find the expression for the voltage across the 160 kΩ resistor in the circuit shown in Fig. 7.22. Let this voltage be denoted v_A, and assume that the reference polarity for the voltage is positive at the upper terminal of the 160 kΩ resistor.

b) Specify the interval of time for which the expression obtained in (a) is valid.

ANSWER: (a) $-60 + 72e^{-100t}$ V; (b) $t \geq 0^+$.

NOTE ◆ *Also try Chapter Problems 7.47 and 7.48.*

7.4 ◆ A General Solution for Step and Natural Responses

The general approach to finding either the natural response or the step response of the first-order *RL* and *RC* circuits shown in Fig. 7.24 is based on their differential equations having the same form (compare Eq. 7.48

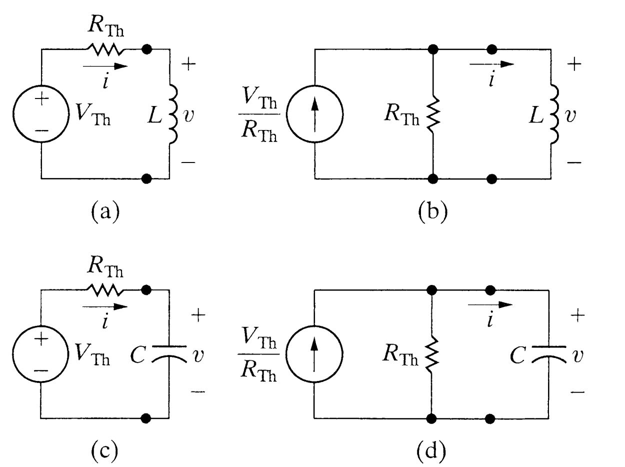


Figure 7.24 Four possible first-order circuits.
(a) An inductor connected to a Thévenin equivalent.
(b) An inductor connected to a Norton equivalent.
(c) A capacitor connected to a Thévenin equivalent.
(d) A capacitor connected to a Norton equivalent.

ASSESSING OBJECTIVE 3

◆ **Know how to analyze circuits with sequential switching**

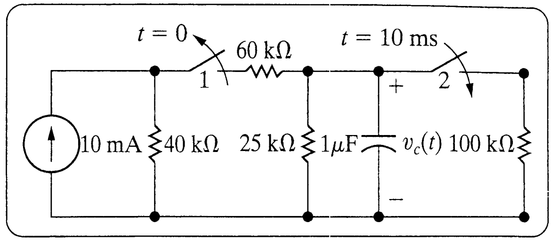
7.7 In the circuit shown, switch 1 has been closed and switch 2 has been open for a long time. At $t = 0$, switch 1 is opened. Then 10 ms later, switch 2 is closed. Find

a) $v_c(t)$ for $0 \leq t \leq 0.01$ s

b) $v_c(t)$ for $t \geq 0.01$ s

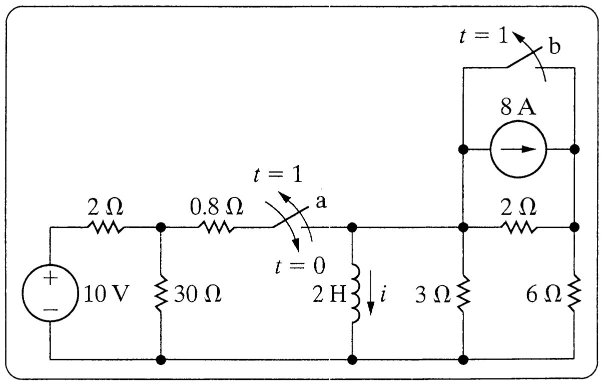
c) the total energy dissipated in the 25 kΩ resistor

d) the total energy dissipated in the 100 kΩ resistor



ANSWER: (a) $80e^{-40t}$ V; (b) $53.63e^{-50(t-0.01)}$ V; (c) 2.91 mJ; (d) 0.29 mJ.

7.8 Switch a in the circuit shown has been open for a long time, and switch b has been closed for a long time. Switch a is closed at $t = 0$ and, after remaining closed for 1 s, is opened again. Switch b is opened simultaneously, and both switches remain open indefinitely. Determine the expression for the inductor current i that is valid when (a) $0 \leq t \leq 1$ s and (b) $t \geq 1$ s.



ANSWER: (a) $(3 - 3e^{-0.5t})$ A, $0 \leq t \leq 1$ s;
(b) $(-4.8 + 5.98e^{-1.25(t-1)})$ A, $t \geq 1$ s.

NOTE ◆ *Also try Chapter Problems 7.69 and 7.70.*

7.6 ◆ Unbounded Response

A circuit response may grow, rather than decay, exponentially with time. This type of response, called an **unbounded response**, is possible if the circuit contains dependent sources. In that case, the Thévenin equivalent

Note that we can convert the integrating amplifier to a differentiating amplifier by interchanging the input resistance R_s and the feedback capacitor C_f. Then

$$v_o = -R_s C_f \frac{dv_s}{dt}. \tag{7.70}$$

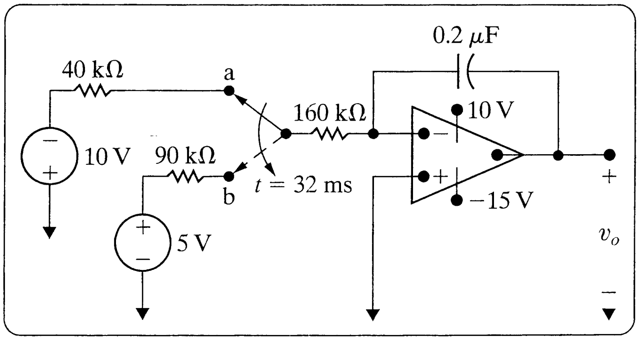
We leave the derivation of Eq. 7.70 as an exercise for you. The differentiating amplifier is seldom used because in practice it is a source of unwanted or noisy signals.

Finally, we can design both integrating- and differentiating- amplifier circuits by using an inductor instead of a capacitor. However, fabricating capacitors for integrated-circuit devices is much easier, so inductors are rarely used in integrating amplifiers.

ASSESSING OBJECTIVE 4

◆ **Be able to analyze op amp circuits containing resistors and a single capacitor**

7.9 There is no energy stored in the capacitor at the time the switch in the circuit makes contact with terminal a. The switch remains at position a for 32 ms and then moves instantaneously to position b. How many milliseconds after making contact with terminal a does the op amp saturate?

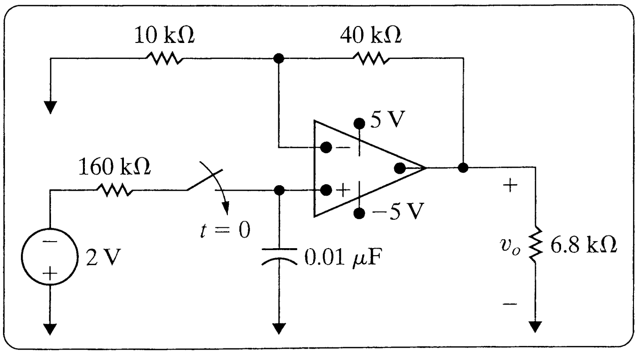


ANSWER: 262 ms.

7.10

a) When the switch closes in the circuit shown, there is no energy stored in the capacitor. How long does it take to saturate the op amp?

b) Repeat (a) with an initial voltage on the capacitor of 1 V, positive at the upper terminal.



ANSWER: (a) 1.11 ms; (b) 1.76 ms

NOTE ◆ *Also try Chapter Problems 7.89 and 7.90.*

Practical Perspective

A Flashing Light Circuit

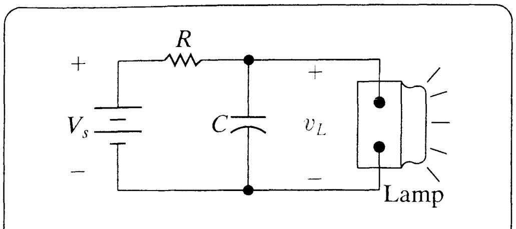


Figure 7.45 A flashing light circuit.

We are now ready to analyze the flashing light circuit introduced at the start of this chapter and shown in Fig. 7.45. The lamp in this circuit starts to conduct whenever the lamp voltage reaches a value $V_{\max}$. During the time the lamp conducts, it can be modeled as a resistor whose resistance is R_L. The lamp will continue to conduct until the lamp voltage drops to the value $V_{\min}$. When the lamp is not conducting, it behaves as an open circuit.

Before we develop the analytical expressions that describe the behavior of the circuit, let us develop a feel for how the circuit works by noting the following. First, when the lamp behaves as an open circuit, the dc voltage source will charge the capacitor via the resistor R toward a value of V_s volts. However, once the lamp voltage reaches $V_{\max}$, it starts conducting and the capacitor will start to discharge toward the Thévenin voltage seen from the terminals of the capacitor. But once the capacitor voltage reaches the cutoff voltage of the lamp ($V_{\min}$), the lamp will act as an open circuit and the capacitor will start to recharge. This cycle of charging and discharging the capacitor is summarized in the sketch shown in Fig. 7.46.

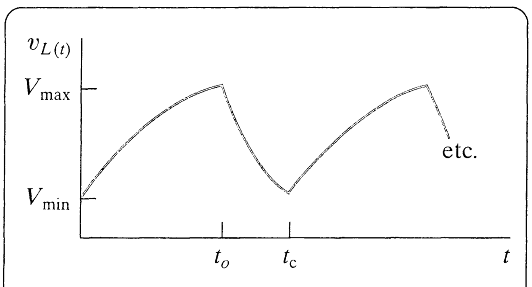


Figure 7.46 Lamp voltage versus time for the circuit in Fig. 7.45.

In drawing Fig. 7.46 we have chosen $t = 0$ at the instant the capacitor starts to charge. The time t_o represents the instant the lamp starts to conduct, and t_c is the end of a complete cycle. We should also mention that in constructing Fig. 7.46 we have assumed the circuit has reached the repetitive stage of its operation. Our design of the flashing light circuit requires we develop the equation for $v_L(t)$ as a function of $V_{\max}$, $V_{\min}$, V_s, R, C, and R_L for the intervals 0 to t_o and t_o to t_c.

To begin the analysis, we assume that the circuit has been in operation for a long time. Let $t = 0$ at the instant when the lamp stops conducting. Thus, at $t = 0$, the lamp is modeled as an open circuit, and the voltage drop across the lamp is $V_{\min}$, as shown in Fig. 7.47.

From the circuit, we find

$$v_L(\infty) = V_s,$$

$$v_L(0) = V_{\min},$$

$$\tau = RC.$$

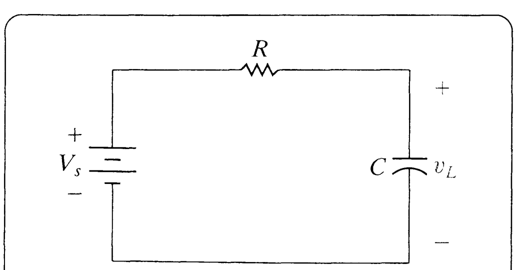


Figure 7.47 The flashing light circuit at $t = 0$, when the lamp is not conducting.

Thus, when the lamp is not conducting,

$$v_L(t) = V_s + (V_{\min} - V_s)e^{-t/RC}.$$

How long does it take before the lamp is ready to conduct? We can find this time by setting the expression for $v_L(t)$ equal to $V_{\max}$ and solving for t. If we call this value t_o, then

$$t_o = RC \ln \frac{V_{\min} - V_s}{V_{\max} - V_s}.$$

When the lamp begins conducting, it can be modeled as a resistance R_L, as seen in Fig. 7.48. In order to find the expression for the voltage drop across the capacitor in this circuit, we need to find the Thévenin equivalent as seen by the capacitor. We leave to you to show, in Problem 7.106, that when the lamp is conducting,

$$v_L(t) = V_{\text{Th}} + (V_{\max} - V_{\text{Th}})e^{-(t-t_o)/\tau}$$

where

$$V_{\text{Th}} = \frac{R_L}{R + R_L} V_s$$

and

$$\tau = \frac{R R_L C}{R + R_L}.$$

We can determine how long the lamp conducts by setting the above expression for $v_L(t)$ to $V_{\min}$ and solving for $(t_c - t_o)$, giving

$$(t_c - t_o) = \frac{R R_L C}{R + R_L} \ln \frac{V_{\max} - V_{\text{Th}}}{V_{\min} - V_{\text{Th}}}.$$

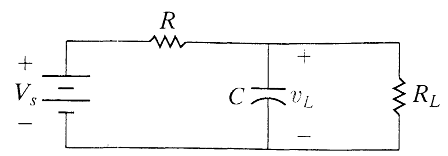


Figure 7.48 The flashing light circuit at $t = t_o$, when the lamp is conducting.

NOTE ◆ *Assess your understanding of this Practical Perspective by trying Chapter Problems 7.103–7.105.*

SUMMARY

- A first-order circuit may be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor. (See page 264.)
- The **natural response** is the currents and voltages that exist when stored energy is released to a circuit that contains no independent sources. (See page 262.)
- The **time constant** of an *RL* circuit equals the equivalent inductance divided by the Thévenin resistance as viewed from the terminals of the equivalent inductor. (See page 267.)
- The **time constant** of an *RC* circuit equals the equivalent capacitance times the Thévenin resistance as viewed from the terminals of the equivalent capacitor. (See page 273.)
- The **step response** is the currents and voltages that result from abrupt changes in dc sources connected to a circuit. Stored energy may or may not be present at the time the abrupt changes take place. (See page 277.)
- The solution for either the natural or step response of both *RL* and *RC* circuits involves finding the initial and final value of the current or voltage of interest and the time constant of the circuit. Equations 7.59 and 7.60 summarize this approach. (See page 286.)
- **Sequential switching** in first-order circuits is analyzed by dividing the analysis into time intervals corresponding to specific switch positions. Initial values for a particular interval are determined from the solution corresponding to the immediately preceding interval. (See page 293.)
- An **unbounded response** occurs when the Thévenin resistance is negative, which is possible when the first-order circuit contains dependent sources. (See page 297.)
- An integrating amplifier consists of an ideal op amp, a capacitor in the negative feedback branch, and a resistor in series with the signal source. It outputs the integral of the signal source, within specified limits that avoid saturating the op amp. (See page 299.)

PROBLEMS

7.1 The switch in the circuit in Fig. P7.1 has been closed for a long time before opening at $t = 0$.

a) Find $i_1(0^-)$ and $i_2(0^-)$.

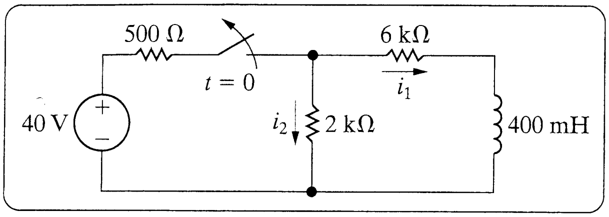
b) Find $i_1(0^+)$ and $i_2(0^+)$.

c) Find $i_1(t)$ for $t \geq 0$.

d) Find $i_2(t)$ for $t \geq 0^+$.

e) Explain why $i_2(0^-) \neq i_2(0^+)$.

Figure P7.1



7.2 In the circuit shown in Fig. P7.2, the switch makes contact with position b just before breaking contact with position a. As already mentioned, this is known as a make-before-break switch and is designed so that the switch does not interrupt the current in an inductive circuit. The interval of time between "making" and "breaking" is assumed to be negligible. The switch has been in the a position for a long time. At $t = 0$ the switch is thrown from position a to position b.

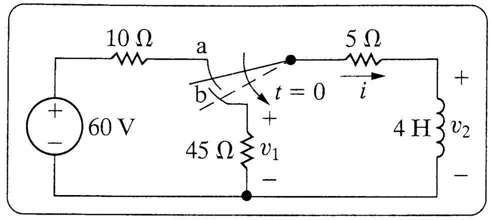
a) Determine the initial current in the inductor.

b) Determine the time constant of the circuit for $t > 0$.

c) Find i, v_1, and v_2 for $t \geq 0$.

d) What percentage of the initial energy stored in the inductor is dissipated in the 45 Ω resistor 40 ms after the switch is thrown from position a to position b?

Figure P7.2



7.3 The switch shown in Fig. P7.3 has been open a long time before closing at $t = 0$.

a) Find $i_o(0^-)$.

b) Find $i_L(0^-)$.

c) Find $i_o(0^+)$.

d) Find $i_L(0^+)$.

e) Find $i_o(\infty)$.

f) Find $i_L(\infty)$.

g) Write the expression for $i_L(t)$ for $t \geq 0$.

h) Find $v_L(0^-)$.

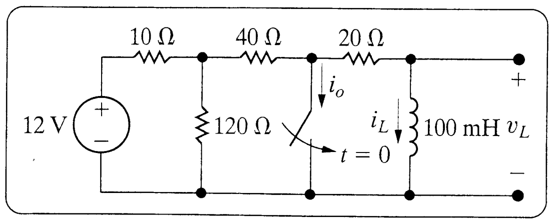
i) Find $v_L(0^+)$.

j) Find $v_L(\infty)$.

k) Write the expression for $v_L(t)$ for $t \geq 0^+$.

l) Write the expression for $i_o(t)$ for $t \geq 0^+$.

Figure P7.3



7.4 In the circuit in Fig. P7.4, the voltage and current expressions are

$$v = 400e^{-5t}\ \text{V}, \quad t \geq 0^+;$$

$$i = 10e^{-5t}\ \text{A}, \quad t \geq 0.$$

Find

a) R.

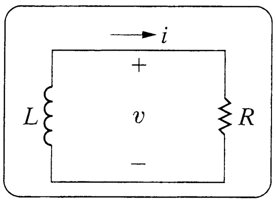
b) τ (in milliseconds).

c) L.

d) the initial energy stored in the inductor.

e) the time (in milliseconds) it takes to dissipate 80% of the initial stored energy.

Figure P7.4



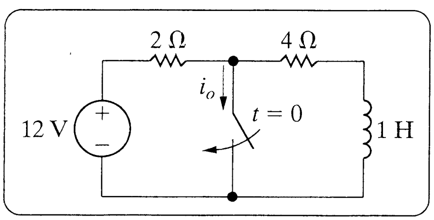
7.5 **P** The switch in the circuit in Fig. P7.5 has been open for a long time. At $t = 0$ the switch is closed.

a) Determine $i_o(0^+)$ and $i_o(\infty)$.

b) Determine $i_o(t)$ for $t \geq 0^+$.

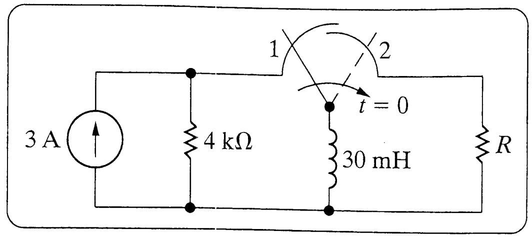
c) How many milliseconds after the switch has been closed will the current in the switch equal 5 A?

Figure P7.5



7.6 The switch in the circuit seen in Fig. P7.6 has been in position 1 for a long time. At $t = 0$, the switch moves instantaneously to position 2. Find the value of R so that 1/5th of the initial energy stored in the 30 mH inductor is dissipated in R in 15 μs.

Figure P7.6



7.7 In the circuit in Fig. P7.6, let I_g represent the dc current source, σ represent the fraction of initial energy stored in the inductor that is dissipated in t_o seconds, and L represent the inductance.

a) Show that

$$R = \frac{L \ln[1/(1-\sigma)]}{2t_o}.$$

b) Test the expression derived in (a) by using it to find the value of R in Problem 7.6.

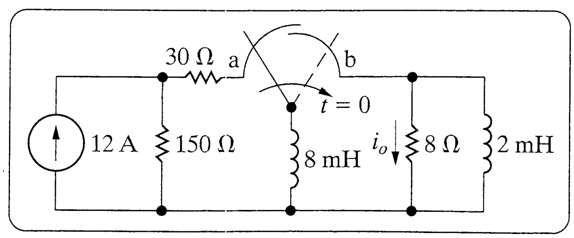
7.8 **P** In the circuit shown in Fig. P7.8, the switch has been in position a for a long time. At $t = 0$, it moves instantaneously from a to b.

a) Find $i_o(t)$ for $t \geq 0$.

b) What is the total energy delivered to the 8 Ω resistor?

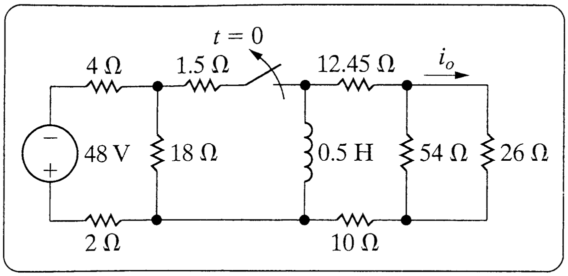
c) How many time constants does it take to deliver 95% of the energy found in (b)?

Figure P7.8



7.9 The switch in the circuit in Fig. P7.9 has been closed a long time. At $t = 0$ it is opened. Find $i_o(t)$ for $t \geq 0$.

Figure P7.9



7.10 Assume that the switch in the circuit in Fig. P7.9 has been open for one time constant. At this instant, what percentage of the total energy stored in the 0.5 H inductor has been dissipated in the 54 Ω resistor?

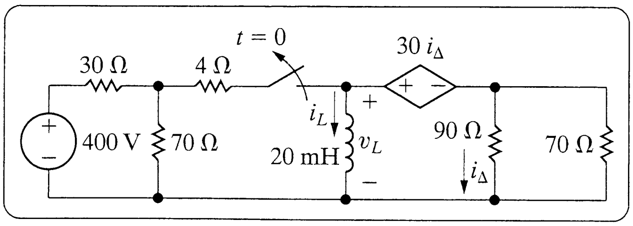
7.11 The switch in Fig. P7.11 has been closed for a long time before opening at $t = 0$. Find

a) $i_L(t),\ t \geq 0$.

b) $v_L(t),\ t \geq 0^+$.

c) $i_\Delta(t),\ t \geq 0^+$.

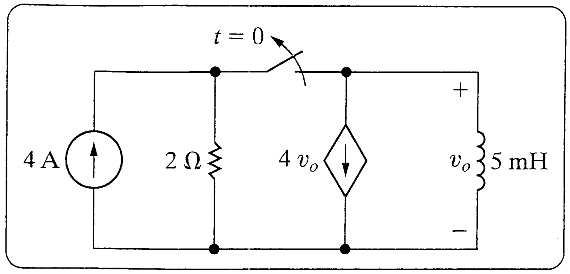
Figure P7.11



7.12 What percentage of the initial energy stored in the inductor in the circuit in Fig. P7.11 is dissipated by the current-controlled voltage source?

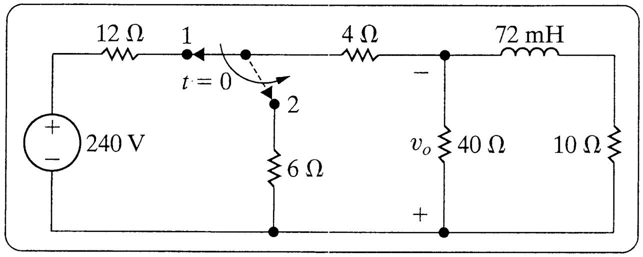
7.13 The switch in the circuit in Fig. P7.13 has been closed for a long time before opening at $t = 0$. Find $v_o(t)$ for $t \geq 0^+$.

Figure P7.13



7.14 The switch in the circuit in Fig. P7.14 has been in position 1 for a long time. At $t = 0$, the switch moves instantaneously to position 2. Find $v_o(t)$ for $t \geq 0^+$.

Figure P7.14



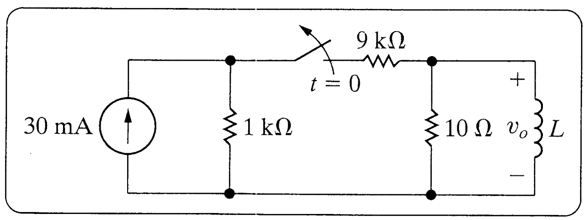
7.15 For the circuit of Fig. P7.14, what percentage of the initial energy stored in the inductor is eventually dissipated in the 40 Ω resistor?

7.16 In the circuit in Fig. P7.16, the switch has been closed for a long time before opening at $t = 0$.

a) Find the value of L so that $v_o(t)$ equals 0.5 $v_o(0^+)$ when $t = 1$ ms.

b) Find the percentage of the stored energy that has been dissipated in the 10 Ω resistor when $t = 1$ ms.

Figure P7.16



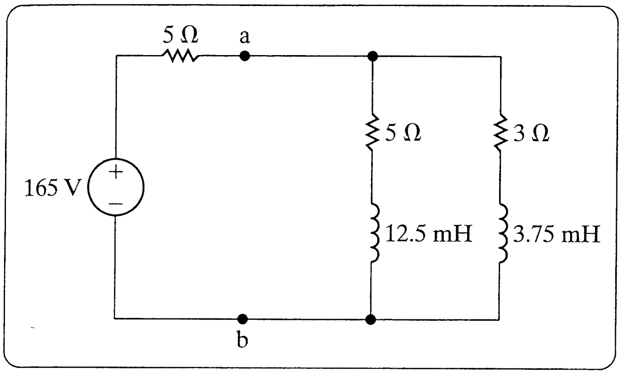
7.17 The 165 V, 5 Ω source in the circuit in Fig. P7.17 is inadvertently short-circuited at its terminals a,b. At the time the fault occurs, the circuit has been in operation for a long time.

a) What is the initial value of the current i_{ab} in the short-circuit connection between terminals a,b?

b) What is the final value of the current i_{ab}?

c) How many microseconds after the short circuit has occurred is the current in the short equal to 19 A?

Figure P7.17

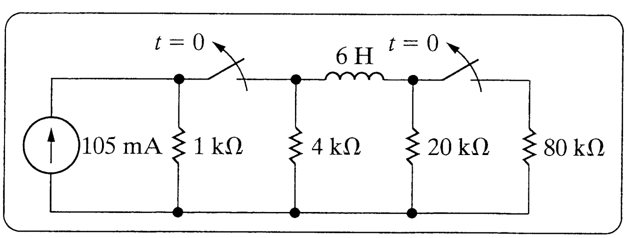


7.18 The two switches in the circuit seen in Fig. P7.18 are synchronized. The switches have been closed for a long time before opening at $t = 0$.

a) How many microseconds after the switches are open is the energy dissipated in the 4 kΩ resistor 10% of the initial energy stored in the 6 H inductor?

b) At the time calculated in (a), what percentage of the total energy stored in the inductor has been dissipated?

Figure P7.18

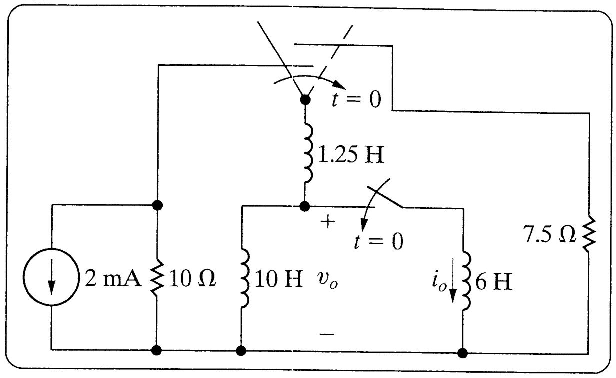


7.19 The two switches shown in the circuit in Fig. P7.19 operate simultaneously. Prior to $t = 0$ each switch has been in its indicated position for a long time. At $t = 0$ the two switches move instantaneously to their new positions. Find

a) $v_o(t), t \geq 0^+$.

b) $i_o(t), t \geq 0$.

Figure P7.19



7.20 For the circuit seen in Fig. P7.19, find

a) the total energy dissipated in the 7.5 Ω resistor.

b) the energy trapped in the ideal inductors.

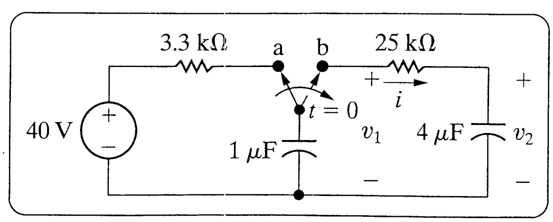
7.21 The switch in the circuit in Fig. P7.21 has been in position a for a long time. At $t = 0$, the switch is thrown to position b. Calculate

a) i, v_1, and v_2 for $t \geq 0^+$.

b) the energy stored in the capacitor at $t = 0$.

c) the energy trapped in the circuit and the total energy dissipated in the 25 kΩ resistor if the switch remains in position b indefinitely.

Figure P7.21

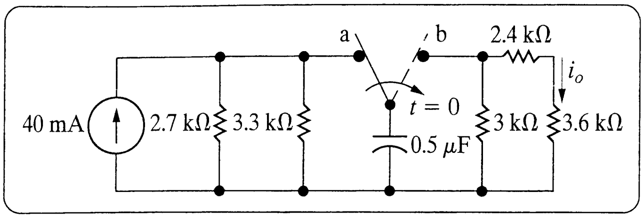


7.22 The switch in the circuit in Fig. P7.22 has been in position a for a long time. At $t = 0$, the switch is thrown to position b.

a) Find $i_o(t)$ for $t \geq 0^+$.

b) What percentage of the initial energy stored in the capacitor is dissipated in the 3 kΩ resistor 500 μs after the switch has been thrown?

Figure P7.22



7.23 In the circuit in Fig. P7.23 the voltage and current expressions are

$$v = 48e^{-25t} \text{ V}, \quad t \geq 0;$$

$$i = 12e^{-25t} \text{ mA}, \quad t \geq 0^+.$$

Find

a) R.

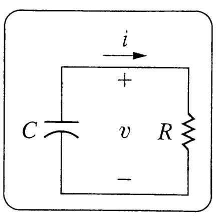
b) C.

c) τ (in milliseconds).

d) the initial energy stored in the capacitor.

e) the amount of energy that has been dissipated in the resistor 60 ms after the voltage has begun to decay.

Figure P7.23



7.24 The switch in the circuit in Fig. P7.24 is closed at $t = 0$ after being open for a long time.

a) Find $i_1(0^-)$ and $i_2(0^-)$.

b) Find $i_1(0^+)$ and $i_2(0^+)$.

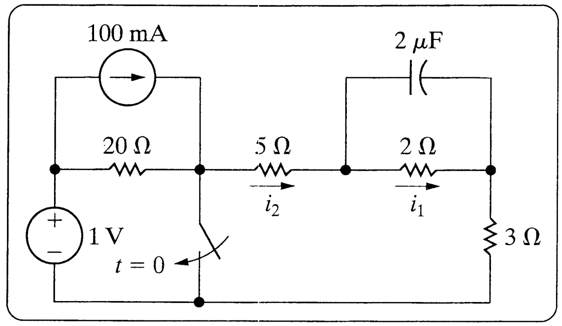
c) Explain why $i_1(0^-) = i_1(0^+)$.

d) Explain why $i_2(0^-) \neq i_2(0^+)$.

e) Find $i_1(t)$ for $t \geq 0$.

f) Find $i_2(t)$ for $t \geq 0^+$.

Figure P7.24

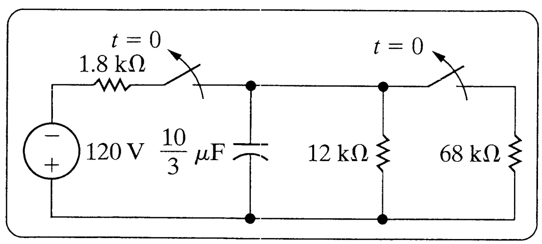


7.25 In the circuit shown in Fig. P7.25, both switches operate together; that is, they either open or close at the same time. The switches are closed a long time before opening at $t = 0$.

a) How many microjoules of energy have been dissipated in the 12 kΩ resistor 12 ms after the switches open?

b) How long does it take to dissipate 75% of the initially stored energy?

Figure P7.25

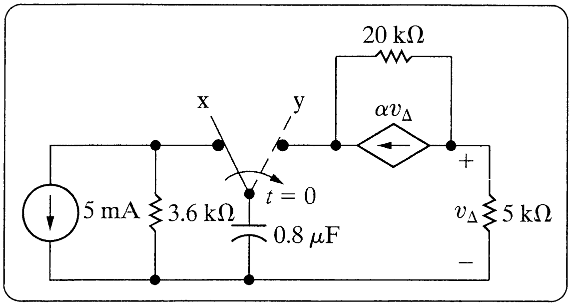


7.26 The switch in the circuit seen in Fig. P7.26 has been in position x for a long time. At $t = 0$, the switch moves instantaneously to position y.

a) Find α so that the time constant for $t > 0$ is 40 ms.

b) For the α found in (a), find v_Δ.

Figure P7.26

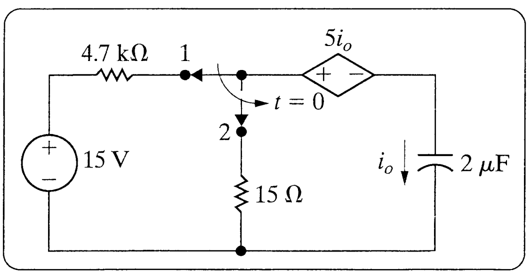


7.27 a) In Problem 7.26, how many microjoules of energy are generated by the dependent current source during the time the capacitor discharges to 0 V?

b) Show that for $t \geq 0$ the total energy stored and generated in the capacitive circuit equals the total energy dissipated.

7.28 P The switch in the circuit in Fig. P7.28 has been in position 1 for a long time before moving to position 2 at $t = 0$. Find $i_o(t)$ for $t \geq 0^+$.

Figure P7.28



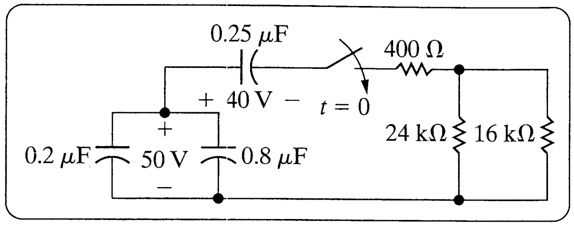
7.29 At the time the switch is closed in the circuit in Fig. P7.29, the voltage across the paralleled capacitors is 50 V and the voltage on the 0.25 μF capacitor is 40 V.

a) What percentage of the initial energy stored in the three capacitors is dissipated in the 24 kΩ resistor?

b) Repeat (a) for the 400 Ω and 16 kΩ resistors.

c) What percentage of the initial energy is trapped in the capacitors?

Figure P7.29



7.30 At the time the switch is closed in the circuit shown in Fig. P7.30, the capacitors are charged as shown.

a) Find $v_o(t)$ for $t \geq 0^+$.

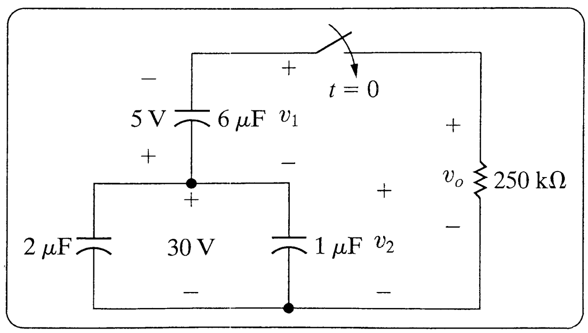
b) What percentage of the total energy initially stored in the three capacitors is dissipated in the 250 kΩ resistor?

c) Find $v_1(t)$ for $t \geq 0$.

d) Find $v_2(t)$ for $t \geq 0$.

e) Find the energy (in millijoules) trapped in the ideal capacitors.

Figure P7.30

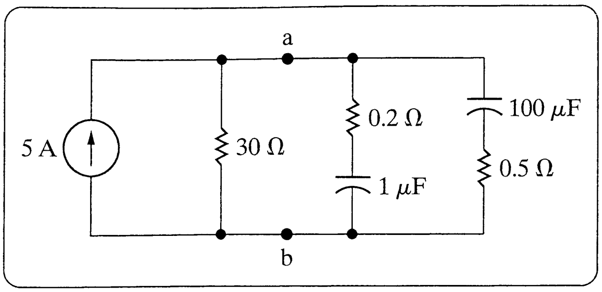


7.31 After the circuit in Fig. P7.31 has been in operation for a long time, a screwdriver is inadvertently connected across the terminals a,b. Assume the resistance of the screwdriver is negligible.

a) Find the current in the screwdriver at $t = 0^+$ and $t = \infty$.

b) Derive the expression for the current in the screwdriver for $t \geq 0^+$.

Figure P7.31



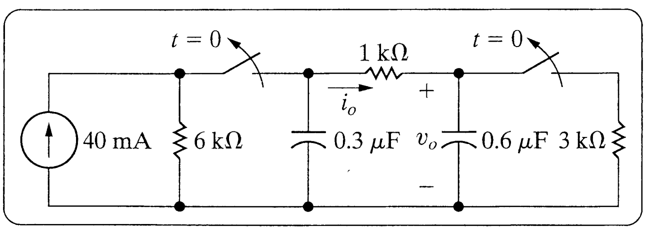
7.32 Both switches in the circuit in Fig. P7.32 have been closed for a long time. At $t = 0$, both switches open simultaneously.

a) Find $i_o(t)$ for $t \geq 0^+$.

b) Find $v_o(t)$ for $t \geq 0$.

c) Calculate the energy (in microjoules) trapped in the circuit.

Figure P7.32

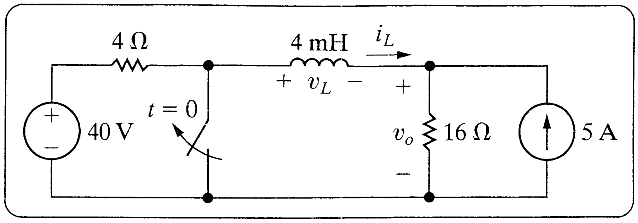


7.33 The switch in the circuit shown in Fig. P7.33 has been closed for a long time before opening at $t = 0$.

a) Find the numerical expressions for $i_L(t)$ and $v_o(t)$ for $t \geq 0$.

b) Find the numerical values of $v_L(0^+)$ and $v_o(0^+)$.

Figure P7.33

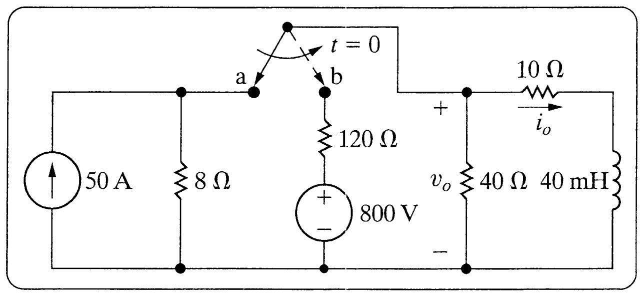


7.34 The switch in the circuit shown in Fig. P7.34 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

a) Find the numerical expression for $i_o(t)$ when $t \geq 0$.

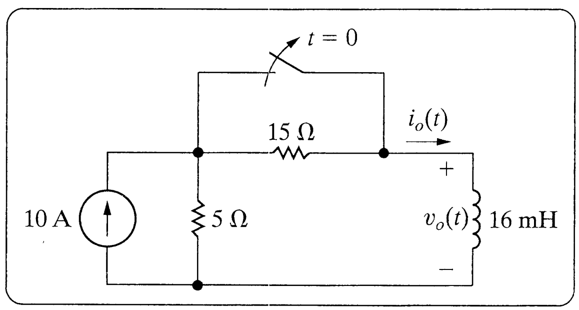
b) Find the numerical expression for $v_o(t)$ for $t \geq 0^+$.

Figure P7.34



7.35 The switch in the circuit seen in Fig. P7.35 has been closed for a long time. The switch opens at $t = 0$. Find the numerical expressions for $i_o(t)$ and $v_o(t)$ when $t \geq 0^+$.

Figure P7.35



7.36 The switch in the circuit shown in Fig. P7.36 has been closed for a long time. The switch opens at $t = 0$. For $t \geq 0^+$:

a) Find $v_o(t)$ as a function of I_g, R_1, R_2, and L.

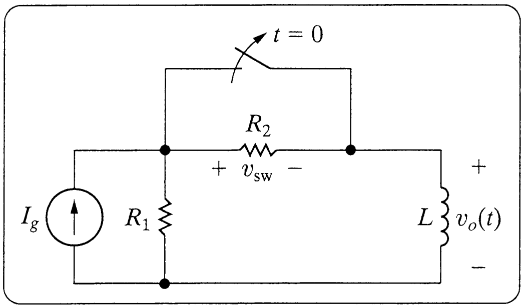
b) Verify your expression by using it to find $v_o(t)$ in the circuit of Fig. P7.35.

c) Explain what happens to $v_o(t)$ as R_2 gets larger and larger.

d) Find v_{SW} as a function of I_g, R_1, R_2, and L.

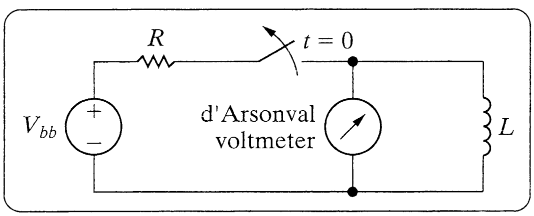
e) Explain what happens to v_{SW} as R_2 gets larger and larger.

Figure P7.36



7.37 The switch in the circuit in Fig. P7.37 has been closed for a long time. A student abruptly opens the switch and reports to her instructor that when the switch opened, an electric arc with noticeable persistence was established across the switch, and at the same time the voltmeter placed across the coil was damaged. On the basis of your analysis of the circuit in Problem 7.36, can you explain to the student why this happened?

Figure P7.37



7.38 The current and voltage at the terminals of the inductor in the circuit in Fig. 7.16 are

$$i(t) = (4 + 4e^{-40t})\ \text{A}, \quad t \geq 0;$$

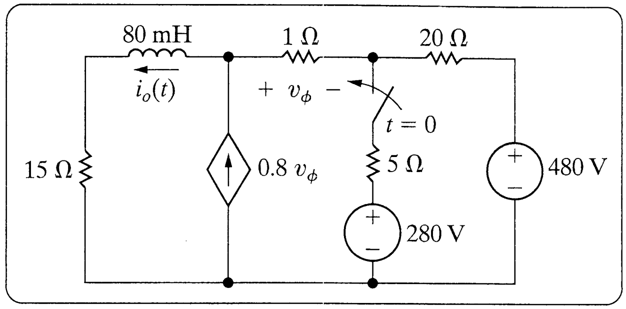
$$v(t) = -80e^{-40t}\ \text{V}, \quad t \geq 0^+.$$

a) Specify the numerical values of V_s, R, I_o, and L.

b) How many milliseconds after the switch has been closed does the energy stored in the inductor reach 9 J?

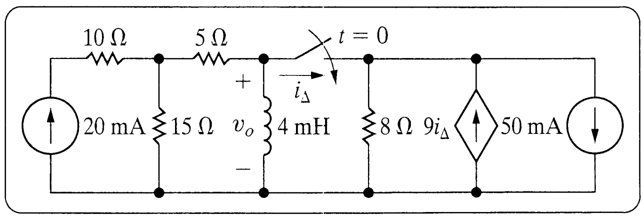
7.39 The switch in the circuit in Fig. P7.39 has been open a long time before closing at $t = 0$. Find $i_o(t)$ for $t \geq 0$.

Figure P7.39



7.40 The switch in the circuit in Fig. P7.40 has been open a long time before closing at $t = 0$. Find $v_o(t)$ for $t \geq 0^+$.

Figure P7.40

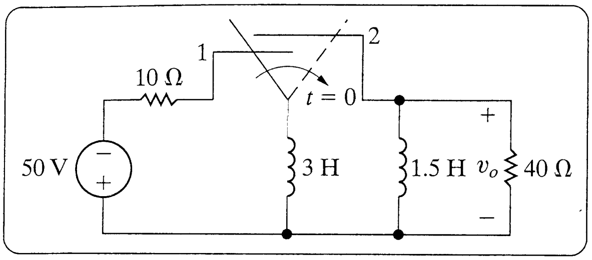


7.41 a) Derive Eq. 7.47 by first converting the Thévenin equivalent in Fig. 7.16 to a Norton equivalent and then summing the currents away from the upper node, using the inductor voltage v as the variable of interest.

b) Use the separation of variables technique to find the solution to Eq. 7.47. Verify that your solution agrees with the solution given in Eq. 7.42.

7.42 The switch in the circuit in Fig. P7.42 has been in position 1 for a long time. At $t = 0$ it moves instantaneously to position 2. How many milliseconds after the switch operates does v_o equal 100 V?

Figure P7.42



7.43 For the circuit in Fig. P7.42, find (in joules):

a) the total energy dissipated in the 40 Ω resistor

b) the energy trapped in the inductors

c) the initial energy stored in the inductors

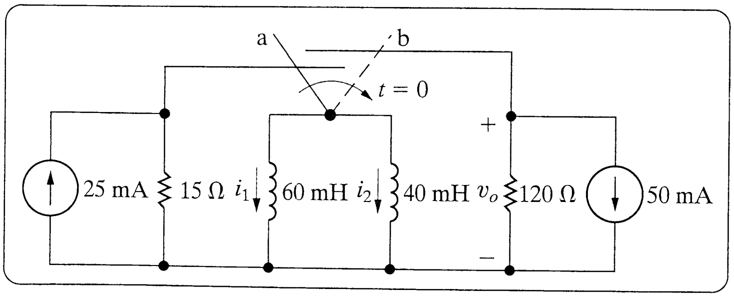
7.44 The make-before-break switch in the circuit of Fig. P7.44 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b. Find

a) $v_o(t)$, $t \geq 0^+$.

b) $i_1(t)$, $t \geq 0$.

c) $i_2(t)$, $t \geq 0$.

Figure P7.44

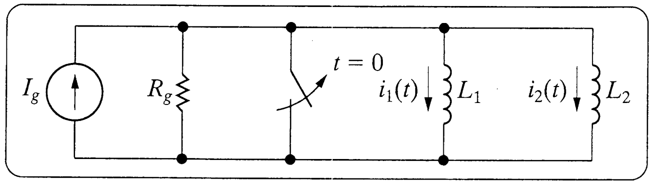


7.45 There is no energy stored in the inductors L_1 and L_2 at the time the switch is opened in the circuit shown in Fig. P7.45.

a) Derive the expressions for the currents $i_1(t)$ and $i_2(t)$ for $t \geq 0$.

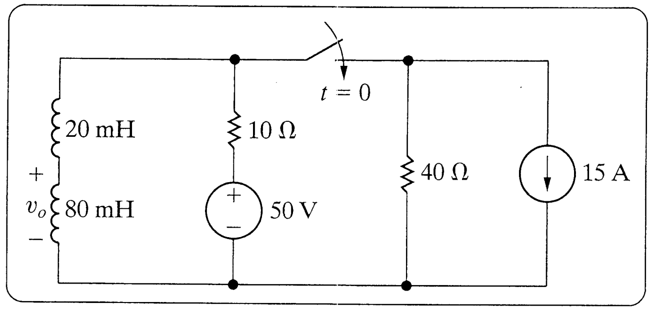
b) Use the expressions derived in (a) to find $i_1(\infty)$ and $i_2(\infty)$.

Figure P7.45



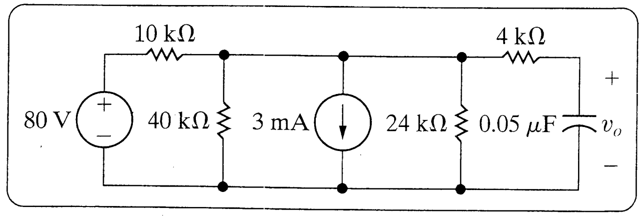
7.46 The switch in the circuit in Fig. P7.46 has been open a long time before closing at $t = 0$. Find $v_o(t)$ for $t \geq 0^+$.

Figure P7.46



7.47 The circuit in Fig. P7.47 has been in operation for a long time. At $t = 0$, the voltage source reverses polarity and the current source drops from 3 mA to 2 mA. Find $v_o(t)$ for $t \geq 0$.

Figure P7.47



7.48 The switch in the circuit shown in Fig. P7.48 has been closed a long time before opening at $t = 0$. For $t \geq 0^+$, find

a) $v_o(t)$.

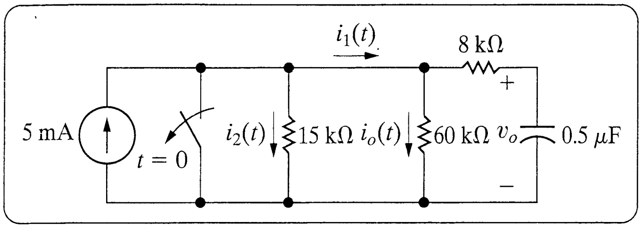
b) $i_o(t)$.

c) $i_1(t)$.

d) $i_2(t)$.

e) $i_1(0^+)$.

Figure P7.48



7.49 The current and voltage at the terminals of the capacitor in the circuit in Fig. 7.21 are

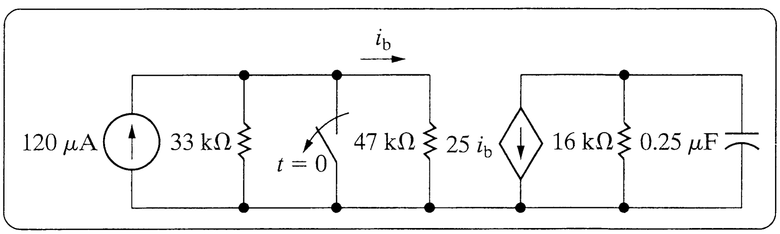
$$i(t) = 3e^{-2500t} \text{ mA}, \qquad t \geq 0^+;$$

$$v(t) = (40 - 24e^{-2500t}) \text{ V}, \quad t \geq 0.$$

a) Specify the numerical values of I_s, V_o, R, C, and τ.

b) How many microseconds after the switch has been closed does the energy stored in the capacitor reach 81% of its final value?

Figure P7.52

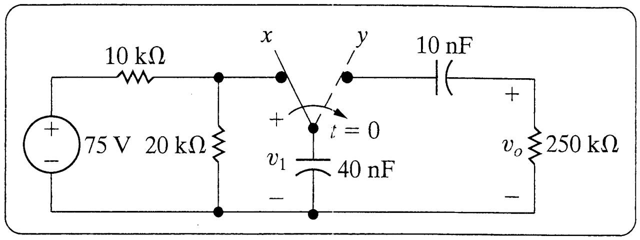


7.50 The switch in the circuit in Fig. P7.50 has been in position x for a long time. The initial charge on the 10 nF capacitor is zero. At $t = 0$, the switch moves instantaneously to position y.

a) Find $v_o(t)$ for $t \geq 0^+$.

b) Find $v_1(t)$ for $t \geq 0$.

Figure P7.50



7.51 For the circuit in Fig. P7.50, find (in microjoules)

a) the energy delivered to the 250 kΩ resistor.

b) the energy trapped in the capacitors.

c) the initial energy stored in the capacitors.

7.52 The switch in the circuit shown in Fig. P7.52 opens at $t = 0$ after being closed for a long time. How many milliseconds after the switch opens is the energy stored in the capacitor 36% of its final value?

7.53 The switch in the circuit shown in Fig. P7.53 has been closed a long time before opening at $t = 0$.

a) What is the initial value of $i_o(t)$?

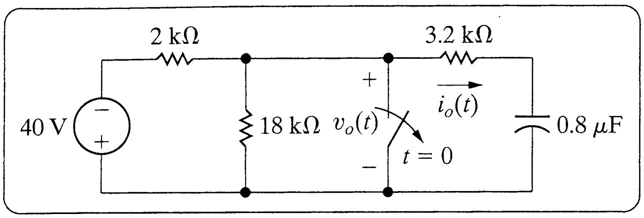
b) What is the final value of $i_o(t)$?

c) What is the time constant of the circuit for $t \geq 0$?

d) What is the numerical expression for $i_o(t)$ when $t \geq 0^+$?

e) What is the numerical expression for $v_o(t)$ when $t \geq 0^+$?

Figure P7.53



7.54 The switch in the circuit seen in Fig. P7.54 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b. For $t \geq 0^+$, find

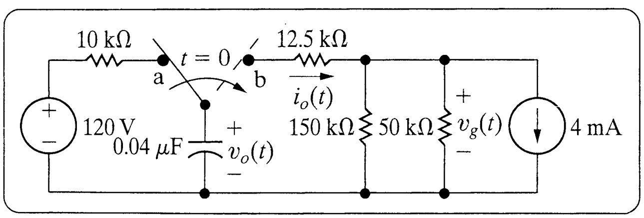
a) $v_o(t)$.

b) $i_o(t)$.

c) $v_g(t)$.

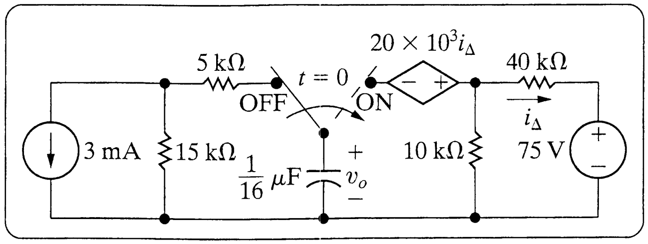
d) $v_g(0^+)$.

Figure P7.54



7.55 The switch in the circuit shown in Fig. P7.55 has been in the OFF position for a long time. At $t = 0$, the switch moves instantaneously to the ON position. Find $v_o(t)$ for $t \geq 0$.

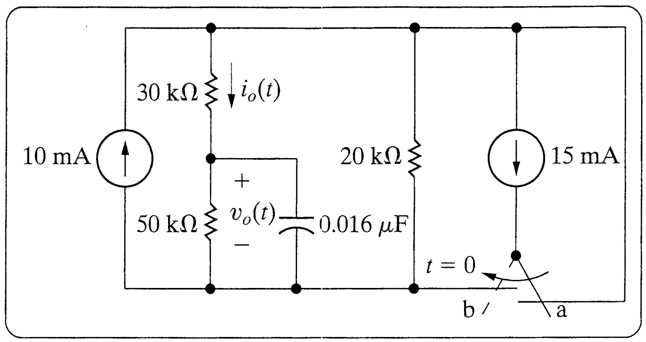
Figure P7.55



7.56 Assume that the switch in the circuit of Fig. P7.55 has been in the ON position for a long time before switching instantaneously to the OFF position at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

7.57 The switch in the circuit seen in Fig. P7.57 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b. Find $v_o(t)$ and $i_o(t)$ for $t \geq 0^+$.

Figure P7.57

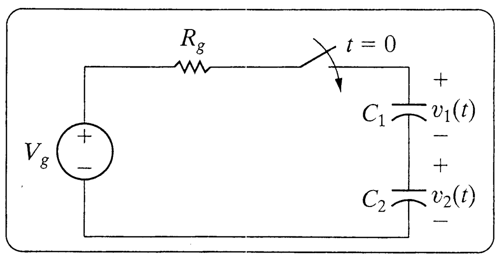


7.58 There is no energy stored in the capacitors C_1 and C_2 at the time the switch is closed in the circuit seen in Fig. P7.58.

a) Derive the expressions for $v_1(t)$ and $v_2(t)$ for $t \geq 0$.

b) Use the expressions derived in (a) to find $v_1(\infty)$ and $v_2(\infty)$.

Figure P7.58



7.59 a) Derive Eq. 7.52 by first converting the Norton equivalent circuit shown in Fig. 7.21 to a Thévenin equivalent and then summing the voltages around the closed loop, using the capacitor current i as the relevant variable.

b) Use the separation of variables technique to find the solution to Eq. 7.52. Verify that your solution agrees with that of Eq. 7.53.

7.60 **P** The switch in the circuit of Fig. P7.60 has been in position a for a long time. At $t = 0$, it moves instantaneously to position b. For $t \geq 0^+$, find

a) $v_o(t)$.

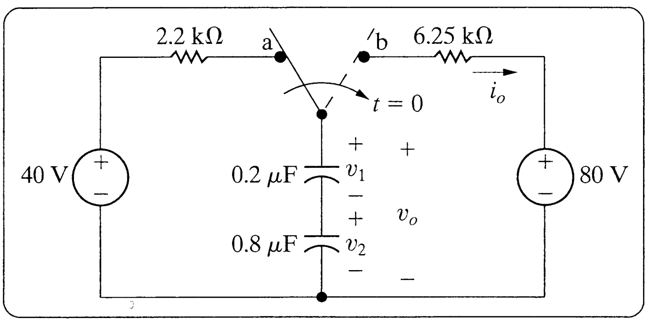
b) $i_o(t)$.

c) $v_1(t)$.

d) $v_2(t)$.

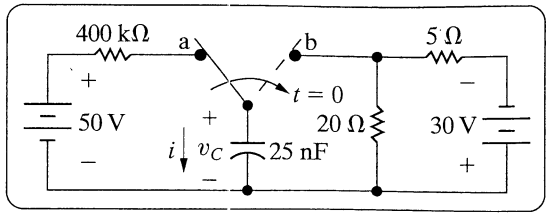
e) the energy trapped in the capacitors as $t \to \infty$.

Figure P7.60



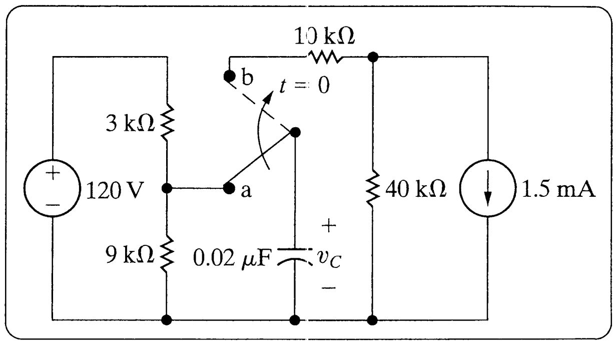
7.61 Assume that the switch in the circuit of Fig. P7.61 has been in position a for a long time and that at $t = 0$ it is moved to position b. Find (a) $v_C(0^+)$; (b) $v_C(\infty)$; (c) τ for $t > 0$; (d) $i(0^+)$; (e) v_C, $t \geq 0$; and (f) i, $t \geq 0^+$.

Figure P7.61



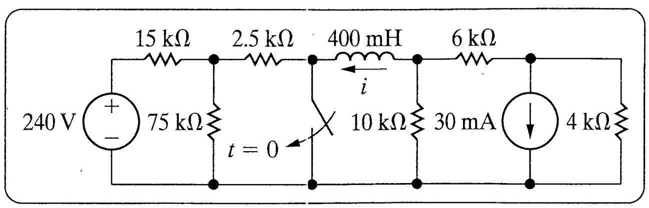
7.62 The switch in the circuit of Fig. P7.62 has been in position a for a long time. At $t = 0$ the switch is moved to position b. Calculate (a) the initial voltage on the capacitor; (b) the final voltage on the capacitor; (c) the time constant (in microseconds) for $t > 0$; and (d) the length of time (in microseconds) required for the capacitor voltage to reach zero after the switch is moved to position b.

Figure P7.62



7.63 After the switch in the circuit of Fig. P7.63 has been open for a long time, it is closed at $t = 0$. Calculate (a) the initial value of i; (b) the final value of i; (c) the time constant for $t \geq 0$; and (d) the numerical expression for $i(t)$ when $t \geq 0$.

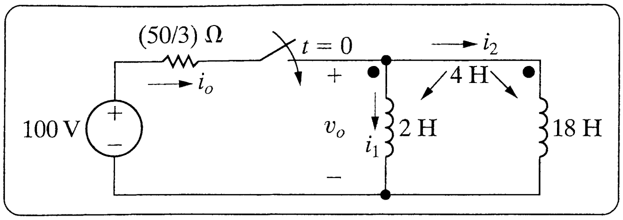
Figure P7.63



7.64 There is no energy stored in the circuit of Fig. P7.64 at the time the switch is closed.

a) Find $i_o(t)$ for $t \geq 0$.
b) Find $v_o(t)$ for $t \geq 0$.
c) Find $i_1(t)$ for $t \geq 0$.
d) Find $i_2(t)$ for $t \geq 0$.
e) Do your answers make sense in terms of known circuit behavior?

Figure P7.64

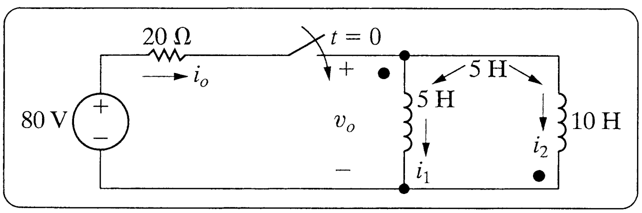


7.65 Repeat (a) and (b) in Example 7.10 if the mutual inductance is reduced to zero.

7.66 There is no energy stored in the circuit in Fig. P7.66 at the time the switch is closed.

a) Find $i_o(t)$ for $t \geq 0$.
b) Find $v_o(t)$ for $t \geq 0$.
c) Find $i_1(t)$ for $t \geq 0$.
d) Find $i_2(t)$ for $t \geq 0$.
e) Do your answers make sense in terms of known circuit behavior?

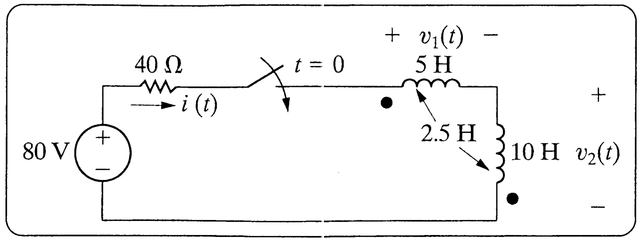
Figure P7.66



7.67 There is no energy stored in the circuit in Fig. P7.67 at the time the switch is closed.

a) Find $i(t)$ for $t \geq 0$.
b) Find $v_1(t)$ for $t \geq 0$.
c) Find $v_2(t)$ for $t \geq 0$.
d) Do your answers make sense in terms of known circuit behavior?

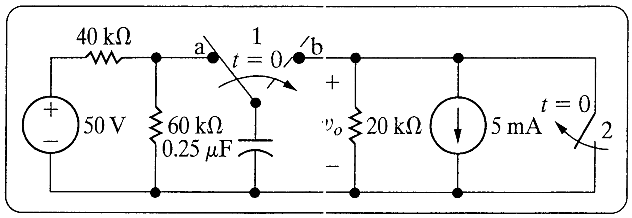
Figure P7.67



7.68 Repeat Problem 7.67 if the dot on the 10 H coil is at the top of the coil.

7.69 The switch in the circuit in Fig. P7.69 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b. At the instant the switch makes contact with terminal b, switch 2 opens. Find $v_o(t)$ for $t \geq 0$.

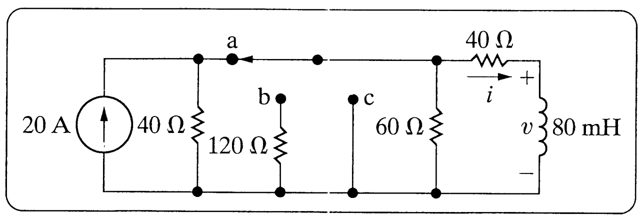
Figure P7.69



7.70 The switch in the circuit shown in Fig. P7.70 has been in position a for a long time. At $t = 0$, the switch is moved to position b, where it remains for 1 ms. The switch is then moved to position c, where it remains indefinitely. Find

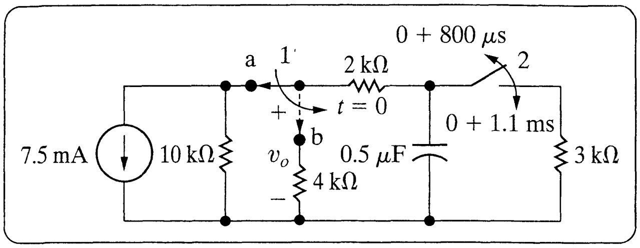
a) $i(0^+)$.
b) $i(200\ \mu s)$.
c) $i(6\ ms)$.
d) $v(1^-\ ms)$.
e) $v(1^+\ ms)$.

Figure P7.70



7.71 In the circuit in Fig. P7.71, switch 1 has been in position a and switch 2 has been closed for a long time. At $t = 0$, switch 1 moves instantaneously to position b. Eight hundred microseconds later, switch 2 opens, remains open for 300 μs, and then recloses. Find v_o 1.5 ms after switch 1 makes contact with terminal b.

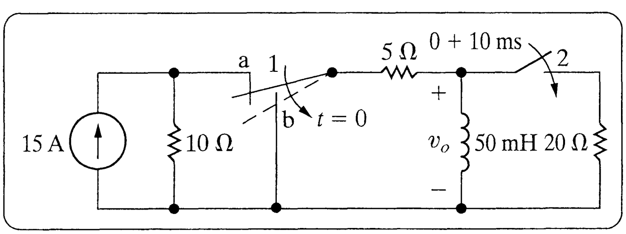
Figure P7.71



7.72 For the circuit in Fig. P7.71, what percentage of the initial energy stored in the 0.5 μF capacitor is dissipated in the 3 kΩ resistor?

7.73 The action of the two switches in the circuit seen in Fig. P7.73 is as follows. For $t < 0$, switch 1 is in position a and switch 2 is open. This state has existed for a long time. At $t = 0$, switch 1 moves instantaneously from position a to position b, while switch 2 remains open. Ten milliseconds after switch 1 operates, switch 2 closes, remains closed for 10 ms, and then opens. Find $v_o(t)$ 25 ms after switch 1 moves to position b.

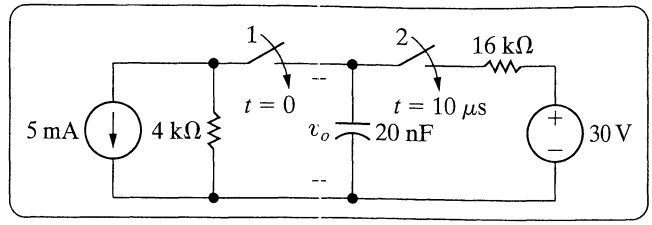
Figure P7.73



7.74 For the circuit in Fig. P7.73, how many milliseconds after switch 1 moves to position b is the energy stored in the inductor 4% of its initial value?

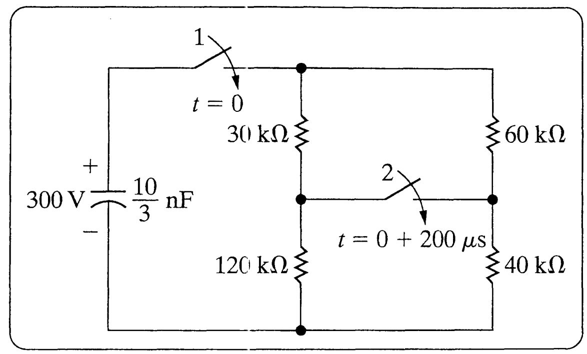
7.75 There is no energy stored in the capacitor in the circuit in Fig. P7.75 when switch 1 closes at $t = 0$. Ten microseconds later, switch 2 closes. Find $v_o(t)$ for $t \geq 0$.

Figure P7.75



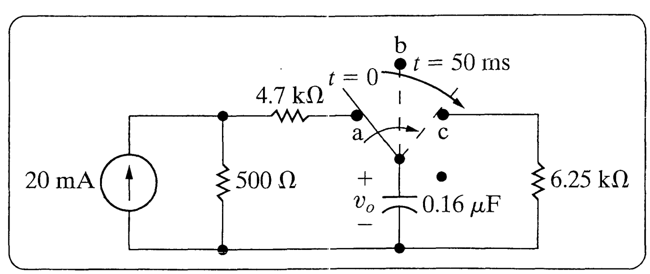
7.76 The capacitor in the circuit seen in Fig. P7.76 has been charged to 300 V. At $t = 0$, switch 1 closes, causing the capacitor to discharge into the resistive network. Switch 2 closes 200 μs after switch 1 closes. Find the magnitude and direction of the current in the second switch 300 μs after switch 1 closes.

Figure P7.76



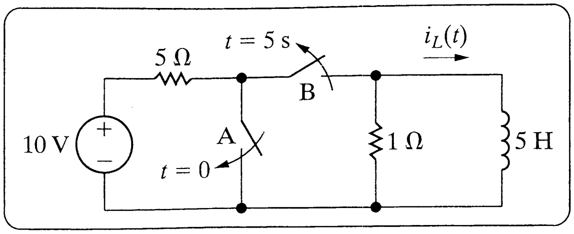
7.77 The switch in the circuit in Fig. P7.77 has been in position a for a long time. At $t = 0$, it moves instantaneously to position b, where it remains for 50 ms before moving instantaneously to position c. Find v_o for $t \geq 0$.

Figure P7.77



7.78 In the circuit in Fig. P7.78, switch A has been open and switch B has been closed for a long time. At $t = 0$, switch A closes. Five seconds after switch A closes, switch B opens. Find $i_L(t)$ for $t \geq 0$.

Figure P7.78



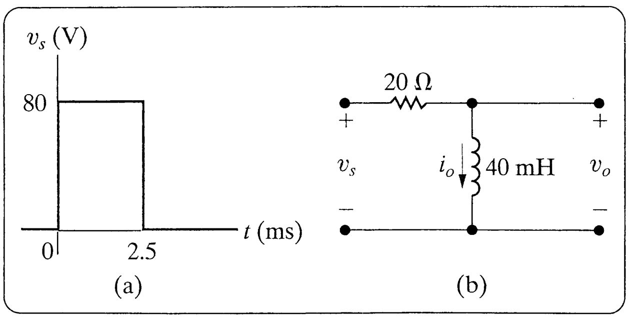
7.79 The voltage waveform shown in Fig. P7.79(a) is applied to the circuit of Fig. P7.79(b). The initial current in the inductor is zero.

a) Calculate $v_o(t)$.

b) Make a sketch of $v_o(t)$ versus t.

c) Find i_o at $t = 5$ ms.

Figure P7.79



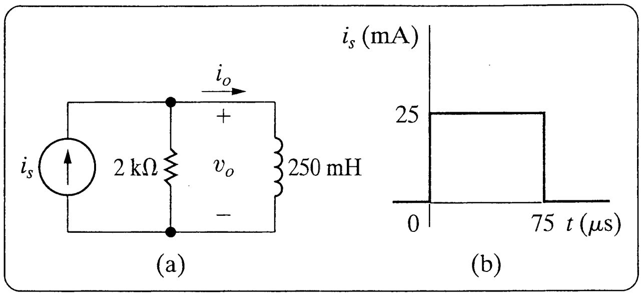
7.80 The current source in the circuit in Fig. P7.80(a) generates the current pulse shown in Fig. P7.80(b). There is no energy stored at $t = 0$.

a) Derive the numerical expressions for $v_o(t)$ for the time intervals $t < 0$, $0 < t < 75$ μs, and 75 μs $< t < \infty$.

b) Calculate v_o (75^- μs) and v_o (75^+ μs).

c) Calculate i_o (75^- μs) and i_o (75^+ μs).

Figure P7.80

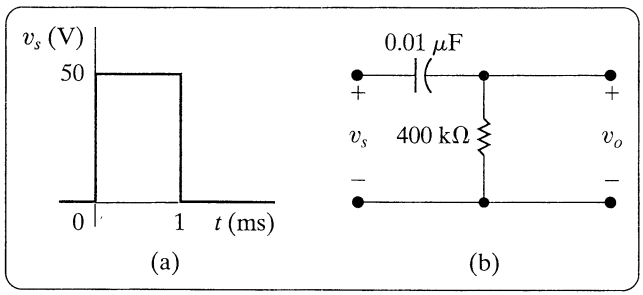


7.81 The voltage waveform shown in Fig. P7.81(a) is applied to the circuit of Fig. P7.81(b). The initial voltage on the capacitor is zero.

a) Calculate $v_o(t)$.

b) Make a sketch of $v_o(t)$ versus t.

Figure P7.81



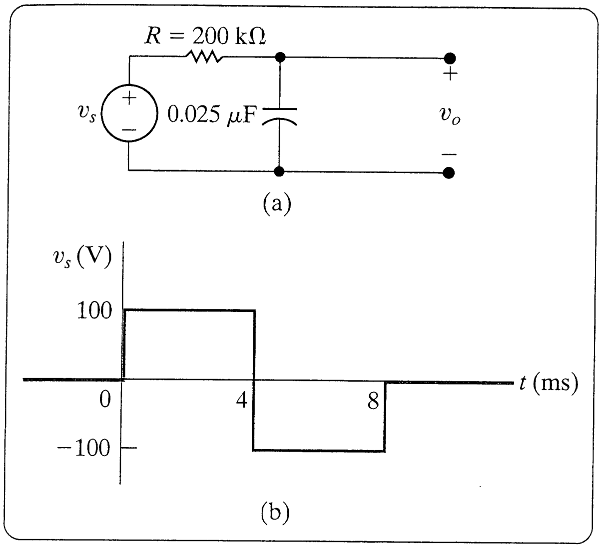
7.82 The voltage signal source in the circuit in Fig. P7.82(a) is generating the signal shown in Fig. P7.82(b). There is no stored energy at $t = 0$.

a) Derive the expressions for $v_o(t)$ that apply in the intervals $t < 0$; $0 \leq t \leq 4$ ms; 4 ms $\leq t \leq 8$ ms; and 8 ms $\leq t \leq \infty$.

b) Sketch v_o and v_s on the same coordinate axes.

c) Repeat (a) and (b) with R reduced to 50 kΩ.

Figure P7.82



7.83 The current source in the circuit in Fig. P7.83(a) generates the current pulse shown in Fig. P7.83(b). There is no energy stored at $t = 0$.

a) Derive the expressions for $i_o(t)$ and $v_o(t)$ for the time intervals $t < 0$; $0 < t < 2$ ms; and 2 ms $< t < \infty$.

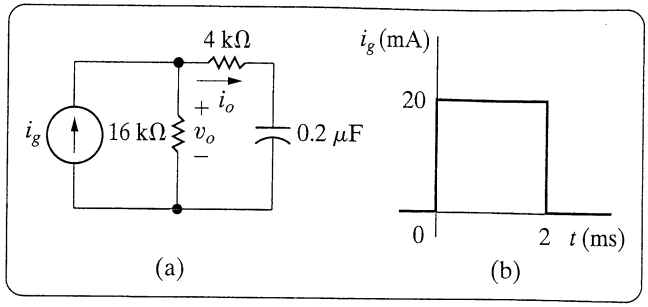
b) Calculate $i_o(0^-)$; $i_o(0^+)$; $i_o(0.002^-)$; and $i_o(0.002^+)$.

c) Calculate $v_o(0^-)$; $v_o(0^+)$; $v_o(0.002^-)$; and $v_o(0.002^+)$.

d) Sketch $i_o(t)$ versus t for the interval -1 ms $< t < 4$ ms.

e) Sketch $v_o(t)$ versus t for the interval -1 ms $< t < 4$ ms.

Figure P7.83

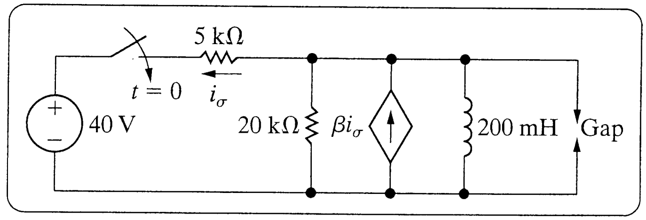


7.84 The gap in the circuit seen in Fig. P7.84 will arc over whenever the voltage across the gap reaches 45 kV. The initial current in the inductor is zero. The value of β is adjusted so the Thévenin resistance with respect to the terminals of the inductor is -5 kΩ.

a) What is the value of β?

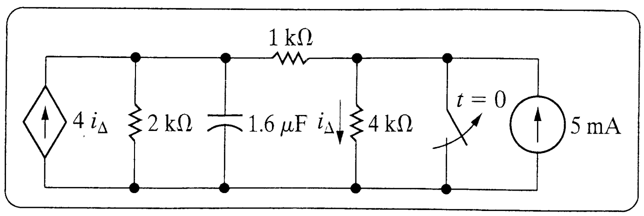
b) How many microseconds after the switch has been closed will the gap arc over?

Figure P7.84



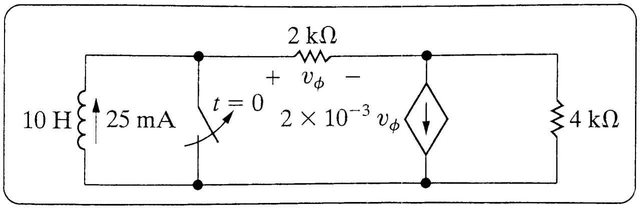
7.85 The switch in the circuit in Fig. P7.85 has been closed for a long time. The maximum voltage rating of the 1.6 μF capacitor is 14,400 V. How long after the switch is opened does the voltage across the capacitor reach the maximum voltage rating?

Figure P7.85



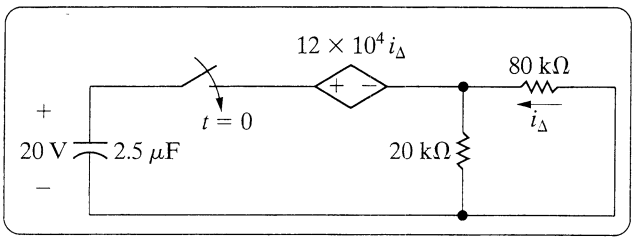
7.86 The inductor current in the circuit in Fig. P7.86 is 25 mA at the instant the switch is opened. The inductor will malfunction whenever the magnitude of the inductor current equals or exceeds 5 A. How long after the switch is opened does the inductor malfunction?

Figure P7.86



7.87 The capacitor in the circuit shown in Fig. P7.87 is charged to 20 V at the time the switch is closed. If the capacitor ruptures when its terminal voltage equals or exceeds 20 kV, how long does it take to rupture the capacitor?

Figure P7.87



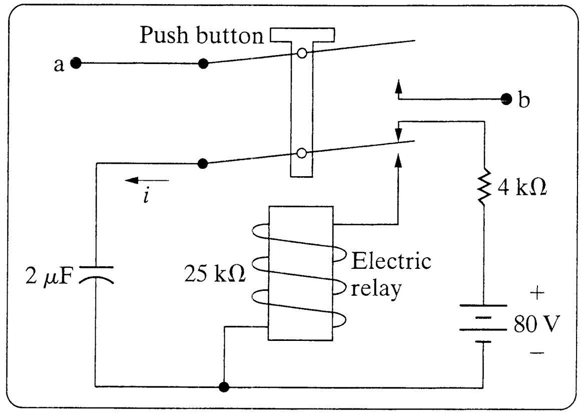
7.88 The circuit shown in Fig. P7.88 is used to close the switch between a and b for a predetermined length of time. The electric relay holds its contact arms down as long as the voltage across the relay coil exceeds 5 V. When the coil voltage equals 5 V, the relay contacts return to their initial position by a mechanical spring action. The switch between a and b is initially closed by momentarily pressing the push button. Assume that the capacitor is fully charged when the push button is first pushed down. The resistance of the relay coil is 25 kΩ, and the inductance of the coil is negligible.

a) How long will the switch between a and b remain closed?

b) Write the numerical expression for i from the time the relay contacts first open to the time the capacitor is completely charged.

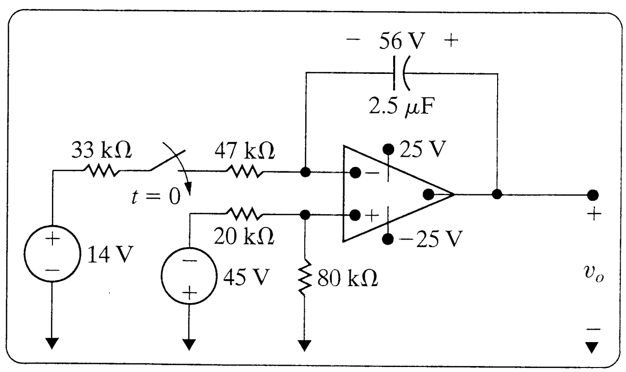
c) How many milliseconds (after the circuit between a and b is interrupted) does it take the capacitor to reach 85% of its final value?

Figure P7.88



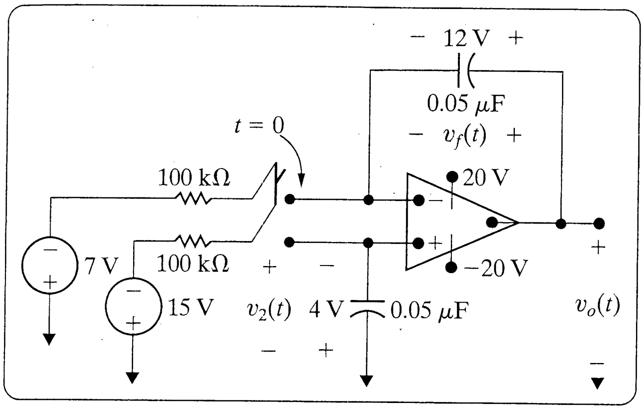
7.89 At the instant the switch of Fig. P7.89 is closed, the voltage on the capacitor is 56 V. Assume an ideal operational amplifier. How many milliseconds after the switch is closed will the output voltage v_o equal zero?

Figure P7.89



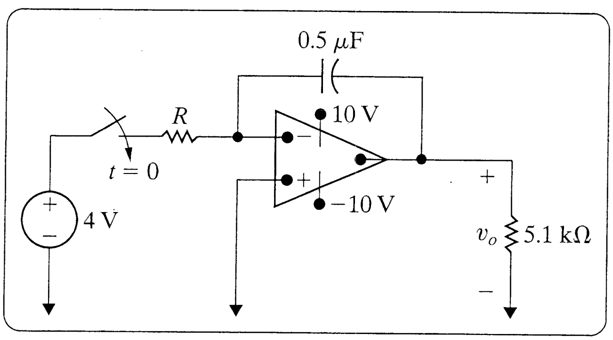
7.90 At the time the double-pole switch in the circuit shown in Fig. P7.90 is closed, the initial voltages on the capacitors are 12 V and 4 V, as shown. Find the numerical expressions for $v_o(t)$, $v_2(t)$, and $v_f(t)$ that are applicable as long as the ideal op amp operates in its linear range.

Figure P7.90



7.91 The energy stored in the capacitor in the circuit shown in Fig. P7.91 is zero at the instant the switch is closed. The ideal operational amplifier reaches saturation in 15 ms. What is the numerical value of R in kilo-ohms?

Figure P7.91



7.92 At the instant the switch is closed in the circuit of Fig. P7.91, the capacitor is charged to 6 V, positive at the right-hand terminal. If the ideal operational amplifier saturates in 40 ms, what is the value of R?

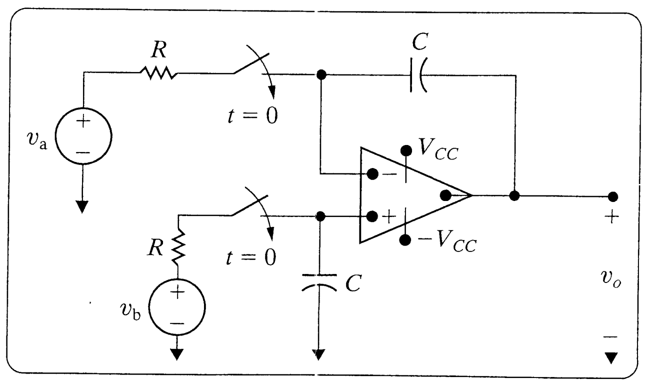
7.93 There is no energy stored in the capacitors in the circuit shown in Fig. P7.93 at the instant the two switches close.

a) Find v_o as a function of v_a, v_b, R, and C.

b) On the basis of the result obtained in (a), describe the operation of the circuit.

c) How long will it take to saturate the amplifier if $v_a = 40$ mV; $v_b = 15$ mV; $R = 50$ kΩ; $C = 10$ nF; and $V_{CC} = 6$ V?

Figure P7.93



7.94 The voltage pulse shown in Fig. P7.94(a) is applied to the ideal integrating amplifier shown in Fig. P7.94(b). Derive the numerical expressions for $v_o(t)$ when $v_o(0) = 0$ for the time intervals

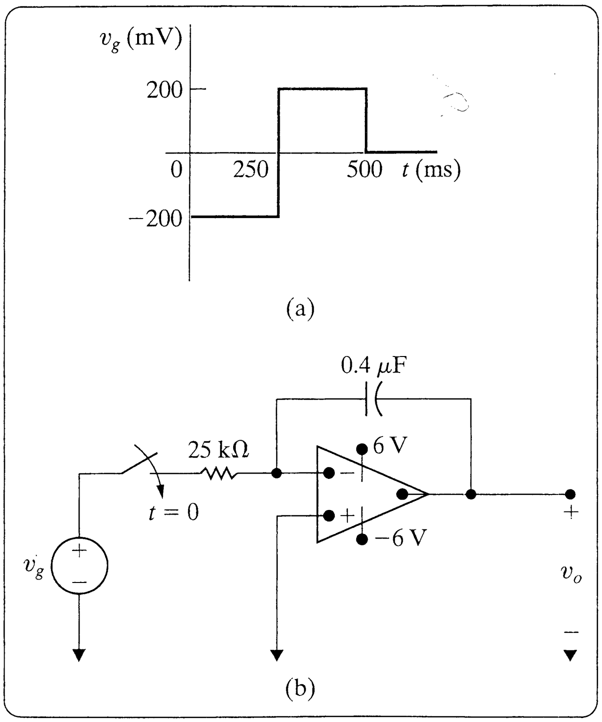
a) $t < 0$.

b) $0 \leq t \leq 250$ ms.

c) $250 \text{ ms} \leq t \leq 500$ ms.

d) $500 \text{ ms} \leq t \leq \infty$.

Figure P7.94



7.95 Repeat Problem 7.94 with a 5 MΩ resistor placed across the 0.4 μF feedback capacitor.

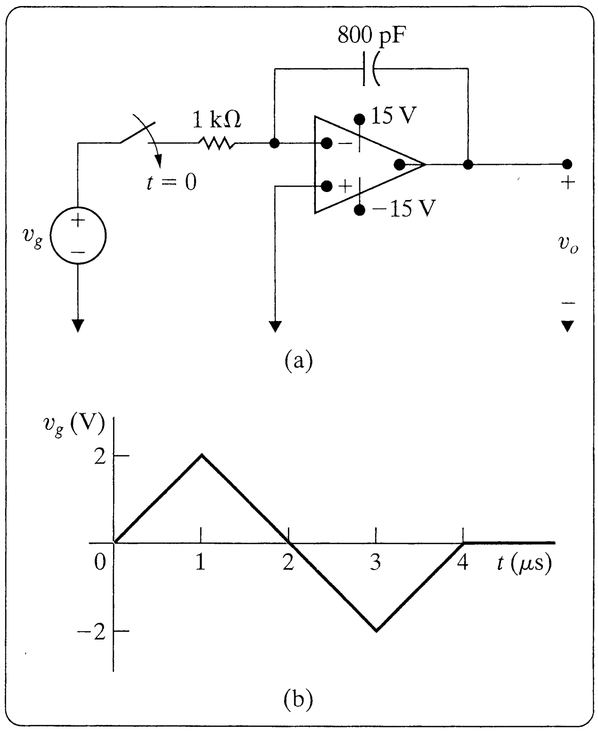
7.96 The voltage source in the circuit in Fig. P7.96(a) is generating the triangular waveform shown in Fig. P7.96(b). Assume the energy stored in the capacitor is zero at $t = 0$.

a) Derive the numerical expressions for $v_o(t)$ for the following time intervals: $0 \leq t \leq 1$ μs; $1 \text{ μs} \leq t \leq 3$ μs; and $3 \text{ μs} \leq t \leq 4$ μs.

b) Sketch the output waveform between 0 and 4 μs.

c) If the triangular input voltage continues to repeat itself for $t > 4$ μs, what would you expect the output voltage to be? Explain.

Figure P7.96



7.97 The circuit shown in Fig. P7.97 is known as an *astable multivibrator* and finds wide application in pulse circuits. The purpose of this problem is to relate the charging and discharging of the capacitors to the operation of the circuit. The key to analyzing the circuit is to understand the behavior of the ideal transistor switches T_1 and T_2. The circuit is designed so that the switches automatically alternate between ON and OFF. When T_1 is OFF, T_2 is ON and vice versa. Thus in the analysis of this circuit, we assume a switch is either ON or OFF. We also assume that the ideal transistor switch can change its state instantaneously. In other words, it can snap from OFF to ON and vice versa. When a transistor switch is ON, (1) the base current i_b is greater than zero, (2) the terminal voltage v_{be} is zero, and (3) the terminal voltage v_{ce} is zero. Thus, when a transistor switch is ON, it presents a short circuit between the terminals b,e and c,e. When a transistor switch is OFF, (1) the terminal voltage v_{be} is negative, (2) the base current is zero, and (3) there is an open circuit between the terminals c,e. Thus when a transistor switch is OFF, it presents an open circuit between the terminals b,e and c,e. Assume that T_2 has been ON and has just snapped OFF, while T_1 has been OFF and has just snapped ON. You may assume that at this instance, C_2 is charged to the supply voltage V_{CC}, and the charge on C_1 is zero. Also assume $C_1 = C_2$ and $R_1 = R_2 = 10R_L$.

a) Derive the expression for v_{be2} during the interval that T_2 is OFF.

b) Derive the expression for v_{ce2} during the interval that T_2 is OFF.

c) Find the length of time T_2 is OFF.

d) Find the value of v_{ce2} at the end of the interval that T_2 is OFF.

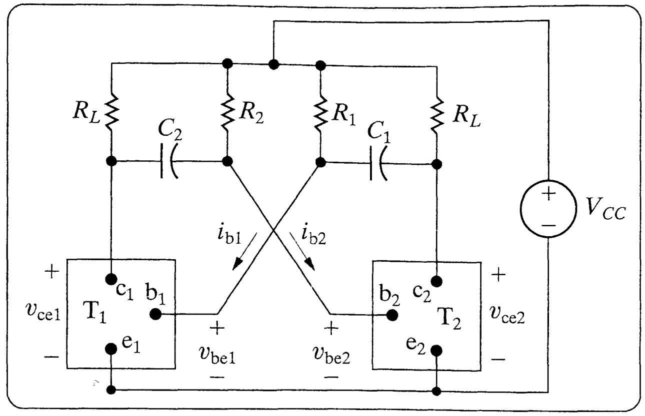
e) Derive the expression for i_{b1} during the interval that T_2 is OFF.

f) Find the value of i_{b1} at the end of the interval that T_2 is OFF.

g) Sketch v_{ce2} versus t during the interval that T_2 is OFF.

h) Sketch i_{b1} versus t during the interval that T_2 is OFF.

Figure P7.97



7.98 The component values in the circuit of Fig. P7.97 are $V_{CC} = 10$ V; $R_L = 1$ kΩ; $C_1 = C_2 = 1$ nF; and $R_1 = R_2 = 14.43$ kΩ.

a) How long is T_2 in the OFF state during one cycle of operation?

b) How long is T_2 in the ON state during one cycle of operation?

c) Repeat (a) for T_1.

d) Repeat (b) for T_1.

e) At the first instant after T_1 turns ON, what is the value of i_{b1}?

f) At the instant just before T_1 turns OFF, what is the value of i_{b1}?

g) What is the value of v_{ce2} at the instant just before T_2 turns ON?

7.99 Repeat Problem 7.98 with $C_1 = 1$ nF and $C_2 = 0.8$ nF. All other component values are unchanged.

7.100 The astable multivibrator circuit in Fig. P7.97 is to satisfy the following criteria: (1) One transistor switch is to be ON for 48 μs and OFF for 36 μs for each cycle; (2) $R_L = 2$ kΩ; (3) $V_{CC} = 5$ V; (4) $R_1 = R_2$; and (5) $6R_L \le R_1 \le 50R_L$. What are the limiting values for the capacitors C_1 and C_2?

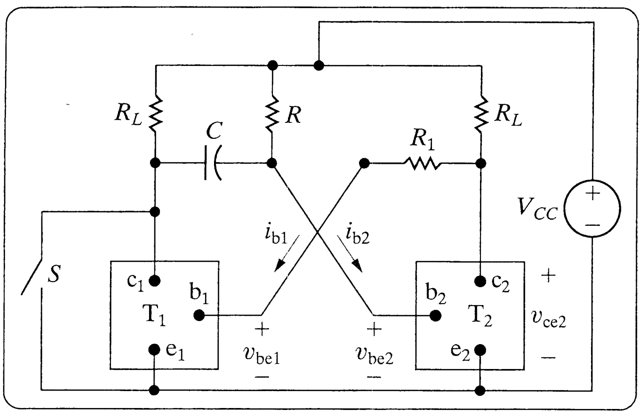
7.101 The circuit shown in Fig. P7.101 is known as a *monostable multivibrator*. The adjective *monostable* is used to describe the fact that the circuit has one stable state. That is, if left alone, the electronic switch T_2 will be ON, and T_1 will be OFF. (The operation of the ideal transistor switch is described in Problem 7.97.) T_2 can be turned OFF by momentarily closing the switch S. After S returns to its open position, T_2 will return to its ON state.

a) Show that if T_2 is ON, T_1 is OFF and will stay OFF.

b) Explain why T_2 is turned OFF when S is momentarily closed.

c) Show that T_2 will stay OFF for $RC \ln 2$ s.

Figure P7.101



7.102 The parameter values in the circuit in Fig. P7.101 are $V_{CC} = 6$ V; $R_1 = 5.0$ kΩ; $R_L = 20$ kΩ; $C = 250$ pF; and $R = 23{,}083$ Ω.

a) Sketch v_{ce2} versus t, assuming that after S is momentarily closed, it remains open until the circuit has reached its stable state. Assume S is closed at $t = 0$. Make your sketch for the interval $-5 \leq t \leq 10$ μs.

b) Repeat (a) for i_{b2} versus t.

7.103 ◆ Suppose the circuit in Fig. 7.45 models a portable flashing light circuit. Assume that four 1.5 V batteries power the circuit, and that the capacitor value is 10 μF. Assume that the lamp conducts when its voltage reaches 4 V and stops conducting when its voltage drops below 1 V. The lamp has a resistance of 20 kΩ when it is conducting and has an infinite resistance when it is not conducting.

a) Suppose we don't want to wait more than 10 s in between flashes. What value of resistance R is required to meet this time constraint?

b) For the value of resistance from (a), how long does the flash of light last?

7.104 ◆ P In the circuit of Fig. 7.45, the lamp starts to conduct whenever the lamp voltage reaches 15 V. During the time when the lamp conducts, it can be modeled as a 10 kΩ resistor. Once the lamp conducts, it will continue to conduct until the lamp voltage drops to 5 V. When the lamp is not conducting, it appears as an open circuit. $V_s = 40$ V; $R = 800$ kΩ; and $C = 25$ μF.

a) How many times per minute will the lamp turn on?

b) The 800 kΩ resistor is replaced with a variable resistor R. The resistance is adjusted until the lamp flashes 12 times per minute. What is the value of R?

7.105 ◆ P In the flashing light circuit shown in Fig. 7.45, the lamp can be modeled as a 1.3 kΩ resistor when it is conducting. The lamp triggers at 900 V and cuts off at 300 V.

a) If $V_s = 1000$ V, $R = 3.7$ kΩ, and $C = 250$ μF, how many times per minute will the light flash?

b) What is the average current in milliamps delivered by the source?

c) Assume the flashing light is operated 24 hours per day. If the cost of power is 5 cents per kilowatt-hour, how much does it cost to operate the light per year?

7.106 ◆ a) Show that the expression for the voltage drop across the capacitor while the lamp is conducting in the flashing light circuit in Fig. 7.48 is given by

$$v_L(t) = V_{\mathrm{Th}} + (V_{\max} - V_{\mathrm{Th}})e^{-(t-t_0)/\tau}$$

where

$$V_{\mathrm{Th}} = \frac{R_L}{R + R_L} V_s$$

$$\tau = \frac{RR_LC}{R + R_L}.$$

b) Show that the expression for the time the lamp conducts in the flashing light circuit in Fig. 7.48 is given by

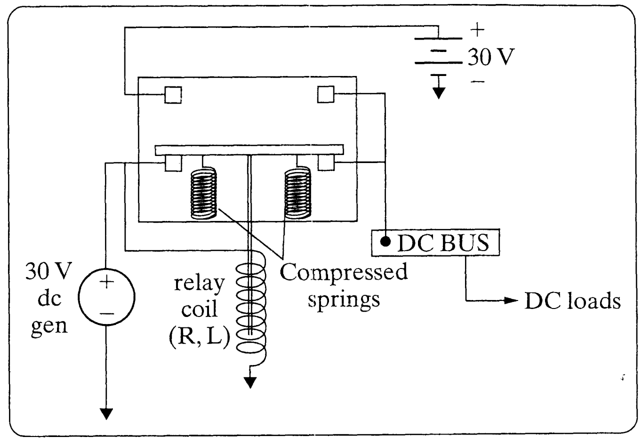
$$(t_c - t_o) = -\frac{RR_LC}{R + R_L} \ln \frac{V_{\max} - V_{\mathrm{Th}}}{V_{\min} - V_{\mathrm{Th}}}.$$

7.107 ◆ The relay shown in Fig. P7.107 connects the 30 V dc generator to the dc bus as long as the relay current is greater than 0.4 A. If the relay current drops to 0.4 A or less, the spring-loaded relay immediately connects the dc bus to the 30 V standby battery. The resistance of the relay winding is 60 Ω. The inductance of the relay winding is to be determined.

a) Assume the prime motor driving the 30 V dc generator abruptly slows down, causing the generated voltage to drop suddenly to 21 V. What value of L will assure that the standby battery will be connected to the dc bus in 0.5 seconds?

b) Using the value of L determined in (a), state how long it will take the relay to operate if the generated voltage suddenly drops to zero.

Figure P7.107



CHAPTER

8 Natural and Step Responses of *RLC* Circuits

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Be able to determine the natural response and the step response of parallel *RLC* circuits.
2. Be able to determine the natural response and the step response of series *RLC* circuits.

In this chapter, discussion of the natural response and step response of circuits containing both inductors and capacitors is limited to two simple structures: the parallel *RLC* circuit and the series *RLC* circuit. Finding the natural response of a parallel *RLC* circuit consists of finding the voltage created across the parallel branches by the release of energy stored in the inductor or capacitor or both. The task is defined in terms of the circuit shown in Fig. 8.1 on page 330. The initial voltage on the capacitor, V_0, represents the initial energy stored in the capacitor. The initial current through the inductor, I_0, represents the initial energy stored in the inductor. If the individual branch currents are of interest, you can find them after determining the terminal voltage.

We derive the step response of a parallel *RLC* circuit by using Fig. 8.2 on page 330. We are interested in the voltage that appears across the parallel branches as a result of the sudden application of a dc current source. Energy may or may not be stored in the circuit when the current source is applied.

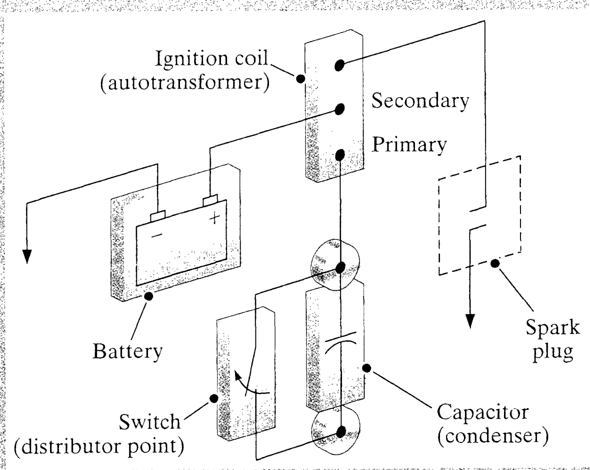
Finding the natural response of a series *RLC* circuit consists of finding the current generated in the series-connected elements by the release of initially stored energy in the inductor, capacitor, or both. The task is defined by the circuit shown in Fig. 8.3 on page 330. As before, the initial inductor current, I_0, and the initial capacitor voltage, V_0, represent the initially stored energy. If any of the individual element voltages are of interest, you can find them after determining the current.

Practical Perspective

An Ignition Circuit

In this chapter we introduce the step response of an *RLC* circuit. An automobile ignition circuit is based on the transient response of an *RLC* circuit. In such a circuit, a switching operation causes a rapid change in the current in an inductive winding known as an ignition coil. The ignition coil consists of two magnetically coupled coils connected in series. This series connection is also known as an autotransformer. The coil connected to the battery is referred to as the primary winding and the coil connected to the spark plug is referred to as the secondary winding. The rapidly changing current in the primary winding induces via magnetic coupling (mutual inductance) a very high voltage in the secondary winding. This voltage, which peaks at from 20 to 40 kV, is used to ignite a spark across the gap of the spark plug. The spark ignites the fuel-air mixture in the cylinder.

A schematic diagram showing the basic components of an ignition system is shown in the accompanying figure. In today's automobile, electronic (as opposed to mechanical) switching is used to cause the rapid change in the primary winding current. An understanding of the electronic switching circuit requires a knowledge of electronic components that is beyond the scope of this text. However, an analysis of the older, conventional ignition circuit will serve as an introduction to the types of problems encountered in the design of a useful circuit.



ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural response and the step response of parallel *RLC* circuits**

8.1 The resistance and inductance of the circuit in Fig. 8.5 are 100 Ω and 20 mH, respectively.

a) Find the value of C that makes the voltage response critically damped.

b) If C is adjusted to give a neper frequency of 5 krad/s, find the value of C and the roots of the characteristic equation.

c) If C is adjusted to give a resonant frequency of 20 krad/s, find the value of C and the roots of the characteristic equation.

ANSWER: (a) 500 nF; (b) $C = 1\ \mu$F, $s_1 = -5000 + j5000$ rad/s, $s_2 = -5000 - j5000$ rad/s; (c) $C = 125$ nF, $s_1 = -5359$ rad/s, $s_2 = -74{,}641$ rad/s.

NOTE ◆ *Also try Chapter Problem 8.1.*

8.2 ◆ The Forms of the Natural Response of a Parallel *RLC* Circuit

So far we have seen that the behavior of a second-order *RLC* circuit depends on the values of s_1 and s_2, which in turn depend on the circuit parameters R, L, and C. Therefore, the first step in finding the natural response is to calculate these values and, relatedly, determine whether the response is over-, under-, or critically damped.

Completing the description of the natural response requires finding two unknown coefficients, such as A_1 and A_2 in Eq. 8.13. The method used to do this is based on matching the solution for the natural response to the initial conditions imposed by the circuit, which are the initial value of the current (or voltage) and the initial value of the first derivative of the current (or voltage). Note that these same initial conditions, plus the final value of the variable, will also be needed when finding the step response of a second-order circuit.

Characteristics of the Underdamped Response

The underdamped response has several important characteristics. First, as the dissipative losses in the circuit decrease, the persistence of the oscillations increases, and the frequency of the oscillations approaches ω_0. In other words, as $R \to \infty$, the dissipation in the circuit in Fig. 8.8 approaches zero because $p = v^2/R$. As $R \to \infty$, $\alpha \to 0$, which tells us that $\omega_d \to \omega_0$. When $\alpha = 0$, the maximum amplitude of the voltage remains constant; thus the oscillation at ω_0 is sustained. In Example 8.4, if R were increased to infinity, the solution for $v(t)$ would become

$$v(t) = 98 \sin 1000t \text{ V}, \quad t \geq 0.$$

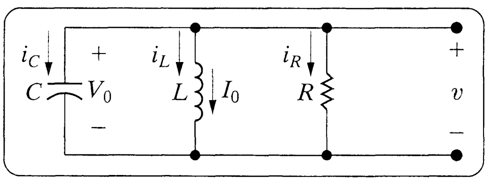
Thus, in this case the oscillation is sustained, the maximum amplitude of the voltage is 98 V, and the frequency of oscillation is 1000 rad/s.

We may now describe qualitatively the difference between an underdamped and an overdamped response. In an underdamped system, the response oscillates, or "bounces," about its final value. This oscillation is also referred to as *ringing*. In an overdamped system, the response approaches its final value without ringing or in what is sometimes described as a "sluggish" manner. When specifying the desired response of a second-order system, you may want to reach the final value in the shortest time possible, and you may not be concerned with small oscillations about that final value. If so, you would design the system components to achieve an underdamped response. On the other hand, you may be concerned that the response not exceed its final value, perhaps to ensure that components are not damaged. In such a case, you would design the system components to achieve an overdamped response, and you would have to accept a relatively slow rise to the final value.

ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural and the step response of parallel *RLC* circuits**

8.4 A 10 mH inductor, a 1 μF capacitor, and a variable resistor are connected in parallel in the circuit shown. The resistor is adjusted so that the roots of the characteristic equation are $-8000 \pm j6000$ rad/s. The initial voltage on the capacitor is 10 V, and the initial current in the inductor is 80 mA. Find (a) R; (b) $dv(0^+)/dt$; (c) B_1 and B_2 in the solution for v; and (d) $i_L(t)$.



ANSWER: (a) 62.5 Ω; (b) −240,000 V/s; (c) $B_1 = 10$ V, $B_2 = -80/3$ V; (d) $i_L(t) = 10e^{-8000t}[8 \cos 6000t + (82/3) \sin 6000t]$ mA when $t \geq 0$.

NOTE ◆ *Also try Chapter Problems 8.7 and 8.17.*

EXAMPLE ◆ 8.5 Finding the Critically Damped Natural Response of a Parallel *RLC* Circuit

a) For the circuit in Example 8.4 (Fig. 8.8), find the value of R that results in a critically damped voltage response.

b) Calculate $v(t)$ for $t \geq 0$.

c) Plot $v(t)$ versus t for $0 \leq t \leq 7$ ms.

SOLUTION

a) From Example 8.4, we know that $\omega_0^2 = 10^6$. Therefore for critical damping,

$$\alpha = 10^3 = \frac{1}{2RC},$$

or

$$R = \frac{10^6}{(2000)(0.125)} = 4000\ \Omega.$$

b) From the solution of Example 8.4, we know that $v(0^+) = 0$ and $dv(0^+)/dt = 98{,}000$ V/s. From Eqs. 8.35 and 8.36, $D_2 = 0$ and $D_1 = 98{,}000$ V/s. Substituting these values for α, D_1, and D_2 into Eq. 8.34 gives

$$v(t) = 98{,}000te^{-1000t}\ \text{V}, \quad t \geq 0.$$

c) Figure 8.10 shows a plot of $v(t)$ versus t in the interval $0 \leq t \leq 7$ ms.

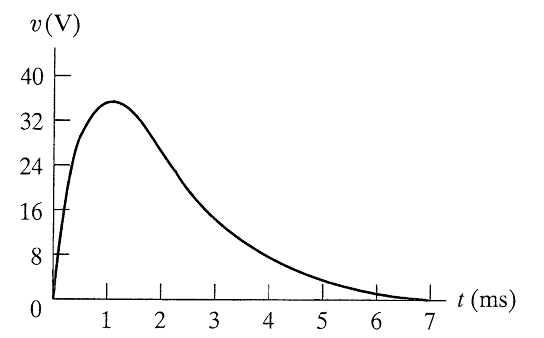


Figure 8.10 The voltage response for Example 8.5.

ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural and the step response of parallel *RLC* circuits**

8.5 The resistor in the circuit in Assessment Problem 8.4 is adjusted for critical damping. The inductance and capacitance values are 0.4 H and 10 μF, respectively. The initial energy stored in the circuit is 25 mJ and is distributed equally between the inductor and capacitor. Find (a) R; (b) V_0; (c) I_0; (d) D_1 and D_2 in the solution for v; and (e) i_R, $t \geq 0^+$.

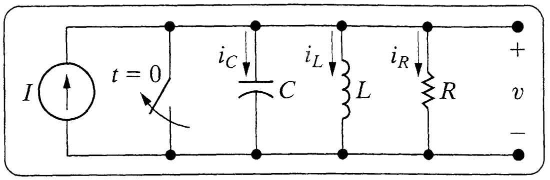
ANSWER: (a) 100 Ω; (b) 50 V; (c) 250 mA; (d) −50,000 V/s, 50 V; (e) $i_R(t) = (-500te^{-500t} + 0.50e^{-500t})$ A, $t \geq 0^+$.

NOTE ◆ *Also try Chapter Problems 8.9 and 8.18.*

ASSESSING OBJECTIVE 1

◆ **Be able to determine the natural response and the step response of parallel *RLC* circuits**

8.6 In the circuit shown, $R = 500\ \Omega$, $L = 0.64$ H, $C = 1\ \mu$F, $I_0 = 0.5$ A, $V_0 = 40$ V, and $I = -1$ A. Find (a) $i_R(0^+)$; (b) $i_C(0^+)$; (c) $di_L(0^+)/dt$; (d) s_1, s_2; (e) $i_L(t)$ for $t \geq 0$; and (f) $v(t)$ for $t \geq 0^+$.



ANSWER: (a) 80 mA; (b) −1.58 A; (c) 62.5 A/s; (d) $(-1000 + j750)$ rad/s, $(-1000 - j750)$ rad/s; (e) $[-1 + e^{-1000t}[1.5\cos 750t + 2.0833\sin 750t]$ A, for $t \geq 0$; (f) $e^{-1000t}(40\cos 750t - 2053.33\sin 750t)$ V, for $t \geq 0^+$.

NOTE ◆ *Also try Chapter Problems 8.24–8.26.*

8.4 ◆ The Natural and Step Response of a Series *RLC* Circuit

The procedures for finding the natural or step responses of a series *RLC* circuit are the same as those used to find the natural or step responses of a parallel *RLC* circuit, because both circuits are described by differential equations that have the same form. We begin by summing the voltages around the closed path in the circuit shown in Fig. 8.14. Thus

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0. \qquad (8.52)$$

We now differentiate Eq. 8.52 once with respect to t to get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0, \qquad (8.53)$$

which we can rearrange as

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0. \qquad (8.54)$$

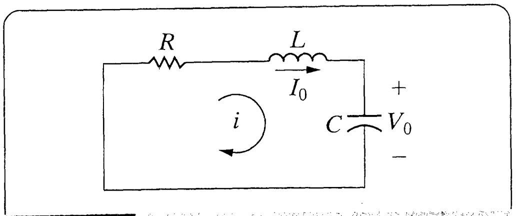


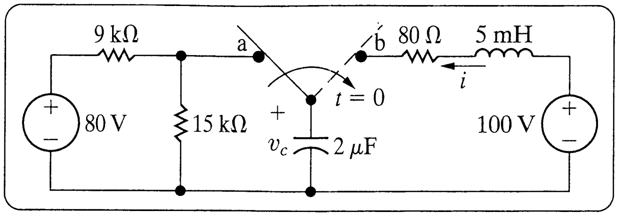
Figure 8.14 A circuit used to illustrate the natural response of a series *RLC* circuit.

Comparing Eq. 8.54 with Eq. 8.3 reveals that they have the same form. Therefore, to find the solution of Eq. 8.54, we follow the same process that led us to the solution of Eq. 8.3.

ASSESSING OBJECTIVE 2

◆ **Be able to determine the natural response and the step response of series *RLC* circuits**

8.7 The switch in the circuit shown has been in position a for a long time. At $t = 0$, it moves to position b. Find (a) $i(0^+)$; (b) $v_C(0^+)$; (c) $di(0^+)/dt$; (d) s_1, s_2; and (e) $i(t)$ for $t \geq 0$.



ANSWER: (a) 0; (b) 50 V; (c) 10,000 A/s; (d) $(-8000 + j6000)$ rad/s, $(-8000 - j6000)$ rad/s; (e) $(1.67e^{-8000t} \sin 6000t)$ A for $t \geq 0$.

8.8 Find $v_C(t)$ for $t \geq 0$ for the circuit in Assessment Problem 8.7.

ANSWER: $[100 - e^{-8000t}(50 \cos 6000t + 66.67 \sin 6000t)]$ V for $t \geq 0$.

NOTE ◆ *Also try Chapter Problems 8.40, 8.42, and 8.46.*

8.5 ◆ A Circuit with Two Integrating Amplifiers

A circuit containing two integrating amplifiers connected in cascade[1] is also a second-order circuit; that is, the output voltage of the second integrator is related to the input voltage of the first by a second-order differential equation. We begin our analysis of a circuit containing two cascaded amplifiers with the circuit shown in Fig. 8.18.

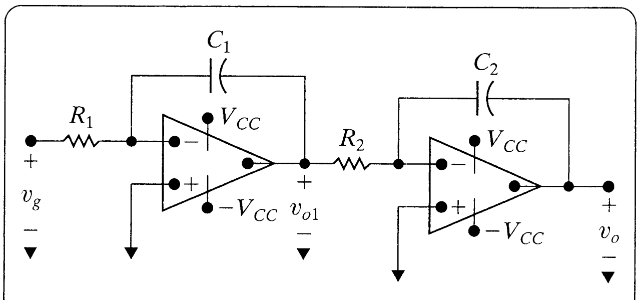


Figure 8.18 Two integrating amplifiers connected in cascade.

[1] In a cascade connection, the output signal of the first amplifier (v_{o1} in Fig. 8.18) is the input signal for the second amplifier.

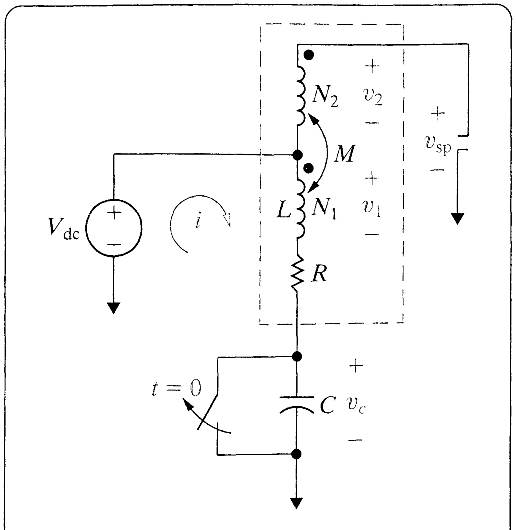


Figure 8.21 The circuit diagram of the conventional automobile ignition system.

Practical Perspective

An Ignition Circuit

Now let us return to the conventional ignition system introduced at the beginning of the chapter. A circuit diagram of the system is shown in Fig. 8.21. Consider the circuit characteristics that provide the energy to ignite the fuel-air mixture in the cylinder. First, the maximum voltage available at the spark plug, v_{sp}, must be high enough to ignite the fuel. Second, the voltage across the capacitor must be limited to prevent arcing across the switch or distributor points. Third, the current in the primary winding of the autotransformer must cause sufficient energy to be stored in the system to ignite the fuel-air mixture in the cylinder. Remember that the energy stored in the circuit at the instant of switching is proportional to the primary current squared, that is, $\omega_0 = \frac{1}{2}Li^2(0)$.

EXAMPLE

a) Find the maximum voltage at the spark plug, assuming the following values in the circuit of Fig. 8.21: $V_{dc} = 12$ V, $R = 4\ \Omega$, $L = 3$ mH, $C = 0.4\ \mu$F, and $a = 100$.

b) What distance must separate the switch contacts to prevent arcing at the time the voltage at the spark plug is maximum?

Solution

a) We analyze the circuit in Fig. 8.21 to find an expression for the spark plug voltage v_{sp}. We limit our analysis to a study of the voltages in the circuit prior to the firing of the spark plug. We assume that the current in the primary winding at the time of switching has its maximum possible value V_{dc}/R, where R is the total resistance in the primary circuit. We also assume that the ratio of the secondary voltage (v_2) to the primary voltage (v_1) is the same as the turns ratio N_2/N_1. We can justify this assumption as follows. With the secondary circuit open, the voltage induced in the secondary winding is

$$v_2 = M\frac{di}{dt}, \tag{8.88}$$

and the voltage induced in the primary winding is

$$v_1 = L\frac{di}{dt}. \tag{8.89}$$

It follows from Eqs. 8.88 and 8.89 that

$$\frac{v_2}{v_1} = \frac{M}{L}. \tag{8.90}$$

It is reasonable to assume that the permeance is the same for the fluxes ϕ_{11} and ϕ_{21} in the iron-core autotransformer; hence Eq. 8.90 reduces to

$$\frac{v_2}{v_1} = \frac{N_1 N_2 \mathcal{P}}{N_1^2 \mathcal{P}} = \frac{N_2}{N_1} = a. \tag{8.91}$$

We are now ready to analyze the voltages in the ignition circuit. The values of R, L, and C are such that when the switch is opened, the primary coil current response is underdamped. Using the techniques developed in Section 8.4 and assuming $t = 0$ at the instant the switch is opened, the expression for the primary coil current is found to be

$$i = \frac{V_{dc}}{R} e^{-\alpha t} \left[\cos \omega_d t + \left(\frac{\alpha}{\omega_d} \right) \sin \omega_d t \right], \tag{8.92}$$

where

$$\alpha = \frac{R}{2L},$$

$$\omega_d = \sqrt{\frac{1}{LC} - \alpha^2}.$$

(See Problem 8.58(a).) The voltage induced in the primary winding of the autotransformer is

$$v_1 = L\frac{di}{dt} = \frac{-V_{dc}}{\omega_d RC} e^{-\alpha t} \sin \omega_d t. \tag{8.93}$$

(See Problem 8.58(b).) It follows from Eq. 8.91 that

$$v_2 = \frac{-aV_{dc}}{\omega_d RC} e^{-\alpha t} \sin \omega_d t. \tag{8.94}$$

The voltage across the capacitor can be derived either by using the relationship

$$v_c = \frac{1}{C} \int_0^t i dx + v_c(0) \tag{8.95}$$

or by summing the voltages around the mesh containing the primary winding:

$$v_c = V_{dc} - iR - L\frac{di}{dt}. \tag{8.96}$$

In either case, we find

$$v_c = V_{dc}[1 - e^{-\alpha t} \cos \omega_d t + K e^{-\alpha t} \sin \omega_d t], \tag{8.97}$$

where

$$K = \frac{1}{\omega_d} \left(\frac{1}{RC} - \alpha \right).$$

(See Problem 8.58(c).) As can be seen from Fig. 8.21, the voltage across the spark plug is

$$v_{sp} = V_{dc} + v_2$$

$$= V_{dc} - \frac{aV_{dc}}{\omega_d RC} e^{-\alpha t} \sin \omega_d t$$

$$= V_{dc} \left[1 - \frac{a}{\omega_d RC} e^{-\alpha t} \sin \omega_d t \right]. \qquad (8.98)$$

To find the maximum value of v_{sp}, we find the smallest positive value of time where dv_{sp}/dt is zero and then evaluate v_{sp} at this instant. The expression for t_{max} is

$$t_{max} = \frac{1}{\omega_d} \tan^{-1} \left(\frac{\omega_d}{\alpha} \right). \qquad (8.99)$$

(See Problem 8.59.) For the component values in the problem statement, we have

$$\alpha = \frac{R}{2L} = \frac{4 \times 10^3}{6} = 666.67 \text{ rad/s},$$

and

$$\omega_d = \sqrt{\frac{10^9}{1.2} - (666.67)^2} = 28{,}859.81 \text{ rad/s}.$$

Substituting these values into Eq. 8.99 gives

$$t_{max} = 53.63 \ \mu\text{s}.$$

Now use Eq. 8.98 to find the maximum spark plug voltage, $v_{sp}(t_{max})$:

$$v_{sp}(t_{max}) = -25{,}975.69 \text{ V}.$$

b) The voltage across the capacitor at t_{max} is obtained from Eq. 8.97 as

$$v_c(t_{max}) = 262.15 \text{ V}.$$

The dielectric strength of air is approximately 3×10^6 V/m, so this result tells us that the switch contacts must be separated by $262.15/3 \times 10^6$, or 87.38, μm to prevent arcing at the points at t_{max}.

In the design and testing of ignition systems, consideration must be given to nonuniform fuel-air mixtures; the widening of the spark plug gap over time due to the erosion of the plug electrodes; the relationship between available spark plug voltage and engine speed; the time it takes the primary current to build up to its initial value after the switch is closed; and the amount of maintenance required to ensure reliable operation.

We can use the preceding analysis of a conventional ignition system to explain why electronic switching has replaced mechanical switching in today's automobiles. First, the current emphasis on fuel economy and exhaust emissions requires a spark plug with a wider gap. This, in turn, requires a higher available spark plug voltage. These higher voltages (up to 40 kV) cannot be achieved with mechanical switching. Electronic switching also permits higher initial currents in the primary winding of the autotransformer. This means the initial stored energy in the system is larger, and hence a wider range of fuel-air mixtures and running conditions can be accommodated. Finally, the electronic switching circuit eliminates the need for the point contacts. This means the deleterious effects of point contact arcing can be removed from the system.

NOTE ◆ *Assess your understanding of the Practical Perspective by trying Chapter Problems 8.60 and 8.61.*

SUMMARY

- The **characteristic equation** for both the parallel and series RLC circuits has the form

$$s^2 + 2\alpha s + \omega_0^2 = 0,$$

where $\alpha = 1/2RC$ for the parallel circuit, $\alpha = R/2L$ for the series circuit, and $\omega_0^2 = 1/LC$ for both the parallel and series circuits. (See pages 332 and 356.)

- The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

(See page 333.)

- The form of the natural and step responses of series and parallel RLC circuits depends on the values of α^2 and ω_0^2; such responses can be **overdamped**, **underdamped**, or **critically damped**. These terms describe the impact of the dissipative element (R) on the response. The **neper frequency**, α, reflects the effect of R. (See page 334.)

- The response of a second-order circuit is overdamped, underdamped, or critically damped as shown in Table 8.2.

TABLE 8.2 The Response of a Second-Order Circuit is Overdamped, Underdamped, or Critically Damped

THE CIRCUIT IS	WHEN	QUALITATIVE NATURE OF THE RESPONSE
Overdamped	$\alpha^2 > \omega_0^2$	The voltage or current approaches its final value without oscillation
Underdamped	$\alpha^2 < \omega_0^2$	The voltage or current oscillates about its final value
Critically damped	$\alpha^2 = \omega_0^2$	The voltage or current is on the verge of oscillating about its final value

PROBLEMS

8.1 The resistance, inductance, and capacitance in a parallel RLC circuit are 1000 Ω, 12.5 H, and 2 μF, respectively.

a) Calculate the roots of the characteristic equation that describe the voltage response of the circuit.

b) Will the response be over-, under-, or critically damped?

c) What value of R will yield a damped frequency of 120 krad/s?

d) What are the roots of the characteristic equation for the value of R found in (c)?

e) What value of R will result in a critically damped response?

8.2 The initial voltage on the 0.1 μF capacitor in the circuit shown in Fig. 8.1 is 24 V. The initial current in the inductor is zero. The voltage response for $t \geq 0$ is

$$v(t) = -8e^{-250t} + 32e^{-1000t} \text{ V}.$$

a) Determine the numerical values of R, L, α, and ω_0.

b) Calculate $i_R(t)$, $i_L(t)$, and $i_C(t)$ for $t \geq 0^+$.

8.3 The circuit elements in the circuit in Fig. 8.1 are $R = 200$ Ω, $C = 0.2$ μF, and $L = 50$ mH. The initial inductor current is -45 mA, and the initial capacitor voltage is 15 V.

a) Calculate the initial current in each branch of the circuit.

b) Find $v(t)$ for $t \geq 0$.

c) Find $i_L(t)$ for $t \geq 0$.

8.4 The resistance in Problem 8.3 is increased to 312.5 Ω. Find the expression for $v(t)$ for $t \geq 0$.

8.5 The resistance in Problem 8.3 is increased to 250 Ω. Find the expression for $v(t)$ for $t \geq 0$.

8.6 The natural response for the circuit shown in Fig. 8.1 is known to be

$$v(t) = 3\left(e^{-100t} + e^{-900t}\right) \text{ V}, \quad t \geq 0.$$

If $L = (40/9)$ H and $C = 2.5$ μF, find $i_L(0^+)$ in milliamperes.

8.7 The natural voltage response of the circuit in Fig. 8.1 is

$$v(t) = 100e^{-20{,}000t}(\cos 15{,}000t - 2\sin 15{,}000t) \text{ V}, \quad t \geq 0,$$

when the capacitor is 0.04 μF. Find (a) L; (b) R; (c) V_0; (d) I_0; and (e) $i_L(t)$.

8.8 The initial value of the voltage v in the circuit in Fig. 8.1 is 15 V, and the initial value of the capacitor current, $i_c(0^+)$, is 45 mA. The expression for the capacitor current is known to be

$$i_c(t) = A_1e^{-200t} + A_2e^{-800t}, \quad t \geq 0^+,$$

when R is 250 Ω. Find

a) the value of α, ω_0, L, C, A_1, and A_2

$$\left(\textit{Hint:} \quad \frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt} = \frac{v(0)}{L} - \frac{1}{R}\frac{i_C(0^+)}{C}\right)$$

b) the expression for $v(t)$, $t \geq 0$,

c) the expression for $i_R(t) \geq 0$,

d) the expression for $i_L(t) \geq 0$.

8.9 The voltage response for the circuit in Fig. 8.1 is known to be

$$v(t) = D_1t\,e^{-500t} + D_2e^{-500t}, \quad t \geq 0.$$

The initial current in the inductor (I_0) is -10 mA, and the initial voltage on the capacitor (V_0) is 8 V. The inductor has an inductance of 4 H.

a) Find the value of R, C, D_1, and D_2.

b) Find $i_C(t)$ for $t \geq 0^+$.

8.10 In the circuit in Fig. 8.1, $R = 12.5\,\Omega$, $L = (50/101)$ H, $C = 0.08$ F, $V_0 = 0$ V, and $I_0 = -4$ A.

a) Find $v(t)$ for $t \geq 0$.
b) Find the first three values of t for which dv/dt is zero. Let these values of t be denoted t_1, t_2, and t_3.
c) Show that $t_3 - t_1 = T_d$.
d) Show that $t_2 - t_1 = T_d/2$.
e) Calculate $v(t_1)$, $v(t_2)$, and $v(t_3)$.
f) Sketch $v(t)$ versus t for $0 \leq t \leq t_2$.

8.11 a) Find $v(t)$ for $t \geq 0$ in the circuit in Problem 8.10 if the 12.5 Ω resistor is removed from the circuit.
b) Calculate the frequency of $v(t)$ in hertz.
c) Calculate the maximum amplitude of $v(t)$ in volts.

8.12 In the circuit shown in Fig. 8.1, a 12.5 H inductor is shunted by a 3.2 nF capacitor, the resistor R is adjusted for critical damping, $V_0 = 100$ V, and $I_0 = 6.4$ mA.

a) Calculate the numerical value of R.
b) Calculate $v(t)$ for $t \geq 0$.
c) Find $v(t)$ when $i_C(t) = 0$.
d) What percentage of the initially stored energy remains stored in the circuit at the instant $i_C(t)$ is 0?

8.13 The resistor in the circuit in Example 8.4 is changed to 3200 Ω.

a) Find the numerical expression for $v(t)$ when $t \geq 0$.
b) Plot $v(t)$ versus t for the time interval $0 \leq t \leq 7$ ms. Compare this response with the one in Example 8.4 ($R = 20\,\text{k}\Omega$) and Example 8.5 ($R = 4\,\text{k}\Omega$). In particular, compare peak values of $v(t)$ and the times when these peak values occur.

8.14 Assume the underdamped voltage response of the circuit in Fig. 8.1 is written as

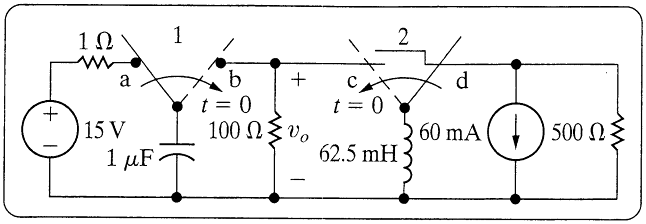
$$v(t) = (A_1 + A_2)e^{-\alpha t}\cos\omega_d t + j(A_1 - A_2)e^{-\alpha t}\sin\omega_d t$$

The initial value of the inductor current is I_0, and the initial value of the capacitor voltage is V_0. Show that A_2 is the conjugate of A_1. (*Hint:* Use the same process as outlined in the text to find A_1 and A_2.)

8.15 Show that the results obtained from Problem 8.14—that is, the expressions for A_1 and A_2—are consistent with Eqs. 8.30 and 8.31 in the text.

8.16 The two switches in the circuit seen in Fig. P8.16 operate synchronously. When switch 1 is in position a, switch 2 is in position d. When switch 1 moves to position b, switch 2 moves to position c. Switch 1 has been in position a for a long time. At $t = 0$, the switches move to their alternate positions. Find $v_o(t)$ for $t \geq 0$.

Figure P8.16

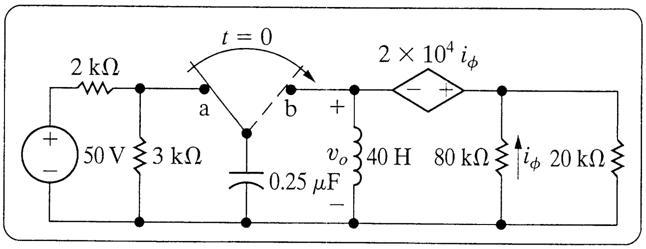


8.17 The resistor in the circuit of Fig. P8.16 is increased from 100 Ω to 200 Ω. Find $v_o(t)$ for $t > 0$.

8.18 The resistor in the circuit of Fig. P8.16 is increased from 100 Ω to 125 Ω. Find $v_o(t)$ for $t \geq 0$.

8.19 The switch in the circuit of Fig. P8.19 has been in position a for a long time. At $t = 0$ the switch moves instantaneously to position b. Find $v_o(t)$ for $t \geq 0$.

Figure P8.19



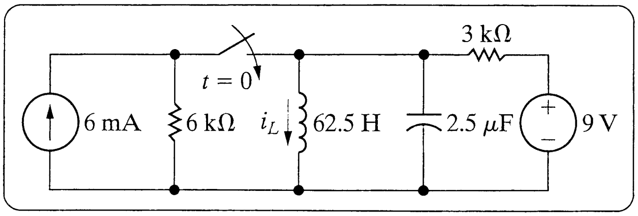
8.20 For the circuit in Example 8.6, find, for $t \geq 0$, (a) $v(t)$; (b) $i_R(t)$; and (c) $i_C(t)$.

8.21 For the circuit in Example 8.7, find, for $t \geq 0$, (a) $v(t)$ and (b) $i_C(t)$.

8.22 For the circuit in Example 8.8, find $v(t)$ for $t \geq 0$.

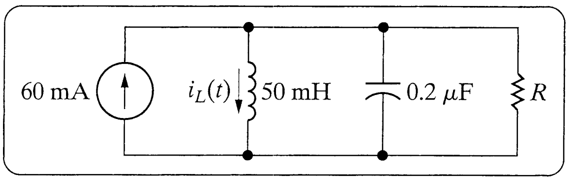
8.23 The switch in the circuit in Fig. P8.23 has been open a long time before closing at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

Figure P8.23



8.24 Assume that at the instant the 66 mA dc current source is applied to the circuit in Fig. P8.24, the initial current in the 50 mH inductor is −45 mA, and the initial voltage on the capacitor is 15 V (positive at the upper terminal). Find the expression for $i_L(t)$ for $t \geq 0$ if R equals 200 Ω.

Figure P8.24

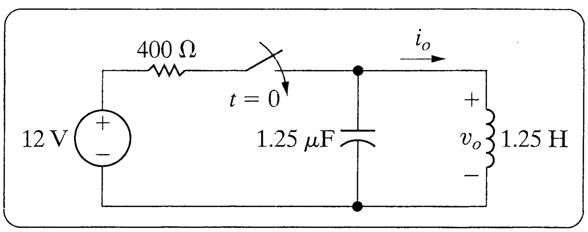


8.25 The resistance in the circuit in Fig. P8.24 is increased to 312.5 Ω. Find $i_L(t)$ for $t \geq 0$.

8.26 The resistance in the circuit in Fig. P8.24 is changed to 250 Ω. Find $i_L(t)$ for $t \geq 0$.

8.27 There is no energy stored in the circuit in Fig. P8.27 when the switch is closed at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

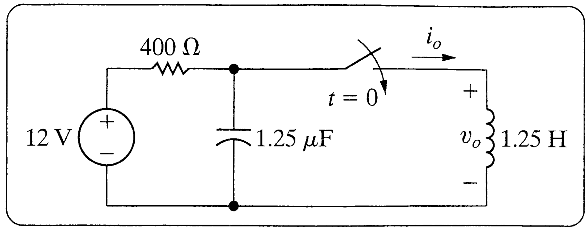
Figure P8.27



8.28
a) For the circuit in Fig. P8.27, find i_o for $t \geq 0$.
b) Show that your solution for i_o is consistent with the solution for v_o in Problem 8.27.

8.29 The switch in the circuit in Fig. P8.29 has been open for a long time before closing at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

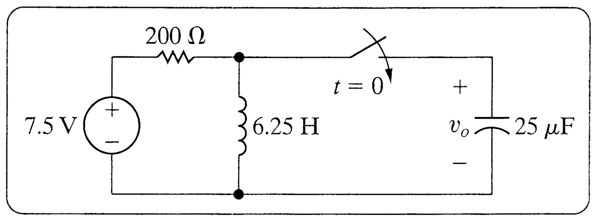
Figure P8.29



8.30
a) For the circuit in Fig. P8.29, find i_o for $t \geq 0$.
b) Show that your solution for i_o is consistent with the solution for v_o in Problem 8.29.

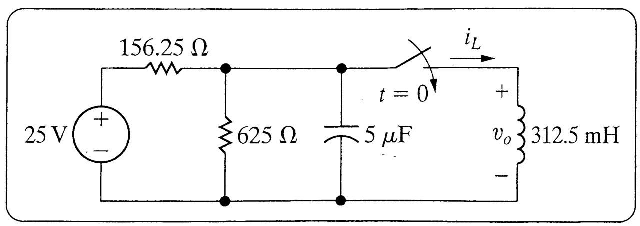
8.31 The switch in the circuit in Fig. P8.31 has been open a long time before closing at $t = 0$. Find v_o for $t \geq 0$.

Figure P8.31



8.32 The switch in the circuit in Fig. P8.32 has been open a long time before closing at $t = 0$. Find
a) $v_o(t)$ for $t \geq 0^+$,
b) $i_L(t)$ for $t \geq 0$.

Figure P8.32



8.33 Use the circuit in Fig. P8.32

a) Find the total energy delivered to the inductor.

b) Find the total energy delivered to the equivalent resistor.

c) Find the total energy delivered to the capacitor.

d) Find the total energy delivered by the equivalent current source.

e) Check the results of parts (a) through (d) against the conservation of energy principle.

8.34 Switches 1 and 2 in the circuit in Fig. P8.34 are synchronized. When switch 1 is opened, switch 2 closes and vice versa. Switch 1 has been open a long time before closing at $t = 0$. Find $i_L(t)$ for $t \geq 0$.

8.35 The initial energy stored in the 50 nF capacitor in the circuit in Fig. P8.35 is 90 μJ. The initial energy stored in the inductor is zero. The roots of the characteristic equation that describes the natural behavior of the current i are $-1000\ \text{s}^{-1}$ and $-4000\ \text{s}^{-1}$.

a) Find the numerical values of R and L.

b) Find the numerical values of $i(0)$ and $di(0)/dt$ immediately after the switch has been closed.

c) Find $i(t)$ for $t \geq 0$.

d) How many microseconds after the switch closes does the current reach its maximum value?

e) What is the maximum value of i in milliamperes?

f) Find $v_L(t)$ for $t \geq 0$.

Figure P8.34

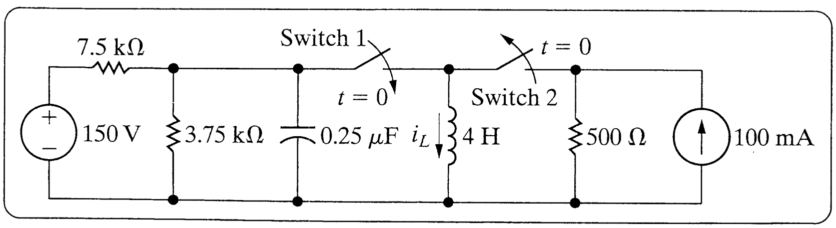
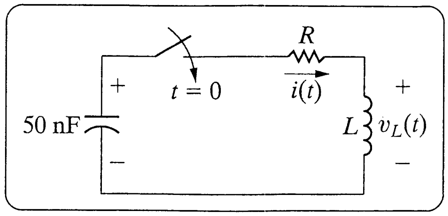


Figure P8.35



8.36 The current in the circuit in Fig. 8.3 is known to be

$$i = B_1 e^{-2000t} \cos 1500t + B_2 e^{-2000t} \sin 1500t, \quad t \geq 0.$$

The capacitor has a value of 80 nF; the initial value of the current is 7.5 mA; and the initial voltage on the capacitor is -30 V. Find the values of R, L, B_1, and B_2.

8.37 Find the voltage across the 80 nF capacitor for the circuit described in Problem 8.36. Assume the reference polarity for the capacitor voltage is positive at the upper terminal.

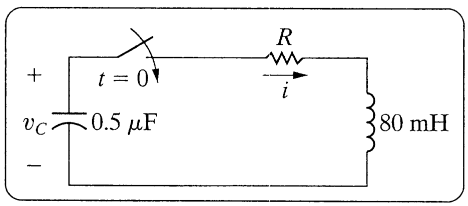
8.38 In the circuit in Fig. P8.38, the resistor is adjusted for critical damping. The initial capacitor voltage is 20 V, and the initial inductor current is 30 mA.

a) Find the numerical value of R.

b) Find the numerical values of i and di/dt immediately after the switch is closed.

c) Find $v_C(t)$ for $t \geq 0$.

Figure P8.38



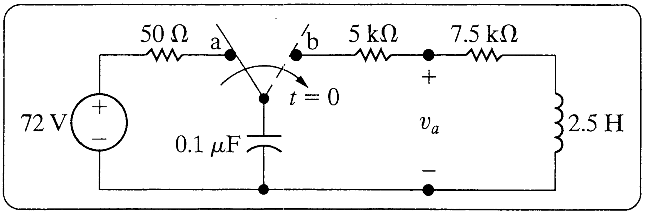
8.39 The switch in the circuit in Fig. P8.39 has been in position a for a long time. At $t = 0$, the switch moves instantaneously to position b.

a) What is the initial value of v_a?

b) What is the initial value of dv_a/dt?

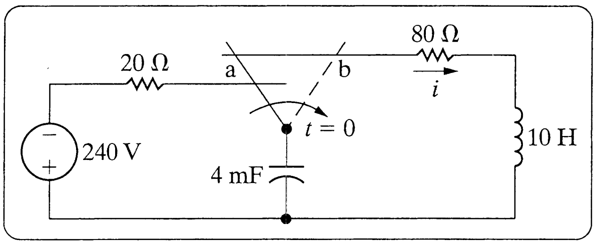
c) What is the numerical expression for $v_a(t)$ for $t \geq 0$?

Figure P8.39



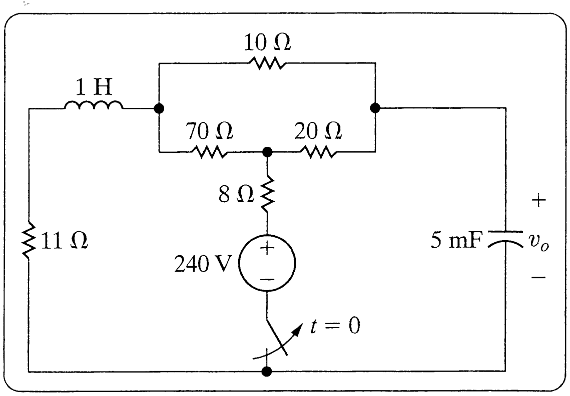
8.40 The make-before-break switch in the circuit shown in Fig. P8.40 has been in position a for a long time. At $t = 0$, the switch is moved instantaneously to position b. Find $i(t)$ for $t \geq 0$.

Figure P8.40



8.41 The switch in the circuit shown in Fig. P8.41 has been closed for a long time. The switch opens at $t = 0$. Find $v_o(t)$ for $t \geq 0$.

Figure P8.41

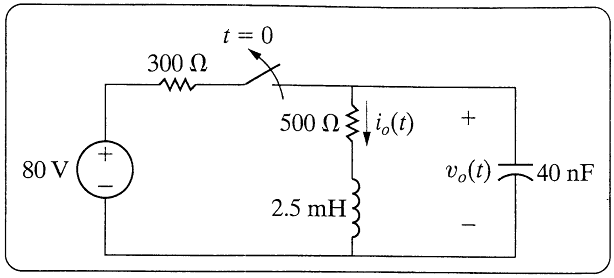


8.42 The switch in the circuit shown in Fig. P8.42 has been closed for a long time. The switch opens at $t = 0$. Find

a) $i_o(t)$ for $t \geq 0$,

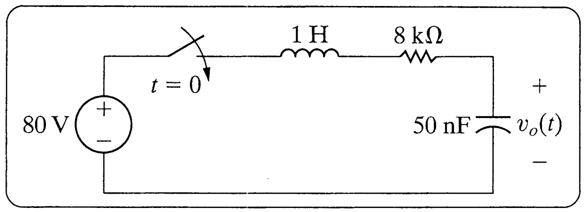
b) $v_o(t)$ for $t \geq 0$.

Figure P8.42



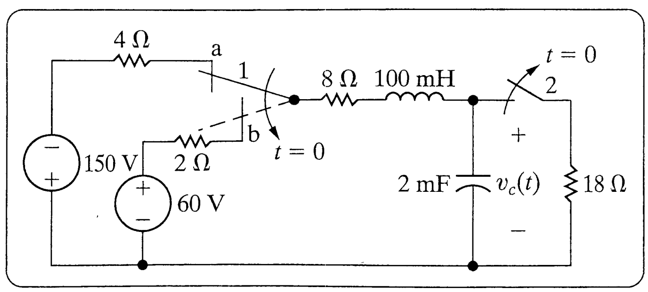
8.43 The initial energy stored in the circuit in Fig. P8.43 is zero. Find $v_o(t)$ for $t \geq 0$.

Figure P8.43



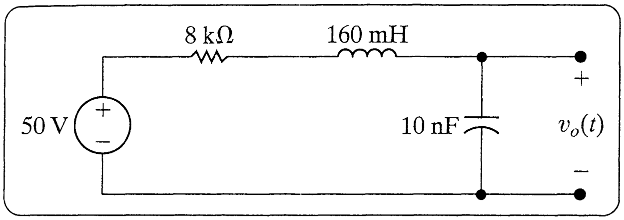
8.44 The two switches in the circuit seen in Fig. P8.44 operate synchronously. When switch 1 is in position a, switch 2 is closed. When switch 1 is in position b, switch 2 is open. Switch 1 has been in position a for a long time. At $t = 0$, it moves instantaneously to position b. Find $v_c(t)$ for $t \geq 0$.

Figure P8.44



8.45 The circuit shown in Fig. P8.45 has been in operation for a long time. At $t = 0$, the voltage suddenly increases to 250 V. Find $v_o(t)$ for $t \geq 0$.

Figure P8.45



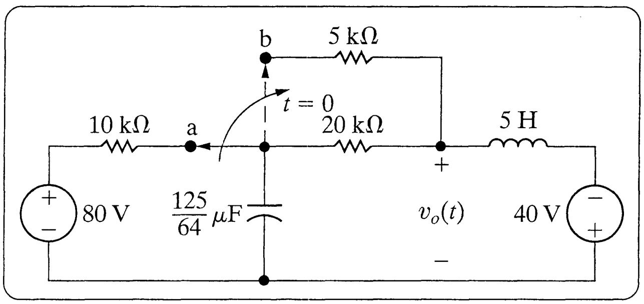
8.46 The switch in the circuit of Fig. P8.46 has been in position a for a long time. At $t = 0$ the switch moves instantaneously to position b. Find

a) $v_o(0^+)$

b) $dv_o(0^+)/dt$

c) $v_o(t)$ for $t \geq 0$.

Figure P8.46



8.47 Assume that the capacitor voltage in the circuit of Fig. 8.15 is underdamped. Also assume that no energy is stored in the circuit elements when the switch is closed.

a) Show that $dv_C/dt = (\omega_0^2/\omega_d)Ve^{-\alpha t}\sin\omega_d t$.

b) Show that $dv_C/dt = 0$ when $t = n\pi/\omega_d$, where $n = 0, 1, 2, \ldots$.

c) Let $t_n = n\pi/\omega_d$, and show that $v_C(t_n) = V - V(-1)^n e^{-\alpha n\pi/\omega_d}$.

d) Show that

$$\alpha = \frac{1}{T_d}\ln\frac{v_C(t_1) - V}{v_C(t_3) - V},$$

where $T_d = t_3 - t_1$.

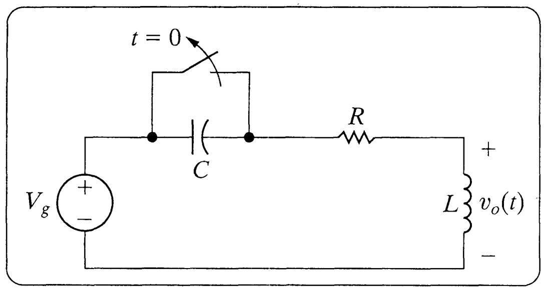
8.48 The voltage across a 0.1 μF capacitor in the circuit of Fig. 8.15 is described as follows: After the switch has been closed for several seconds, the voltage is constant at 100 V. The first time the voltage exceeds 100 V, it reaches a peak of 163.84 V. This occurs $\pi/7$ ms after the switch has been closed. The second time the voltage exceeds 100 V, it reaches a peak of 126.02 V. This second peak occurs $3\pi/7$ ms after the switch has been closed. At the time when the switch is closed, there is no energy stored in either the capacitor or the inductor. Find the numerical values of R and L. (*Hint:* Work Problem 8.47 first.)

8.49 The switch in the circuit shown in Fig. P8.49 has been closed for a long time before it is opened at $t = 0$. Assume that the circuit parameters are such that the response is underdamped.

a) Derive the expression for $v_o(t)$ as a function of v_g, α, ω_d, C, and R for $t \geq 0$.

b) Derive the expression for the value of t when the magnitude of v_o is maximum.

Figure P8.49



8.50 The circuit parameters in the circuit of Fig. P8.49 are $R = 4800\,\Omega$, $L = 64$ mH, $C = 4$ nF, and $v_g = -72$ V.

a) Express $v_o(t)$ numerically for $t \geq 0$.

b) How many microseconds after the switch opens is the inductor voltage maximum?

c) What is the maximum value of the inductor voltage?

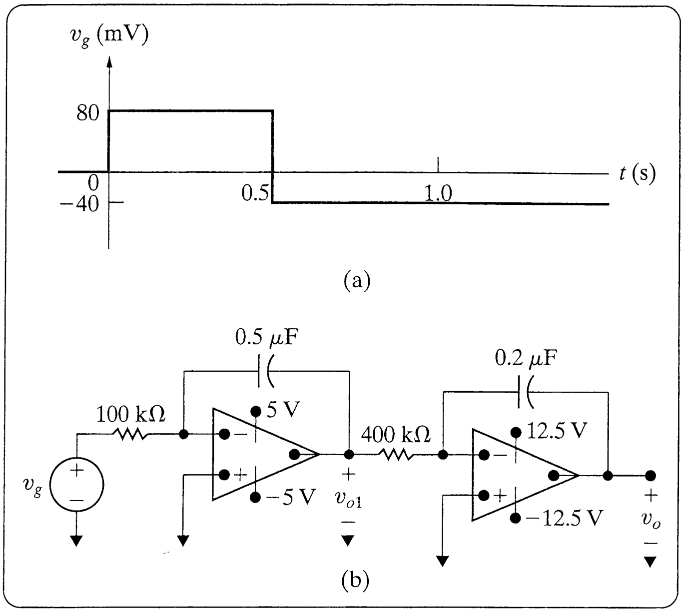
d) Repeat (a)–(c) with R reduced to 480 Ω.

8.51 The voltage signal of Fig. P8.51(a) is applied to the cascaded integrating amplifiers shown in Fig. P8.51(b). There is no energy stored in the capacitors at the instant the signal is applied.

a) Derive the numerical expressions for $v_o(t)$ and $v_{o1}(t)$ for the time intervals $0 \le t \le 0.5$ s and $0.5 \text{ s} \le t \le t_{sat}$.

b) Compute the value of t_{sat}.

Figure P8.51



8.52 The circuit in Fig. P8.51(b) is modified by adding a 1 MΩ resistor in parallel with the 0.5 μF capacitor and a 5 MΩ resistor in parallel with the 0.2 μF capacitor. As in Problem 8.51, there is no energy stored in the capacitors at the time the signal is applied. Derive the numerical expressions for $v_o(t)$ and $v_{o1}(t)$ for the time intervals $0 \le t \le 0.5$ s and $0.5 \text{ s} \le t \le \infty$.

8.53 Show that, if no energy is stored in the circuit shown in Fig. 8.19 at the instant v_g jumps in value, then dv_o/dt equals zero at $t = 0$.

8.54 a) Find the equation for $v_o(t)$ for $0 \le t \le t_{\text{sat}}$ in the circuit shown in Fig. 8.19 if $v_{o1}(0) = 5$ V and $v_o(0) = 8$ V.

b) How long does the circuit take to reach saturation?

8.55 a) Rework Example 8.14 with feedback resisto R_1 and R_2 removed.

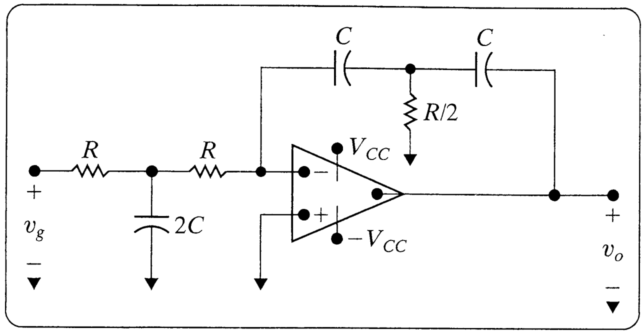
b) Rework Example 8.14 with $v_{o1}(0) = -2$ and $v_o(0) = 4$ V.

8.56 a) Derive the differential equation that relate the output voltage to the input voltage for th circuit shown in Fig. P8.56.

b) Compare the result with Eq. 8.75 when R_1C_1 $R_2C_2 = RC$ in Fig. 8.18.

c) What is the advantage of the circuit shown i Fig. P8.56?

Figure P8.56

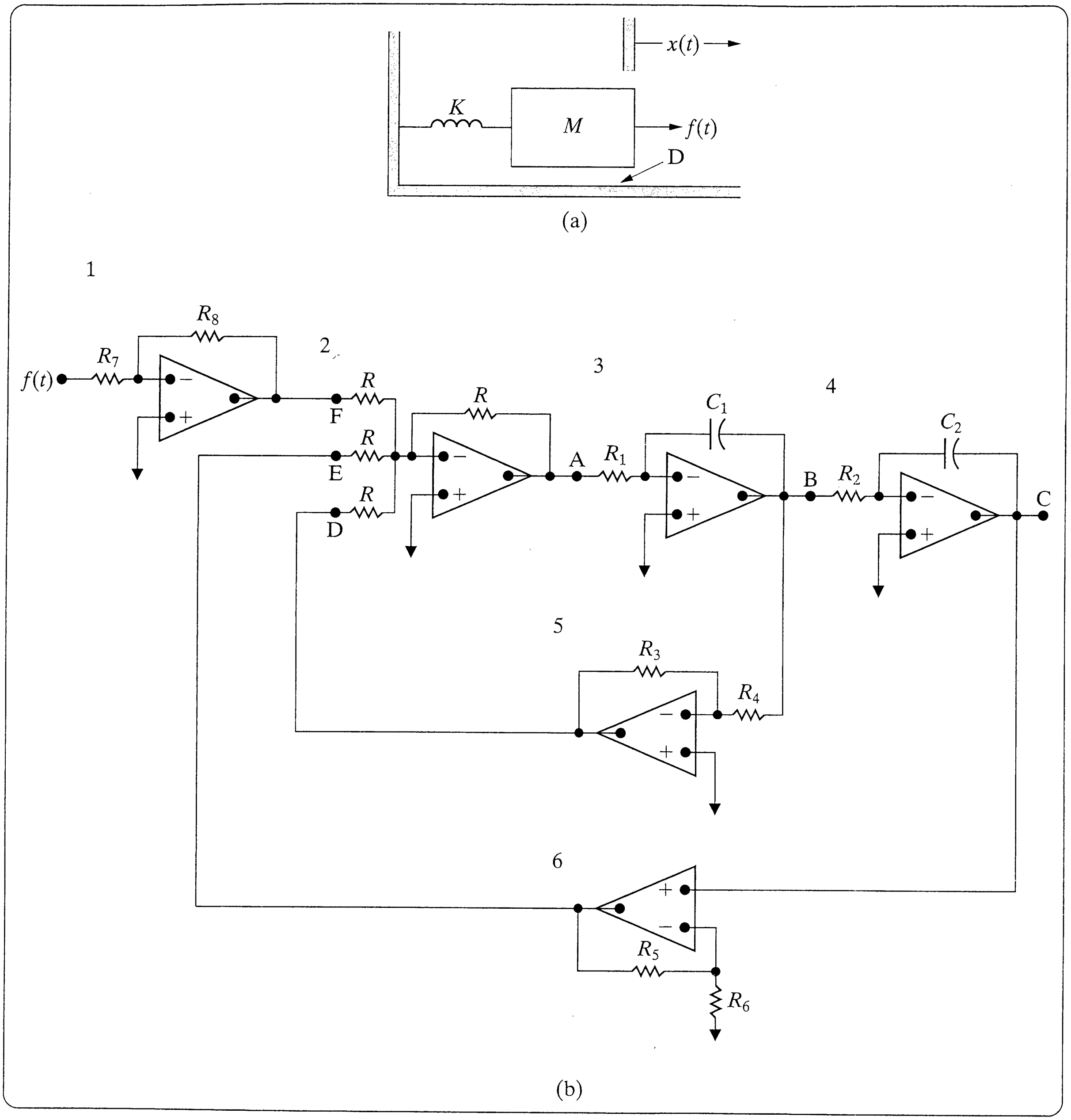


8.57 We now wish to illustrate how several op amp ci cuits can be interconnected to solve a differentia equation.

a) Derive the differential equation for the sprin mass system shown in Fig. P8.57(a). (Se page 378.) Assume that the force exerte by the spring is directly proportional to th spring displacement, that the mass is constan and that the frictional force is directly propo tional to the velocity of the moving mass.

b) Rewrite the differential equation derived i (a) so that the highest order derivative is ex pressed as a function of all the other term in the equation. Now assume that a voltag equal to d^2x/dt^2 is available and by succes sive integrations generates dx/dt and x. W can synthesize the coefficients in the equa tions by scaling amplifiers, and we can com bine the terms required to generate d^2x/dt by using a summing amplifier. With thes ideas in mind, analyze the interconnectio shown in Fig. P8.57(b). In particular, describe the purpose of each shaded area in the circuit and describe the signal at the points labeled B, C, D, E, and F, assuming the signal at A represents d^2x/dt^2. Also discuss the parameters R; R_1, C_1; R_2, C_2; R_3, R_4; R_5, R_6; and R_7, R_8 in terms of the coefficients in the differential equation.

Figure P8.57



8.58 ◆
a) Derive Eq. 8.92.
b) Derive Eq. 8.93.
c) Derive Eq. 8.97.

8.59 ◆ Derive Eq. 8.99.

8.60 ◆
a) Using the same numerical values used in the Practical Perspective example in the text, find the instant of time when the voltage across the capacitor is maximum.
b) Find the maximum value of v_c.
c) Compare the values obtained in (a) and (b) with t_{max} and $v_c(t_{max})$.

8.61 ◆ The values of the parameters in the circuit in Fig. 8.21 are $R = 3\ \Omega$; $L = 5$ mH; $C = 0.25\ \mu$F; $V_{dc} = 12$ V; and $a = 50$. Assume the switch opens when the primary winding current is 4 A.

a) How much energy is stored in the circuit at $t = 0^+$?
b) Assume the spark plug does not fire. What is the maximum voltage available at the spark plug?
c) What is the voltage across the capacitor when the voltage across the spark plug is at its maximum value?

CHAPTER

9

Sinusoidal Steady-State Analysis

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1 Understand phasor concepts and be able to perform a phasor transform and an inverse phasor transform.

2 Be able to transform a circuit with a sinusoidal source into the frequency domain using phasor concepts.

3 Know how to use the following circuit analysis techniques to solve a circuit in the frequency domain:
- Kirchhoff's laws;
- Series, parallel, and delta-to-wye simplifications;
- Voltage and current division;
- Thévenin and Norton equivalents;
- Node-voltage method; and
- Mesh-current method.

4 Be able to analyze circuits containing linear transformers using phasor methods.

5 Understand the ideal transformer constraints and be able to analyze circuits containing ideal transformers using phasor methods.

Thus far, we have focused on circuits with constant sources; in this chapter we are now ready to consider circuits energized by time-varying voltage or current sources. In particular, we are interested in sources in which the value of the voltage or current varies sinusoidally. Sinusoidal sources and their effect on circuit behavior form an important area of study for several reasons. First, the generation, transmission, distribution, and consumption of electric energy occur under essentially sinusoidal steady-state conditions. Second, an understanding of sinusoidal behavior makes it possible to predict the behavior of circuits with nonsinusoidal sources. Third, steady-state sinusoidal behavior often simplifies the design of electrical systems. Thus a designer can spell out specifications in terms of a desired steady-state sinusoidal response and design the circuit or system to meet those characteristics. If the device satisfies the specifications, the designer knows that the circuit will respond satisfactorily to nonsinusoidal inputs.

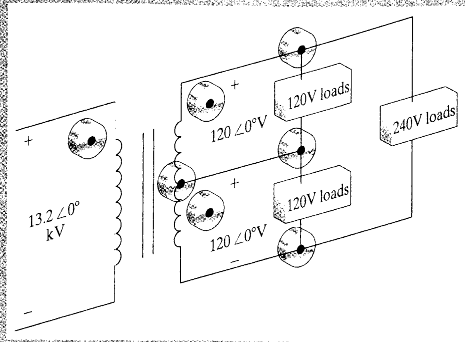
The subsequent chapters of this book are largely based on a thorough understanding of the techniques needed to analyze circuits driven by sinusoidal sources. Fortunately, the circuit analysis and simplification techniques first introduced in Chapters 1–4 work for circuits with sinusoidal as well as dc sources, so some of the material in this chapter will be very familiar to you. The challenges in first approaching sinusoidal analysis include developing the appropriate modeling equations and working in the mathematical realm of complex numbers.

Practical Perspective

A Household Distribution Circuit

Power systems that generate, transmit, and distribute electrical power are designed to operate in the sinusoidal steady state. The standard household distribution circuit used in the United States is the three-wire, 240/120 V circuit shown in the accompanying figure.

The transformer is used to reduce the utility distribution voltage from 13.2 kV to 240 V. The center tap on the secondary winding provides the 120 V service. The operating frequency of power systems in the United States is 60 Hz. Both 50 and 60 Hz systems are found outside the United States. The voltage ratings alluded to above are rms values. The reason for defining an rms value of a time-varying signal is explained in Chapter 10.



Again, we invoke the identity involving the cosine of the sum of two angles and write

$$y = 44.72\cos(\omega t + 33.43^\circ).$$

b) We can solve the problem by using phasors as follows: Because

$$y = y_1 + y_2,$$

then, from Eq. 9.24,

$$\begin{aligned}\mathbf{Y} &= \mathbf{Y}_1 + \mathbf{Y}_2 \\ &= 20 \angle -30^\circ + 40 \angle 60^\circ \\ &= (17.32 - j10) + (20 + j34.64) \\ &= 37.32 + j24.64 \\ &= 44.72 \angle 33.43^\circ.\end{aligned}$$

Once we know the phasor $\mathbf{Y}$, we can write the corresponding trigonometric function for y by taking the inverse phasor transform:

$$\begin{aligned} y &= \mathcal{P}^{-1}\{44.72e^{j33.43}\} = \Re\{44.72e^{j33.43}e^{j\omega t}\} \\ &= 44.72\cos(\omega t + 33.43^\circ).\end{aligned}$$

The superiority of the phasor approach for adding sinusoidal functions should be apparent. Note that it requires the ability to move back and forth between the polar and rectangular forms of complex numbers.

ASSESSING OBJECTIVE 1

◆ **Understand phasor concepts and be able to perform a phasor transform and an inverse phasor transform**

9.1 Find the phasor transform of each trigonometric function:

a) $v = 170\cos(377t - 40^\circ)$ V.

b) $i = 10\sin(1000t + 20^\circ)$ A.

c) $i = [5\cos(\omega t + 36.87^\circ) + 10\cos(\omega t - 53.13^\circ)]$ A.

d) $v = [300\cos(20{,}000\pi t + 45^\circ) - 100\sin(20{,}000\pi t + 30^\circ)]$ mV.

ANSWER: (a) $170 \angle -40^\circ$ V; (b) $10 \angle -70^\circ$ A; (c) $11.18 \angle -26.57^\circ$ A; (d) $339.90 \angle 61.51^\circ$ mV.

9.2 Find the time-domain expression corresponding to each phasor:

a) $\mathbf{V} = 18.6 \angle -54^\circ$ V.

b) $\mathbf{I} = (20 \angle 45^\circ - 50 \angle -30^\circ)$ mA.

c) $\mathbf{V} = (20 + j80 - 30 \angle 15^\circ)$ V.

ANSWER: (a) $18.6\cos(\omega t - 54^\circ)$ V; (b) $48.81\cos(\omega t + 126.68^\circ)$ mA; (c) $72.79\cos(\omega t + 97.08^\circ)$ V.

NOTE ◆ *Also try Chapter Problem 9.12.*

is measured in ohms. Note that, although impedance is a complex number, it is not a phasor. Remember, a phasor is a complex number that shows up as the coefficient of $e^{j\omega t}$. Thus, although all phasors are complex numbers, not all complex numbers are phasors.

Impedance in the frequency domain is the quantity analogous to resistance, inductance, and capacitance in the time domain. The imaginary part of the impedance is called **reactance**. The values of impedance and reactance for each of the component values are summarized in Table 9.1.

And finally, a reminder. If the reference direction for the current in a passive circuit element is in the direction of the voltage rise across the element, you must insert a minus sign into the equation that relates the voltage to the current.

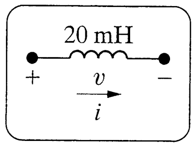
TABLE 9.1 Impedance and Reactance Values

CIRCUIT ELEMENT	IMPEDANCE	REACTANCE
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

ASSESSING OBJECTIVE 2

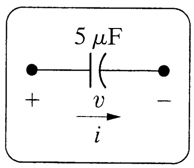
◆ **Be able to transform a circuit with a sinusoidal source into the frequency domain using phasor concepts**

9.3 The current in the 20 mH inductor is $10\cos(10{,}000t + 30°)$ mA. Calculate (a) the inductive reactance; (b) the impedance of the inductor; (c) the phasor voltage **V**; and (d) the steady-state expression for $v(t)$.



ANSWER: (a) 200 Ω; (b) $j200$ Ω; (c) $2\,\angle 120°$ V; (d) $2\cos(10{,}000t + 120°)$ V.

9.4 The voltage across the terminals of the 5 μF capacitor is $30\cos(4000t + 25°)$ V. Calculate (a) the capacitive reactance; (b) the impedance of the capacitor; (c) the phasor current **I**; and (d) the steady-state expression for $i(t)$.



ANSWER: (a) −50 Ω; (b) $-j50$ Ω; (c) $0.6\,\angle 115°$ A; (d) $0.6\cos(4000t + 115°)$ A.

NOTE ◆ *Also try Chapter Problems 9.13 and 9.14.*

Equations 9.35, 9.41, and 9.43 form the basis for circuit analysis in the frequency domain. Note that Eq. 9.35 has the same algebraic form as Ohm's law, and that Eqs. 9.41 and 9.43 state Kirchhoff's laws for phasor quantities. Therefore you may use all the techniques developed for analyzing resistive circuits to find phasor currents and voltages. You need learn no new analytic techniques; the basic circuit analysis and simplification tools covered in Chapters 2–4 can all be used to analyze circuits in the frequency domain. Phasor circuit analysis consists of two fundamental tasks: (1) You must be able to construct the frequency-domain model of a circuit; and (2) you must be able to manipulate complex numbers and/or quantities algebraically. We illustrate these aspects of phasor analysis in the discussion that follows, beginning with series, parallel, and delta-to-wye simplifications.

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.5 Four branches terminate at a common node. The reference direction of each branch current (i_1, i_2, i_3, and i_4) is toward the node. If $i_1 = 100\cos(\omega t + 25°)$ A, $i_2 = 100\cos(\omega t + 145°)$ A, and $i_3 = 100\cos(\omega t - 95°)$ A, find i_4.

ANSWER: $i_4 = 0$.

NOTE ◆ *Also try Chapter Problems 9.15 and 9.16.*

9.6 ◆ Series, Parallel, and Delta-to-Wye Simplifications

The rules for combining impedances in series or parallel and for making delta-to-wye transformations are the same as those for resistors. The only difference is that combining impedances involves the algebraic manipulation of complex numbers.

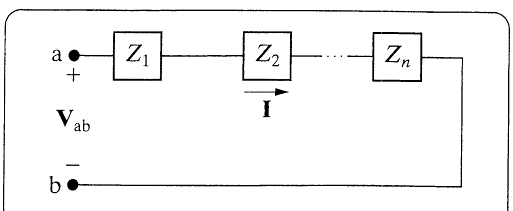


Figure 9.14 Impedances in series.

Combining Impedances in Series and Parallel

Impedances in series can be combined into a single impedance by simply adding the individual impedances. The circuit shown in Fig. 9.14 defines the problem in general terms. The impedances $Z_1, Z_2, \ldots, Z_n$ are connected

The phasor transform of v_s is

$$\mathbf{V}_s = 750 \angle 30^\circ \text{ V}.$$

Figure 9.16 illustrates the frequency-domain equivalent circuit of the circuit shown in Fig. 9.15.

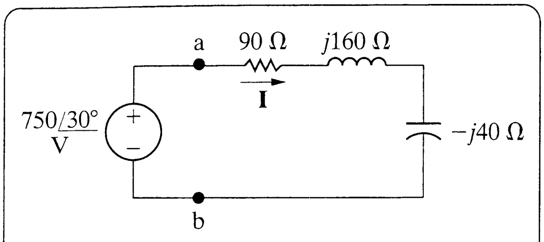


Figure 9.16 The frequency-domain equivalent circuit of the circuit shown in Fig. 9.15.

b) We compute the phasor current simply by dividing the voltage of the voltage source by the equivalent impedance between the terminals a,b. From Eq. 9.45,

$$Z_{\text{ab}} = 90 + j160 - j40$$
$$= 90 + j120 = 150 \angle 53.13^\circ \ \Omega.$$

Thus

$$\mathbf{I} = \frac{750 \angle 30^\circ}{150 \angle 53.13^\circ} = 5 \angle -23.13^\circ \text{ A}.$$

We may now write the steady-state expression for i directly:

$$i = 5\cos(5000t - 23.13^\circ) \text{ A}.$$

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.6 For the circuit in Fig. 9.15, with $\mathbf{V}_s = 125 \angle -60^\circ$ V and $\omega = 5000$ rad/s, find

a) the value of capacitance that yields a steady-state output current i with a phase angle of -105°.

b) the magnitude of the steady-state output current i.

ANSWER: (a) 2.86 μF; (b) 0.982 A.

NOTE ◆ *Also try Chapter Problem 9.17.*

Impedances connected in parallel may be reduced to a single equivalent impedance by the reciprocal relationship

$$\frac{1}{Z_{ab}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_n}. \tag{9.46}$$

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.7 A 20 Ω resistor is connected in parallel with a 5 mH inductor. This parallel combination is connected in series with a 5 Ω resistor and a 25 μF capacitor.

a) Calculate the impedance of this interconnection if the frequency is 2 krad/s.

b) Repeat (a) for a frequency of 8 krad/s.

c) At what finite frequency does the impedance of the interconnection become purely resistive?

d) What is the impedance at the frequency found in (c)?

ANSWER: (a) $9 - j12$ Ω; (b) $21 + j3$ Ω; (c) 4 krad/s; (d) 15 Ω.

9.8 The interconnection described in Assessment Problem 9.7 is connected across the terminals of a voltage source that is generating $v = 150 \cos 4000t$ V. What is the maximum amplitude of the current in the 5 mH inductor?

ANSWER: 7.07 A.

NOTE ◆ *Also try Chapter Problems 9.18–9.20.*

Delta-to-Wye Transformations

The Δ-to-Y transformation that we discussed in Section 3.7 with regard to resistive circuits also applies to impedances. Figure 9.20 defines the Δ-connected impedances along with the Y-equivalent circuit. The Y impedances as functions of the Δ impedances are

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}, \tag{9.51}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}, \tag{9.52}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}. \tag{9.53}$$

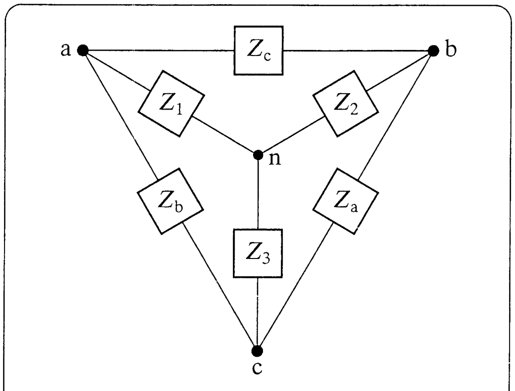


Figure 9.20 The delta-to-wye transformation.

In terms of the branch currents defined in Fig. 9.21,

$$\mathbf{I}_1 = \mathbf{I}_{abn} = 2 + j\frac{8}{3} \text{ A},$$

$$\mathbf{I}_2 = \mathbf{I}_{acn} = \frac{4}{10} + j\frac{8}{15} \text{ A}.$$

We check the calculations of $\mathbf{I}_1$ and $\mathbf{I}_2$ by noting that

$$\mathbf{I}_1 + \mathbf{I}_2 = 2.4 + j3.2 = \mathbf{I}_0.$$

To find the branch currents $\mathbf{I}_3$, $\mathbf{I}_4$, and $\mathbf{I}_5$, we must first calculate the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$. Referring to Fig. 9.21, we note that

$$\mathbf{V}_1 = 120 \angle 0^\circ - (-j4)\mathbf{I}_1 = \frac{328}{3} + j8 \text{ V},$$

$$\mathbf{V}_2 = 120 \angle 0^\circ - (63.2 + j2.4)\mathbf{I}_2 = 96 - j\frac{104}{3} \text{ V}.$$

We now calculate the branch currents $\mathbf{I}_3$, $\mathbf{I}_4$, and $\mathbf{I}_5$:

$$\mathbf{I}_3 = \frac{\mathbf{V}_1 - \mathbf{V}_2}{10} = \frac{4}{3} + j\frac{12.8}{3} \text{ A},$$

$$\mathbf{I}_4 = \frac{\mathbf{V}_1}{20 + j60} = \frac{2}{3} - j1.6 \text{ A},$$

$$\mathbf{I}_5 = \frac{\mathbf{V}_2}{-j20} = \frac{26}{15} + j4.8 \text{ A}.$$

We check the calculations by noting that

$$\mathbf{I}_4 + \mathbf{I}_5 = \frac{2}{3} + \frac{26}{15} - j1.6 + j4.8 = 2.4 + j3.2 = \mathbf{I}_0,$$

$$\mathbf{I}_3 + \mathbf{I}_4 = \frac{4}{3} + \frac{2}{3} + j\frac{12.8}{3} - j1.6 = 2 + j\frac{8}{3} = \mathbf{I}_1,$$

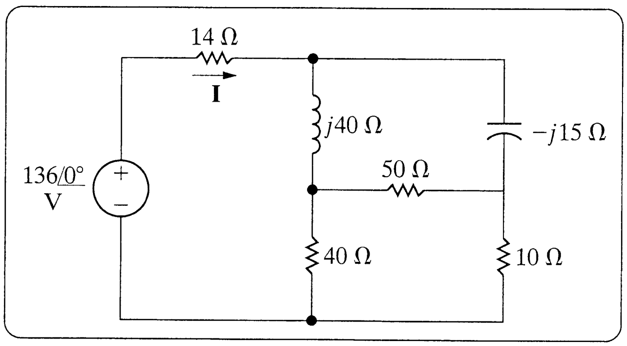
$$\mathbf{I}_3 + \mathbf{I}_2 = \frac{4}{3} + \frac{4}{10} + j\frac{12.8}{3} + j\frac{8}{15} = \frac{26}{15} + j4.8 = \mathbf{I}_5.$$

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.9 Use a Δ-to-Y transformation to find the current **I** in the circuit shown.

ANSWER: $\mathbf{I} = 4 \angle 28.07^\circ$ A.



NOTE ◆ *Also try Chapter Problem 9.35.*

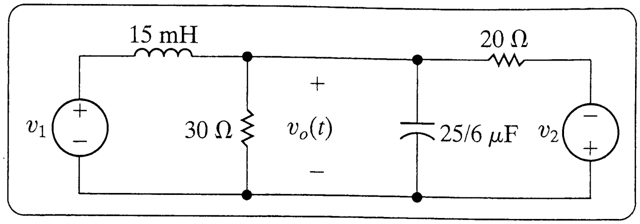
ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.10 Find the steady-state expression for $v_o(t)$ in the circuit shown by using the technique of source transformations. The sinusoidal voltage sources are

$$v_1 = 240\cos(4000t + 53.13^\circ)\text{ V},$$

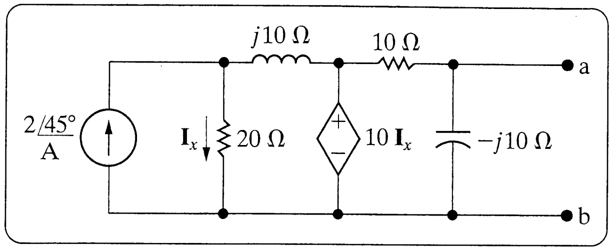
$$v_2 = 96\sin 4000t\text{ V}.$$



ANSWER: $48\cos(4000t + 36.87^\circ)$ V

9.11 Find the Thévenin equivalent with respect to terminals a,b in the circuit shown.

ANSWER: $\mathbf{V}_{\text{Th}} = \mathbf{V}_{\text{ab}} = 0.5\,\angle{-45^\circ}$ V; $Z_{\text{Th}} = 5 - j5\ \Omega$.



NOTE ◆ *Also try Chapter Problems 9.39, 9.42, and 9.43.*

9.8 ◆ The Node-Voltage Method

In Sections 4.2–4.4, we introduced the basic concepts of the node-voltage method of circuit analysis. The same concepts apply when we use the node-voltage method to analyze frequency-domain circuits. Example 9.11 illustrates the solution of such a circuit by the node-voltage technique. Assessment Problem 9.12 and many of the Chapter Problems give you an opportunity to use the node-voltage method to solve for steady-state sinusoidal responses.

To check our work, we note that

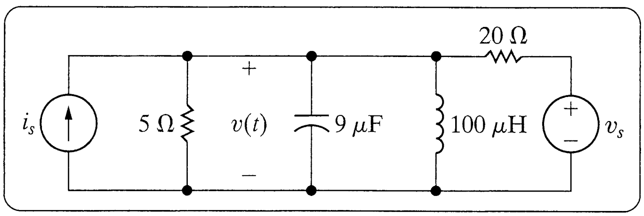
$$\mathbf{I}_a + \mathbf{I}_x = 6.84 - j1.68 + 3.76 + j1.68$$
$$= 10.6 \text{ A},$$
$$\mathbf{I}_x = \mathbf{I}_b + \mathbf{I}_c = -1.44 - j11.92 + 5.2 + j13.6$$
$$= 3.76 + j1.68 \text{ A}.$$

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis techniques to solve a circuit in the frequency domain**

9.12 Use the node-voltage method to find the steady-state expression for $v(t)$ in the circuit shown. The sinusoidal sources are $i_s = 10\cos\omega t$ A and $v_s = 100\sin\omega t$ V, where $\omega = 50$ krad/s.

ANSWER: $v(t) = 31.62\cos(50{,}000t - 71.57°)$ V.



NOTE ◆ *Also try Chapter Problems 9.51 and 9.54.*

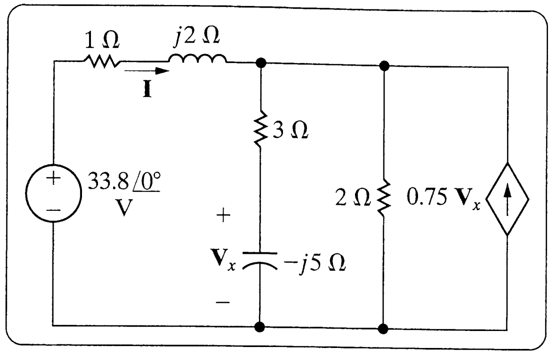
9.9 ◆ The Mesh-Current Method

We can also use the mesh-current method to analyze frequency-domain circuits. The procedures used in frequency-domain applications are the same as those used in analyzing resistive circuits. In Sections 4.5–4.7, we introduced the basic techniques of the mesh-current method; we demonstrate the extension of this method to frequency-domain circuits in Example 9.12.

ASSESSING OBJECTIVE 3

◆ **Know how to use circuit analysis technques to solve a circuit in the frequency domain**

9.13 Use the mesh-current method to find the phasor current **I** in the circuit shown.



ANSWER: $\mathbf{I} = 29 + j2 = 29.07 \angle 3.95^\circ$ A.

NOTE ◆ *Also try Chapter Problems 9.56 and 9.59.*

9.10 ◆ The Transformer

A transformer is a device that is based on magnetic coupling. Transformers are used in both communication and power circuits. In communication circuits, the transformer is used to match impedances and eliminate dc signals from portions of the system. In power circuits, transformers are used to establish ac voltage levels that facilitate the transmission, distribution, and consumption of electrical power. A knowledge of the sinusoidal steady-state behavior of the transformer is required in the analysis of both communication and power systems. In this section, we will discuss the sinusoidal steady-state behavior of the **linear transformer**, which is found primarily in communication circuits. In Section 9.11, we will deal with the **ideal transformer**, which is used to model the ferromagnetic transformer found in power systems.

Before starting we make a useful observation. When analyzing circuits containing mutual inductance use the mesh- or loop-current method for writing circuit equations. The node-voltage method is cumbersome to use when mutual inductance in involved. This is because the currents in the various coils cannot be written by inspection as functions of the node voltages.

The Analysis of a Linear Transformer Circuit

A simple **transformer** is formed when two coils are wound on a single core to ensure magnetic coupling. Figure 9.38 shows the frequency-domain circuit model of a system that uses a transformer to connect a load to a source. In discussing this circuit, we refer to the transformer winding connected to the source as the **primary winding** and the winding connected to the load as the **secondary winding**. Based on this terminology, the transformer circuit

f) The impedance seen looking into the primary terminals of the transformer is the impedance of the primary winding plus the reflected impedance; thus

$$Z_{ab} = 200 + j3600 + 800 + j800 = 1000 + j4400\ \Omega.$$

g) The Thévenin voltage will equal the open circuit value of $\mathbf{V}_{cd}$. The open circuit value of $\mathbf{V}_{cd}$ will equal $j1200$ times the open circuit value of $\mathbf{I}_1$. The open circuit value of $\mathbf{I}_1$ is

$$\mathbf{I}_1 = \frac{300\,\angle 0^\circ}{700 + j3700} = 79.67\,\angle{-79.29^\circ}\ \text{mA}.$$

Therefore,

$$\mathbf{V}_{Th} = j1200(79.67\,\angle{-79.29^\circ}) \times 10^{-3} = 95.60\,\angle 10.71^\circ\ \text{V}.$$

The Thévenin impedance will be equal to the impedance of the secondary winding plus the impedance reflected from the primary when the voltage source is replaced by a short-circuit. Thus

$$Z_{Th} = 100 + j1600 + \left(\frac{1200}{|700 + j3700|}\right)^2 (700 - j3700) = 171.09 + j1224.26\ \Omega.$$

The Thévenin equivalent is shown in Fig. 9.40.

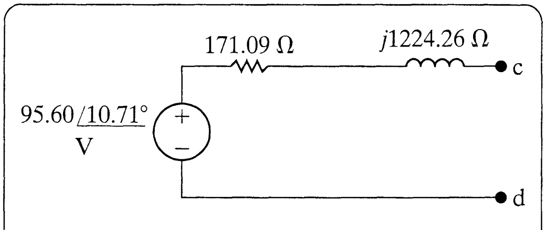
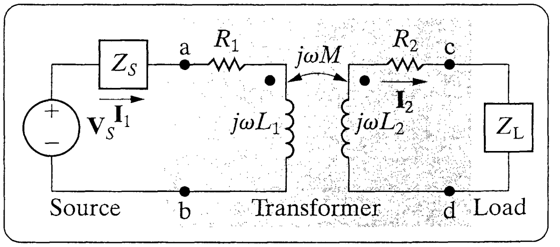


Figure 9.40 The Thévenin equivalent circuit for Example 9.13.

ASSESSING OBJECTIVE 4

◆ **Be able to analyze circuits containing linear transformers using phasor methods**

9.14 A linear transformer couples a load consisting of a 360 Ω resistor in series with a 0.25 H inductor to a sinusoidal voltage source, as shown. The voltage source has an internal impedance of $184 + j0\ \Omega$ and a maximum voltage of 245.20 V, and it is operating at 800‘rad/s. The transformer parameters are $R_1 = 100\ \Omega$, $L_1 = 0.5$ H, $R_2 = 40\ \Omega$, $L_2 = 0.125$ H, and $k = 0.4$. Calculate (a) the reflected impedance; (b) the primary current; and (c) the secondary current.



ANSWER: (a) $10.24 - j7.68\ \Omega$;
(b) $0.5\cos(800t - 53.13^\circ)$ A;
(c) $0.08\cos 800t$ A.

NOTE ◆ *Also try Chapter Problems 9.66 and 9.67.*

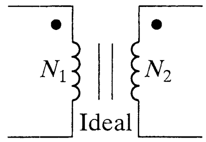


Figure 9.42 The graphic symbol for an ideal transformer.

Figure 9.42 shows the graphic symbol for an ideal transformer. The vertical lines in the symbol represent the layers of magnetic material from which ferromagnetic cores are often made. Thus, the symbol reminds us that coils wound on a ferromagnetic core behave very much like an ideal transformer.

There are several reasons for this. The ferromagnetic material creates a space with high permeance. Thus most of the magnetic flux is trapped inside the core material, establishing tight magnetic coupling between coils that share the same core. High permeance also means high self-inductance, because $L = N^2\mathscr{P}$. Finally, ferromagnetically coupled coils efficiently transfer power from one coil to the other. Efficiencies in excess of 95% are common, so neglecting losses is not a crippling approximation for many applications.

Determining the Polarity of the Voltage and Current Ratios

We now turn to the removal of the magnitude signs from Eqs. 9.76 and 9.77. Note that magnitude signs did not show up in the derivations of Eqs. 9.83 and 9.86. We did not need them there because we had established reference polarities for voltages and reference directions for currents. In addition, we knew the magnetic polarity dots of the two coupled coils.

The rules for assigning the proper algebraic sign to Eqs. 9.76 and 9.77 are as follows:

DOT CONVENTION FOR IDEAL TRANSFORMERS

1. If the coil voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ are both positive or negative at the dot-marked terminal, use a plus sign in Eq. 9.76. Otherwise, use a negative sign.
2. If the coil currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are both directed into or out of the dot-marked terminal, use a minus sign in Eq. 9.77. Otherwise, use a plus sign.

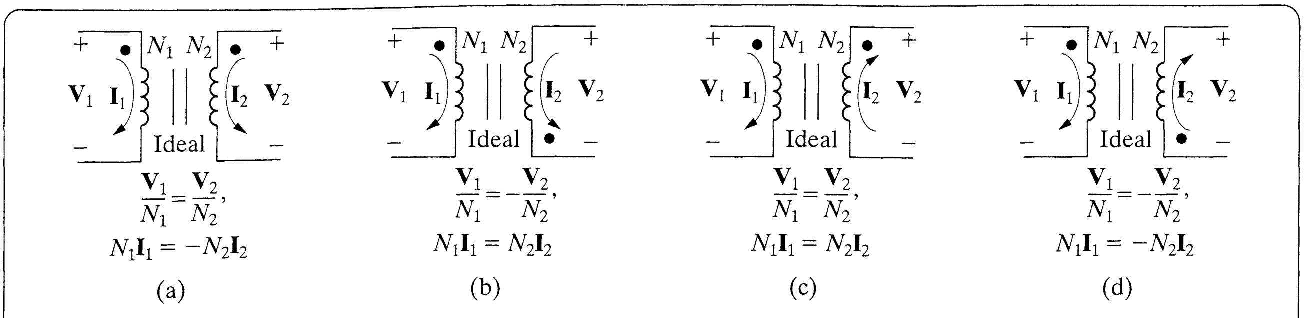


Figure 9.43 Circuits that show the proper algebraic signs for relating the terminal voltages and currents of an ideal transformer.

The four circuits shown in Fig. 9.43 illustrate these rules.

The ratio of the turns on the two windings is an important parameter of the ideal transformer. The turns ratio is defined as either N_1/N_2 or N_2/N_1; both ratios appear in various writings. In this text, we use a to denote the ratio N_2/N_1, or

$$a = \frac{N_2}{N_1}. \tag{9.87}$$

Figure 9.44 shows three ways to represent the turns ratio of an ideal transformer. Figure 9.44(a) shows the number of turns in each coil explicitly. Figure 9.44(b) shows that the ratio N_2/N_1 is 5 to 1, and Fig. 9.44(c) shows that the ratio N_2/N_1 is 1 to $\frac{1}{5}$.

Example 9.14 illustrates the analysis of a circuit containing an ideal transformer.

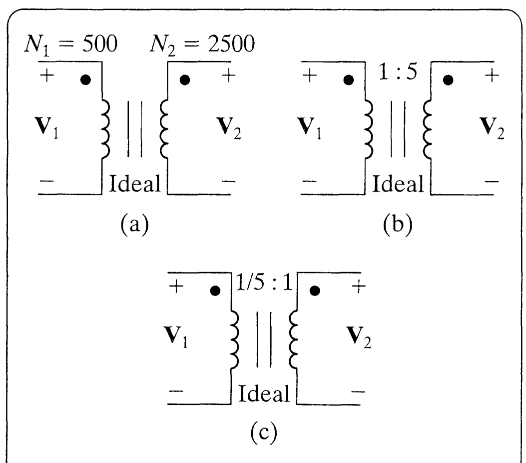


Figure 9.44 Three ways to show that the turns ratio of an ideal transformer is 5.

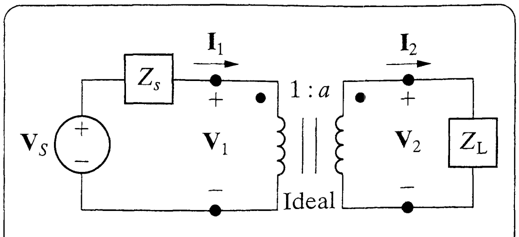


Figure 9.47 Using an ideal transformer to couple a load to a source.

The Use of an Ideal Transformer for Impedance Matching

Ideal transformers can also be used to raise or lower the impedance level of a load. The circuit shown in Fig. 9.47 illustrates this. The impedance seen by the practical voltage source ($\mathbf{V}_s$ in series with Z_s) is $\mathbf{V}_1/\mathbf{I}_1$. The voltage and current at the terminals of the load impedance ($\mathbf{V}_2$ and $\mathbf{I}_2$) are related to $\mathbf{V}_1$ and $\mathbf{I}_1$ by the transformer turns ratio; thus

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{a}, \tag{9.88}$$

and

$$\mathbf{I}_1 = a\mathbf{I}_2. \tag{9.89}$$

Therefore the impedance seen by the practical source is

$$Z_{\text{IN}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{a^2}\frac{\mathbf{V}_2}{\mathbf{I}_2}, \tag{9.90}$$

but the ratio $\mathbf{V}_2/\mathbf{I}_2$ is the load impedance Z_{L}, so Eq. 9.90 becomes

$$Z_{\text{IN}} = \frac{1}{a^2}Z_{\text{L}}. \tag{9.91}$$

Thus, the ideal transformer's secondary coil reflects the load impedance back to the primary coil, with the scaling factor $1/a^2$.

Note that the ideal transformer changes the magnitude of Z_{L} but does not affect its phase angle. Whether Z_{IN} is greater or less than Z_{L} depends on the turns ratio a.

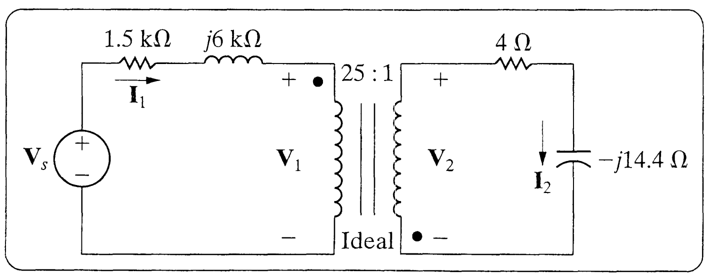
The ideal transformer—or its practical counterpart, the ferromagnetic core transformer—can be used to match the magnitude of Z_{L} to the magnitude of Z_s. We will discuss why this may be desirable in Chapter 10.

ASSESSING OBJECTIVE 5

◆ **Be able to analyze circuits with ideal transformers**

9.15 The source voltage in the phasor domain circuit in the accompanying figure is $25 \angle 0^\circ$ kV. Find the amplitude and phase angle of $\mathbf{V}_2$ and $\mathbf{I}_2$.

ANSWER: $\mathbf{V}_2 = 1868.15 \angle 142.39^\circ$ V;
$\mathbf{I}_2 = 125 \angle 216.87^\circ$ A.



NOTE ◆ *Also try Chapter Problem 9.71.*

Practical Perspective

A Household Circuit

Let us return to the household distribution circuit introduced at the beginning of the chapter. We will modify the circuit slightly by adding resistance to each conductor on the secondary side of the transformer to simulate more accurately the residential wiring conductors. The modified circuit is shown in Fig. 9.58. In Problem 9.85 you will calculate the six branch currents on the secondary side of the distribution transformer and then show how to calculate the current in the primary winding.

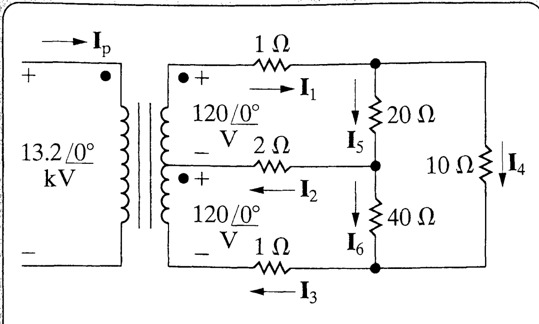


Figure 9.58 Distribution circuit.

NOTE ◆ *Assess your understanding of this Practical Perspective by trying Chapter Problems 9.85 and 9.86.*

SUMMARY

- The general equation for a **sinusoidal source** is

$$v = V_m \cos(\omega t + \phi) \text{ (voltage source)},$$

or

$$i = I_m \cos(\omega t + \phi) \text{ (current source)},$$

where V_m (or I_m) is the maximum amplitude, ω is the frequency, and ϕ is the phase angle. (See page 382.)

- The frequency, ω, of a sinusoidal response is the same as the frequency of the sinusoidal source driving the circuit. The amplitude and phase angle of the response are usually different from those of the source. (See page 387.)

- The best way to find the steady-state voltages and currents in a circuit driven by sinusoidal sources is to perform the analysis in the frequency domain. The following mathematical transforms allow us to move between the time and frequency domains.

- The phasor transform (from the time domain to the frequency domain):

$$\mathbf{V} = V_m e^{j\phi} = \mathcal{P}\{V_m \cos(\omega t + \phi)\}.$$

- The inverse phasor transform (from the frequency domain to the time domain):

$$\mathcal{P}^{-1}\{V_m e^{j\phi}\} = \Re\{V_m e^{j\phi} e^{j\omega t}\}.$$

(See pages 388–389.)

- When working with sinusoidally varying signals, remember that voltage leads current by 90° at the terminals of an inductor, and current leads voltage by 90° at the terminals of a capacitor. (See pages 394–395.)

- **Impedance** (Z) plays the same role in the frequency domain as resistance, inductance, and capacitance play in the time domain. Specifically, the relationship between

TABLE 9.3 Impedance and Related Values

ELEMENT	IMPEDANCE (Z)	REACTANCE	ADMITTANCE (Y)	SUSCEPTANCE
Resistor	R (resistance)	—	G (conductance)	—
Capacitor	$j(-1/\omega C)$	$-1/\omega C$	$j\omega C$	ωC
Inductor	$j\omega L$	ωL	$j(-1/\omega L)$	$-1/\omega L$

phasor current and phasor voltage for resistors, inductors, and capacitors is

$$\mathbf{V} = Z\mathbf{I},$$

where the reference direction for $\mathbf{I}$ obeys the passive sign convention. The reciprocal of impedance is **admittance** (Y), so another way to express the current-voltage relationship for resistors, inductors, and capacitors in the frequency domain is

$$\mathbf{V} = \mathbf{I}/Y.$$

(See pages 395 and 401.)

- All of the circuit analysis techniques developed in Chapters 2–4 for resistive circuits also apply to sinusoidal steady-state circuits in the frequency domain. These techniques include KVL, KCL, series, and parallel combinations of impedances, voltage and current division, node voltage and mesh current methods, source transformations and Thévenin and Norton equivalents.
- The two-winding **linear transformer** is a coupling device made up of two coils wound on the same nonmagnetic core. **Reflected impedance** is the impedance of the secondary circuit as seen from the terminals of the primary circuit or vice versa. The reflected impedance of a linear transformer seen from the primary side is the conjugate of the self-impedance of the secondary circuit scaled by the factor $(\omega M/|Z_{22}|)^2$. (See pages 415 and 417.)
- The two-winding **ideal transformer** is a linear transformer with the following special properties: perfect coupling ($k = 1$), infinite self-inductance in each coil ($L_1 = L_2 = \infty$), and lossless coils ($R_1 = R_2 = 0$). The circuit behavior is governed by the turns ratio $a = N_2/N_1$. In particular, the volts per turn is the same for each winding, or

$$\frac{\mathbf{V}_1}{N_1} = \pm\frac{\mathbf{V}_2}{N_2},$$

and the ampere turns are the same for each winding, or

$$N_1\mathbf{I}_1 = \pm N_2\mathbf{I}_2.$$

(See page 420.)

PROBLEMS

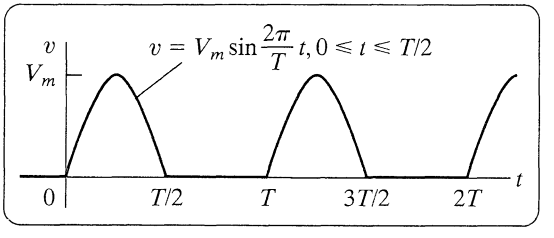
9.1 A sinusoidal voltage is given by the expression

$$v = 10\cos(3769.91t - 53.13°).$$

Find (a) f in hertz; (b) T in milliseconds; (c) V_m; (d) $v(0)$; (e) ϕ in degrees and radians; (f) the smallest positive value of t at which $v = 0$; and (g) the smallest positive value of t at which $dv/dt = 0$.

9.2 Find the rms value of the half-wave rectified sinusoidal voltage shown.

Figure P9.2



9.3 Consider the sinusoidal voltage

$$v(t) = 40\cos(100\pi t + 60^\circ)\text{ V}.$$

a) What is the maximum amplitude of the voltage?

b) What is the frequency in hertz?

c) What is the frequency in radians per second?

d) What is the phase angle in radians?

e) What is the phase angle in degrees?

f) What is the period in milliseconds?

g) What is the first time after $t = 0$ that $v = -40$ V?

h) The sinusoidal function is shifted 10/3 ms to the right along the time axis. What is the expression for $v(t)$?

i) What is the minimum number of milliseconds that the function must be shifted to the right if the expression for $v(t)$ is $40\sin 100\pi t$ V?

j) What is the minimum number of milliseconds that the function must be shifted to the left if the expression for $v(t)$ is $40\cos 100\pi t$ V?

9.4 In a single graph, sketch $v = 100\cos(\omega t + \phi)$ versus ωt for $\phi = -60^\circ, -30^\circ, 0^\circ, 30^\circ$, and 60°.

a) State whether the voltage function is shifting to the right or left as ϕ becomes more positive.

b) What is the direction of shift if ϕ changes from 0 to 30°?

9.5 A sinusoidal voltage is zero at $t = -2\pi/3$ ms and increasing at a rate of 80,000 V/s. The maximum amplitude of the voltage is 80 V.

a) What is the frequency of v in radians per second?

b) What is the expression for v?

9.6 At $t = -2$ ms, a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at $t = 8$ ms. It is also known that the voltage is 80.9 V at $t = 0$.

a) What is the frequency of v in hertz?

b) What is the expression for v?

9.7 Show that

$$\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi)dt = \frac{V_m^2 T}{2}$$

9.8 The rms value of the sinusoidal voltage supplied to the convenience outlet of a U.S. home is 120 V. What is the maximum value of the voltage at the outlet?

9.9 The voltage applied to the circuit shown in Fig. 9.5 at $t = 0$ is $20\cos(800t + 25^\circ)$. The circuit resistance is 80 Ω, and the initial current in the 75 mH inductor is zero.

a) Find $i(t)$ for $t \geq 0$.

b) Write the expressions for the transient and steady-state components of $i(t)$.

c) Find the numerical value of i after the switch has been closed for 1.875 ms.

d) What are the maximum amplitude, frequency (in radians per second), and phase angle of the steady-state current?

e) By how many degrees are the voltage and the steady-state current out of phase?

9.10 a) Verify that Eq. 9.9 is the solution of Eq. 9.8. This can be done by substituting Eq. 9.9 into the left-hand side of Eq. 9.8 and then noting that it equals the right-hand side for all values of $t > 0$. At $t = 0$, Eq. 9.9 should reduce to the initial value of the current.

b) Because the transient component vanishes as time elapses and because our solution must satisfy the differential equation for all values of t, the steady-state component, by itself, must also satisfy the differential equation. Verify this observation by showing that the steady-state component of Eq. 9.9 satisfies Eq. 9.8.

9.11 Use the concept of the phasor to combine the following sinusoidal functions into a single trigonometric expression:

a) $y = 50\cos(500t + 60^\circ) + 100\cos(500t - 30^\circ)$,

b) $y = 200\cos(377t + 50^\circ) - 100\sin(377t + 150^\circ)$,

c) $y = 80\cos(100t + 30^\circ) - 100\sin(100t - 135^\circ) + 50\cos(100t - 90^\circ)$, and

d) $y = 250\cos\omega t + 250\cos(\omega t + 120^\circ) + 250\cos(\omega t - 120^\circ)$.

9.12 A 1000 Hz sinusoidal voltage with a maximum amplitude of 200 V at $t = 0$ is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 25 A.

a) What is the frequency of the inductor current?

b) What is the phase angle of the voltage?

c) What is the phase angle of the current?

d) What is the inductive reactance of the inductor?

e) What is the inductance of the inductor in millihenrys?

f) What is the impedance of the inductor?

9.13 A 50 kHz sinusoidal voltage has zero phase angle and a maximum amplitude of 10 mV. When this voltage is applied across the terminals of a capacitor, the resulting steady-state current has a maximum amplitude of 628.32 μA.

a) What is the frequency of the current in radians per second?

b) What is the phase angle of the current?

c) What is the capacitive reactance of the capacitor?

d) What is the capacitance of the capacitor in microfarads?

e) What is the impedance of the capacitor?

9.14 **P** A 10 Ω resistor and a 5 μF capacitor are connected in parallel. This parallel combination is also in parallel with the series combination of a 8 Ω resistor and an 300 μH inductor. These three parallel branches are driven by a sinusoidal current source whose current is $922 \cos(20{,}000t + 30^\circ)$ A.

a) Draw the frequency-domain equivalent circuit.

b) Reference the voltage across the current source as a rise in the direction of the source current, and find the phasor voltage.

c) Find the steady-state expression for $v(t)$.

9.15 **P** A 40 Ω resistor, a 5 mH inductor, and a 1.25 μF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is $600 \cos(8000t + 20^\circ)$ V.

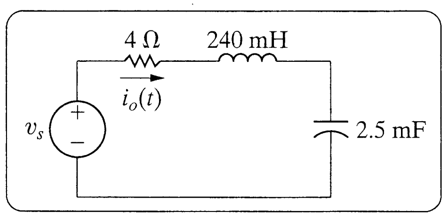
a) Draw the frequency-domain equivalent circuit.

b) Reference the current in the direction of the voltage rise across the source, and find the phasor current.

c) Find the steady-state expression for $i(t)$.

9.16 **P** Find the steady-state expression for $i_o(t)$ in the circuit in Fig. P9.16 if $v_s = 100 \sin 50t$ mV.

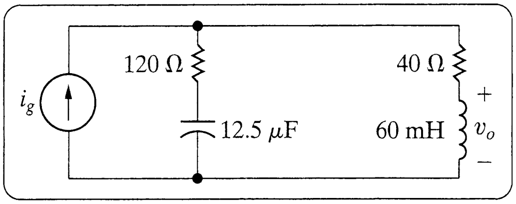
Figure P9.16



9.17 Three branches having impedances of $3 + j4$ Ω, $16 - j12$ Ω, and $-j4$ Ω, respectively, are connected in parallel. What are the equivalent (a) admittance, (b) conductance, and (c) susceptance of the parallel connection in millisiemens? (d) If the parallel branches are excited from a sinusoidal current source where $i = 8 \cos \omega t$ A, what is the maximum amplitude of the current in the purely capacitive branch?

9.18 Find the steady-state expression for v_o in the circuit of Fig. P9.18 if $i_g = 0.5\cos 2000t$ A.

Figure P9.18



9.19 The expressions for the steady-state voltage and current at the terminals of the circuit seen in Fig. P9.19 are

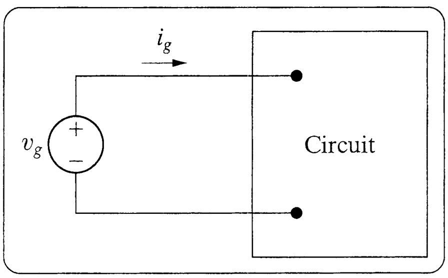
$$v_g = 300\cos(5000\pi t + 78^\circ)\text{ V},$$

$$i_g = 6\sin(5000\pi t + 123^\circ)\text{ A}$$

a) What is the impedance seen by the source?

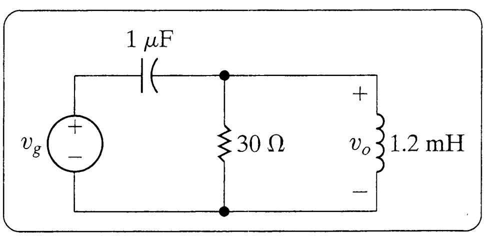
b) By how many microseconds is the current out of phase with the voltage?

Figure P9.19



9.20 **P** The circuit in Fig. P9.20 is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g = 40\cos 50{,}000t$ V.

Figure P9.20

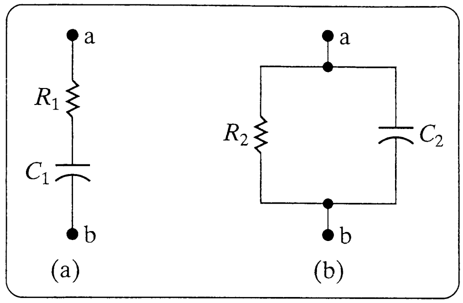


9.21 a) Show that at a given frequency ω, the circuits in Fig. P9.21(a) and (b) will have the same impedance between the terminals a,b if

$$R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2},$$

$$C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}.$$

Figure P9.21



b) Find the values of resistance and capacitance that when connected in series will have the same impedance at 40 krad/s as that of a 1000 Ω resistor connected in parallel with a 50 nF capacitor.

9.22 a) Show that at a given frequency ω, the circuits in Fig P9.21(a) and (b) will have the same impedance between the terminals a,b if

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2},$$

$$C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}.$$

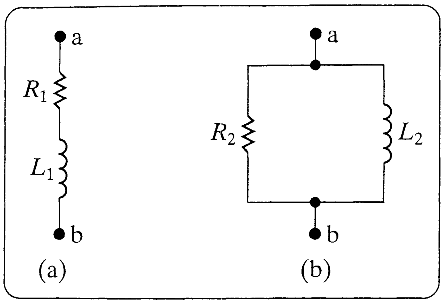
(*Hint:* The two circuits will have the same impedance if they have the same admittance.)

b) Find the values of resistance and capacitance that when connected in parallel will give th same impedance at 50 krad/s as that of a 1 kΩ resistor connected in series with a capacitanc of 40 nF.

9.23 a) Show that, at a given frequency ω, the circuits in Fig. P9.23(a) and (b) will have the same impedance between the terminals a,b if

$$R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2}, \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}.$$

Figure P9.23



b) Find the values of resistance and inductance that when connected in series will have the same impedance at 4 krad/s as that of a 5 kΩ resistor connected in parallel with a 1.25 H inductor.

9.24 a) Show that at a given frequency ω, the circuits in Fig. P9.23(a) and (b) will have the same impedance between the terminals a,b if

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1}, \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}.$$

(*Hint:* The two circuits will have the same impedance if they have the same admittance.)

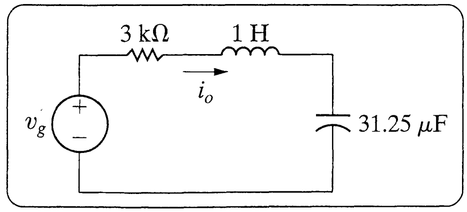
b) Find the values of resistance and inductance that when connected in parallel will have the same impedance at 1 krad/s as a 8 kΩ resistor connected in series with a 4 H inductor.

9.25 The circuit shown in Fig. P9.25 is operating in the sinusoidal steady state. Find the value of ω if

$$i_o = 100 \sin(\omega t + 173.13°) \text{ mA},$$

$$v_g = 500 \cos(\omega t + 30°) \text{ V}.$$

Figure P9.25

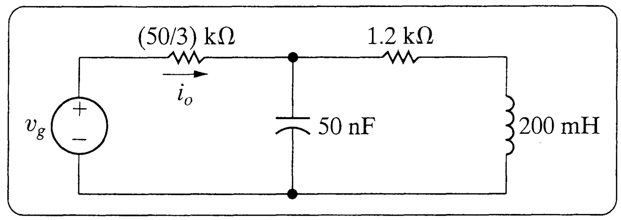


9.26 P The frequency of the sinusoidal voltage source in the circuit in Fig. P9.26 is adjusted until the current i_o in phase with v_g.

a) Find the frequency in hertz.

b) Find the steady-state expression for i_o (at the frequency found in [a]) if $v_g = 30 \cos \omega t$ V.

Figure P9.26

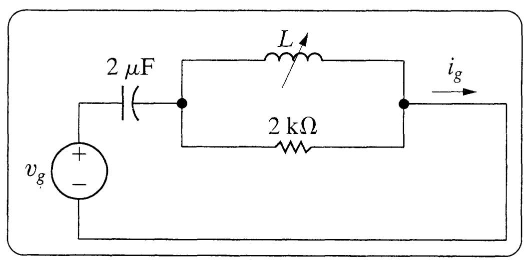


9.27 P The circuit shown in Fig. P9.27 is operating in the sinusoidal steady state. The inductor is adjusted until the current i_g is in phase with the sinusoidal voltage v_g.

a) Specify the inductance in henries if $v_g = 100 \cos 500t$ V.

b) Give the steady-state expression for i_g when L has the value found in (a).

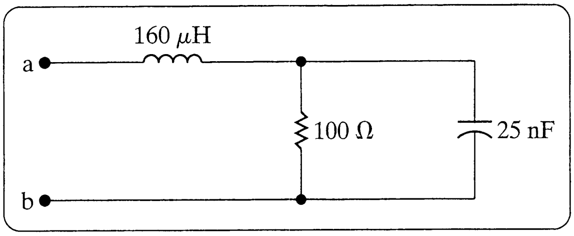
Figure P9.27



9.28 a) For the circuit shown in Fig. P9.28, find the frequency (in radians per second) at which the impedance Z_{ab} is purely resistive.

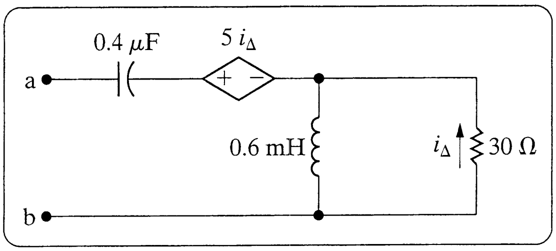
b) Find the value of Z_{ab} at the frequency of (a).

Figure P9.28



9.29 Find Z_{ab} in the circuit shown in Fig. P9.29 when the circuit is operating at a frequency of 100 krad/s.

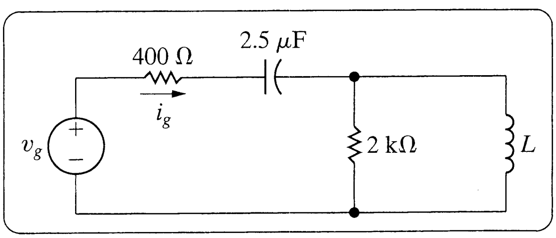
Figure P9.29



9.30 a) The source voltage in the circuit in Fig. P9.30 is $v_g = 200 \cos 500t$ V. Find the values of L such that i_g is in phase with v_g when the circuit is operating in the steady state.

b) For the values of L found in (a), find the steady-state expressions for i_g.

Figure P9.30

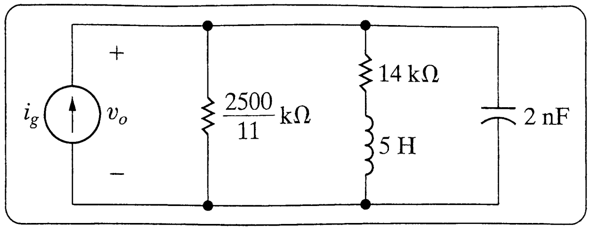


9.31 The frequency of the sinusoidal current source in the circuit in Fig. P9.31 is adjusted until v_o is in phase with i_g.

a) What is the value of ω in radians per second?

b) If $i_g = 0.25 \cos \omega t$ mA (where ω is the frequency found in [a]), what is the steady-state expression for v_o?

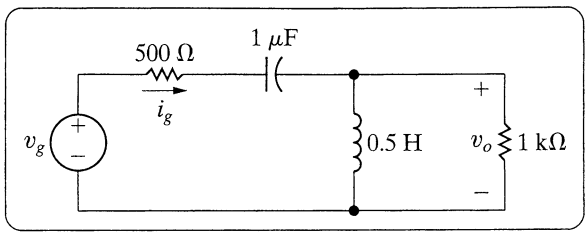
Figure P9.31



9.32 a) The frequency of the source voltage in the circuit in Fig. P9.32 is adjusted until i_g is in phase with v_g. What is the value of ω in radians per second?

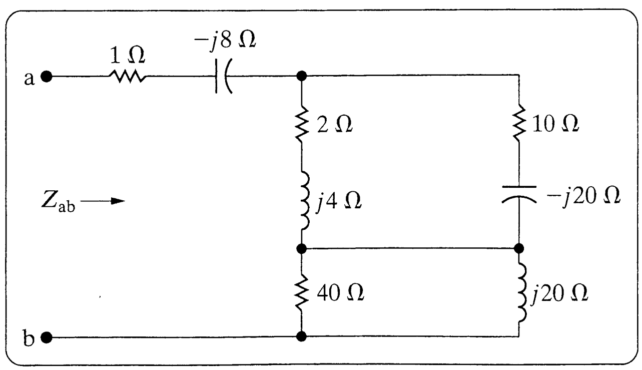
b) If $v_g = 20 \cos \omega t$ V (where ω is the frequency found in [a]), what is the steady-state expression for v_o?

Figure P9.32



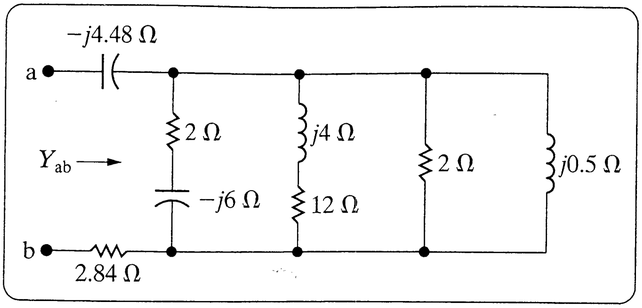
9.33 Find the impedance Z_{ab} in the circuit seen in Fig. P9.33. Express Z_{ab} in both polar and rectangular form.

Figure P9.33



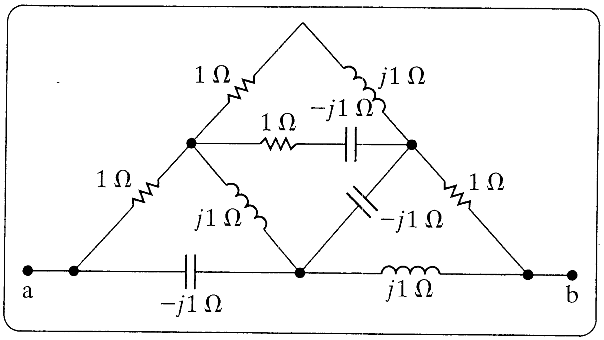
9.34 Find the admittance Y_{ab} in the circuit seen in Fig. P9.34. Express Y_{ab} in both polar and rectangular form. Give the value of Y_{ab} in millisiemens.

Figure P9.34



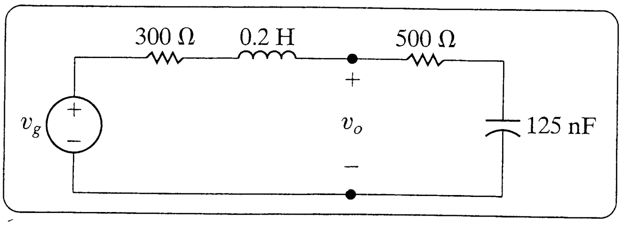
9.35 Find Z_{ab} for the circuit shown in Fig P9.35.

Figure P9.35



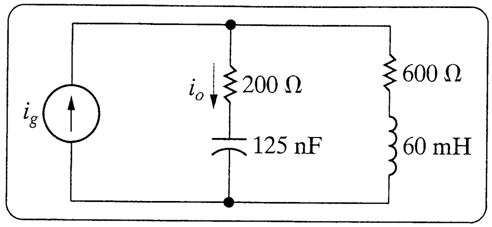
9.36 Use the concept of voltage division to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.36 if $v_g = 100 \cos 8000t$ V.

Figure P9.36



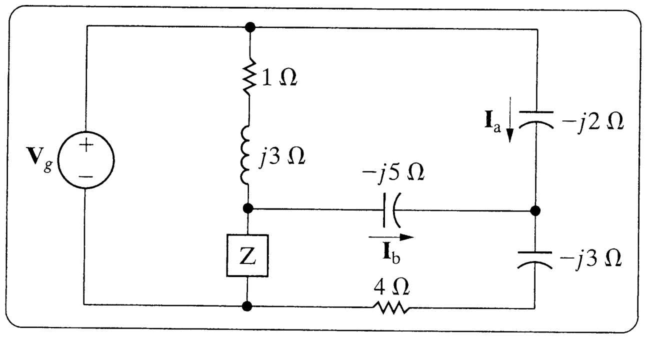
9.37 Use the concept of current division to find the steady-state expression for i_o in the circuit in Fig. P9.37 if $i_g = 400 \cos 20{,}000t$ mA.

Figure P9.37



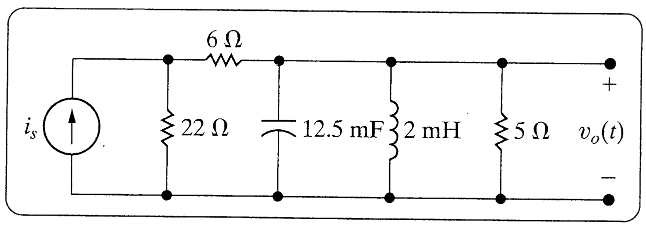
9.38 Find $\mathbf{I}_b$ and Z in the circuit shown in Fig. P9.38 if $\mathbf{V}_g = 25 \angle 0°$ V and $\mathbf{I}_a = 5 \angle 90°$ A.

Figure P9.38



9.39 The circuit in Fig. P9.39 is operating in the sinusoidal steady state. Find $v_o(t)$ if $i_s(t) = 3 \cos 200t$ mA.

Figure P9.39



9.40 The phasor current $\mathbf{I}_a$ in the circuit shown in Fig. P9.40 is $2 \angle 0^\circ$ A.

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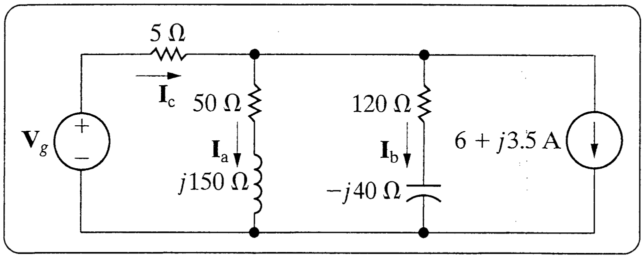
a) Find $\mathbf{I}_a$, $\mathbf{I}_c$, and $\mathbf{V}_g$.

b) If $\omega = 800$ rad/s, write the expressions for $i_b(t)$, $i_c(t)$, and $v_g(t)$.

Figure P9.40



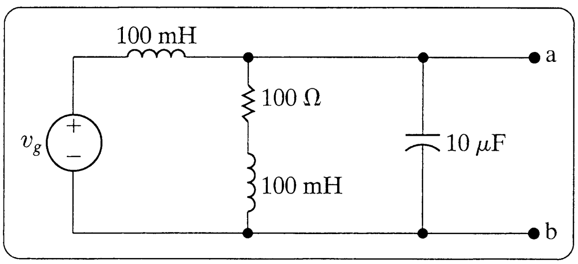
9.41 The sinusoidal voltage source in the circuit in Fig. P9.41 is developing a voltage equal to $247.49 \cos(1000t + 45^\circ)$ V.

a) Find the Thévenin voltage with respect to the terminals a,b.

b) Find the Thévenin impedance with respect to the terminals a,b.

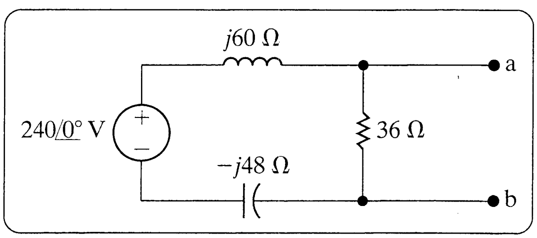
c) Draw the Thévenin equivalent.

Figure P9.41



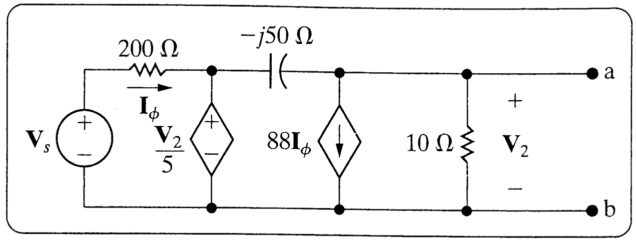
9.42 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.42.

Figure P9.42



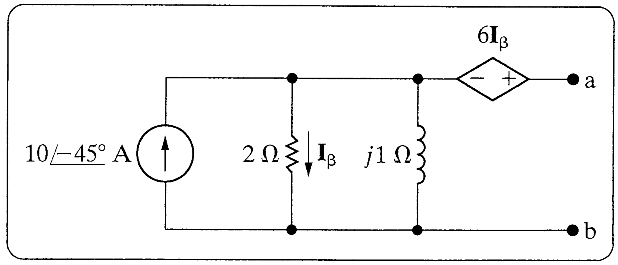
9.43 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.43 when $\mathbf{V}_s = 5 \angle 0^\circ$ V.

Figure P9.43



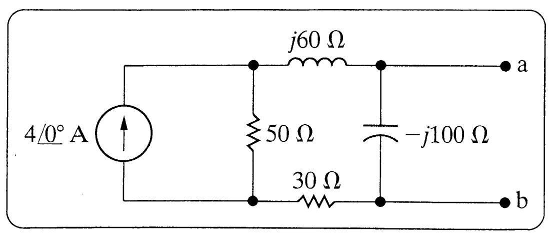
9.44 Find the Norton equivalent with respect to terminals a,b in the circuit of Fig. P9.44.

Figure P9.44



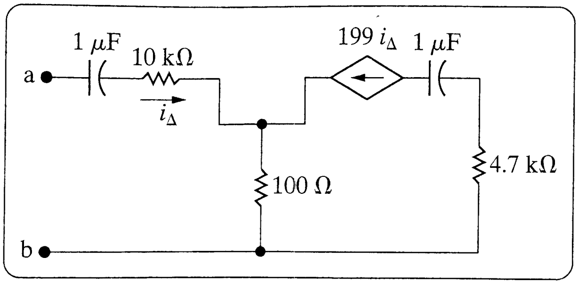
9.45 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.45.

Figure P9.45



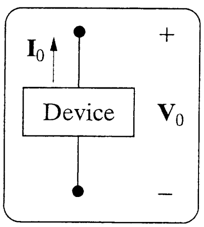
9.46 Find the Thévenin impedance seen looking into the terminals a,b of the circuit in Fig. P9.46 if the frequency of operation is $(200/\pi)$ Hz.

Figure P9.46



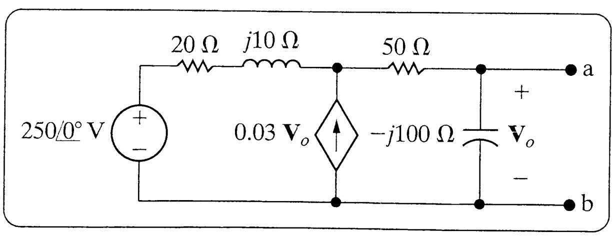
9.47 The device in Fig. P9.47 is represented in the frequency domain by a Norton equivalent. When a resistor having an impedance of 5 kΩ is connected across the device, the value of $\mathbf{V}_0$ is $5 - j15$ V. When a capacitor having an impedance of $-j3$ kΩ is connected across the device, the value of $\mathbf{I}_0$ is $4.5 - j6$ mA. Find the Norton current $\mathbf{I}_N$ and the Norton impedance Z_N.

Figure P9.47



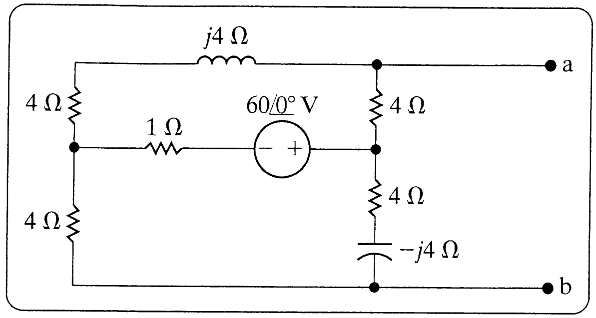
9.48 Find the Thévenin equivalent circuit with respect to the terminals a,b of the circuit shown in Fig. P9.48.

Figure P9.48



9.49 Find the Thévenin equivalent circuit with respect to the terminals a,b for the circuit shown in Fig. P9.49.

Figure P9.49



9.50 The circuit shown in Fig. P9.50 is operating at a frequency of 10 rad/s. Assume α is real and lies between −10 and +10, that is, $-10 \leq \alpha \leq 10$.

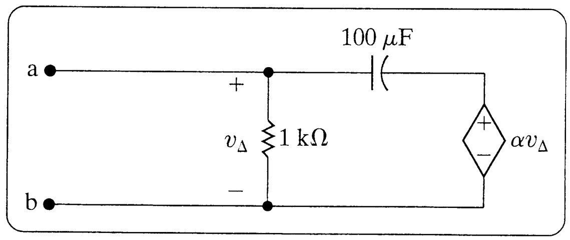
a) Find the value of α so that the Thévenin impedance looking into the terminals a,b is purely resistive.

b) What is the value of the Thévenin impedance for the α found in (a)?

c) Can α be adjusted so that the Thévenin impedance equals $500 - j500$ Ω? If so, what is the value of α?

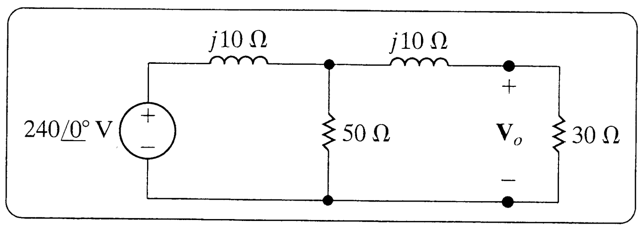
d) For what values of α will the Thévenin impedance be inductive?

Figure P9.50



9.51 Use the node-voltage method to find $\mathbf{V}_o$ in the circuit in Fig. P9.51.

Figure P9.51



9.52 Use the node-voltage method to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.52 if

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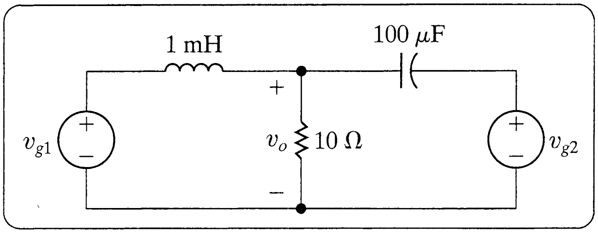
$$v_{g1} = 20\cos(2000t - 36.87^\circ)\text{ V},$$

$$v_{g2} = 50\sin(2000t - 16.26^\circ)\text{ V}.$$

Figure P9.52



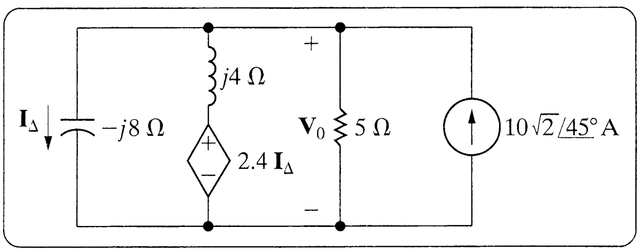
9.53 Use source transformations to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.52.

9.54 Use the mesh-current method to find the steady-state expression for $v_o(t)$ in the circuit in Fig. P9.52.

9.55 Use the principle of superposition to find the steady-state expression for the voltage $v_o(t)$ in the circuit in Fig. P9.52.

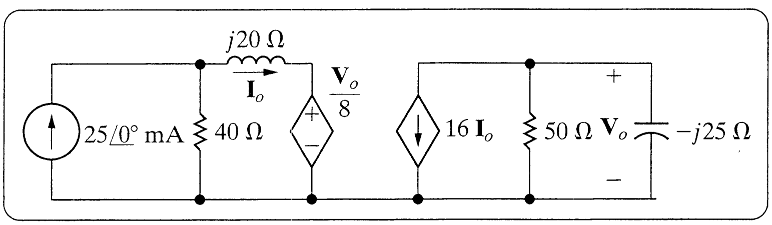
9.56 Use the node-voltage method to find the phasor voltage $\mathbf{V}_o$ in the circuit shown in Fig. P9.56. Express the voltage in both polar and rectangular form.

Figure P9.56



9.57 Use the node-voltage method to find $\mathbf{V}_o$ and $\mathbf{I}_o$ in the circuit seen in Fig. P9.57.

Figure P9.57

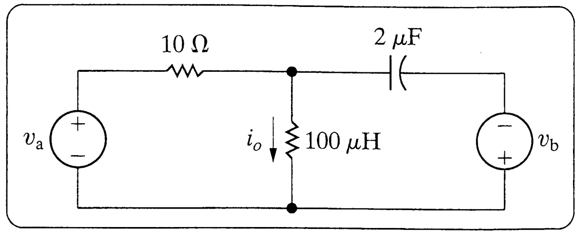


9.58 Use the mesh-current method to find the steady-state expression for $i_o(t)$ in the circuit in Fig. P9.58 if

$$v_a = 100\cos 50{,}000t\text{ V},$$

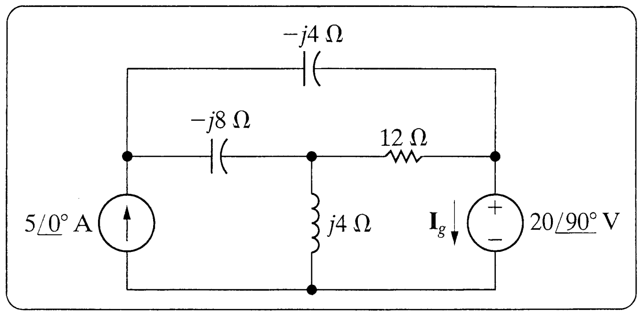
$$v_b = 100\sin(50{,}000t + 180^\circ)\text{ V}.$$

Figure P9.58



9.59 Use the mesh-current method to find the phasor current $\mathbf{I}_g$ in the circuit shown in Fig. P9.59.

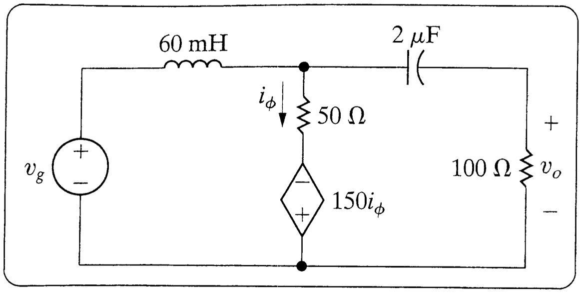
Figure P9.59



9.60 Use the node-voltage method to find the phasor voltage across the $-j4\ \Omega$ capacitor in the circuit in Fig. P9.59. Assume the voltage is positive at the left-hand terminal of the capacitor.

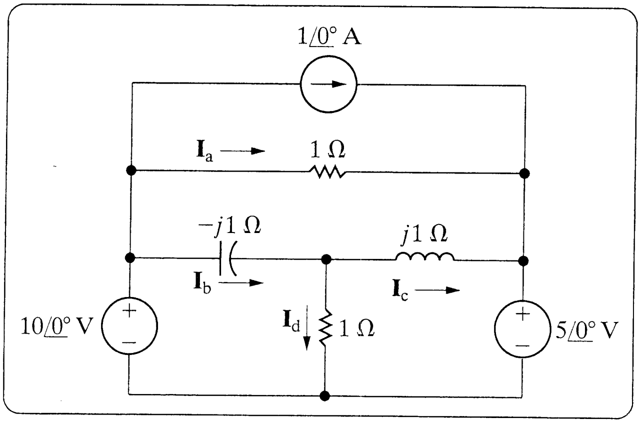
9.61 P Use the mesh-current method to find the steady-state expression for v_o in the circuit seen in Fig. P9.61 if v_g equals $400 \cos 5000t$ V.

Figure P9.61



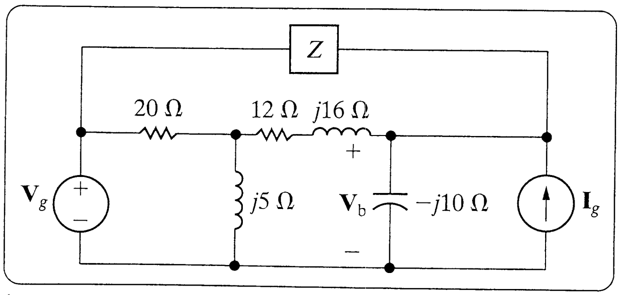
9.62 Use the mesh-current method to find the branch currents $\mathbf{I}_a$, $\mathbf{I}_b$, $\mathbf{I}_c$, and $\mathbf{I}_d$ in the circuit shown in Fig. P9.62.

Figure P9.62



9.63 Find the value of Z in the circuit seen in Fig. P9.63 if $\mathbf{V}_g = 100 - j50$ V, $\mathbf{I}_g = 30 + j20$ A, and $\mathbf{V}_b = 140 + j30$ V.

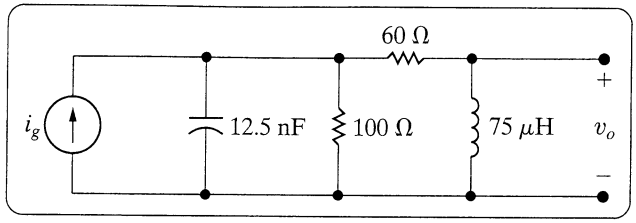
Figure P9.63



9.64 P a) For the circuit shown in Fig. P9.64, find the steady-state expression for v_o if $i_g = 2 \cos(16 \times 10^5 t)$ A.

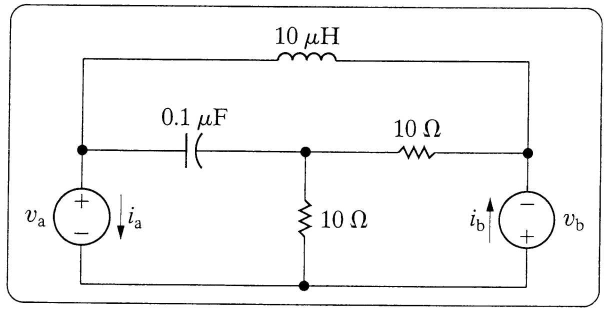
b) By how many nanoseconds does v_o lag i_g?

Figure P9.64



9.65 P Find the steady-state expressions for the branch currents i_a and i_b in the circuit seen in Fig. P9.65 if $v_a = 50 \sin 10^6 t$ V and $v_b = 25 \cos(10^6 t + 90°)$ V.

Figure P9.65

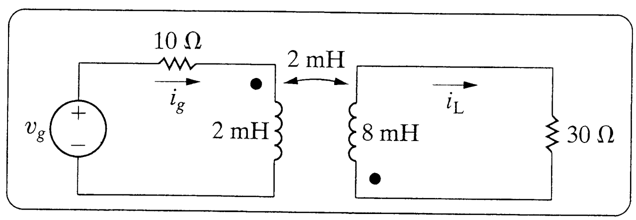


9.66 P a) Find the steady-state expressions for the currents i_g and i_L in the circuit in Fig. P9.66 when $v_g = 70 \cos 5000t$ V.

b) Find the coefficient of coupling.

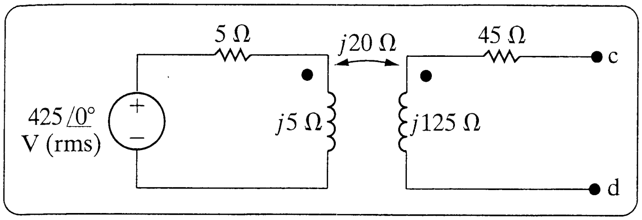
c) Find the energy stored in the magnetically coupled coils at $t = 100\pi\ \mu$s and $t = 200\pi\ \mu$s.

Figure P9.66



9.67 For the circuit in Fig. P9.67, find the Thévenin equivalent with respect to the terminals c,d.

Figure P9.67

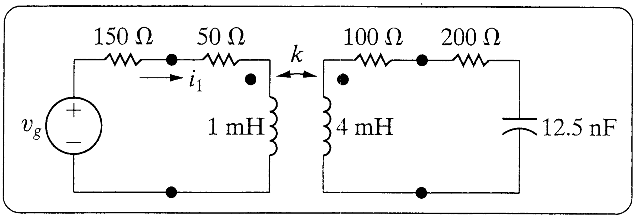


9.68 The sinusoidal voltage source in the circuit seen in Fig. P9.68 is operating at a frequency of 200 krad/s. The coefficient of coupling is adjusted until the peak amplitude of i_1 is maximum.

a) What is the value of k?

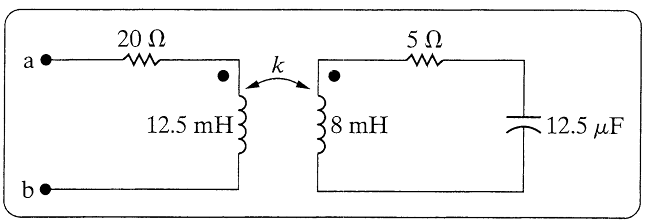
b) What is the peak amplitude of i_1 if $v_g = 560\cos(2 \times 10^5 t)$ V?

Figure P9.68



9.69 The value of k in the circuit in Fig. P9.69 is adjusted so that Z_{ab} is purely resistive when $\omega = 4$ krad/s. Find Z_{ab}.

Figure P9.69



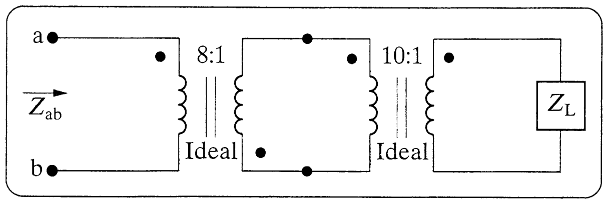
9.70 A series combination of a 300 Ω resistor and a 100 mH inductor is connected to a sinusoidal voltage source by a linear transformer. The source is operating at a frequency of 1 krad/s. At this frequency, the internal impedance of the source is $100 + j13.74$ Ω. The rms voltage at the terminals of the source is 50 V when it is not loaded. The parameters of the linear transformer are $R_1 = 41.68$ Ω, $L_1 = 180$ mH, $R_2 = 500$ Ω, $L_2 = 500$ mH, and $M = 270$ mH.

a) What is the value of the impedance reflected into the primary?

b) What is the value of the impedance seen from the terminals of the practical source?

9.71 Find the impedance Z_{ab} in the circuit in Fig. P9.71 if $Z_L = 80 \angle 60^\circ$ Ω.

Figure P9.71



9.72 At first glance, it may appear from Eq. 9.69 that an inductive load could make the reactance seen looking into the primary terminals (i.e., X_{ab}) look capacitive. Intuitively, we know this is impossible. Show that X_{ab} can never be negative if X_L is an inductive reactance.

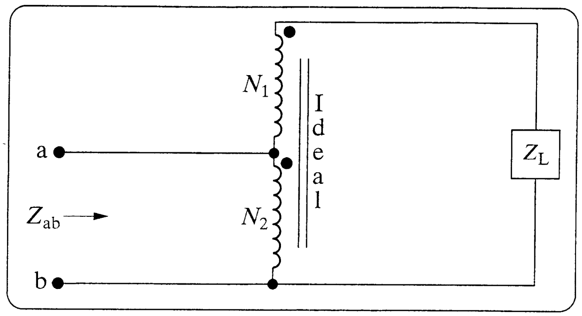
9.73 a) Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.73 is given by the expression

$$Z_{ab} = \frac{Z_L}{\left(1 + \frac{N_1}{N_2}\right)^2}.$$

b) Show that if the polarity terminal of either one of the coils is reversed that

$$Z_{ab} = \frac{Z_L}{\left(1 - \frac{N_1}{N_2}\right)^2}.$$

Figure P9.73



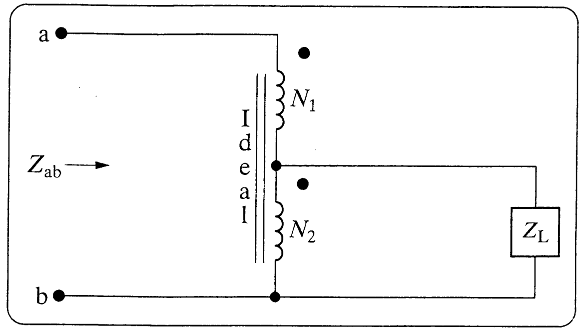
9.74 a) Show that the impedance seen looking into the terminals a,b in the circuit in Fig. P9.74 is given by the expression

$$Z_{ab} = \left(1 + \frac{N_1}{N_2}\right)^2 Z_L.$$

b) Show that if the polarity terminals of either one of the coils is reversed,

$$Z_{ab} = \left(1 - \frac{N_1}{N_2}\right)^2 Z_L.$$

Figure P9.74



9.75 The parameters in the circuit shown in Fig. 9.53 are $R_1 = 0.1\ \Omega$, $\omega L_1 = 0.8\ \Omega$, $R_2 = 24\ \Omega$, $\omega L_2 = 32\ \Omega$, and $\mathbf{V}_L = 240 + j0$ V.

a) Calculate the phasor voltage $\mathbf{V}_s$.

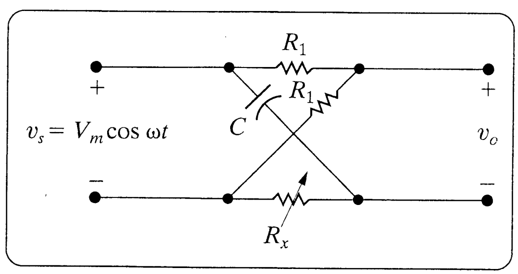
b) Connect a capacitor in parallel with the inductor, hold $\mathbf{V}_L$ constant, and adjust the capacitor until the magnitude of $\mathbf{I}$ is a minimum. What is the capacitive reactance? What is the value of $\mathbf{V}_s$?

c) Find the value of the capacitive reactance that keeps the magnitude of $\mathbf{I}$ as small as possible and that at the same time makes

$$|\mathbf{V}_s| = |\mathbf{V}_L| = 240 \text{ V}.$$

9.76 P Show by using a phasor diagram what happens to the magnitude and phase angle of the voltage v_o in the circuit in Fig. P9.76 as R_x is varied from zero to infinity. The amplitude and phase angle of the source voltage are held constant as R_x varies.

Figure P9.76



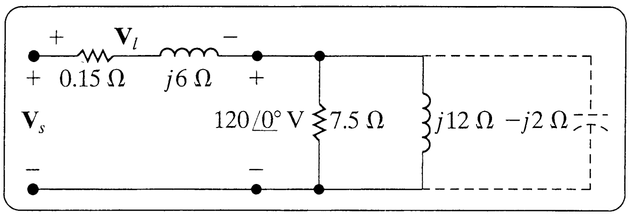
9.77 a) For the circuit shown in Fig. P9.77, compute $\mathbf{V}_s$ and $\mathbf{V}_l$.

b) Construct a phasor diagram showing the relationship between $\mathbf{V}_s$, $\mathbf{V}_l$, and the load voltage of $120\,\underline{/0^\circ}$ V.

c) Repeat parts (a) and (b), given that the load resistance changes from 7.5 Ω to 2.5 Ω and the load reactance changes from 12 Ω to 4 Ω. Assume that the load voltage remains constant at $120\,\underline{/0^\circ}$ V. How much must the amplitude of $\mathbf{V}_s$ be increased in order to maintain the load voltage at 120 V?

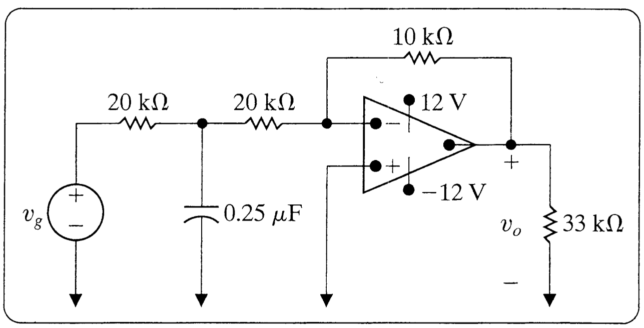
d) Repeat part (c), given that at the same time the load resistance and reactance changes, a capacitive reactance of −2 Ω is connected across the load terminals.

Figure P9.77



9.78 The sinusoidal voltage source in the circuit shown in Fig. P9.78 is generating the voltage $v_g = 4\cos 200t$ V. If the op amp is ideal, what is the steady-state expression for $v_o(t)$?

Figure P9.78



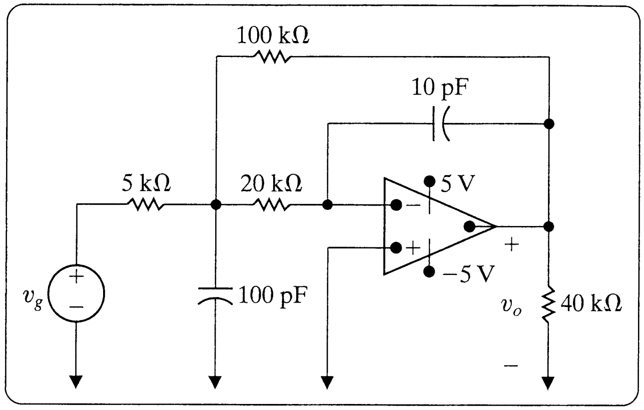
9.79 The 0.25 μF capacitor in the circuit seen in Fig. P9.78 is replaced with a variable capacitor. The capacitor is adjusted until the output voltage leads the input voltage by 135°.

a) Find the value of C in microfarads.

b) Write the steady-state expression for $v_o(t)$ when C has the value found in (a).

9.80 The op amp in the circuit seen in Fig. P9.80 is ideal. Find the steady-state expression for $v_o(t)$ when $v_g = 2\cos 10^6 t$ V.

Figure P9.80

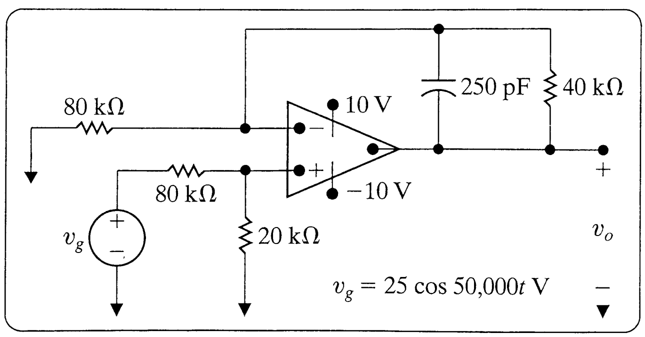


9.81 The op amp in the circuit in Fig. P9.81 is ideal.

a) Find the steady-state expression for $v_o(t)$.

b) How large can the amplitude of v_g be before the amplifier saturates?

Figure P9.81

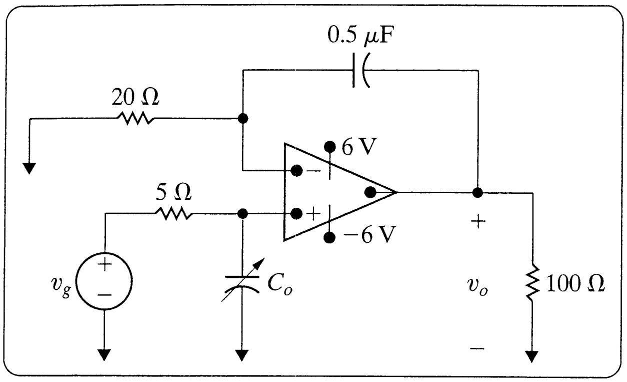


9.82 The operational amplifier in the circuit shown in Fig. P9.82 is ideal. The voltage of the ideal sinusoidal source is $v_g = 6 \cos 10^5 t$ V.

a) How small can C_o be before the steady-state output voltage no longer has a pure sinusoidal waveform?

b) For the value of C_o found in (a), write the steady-state expression for v_o.

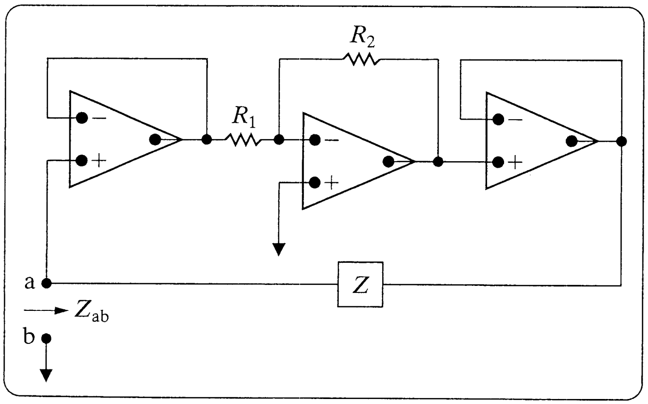
Figure P9.82



9.83 a) Find the input impedance Z_{ab} for the circuit in Fig. P9.83. Express Z_{ab} as a function of Z and K where $K = (R_2/R_1)$.

b) If Z is a pure capacitive element, what is the capacitance seen looking into the terminals a,b?

Figure P9.83



9.84 You may have the opportunity as an engineering graduate to serve as an expert witness in lawsuits involving either personal injury or property damage. As an example of the type of problem on which you may be asked to give an opinion, consider the following event. At the end of a day of fieldwork, a farmer returns to his farmstead, checks his hog-confinement building, and finds to his dismay that the hogs are dead. The problem is traced to a blown fuse that caused a 240 V fan motor to stop. The loss of ventilation led to the suffocation of the livestock. The interrupted fuse is located in the main switch that connects the farmstead to the electrical service. Before the insurance company settles the claim, it wants to know if the electric circuit supplying the farmstead functioned properly. The lawyers for the insurance company are puzzled because the farmer's wife, who was in the house on the day of the accident convalescing from minor surgery, was able to watch TV during the afternoon. Furthermore, when she went to the kitchen to start preparing the evening meal, the electric clock indicated the correct time. The lawyers have hired you to explain (1) why the electric clock in the kitchen and the television set in the living room continued to operate after the fuse in the main switch blew and (2) why the second fuse in the main switch didn't blow after the fan motor stalled. After ascertaining the loads on the three-wire distribution circuit prior to the interruption of fuse A, you are able to construct the circuit model shown in Fig. P9.84 on page 447. The impedances of the line conductors and the neutral conductor are assumed negligible.

a) Calculate the branch currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, $\mathbf{I}_4$, $\mathbf{I}_5$, and $\mathbf{I}_6$ prior to the interruption of fuse A.

b) Calculate the branch currents after the interruption of fuse A. Assume the stalled fan motor behaves as a short circuit.

c) Explain why the clock and television set were not affected by the momentary short circuit that interrupted fuse A.

d) Assume the fan motor is equipped with a thermal cutout designed to interrupt the motor circuit if the motor current becomes excessive. Would you expect the thermal cutout to operate? Explain.

e) Explain why fuse B is not interrupted when the fan motor stalls.

9.85 ◆
a) Calculate the branch currents $\mathbf{I}_1$–$\mathbf{I}_6$ in the circuit in Fig. 9.58.
b) Find the primary current $\mathbf{I}_p$.

9.86 ◆ Suppose the 40 Ω resistance in the distribution circuit in Fig. 9.58 is replaced by a 20 Ω resistance.
a) Recalculate the branch current in the 2 Ω resistor, $\mathbf{I}_2$.
b) Recalculate the primary current, $\mathbf{I}_p$.
c) On the basis of your answers, is it desirable to have the resistance of the two 120 V loads be equal?

9.87 ◆ A residential wiring circuit is shown in Fig. P9.87. In this model, the resistor R_3 is used to model a 240 V appliance (such as an electric range), and the resistors R_1 and R_2 are used to model 120 V appliances (such as a lamp, toaster, and iron). The branches carrying $\mathbf{I}_1$ and $\mathbf{I}_2$ are modeling what electricians refer to as the hot conductors in the circuit, and the branch carrying $\mathbf{I}_n$ is modeling the neutral conductor. Our purpose in analyzing the circuit is to show the importance of the neutral conductor in the satisfactory operation of the circuit. You are to choose the method for analyzing the circuit.
a) Show that $\mathbf{I}_n$ is zero if $R_1 = R_2$.
b) Show that $\mathbf{V}_1 = \mathbf{V}_2$ if $R_1 = R_2$.
c) Open the neutral branch and calculate $\mathbf{V}_1$ and $\mathbf{V}_2$ if $R_1 = 60\ \Omega$, $R_2 = 600\ \Omega$, and $R_3 = 10\ \Omega$.
d) Close the neutral branch and repeat (c).
e) On the basis of your calculations, explain why the neutral conductor is never fused in such a manner that it could open while the hot conductors are energized.

9.88 ◆
a) Find the primary current $\mathbf{I}_p$ for (c) and (d) in Problem 9.87.
b) Do your answers make sense in terms of known circuit behavior?

Figure P9.84

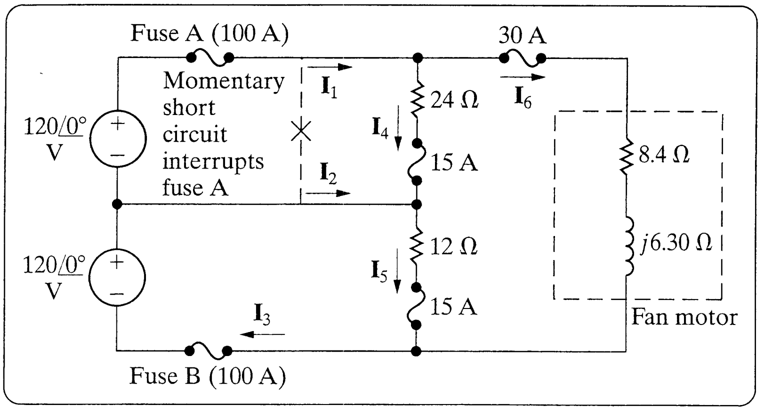
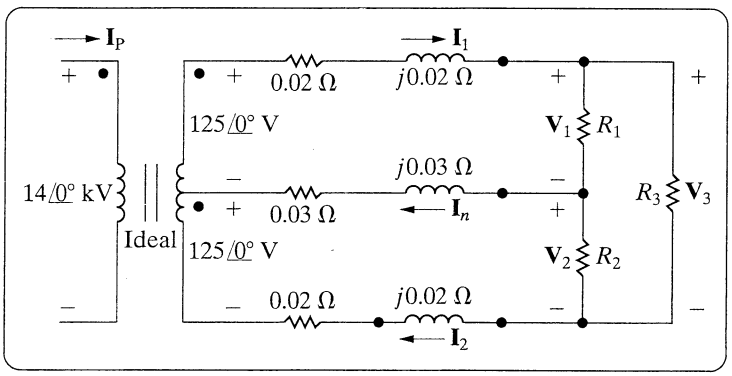


Figure P9.87



CHAPTER

14 Introduction to Frequency Selective Circuits

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Know the *RL* and *RC* circuit configurations that act as low-pass filters and be able to design *RL* and *RC* circuit component values to meet a specified cutoff frequency.
2. Know the *RL* and *RC* circuit configurations that act as high-pass filters and be able to design *RL* and *RC* circuit component values to meet a specified cutoff frequency.
3. Know the *RLC* circuit configurations that act as bandpass filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a bandpass filter, and be able to design *RLC* circuit component values to meet design specifications.
4. Know the *RLC* circuit configurations that act as bandreject filters, understand the definition of and relationship among the center frequency, cutoff frequencies, bandwidth, and quality factor of a bandreject filter, and be able to design *RLC* circuit component values to meet design specifications.

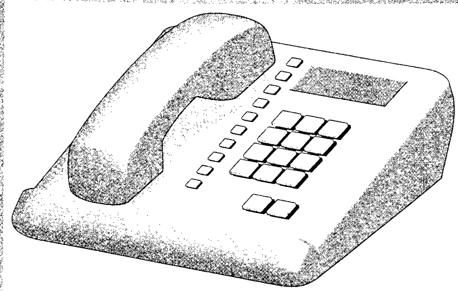
Up to this point in our analysis of circuits with sinusoidal sources, the source frequency was held constant. In this chapter, we analyze the effect of varying source frequency on circuit voltages and currents. The result of this analysis is the **frequency response** of a circuit.

We've seen in previous chapters that a circuit's response depends on the types of elements in the circuit, the way the elements are connected, and the impedance of the elements. Although varying the frequency of a sinusoidal source does not change the element types or their connections, it does alter the impedance of capacitors and inductors, because the impedance of these elements is a function of frequency. As we will see, the careful choice of circuit elements, their values, and their connections to other elements enables us to construct circuits that pass to the output only those input signals that reside in a desired range of frequencies. Such circuits are called **frequency-selective circuits**. Many devices that communicate via electric signals, such as telephones, radios, televisions, and satellites, employ frequency-selective circuits.

Frequency-selective circuits are also called **filters** because of their ability to filter out certain input signals on the basis of frequency. Figure 14.1 on page 652 represents this ability in a simplistic way. To be more accurate, we should note that no practical frequency-selective circuit can perfectly or completely filter out selected frequencies. Rather, filters **attenuate**—that is, weaken or lessen

Practical Perspective

Pushbutton Telephone Circuits



In this chapter, we examine circuits in which the source frequency varies. The behavior of these circuits varies as the source frequency varies, because the impedance of the reactive components is a function of the source frequency. These frequency-dependent circuits are called **filters** and are used in many common electrical devices. In radios, filters are used to select one radio station's signal while rejecting the signals from others transmitting at different frequencies. In stereo systems, filters are used to adjust the relative strengths of the low- and high-frequency components of the audio signal. Filters are also used throughout telephone systems.

A pushbutton telephone produces tones that you hear when you press a button. You may have wondered about these tones. How are they used to tell the telephone system which button was pushed? Why are tones used at all? Why do the tones sound musical? How does the phone system tell the difference between button tones and the normal sounds of people talking or singing?

The telephone system was designed to handle audio signals—those with frequencies between 300 Hz and 3 kHz. Thus, all signals from the system to the user have to be audible—including the dial tone and the busy signal. Similarly, all signals from the user to the system have to be audible, including the signal that the user has pressed a button. It is important to distinguish button signals from the normal audio signal, so a dual-tone-multiple-frequency (DTMF) design is employed. When a number button is pressed, a unique pair of sinusoidal tones with very precise frequencies is sent by the phone to the telephone system. The DTMF frequency and timing specifications make it unlikely that a human voice could produce the exact tone pairs, even if the person were trying. In the central telephone facility, electric circuits monitor the audio signal, listening for the tone pairs that signal a number. In the Practical Perspective example at the end of the chapter, we will examine the design of the DTMF filters used to determine which button has been pushed.

Figure 14.9 summarizes the two low-pass filter circuits we have examined. Look carefully at the transfer functions. Notice how similar in form they are—they differ only in the terms that specify the cutoff frequency. In fact, we can state a general form for the transfer functions of these two low-pass filters:

TRANSFER FUNCTION FOR A LOW-PASS FILTER

$$H(s) = \frac{\omega_c}{s + \omega_c}. \tag{14.13}$$

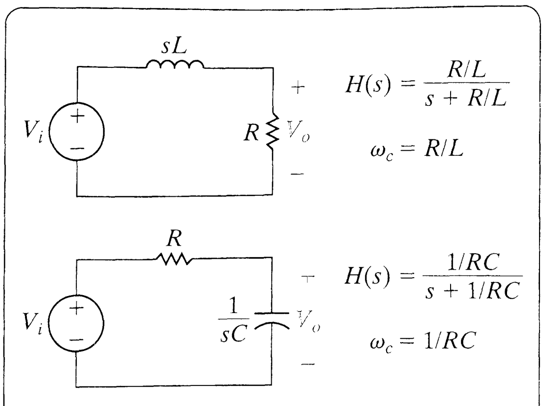


Figure 14.9 Two low-pass filters, the series *RL* and the series *RC*, together with their transfer functions and cutoff frequencies.

Any circuit with the voltage ratio in Eq. 14.13 would behave as a low-pass filter with a cutoff frequency of ω_c. The problems at the end of the chapter give you other examples of circuits with this voltage ratio.

Relating the Frequency Domain to the Time Domain

Finally, you might have noticed one other important relationship. Remember our discussion of the natural responses of the first-order RL and RC circuits in Chapter 6. An important parameter for these circuits is the time constant, τ, which characterizes the shape of the time response. For the RL circuit, the time constant has the value L/R (Eq. 7.14); for the RC circuit, the time constant is RC (Eq. 7.24). Compare the time constants to the cutoff frequencies for these circuits and notice that

$$\tau = 1/\omega_c. \tag{14.14}$$

This result is a direct consequence of the relationship between the time response of a circuit and its frequency response, as revealed by the Laplace transform. The discussion of memory and weighting as represented in the convolution integral of Section 13.6 shows that as $\omega_c \to \infty$, the filter has no memory, and the output approaches a scaled replica of the input; that is, no filtering has occurred. As $\omega_c \to 0$, the filter has increased memory and the output voltage is a distortion of the input, because filtering has occurred.

ASSESSING OBJECTIVE 1

◆ **Know the *RL* and *RC* circuit configurations that act as low-pass filters**

14.1 A series RC low-pass filter requires a cutoff frequency of 8 kHz. Use $R = 10$ kΩ and compute the value of C required.

ANSWER: 1.99 nF.

14.2 A series RL low-pass filter with a cutoff frequency of 2 kHz is needed. Using $R = 5$ kΩ, compute (a) L; (b) $|H(j\omega)|$ at 50 kHz; and (c) $\theta(j\omega)$ at 50 kHz.

ANSWER: (a) 0.40 H; (b) 0.04; (c) $-87.71°$.

NOTE ◆ *Also try Chapter Problems 14.1 and 14.2.*

cutoff frequency of a filter circuit and the time constant of that same circuit, we should expect the cutoff frequency to be a characteristic parameter of the circuit whose value depends only on the circuit components, their values, and the way they are connected.

ASSESSING OBJECTIVE 2

◆ **Know the *RL* and *RC* circuit configurations that act as high-pass filters**

14.3 A series RL high-pass filter has $R = 5$ kΩ and $L = 3.5$ mH. What is ω_c for this filter?

ANSWER: 1.43 Mrad/s.

14.4 A series RC high-pass filter has $C = 1\ \mu$F. Compute the cutoff frequency for the following values of R: (a) 100 Ω; (b) 5 kΩ; and (c) 30 kΩ.

ANSWER: (a) 10 krad/s; (b) 200 rad/s; (c) 33.33 rad/s.

14.5 Compute the transfer function of a series RC low-pass filter that has a load resistor R_L in parallel with its capacitor.

ANSWER: $H(s) = \dfrac{\frac{1}{RC}}{s + \frac{1}{KRC}}$, where $K = \dfrac{R_L}{R + R_L}$.

NOTE ◆ *Also try Chapter Problems 14.9 and 14.10.*

14.4 ◆ Bandpass Filters

The next filters we examine are those that pass voltages within a band of frequencies to the output while filtering out voltages at frequencies outside this band. These filters are somewhat more complicated than the low-pass and high-pass filters of the previous sections. As we have already seen in Fig. 14.3(c), ideal bandpass filters have two cutoff frequencies, ω_{c1} and ω_{c2}, which identify the passband. For realistic bandpass filters, these cutoff frequencies are again defined as the frequencies for which the magnitude of the transfer function equals $(1/\sqrt{2})H_{\max}$.

Center Frequency, Bandwidth, and Quality Factor

There are three other important parameters that characterize a bandpass filter. The first is the **center frequency**, ω_o, defined as the frequency for which a circuit's transfer function is purely real. Another name for the center frequency is the **resonant frequency**. This is the same name given

ASSESSING OBJECTIVE 3

◆ **Know the *RLC* circuit configurations that act as bandpass filters**

14.6 Using the circuit in Fig. 14.19(a), compute the values of R and L to give a bandpass filter with a center frequency of 12 kHz and a quality factor of 6. Use a 0.1 μF capacitor.

ANSWER: $L = 1.76$ mH, $R = 22.10\ \Omega$.

14.7 Using the circuit in Fig. 14.22, compute the values of L and C to give a bandpass filter with a center frequency of 2 kHz and a bandwidth of 500 Hz. Use a 250 Ω resistor.

ANSWER: $L = 4.97$ mH, $C = 1.27\ \mu$F.

14.8 Recalculate the component values for the circuit in Example 14.6(d) so that the frequency response of the resulting circuit is unchanged using a 0.2 μF capacitor.

ANSWER: $L = 5.07$ mH, $R = 3.98$ kΩ.

14.9 Recalculate the component values for the circuit in Example 14.6(d) so that the quality factor of the resulting circuit is unchanged but the center frequency has been moved to 2 kHz. Use a 0.2 μF capacitor.

ANSWER: $R = 9.95$ kΩ, $L = 31.66$ mH.

NOTE ◆ *Also try Chapter Problems 14.15 and 14.16.*

14.5 ◆ Bandreject Filters

We turn now to the last of the four filter categories—the bandreject filter. This filter passes source voltages outside the band between the two cutoff frequencies to the output (the passband), and attenuates source voltages before they reach the output at frequencies between the two cutoff frequencies (the stopband). Bandpass filters and bandreject filters thus perform complementary functions in the frequency domain.

Bandreject filters are characterized by the same parameters as bandpass filters: the two cutoff frequencies, the center frequency, the bandwidth, and the quality factor. Again, only two of these five parameters can be specified independently.

ASSESSING OBJECTIVE 4

◆ **Know the *RLC* circuit configurations that act as bandreject filters**

14.10 Design the component values for the series *RLC* bandreject filter shown in Fig. 14.28(a) so that the center frequency is 4 kHz and the quality factor is 5. Use a 500 nF capacitor.

ANSWER: $L = 3.17$ mH, $R = 14.92\ \Omega$.

14.11 Recompute the component values for Drill Exercise 14.10 to achieve a bandreject filter with a center frequency of 20 kHz. The filter has a 100 Ω resistor. The quality factor remains at 5.

ANSWER: $L = 3.98$ mH, $C = 15.92$ nF.

NOTE ◆ *Also try Chapter Problems 14.24 and 14.25.*

Practical Perspective

Pushbutton Telephone Circuits

In the Practical Perspective at the start of this chapter, we described the dual-tone-multiple-frequency (DTMF) system used to signal that a button has been pushed on a pushbutton telephone. A key element of the DTMF system is the DTMF receiver—a circuit that decodes the tones produced by pushing a button and determines which button was pushed.

In order to design a DTMF reciever, we need a better understanding of the DTMF system. As you can see from Fig. 14.32, the buttons on the telephone are organized into rows and columns. The pair of tones generated by pushing a button depends on the button's row and column. The button's row determines its low-frequency tone, and the button's column determines its high- frequency tone.[1] For example, pressing the "6" button produces sinusoidal tones with the frequencies 770 Hz and 1477 Hz.

At the telephone switching facility, bandpass filters in the DTMF receiver first detect whether tones from both the low-frequency and high-frequency groups are simultaneously present. This test rejects many extraneous audio signals that are not DTMF. If tones are present in both bands, other filters are used to select among the possible tones in each band so that the frequencies can be decoded into a unique button signal. Additional tests are performed to prevent false button detection. For example, only one tone per frequency band is allowed; the high- and low-band frequencies must start and stop within a few milliseconds of one another to be considered valid; and the high- and low-band signal amplitudes must be sufficiently close to each other.

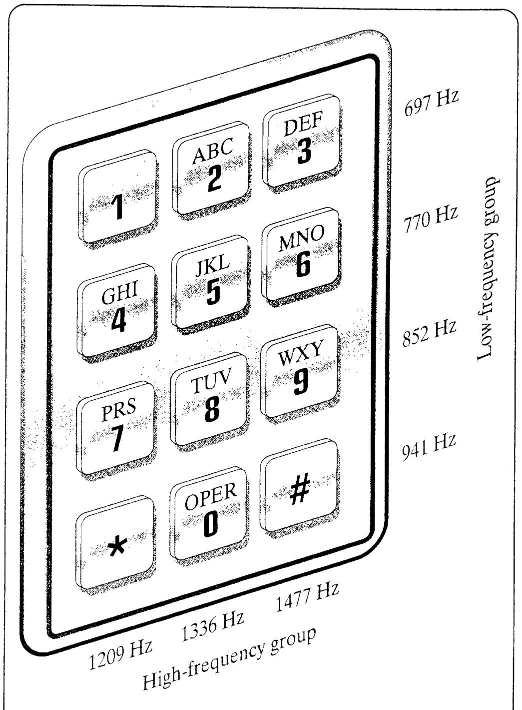


Figure 14.32 Tones generated by the rows and columns of telephone pushbuttons.

[1] A fourth high-frequency tone is reserved at 1633 Hz. This tone is used infrequently and is not produced by a standard 12-button telephone.

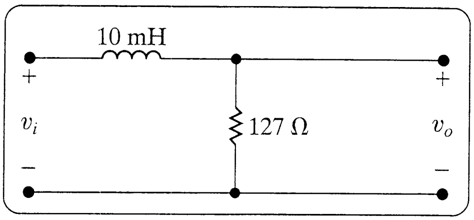
PROBLEMS

14.1 a) Find the cutoff frequency in hertz for the RL filter shown in Fig. P14.1.

b) Calculate $H(j\omega)$ at ω_c, $0.2\omega_c$, and $5\omega_c$.

c) If $v_i = 10\cos\omega t$ V, write the steady-state expression for v_o when $\omega = \omega_c$, $\omega = 0.2\omega_c$, and $\omega = 5\omega_c$.

Figure P14.1



14.2 Use a 5 mH inductor to design a low-pass, RL, passive filter with a cutoff frequency of 1 kHz.

a) Specify the value of the resistor.

b) A load having a resistance of 270 Ω is connected across the output terminals of the filter. What is the corner, or cutoff, frequency of the loaded filter in hertz?

14.3 A resistor, denoted as R_l, is added in series with the inductor in the circuit in Fig. 14.3(a). The new low-pass filter circuit is shown in Fig. P14.3.

a) Derive the expression for $H(s)$ where $H(s) = V_o/V_i$.

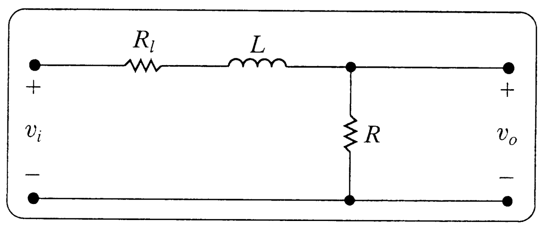
b) At what frequency will the magnitude of $H(j\omega)$ be maximum?

c) What is the maximum value of the magnitude of $H(j\omega)$?

d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum value divided by $\sqrt{2}$?

e) Assume a resistance of 75 Ω is added in series with the 250 mH inductor in the circuit in Fig. P14.1. Find ω_c, $H(j0)$, $H(j\omega_c)$, $H(j0.3\omega_c)$, and $H(j3\omega_c)$.

Figure P14.3

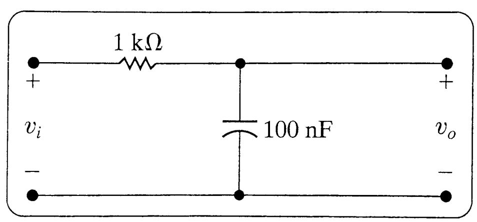


14.4 a) Find the cutoff frequency (in hertz) of the low-pass filter shown in Fig. P14.4.

b) Calculate $H(j\omega)$ at ω_c, $0.1\omega_c$, and $10\omega_c$.

c) If $v_i = 200\cos\omega t$ mV, write the steady-state expression for v_o when $\omega = \omega_c$, $0.1\omega_c$, and $10\omega_c$.

Figure P14.4



14.5 A resistor denoted as R_L is connected in parallel with the capacitor in the circuit in Fig. 14.7. The loaded low-pass filter circuit is shown in Fig. P14.5.

a) Derive the expression for the voltage transfer function V_o/V_i.

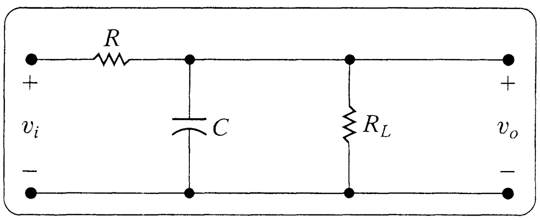
b) At what frequency will the magnitude of $H(j\omega)$ be maximum?

c) What is the maximum value of the magnitude of $H(j\omega)$?

d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum value divided by $\sqrt{2}$?

e) Assume a resistance of 10 $k\Omega$ is added in parallel with the 100 nF capacitor in the circuit in Fig. P14.4. Find ω_c, $H(j0)$, $H(j\omega_c)$, $H(j0.1\omega_c)$, and $H(j10\omega_c)$.

Figure P14.5



14.6 ❖ P Use a 0.5 μF capacitor to design a low-pass passive filter with a cutoff frequency of 50 krad/s.

a) Specify the cutoff frequency in hertz.

b) Specify the value of the filter resistor.

c) Assume the cutoff frequency cannot increase by more than 5%. What is the smallest value of load resistance that can be connected across the output terminals of the filter?

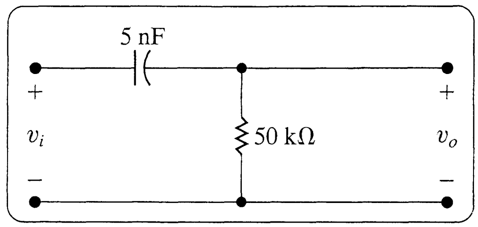
d) If the resistor found in (c) is connected across the output terminals, what is the magnitude of $H(j\omega)$ when $\omega = 0$?

14.7 a) Find the cutoff frequency (in hertz) for the high-pass filter shown in Fig. P14.7.

b) Find $H(j\omega)$ at ω_c, $0.2\omega_c$, and $5\omega_c$.

c) If $v_i = 500 \cos \omega t$ mV, write the steady-state expression for v_o when $\omega = \omega_c$, $\omega = 0.2\omega_c$, and $\omega = 5\omega_c$.

Figure P14.7



14.8 A resistor, denoted as R_c, is connected in series with the capacitor in the circuit in Fig. 14.10(a). The new high-pass filter circuit is shown in Fig. P14.8.

a) Derive the expression for $H(s)$ where $H(s) = V_o/V_i$.

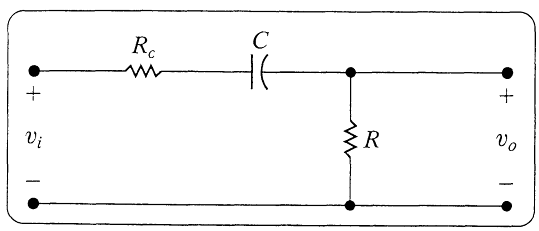
b) At what frequency will the magnitude of $H(j\omega)$ be maximum?

c) What is the maximum value of the magnitude of $H(j\omega)$?

d) At what frequency will the magnitude of $H(j\omega)$ equal its maximum value divided by $\sqrt{2}$?

e) Assume a resistance of 12.5 kΩ is connected in series with the 5 nF capacitor in the circuit in Fig. P14.7. Calculate ω_c, $H(j\omega_c)$, $H(j0.2\omega_c)$, and $H(j5\omega_c)$.

Figure P14.8



14.9 Using a 100 nF capacitor, design a high-pass passive filter with a cutoff frequency of 300 Hz.

a) Specify the value of R in kilohms.

b) A 47 kΩ resistor is connected across the output terminals of the filter. What is the cutoff frequency, in hertz, of the loaded filter?

14.10 Using a 5 mH inductor, design a high-pass, RL, passive filter with a cutoff frequency of 25 krad/s.

a) Specify the value of the resistance.

b) Assume the filter is connected to a pure resistive load. The cutoff frequency is not to drop below 24 krad/s. What is the smallest load resistor that can be connected across the output terminals of the filter?

14.11 Show that the alternative forms for the cutoff frequencies of a bandpass filter, given in Eqs. 14.36 and 14.37, can be derived from Eqs. 14.34 and 14.35.

14.12 Calculate the center frequency, the bandwidth, and the quality factor of a bandpass filter that has an upper cutoff frequency of 121 krad/s and a lower cutoff frequency of 100 krad/s.

14.13 A bandpass filter has a center, or resonant, frequency of 50 krad/s and a quality factor of 4. Find the bandwidth, the upper cutoff frequency, and the lower cutoff frequency. Express all answers in kilohertz.

14.14 Use a 5 nF capacitor to design a series RLC bandpass filter, as shown at the top of Fig. 14.27. The center frequency of the filter is 8 kHz, and the quality factor is 2.

a) Specify the values of R and L.

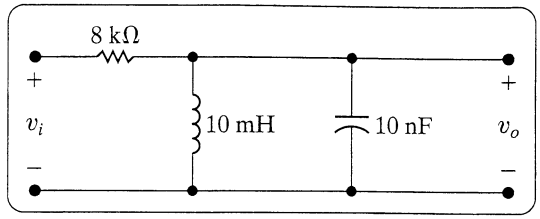
b) What is the lower cutoff frequency in kilohertz?

c) What is the upper cutoff frequency in kilohertz?

d) What is the bandwidth of the filter in kilohertz?

14.15 For the bandpass filter shown in Fig. P14.15, find (a) ω_o, (b) f_o, (c) Q, (d) ω_{c1}, (e) f_{c1}, (f) ω_{c2}, (g) f_{c2}, and (h) β.

Figure P14.15



14.16 Using a 50 nF capacitor in the bandpass circuit shown in Fig. 14.22, design a filter with a quality factor of 5 and a center frequency of 20 krad/s.

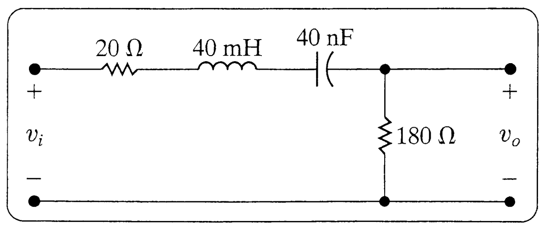
a) Specify the numerical values of R and L.

b) Calculate the upper and lower cutoff frequencies in kilohertz.

c) Calculate the bandwidth in hertz.

14.17 For the bandpass filter shown in Fig. P14.17, calculate the following: (a) f_o; (b) Q; (c) f_{c1}; (d) f_{c2}; and (e) β.

Figure P14.17



14.18 The input voltage in the circuit in Fig. P14.17 is $500 \cos \omega t$ mV. Calculate the output voltage when (a) $\omega = \omega_o$; (b) $\omega = \omega_{c1}$; and (c) $\omega = \omega_{c2}$.

14.19 A block diagram of a system consisting of a sinusoidal voltage source, an RLC series bandpass filter, and a load is shown in Fig. P14.19. The internal impedance of the sinusoidal source is $80 + j0\ \Omega$, and the impedance of the load is $480 + j0\ \Omega$.

The RLC series bandpass filter has a 20 nF capacitor, a center frequency of 50 krad/s, and a quality factor of 6.25.

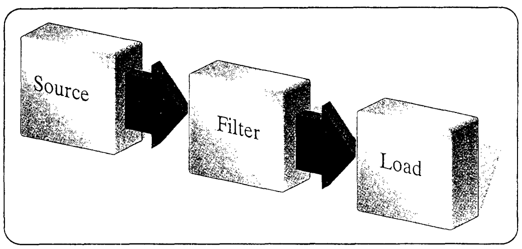
a) Draw a circuit diagram of the system.

b) Specify the numerical values of L and R for the filter section of the system.

c) What is the quality factor of the interconnected system?

d) What is the bandwidth (in hertz) of the interconnected system?

Figure P14.19



14.20 The purpose of this problem is to investigate how a resistive load connected across the output terminals of the bandpass filter shown in Fig. 14.19 affects the quality factor and hence the bandwidth of the filtering system. The loaded filter circuit is shown in Fig. P14.20.

a) Calculate the transfer function V_o/V_i for the circuit shown in Fig. P14.20.

b) What is the expression for the bandwidth of the system?

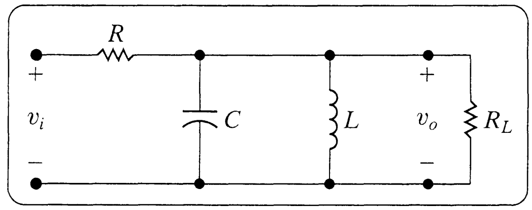
c) What is the expression for the loaded bandwidth (β_L) as a function of the unloaded bandwidth (β_U)?

d) What is the expression for the quality factor of the system?

e) What is the expression for the loaded quality factor (Q_L) as a function of the unloaded quality factor (Q_U)?

f) What are the expressions for the cutoff frequencies ω_{c1} and ω_{c2}?

Figure P14.20



14.21 Consider the circuit shown in Fig. P14.21.

a) Find ω_o.

b) Find β.

c) Find Q.

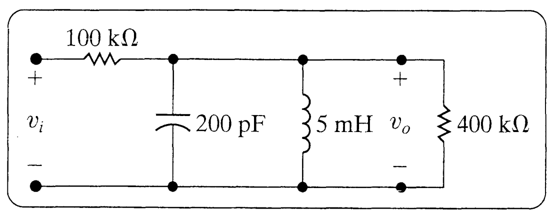
d) Find the steady-state expression for v_o when $v_i = 250 \cos \omega_o t$ mV.

e) Show that if R_L is expressed in kilohms the Q of the circuit in Fig. P14.21 is

$$Q = \frac{20}{1 + 100/R_L}$$

f) Plot Q versus R_L for $20\ \text{k}\Omega \le R_L \le 2\ \text{M}\Omega$.

Figure P14.21



14.22 The parameters in the circuit in Fig. P14.21 are $R = 2.4\ \text{k}\Omega$, $C = 50$ pF, and $L = 2\ \mu$H. The quality factor of the circuit is not to drop below 7.5. What is the smallest permissible value of the load resistor R_L?

14.23
a) Show (via a qualitative analysis) that the circuit in Fig. P14.23 is a bandreject filter.

b) Support the qualitative analysis of (a) by finding the voltage transfer function of the filter.

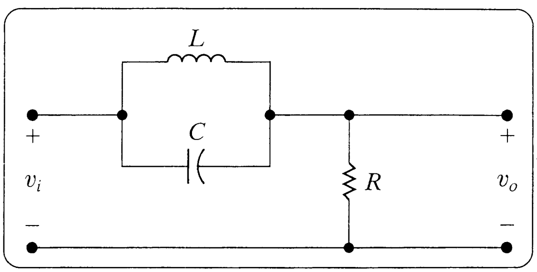
c) Derive the expression for the center frequency of the filter.

d) Derive the expressions for the cutoff frequencies ω_{c1} and ω_{c2}.

e) What is the expression for the bandwidth of the filter?

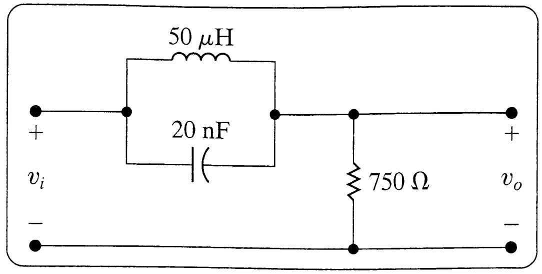
f) What is the expression for the quality factor of the circuit?

Figure P14.23



14.24 For the bandreject filter in Fig. P14.24, calculate (a) ω_o; (b) f_o; (c) Q; (d) ω_{c1}; (e) f_{c1}; (f) ω_{c2}; (g) f_{c2}; and (h) β in kilohertz.

Figure P14.24



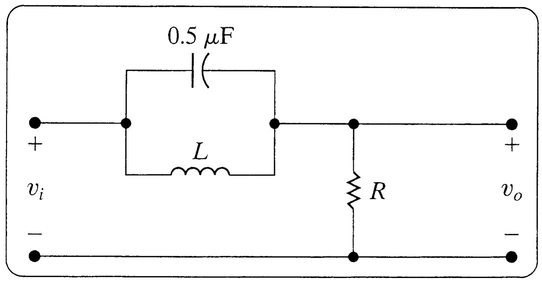
14.25 Use a 0.5 μF capacitor to design a bandreject filter, as shown in Fig. P14.25. The filter has a center frequency of 4 kHz and a quality factor of 5.

a) Specify the numerical values of R and L.

b) Calculate the upper and lower corner, or cutoff, frequencies in kilohertz.

c) Calculate the filter bandwidth in hertz.

Figure P14.25



14.26 Assume the bandreject filter in Problem 14.25 is loaded with a 1 kΩ resistor.

a) What is the quality factor of the loaded circuit?

b) What is the bandwidth (in kilohertz) of the loaded circuit?

c) What is the upper cutoff frequency in kilohertz?

d) What is the lower cutoff frequency in kilohertz?

14.27 The purpose of this problem is to investigate how a resistive load connected across the output terminals of the bandreject filter shown in Fig. 14.28(a) affects the behavior of the filter. The loaded filter circuit is shown in Fig. P14.27.

a) Find the voltage transfer function V_o/V_i.

b) What is the expression for the center frequency?

c) What is the expression for the bandwidth?

d) What is the expression for the quality factor?

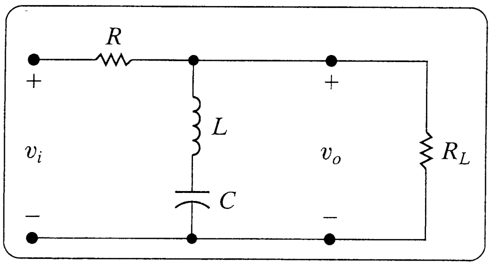
e) Evaluate $H(j\omega_o)$.

f) Evaluate $H(j0)$.

g) Evaluate $H(j\infty)$.

h) What are the expressions for the corner frequencies ω_{c1} and ω_{c2}?

Figure P14.27



14.28 P The parameters in the circuit in Fig. P14.27 are $R = 30\ \Omega$, $L = 1\ \mu\text{H}$, $C = 4$ pF, and $R_L = 150\ \Omega$.

a) Find ω_o, β (in megahertz), and Q.

b) Find $H(j0)$ and $H(j\infty)$.

c) Find f_{c2} and f_{c1}.

d) Show that if R_L is expressed in ohms the Q of the circuit is

$$Q = \frac{50}{3}[1 + (30/R_L)].$$

e) Plot Q versus R_L for $10 \le R_L \le 300\ \Omega$.

14.29 P The load in the bandreject filter circuit shown in Fig. P14.27 is 36 kΩ. The center frequency of the filter is 1 Mrad/s, and the capacitor is 400 pF. At very low and very high frequencies, the amplitude of the sinusoidal output voltage should be at least 96% of the amplitude of the sinusoidal input voltage.

a) Specify the numerical values of R and L.

b) What is the quality factor of the circuit?

14.30 Given the following voltage transfer function:

$$H(s) = \frac{V_o}{V_i} = \frac{10^{10}}{s^2 + 50{,}000s + 10^{10}}.$$

a) At what frequencies (in radians per second) is the ratio of V_o/V_i equal to unity?

b) At what frequency is the ratio maximum?

14.31 ◆ ❖ Design a series RLC bandpass filter (see Fig. 14.27) for detecting the low-frequency tone generated by pushing a telephone button as shown in Fig. 14.32.

a) Calculate the values of L and C that place the cutoff frequencies at the edges of the DTMF low-frequency band. Note that the resistance in standard telephone circuits is always $R = 600\ \Omega$.

b) What is the output amplitude of this circuit at each of the low-band frequencies, relative to the peak amplitude of the bandpass filter?

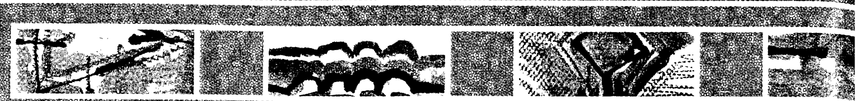
c) What is the output amplitude of this circuit at the lowest of the high-band frequencies?

14.32 ◆ ❖ Design a DTMF high-band bandpass filter similar to the low-band filter design in Problem 14.31. Be sure to include the fourth high-frequency tone, 1633 Hz, in your design. What is the response amplitude of your filter to the highest of the low-frequency DTMF tones?

14.33 ◆ ❖ The 20 Hz signal that rings a telephone's bell has to have a very large amplitude to produce a loud enough bell signal. How much larger can the ringing signal amplitude be, relative to the low-bank DTMF signal, so that the response of the filter in Problem 14.31 is no more than half as large as the largest of the DTMF tones?

CHAPTER

15 Active Filter Circuits



CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Know the op amp circuits that behave as first-order low-pass and high-pass filters and be able to calculate component values for these circuits to meet specifications of cutoff frequency and passband gain.
2. Be able to design filter circuits starting with a prototype circuit and use scaling to achieve desired frequency response characteristics and component values.
3. Understand how to use cascaded first- and second-order Butterworth filters to implement low-pass, high-pass, bandpass, and bandreject filters of any order.
4. Be able to use the design equations to calculate component values for prototype narrowband, bandpass, and bandreject filters to meet desired filter specifications.

Up to this point, we have considered only passive filter circuits, that is, filter circuits consisting of resistors, inductors, and capacitors. There are areas of application, however, where active circuits, those that employ op amps, have certain advantages over passive filters. For instance, active circuits can produce bandpass and bandreject filters without using inductors. This is desirable because inductors are usually large, heavy, and costly, and they may introduce electromagnetic field effects that compromise the desired frequency response characteristics.

Examine the transfer functions of all the filter circuits from Chapter 14 and you will notice that the maximum magnitude does not exceed 1. Even though passive resonant filters can achieve voltage and current amplification at the resonant frequency, passive filters in general are incapable of amplification, because the output magnitude does not exceed the input magnitude. This is not a surprising observation, as many of the transfer functions in Chapter 14 were derived using voltage or current division. Active filters provide a control over amplification not available in passive filter circuits.

Finally, recall that both the cutoff frequency and the passband magnitude of passive filters were altered with the addition of a resistive load at the output of the filter. This is not the case with active filters, due to the properties of op amps. Thus, we use active circuits to implement filter designs when gain, load variation, and physical size are important parameters in the design specifications.

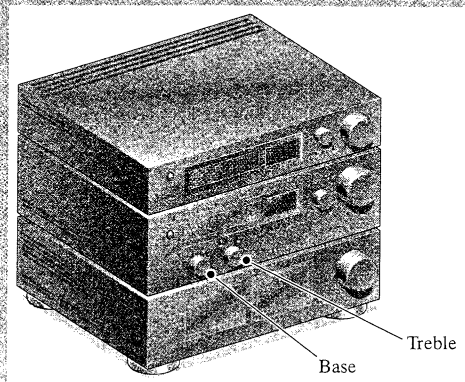
Practical Perspective

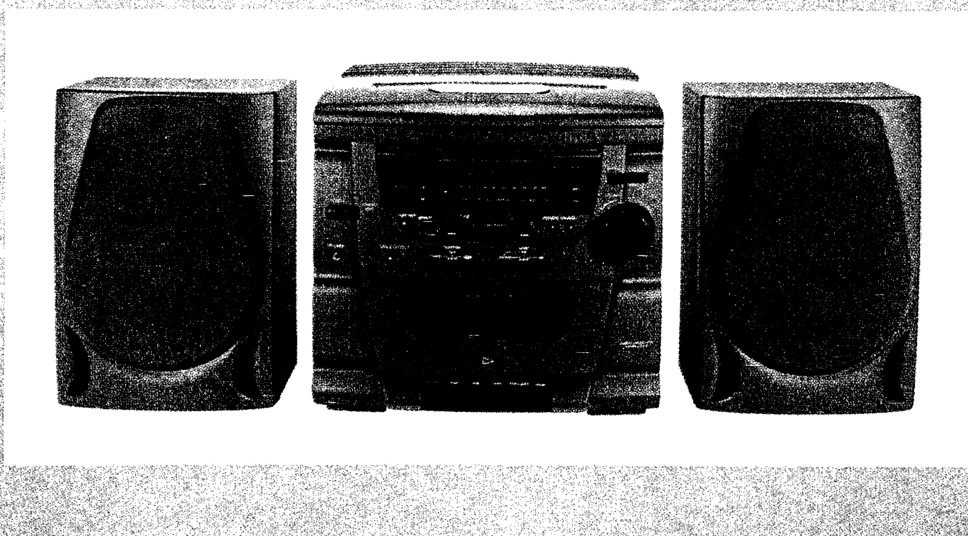
Bass Volume Control

In this chapter, we continue to examine circuits that are frequency selective. As described in Chapter 14, this means that the behavior of the circuit depends on the frequency of its sinusoidal input. Most of the circuits presented here fall into one of the four categories identified in Chapter 14—low-pass filters, high-pass filters, bandpass filters, and bandreject filters. But whereas the circuits in Chapter 14 were constructed using sources, resistors, capacitors, and inductors, the circuits in this chapter employ op amps. We shall soon see what advantages are conferred to a filter circuit constructed using op amps.

Audio electronic systems such as radios, tape players, and CD players often provide separate volume controls labeled "treble" and "bass." These controls permit the user to select the volume of high frequency audio signals ("treble") independent of the volume of low frequency audio signals ("bass"). The ability to independently adjust the amount of amplification (boost) or attenuation (cut) in these two frequency bands allows a listener to customize the sound with more precision than would be provided with a single volume control. Hence the boost and cut control circuit is also referred to as a tone control circuit.

The Practical Perspective example at the end of this chapter presents a circuit that implements bass volume control using a single op amp together with resistors and capacitors. An adjustable resistor supplies the necessary control over the amplification in the bass frequency range.





ASSESSING OBJECTIVE 1

◆ **Know the op amp circuits that behave as first order low-pass and high-pass filters and be able to calculate their component values**

15.1 Compute the values for R_2 and C that yield a high-pass filter with a passband gain of 1 and a cutoff frequency of 1 rad/s if R_1 is 1 Ω. (*Note:* This is the prototype high-pass filter.)

ANSWER: $R_2 = 1\ \Omega, C = 1$ F.

15.2 Compute the resistor values needed for the low-pass filter circuit in Fig. 15.1 to produce the transfer function

$$H(s) = \frac{-20{,}000}{s + 5000}.$$

Use a 5 μF capacitor.

ANSWER: $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$.

NOTE ◆ *Also try Chapter Problems 15.4 and 15.5.*

15.2 ◆ Scaling

In the design and analysis of both passive and active filter circuits, working with element values such as 1 Ω, 1 H, and 1 F is convenient. Although these values are unrealistic for specifying practical components, they greatly simplify computations. After making computations using convenient values of R, L, and C, the designer can transform the convenient values into realistic values using the process known as **scaling**.

There are two types of scaling: **magnitude** and **frequency**. We scale a circuit in magnitude by multiplying the impedance at a given frequency by the scale factor k_m. Thus we multiply all resistors and inductors by k_m and all capacitors by $1/k_m$. If we let unprimed variables represent the initial values of the parameters, and we let primed variables represent the scaled values of the variables, we have

$$R' = k_m R, \quad L' = k_m L, \quad \text{and} \quad C' = C/k_m. \tag{15.7}$$

where the primed variable has the new value and the unprimed variable has the old value of the cutoff frequency. Then compute the magnitude scale factor that, together with $k_f = 6283.185$, will scale the capacitor to 0.01 μF:

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{1}{(6283.185)(10^{-8})} = 15{,}915.5.$$

Since resistors are scaled only by using magnitude scaling,

$$R_1' = R_2' = k_m R = (15{,}915.5)(1) = 15{,}915.5\ \Omega.$$

Finally, we need to meet the passband gain specification. We can adjust the scaled values of either R_1 or R_2, because $K = R_2/R_1$. If we adjust R_2, we will change the cutoff frequency, because $\omega_c = 1/R_2C$. Therefore, we can adjust the value of R_1 to alter only the passband gain:

$$R_1 = R_2/K = (15{,}915.5)/(5) = 3183.1\ \Omega.$$

The final component values are

$$R_1 = 3183.1\ \Omega, \quad R_2 = 15{,}915.5\ \Omega, \quad C = 0.01\ \mu\text{F}.$$

The transfer function of the filter is given by

$$H(s) = \frac{-31{,}415.93}{s + 6283.185}.$$

The Bode plot of the magnitude of this transfer function is shown in Fig. 15.8.

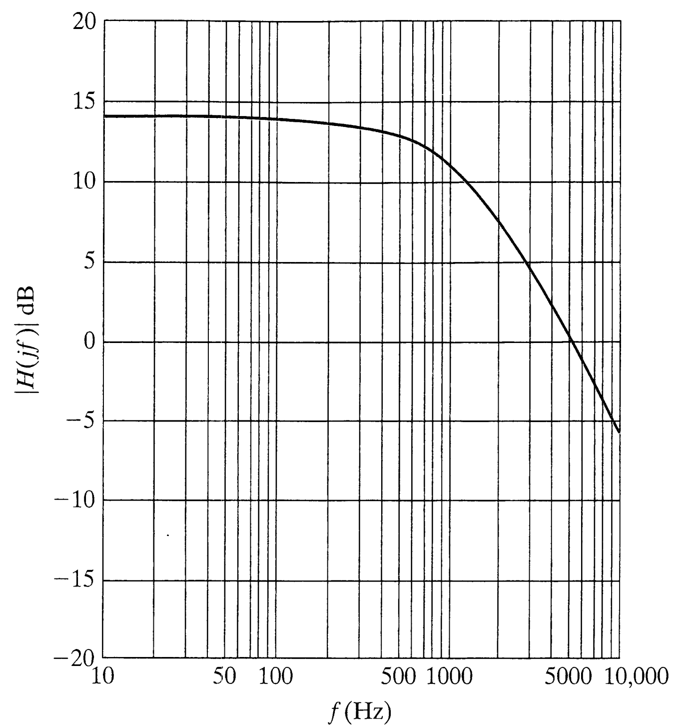


Figure 15.8 The Bode magnitude plot of the low-pass filter from Example 15.4

ASSESSING OBJECTIVE 2

◆ **Be able to design filter circuits starting with a prototype and use scaling to achieve desired frequency response and component values**

15.3 What magnitude and frequency scale factors will transform the prototype high-pass filter into a high-pass filter with a 0.5 μF capacitor and a cutoff frequency of 10 kHz?

ANSWER: $k_f = 62{,}831.85$, $k_m = 31.831$.

NOTE ◆ *Also try Chapter Problems 15.9 and 15.10.*

We can use frequency and magnitude scaling to design a Butterworth high-pass filter with practical component values and a cutoff frequency other than 1 rad/s. Adding an inverting amplifier to the cascade will accommodate designs with nonunity passband gains. The problems at the end of the chapter include several Butterworth high-pass filter designs.

Now that we can design both nth-order low-pass and high-pass Butterworth filters with arbitrary cutoff frequencies and passband gains, we can combine these filters in cascade (as we did in Section 15.3) to produce nth-order Butterworth bandpass filters. We can combine these filters in parallel with a summing amplifier (again, as we did in Section 15.3) to produce nth-order Butterworth bandreject filters. This chapter's problems also include Butterworth bandpass and bandreject filter designs.

ASSESSING OBJECTIVE 3

◆ **Understand how to use cascaded first- and second-order Butterworth filters**

15.4 For the circuit in Fig. 15.25, find values of R_1 and R_2 that yield a second-order prototype Butterworth high-pass filter.

ANSWER: $R_1 = 0.707\ \Omega,\ R_2 = 1.41\ \Omega.$

NOTE ◆ *Also try Chapter Problems 15.28, 15.31 and 15.32.*

15.5 ◆ Narrowband Bandpass and Bandreject Filters

The cascade and parallel component designs for synthesizing bandpass and bandreject filters from simpler low-pass and high-pass filters have the restriction that only broadband, or low-Q, filters will result. (The Q, of course, stands for *quality factor*.) This limitation is due principally to the fact that the transfer functions for cascaded bandpass and parallel bandreject filters have discrete real poles. The synthesis techniques work best for cutoff frequencies that are widely separated and therefore yield the lowest quality factors. But the largest quality factor we can achieve with discrete real poles arises when the cutoff frequencies, and thus the pole locations, are the same. Consider the transfer function that results:

$$H(s) = \left(\frac{-\omega_c}{s+\omega_c}\right)\left(\frac{-s}{s+\omega_c}\right)$$

$$= \frac{s\omega_c}{s^2 + 2\omega_c s + \omega_c^2}$$

$$= \frac{0.5\beta s}{s^2 + \beta s + \omega_c^2}. \qquad (15.50)$$

ASSESSING OBJECTIVE 4

◆ **Be able to use design equations to calculate component values for prototype narrowband, bandpass, and bandreject filters**

15.5 Design an active bandpass filter with $Q = 8$, $K = 5$, and $\omega_o = 1000$ rad/s. Use 1 μF capacitors, and specify the values of all resistors.

ANSWER: $R_1 = 1.6$ kΩ, $R_2 = 65.04$ Ω, $R_3 = 16$ kΩ.

15.6 Design an active unity-gain bandreject filter with $\omega_o = 1000$ rad/s and $Q = 4$. Use 2 μF capacitors in your design, and specify the values of R and σ.

ANSWER: $R = 500$ Ω, $\sigma = 0.9375$.

NOTE ◈ *Also try Chapter Problem 15.53.*

Practical Perspective

Bass Volume Control

We now look at an op amp circuit that can be used to control the amplification of an audio signal in the bass range. The audio range consists of signals having frequencies from 20 Hz to 20 kHz. The bass range includes frequencies up to 300 Hz. The volume control circuit and its frequency response are shown in Fig. 15.32. The particular response curve in the family of response curves is selected by adjusting the potentiometer setting in Fig. 15.32(a).

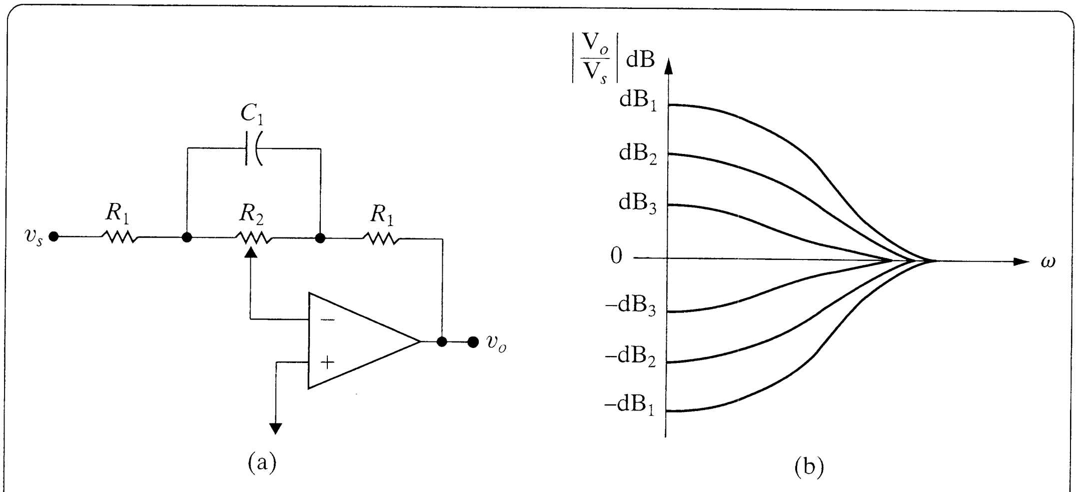


Figure 15.32 (a) Bass volume control circuit; (b) Bass volume control circuit frequency response.

In studying the frequency response curves in Fig. 15.32(b) note the following. First, the gain in dB can be either positive or negative. If the gain is positive a signal in the bass range is amplified or boosted. If the gain is negative the signal is attenuated or cut. Second, it is possible to select a response characteristic that yields unity gain (zero dB) for all frequencies in the bass range. As we shall see, if the potentiometer is set at its midpoint, the circuit will have no effect on signals in the bass range. Finally, as the frequency increases, all the characteristic responses approach zero dB or unity gain. Hence the volume control circuit will have no effect on signals in the upper end or treble range of the audio frequencies.

The first step in analyzing the frequency response of the circuit in Fig. 15.32(a) is to calculate the transfer function V_o/V_s. To facilitate this calculation the s-domain equivalent circuit is given in Fig. 15.33. The node voltages V_a and V_b have been labeled in the circuit to support node voltage analysis. The position of the potentiometer is determined by the numerical value of α, as noted in Fig. 15.33.

To find the transfer function we write the three node voltage equations that describe the circuit and then solve the equations for the voltage ratio V_o/V_s. The node voltage equations are

$$\frac{V_a}{(1-\alpha)R_2} + \frac{V_a - V_s}{R_1} + (V_a - V_b)sC_1 = 0;$$

$$\frac{V_b}{\alpha R_2} + (V_b - V_a)sC_1 + \frac{V_b - V_o}{R_1} = 0;$$

$$\frac{V_a}{(1-\alpha)R_2} + \frac{V_b}{\alpha R_2} = 0.$$

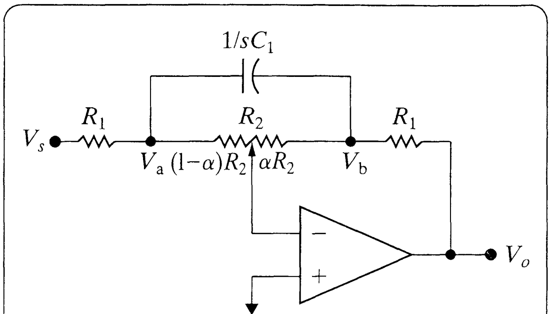


Figure 15.33 The s-domain circuit for the bass volume control. Note that α determines the potentiometer setting, so $0 \leq \alpha \leq 1$.

These three node-voltage equations can be solved to find V_o as a function of V_s and hence the transfer function $H(s)$:

$$H(s) = \frac{V_o}{V_s} = \frac{-(R_1 + \alpha R_2 + R_1 R_2 C_1 s)}{R_1 + (1-\alpha)R_2 + R_1 R_2 C_1 s}.$$

It follows directly that

$$H(j\omega) = \frac{-(R_1 + \alpha R_2 + j\omega R_1 R_2 C_1)}{[R_1 + (1-\alpha)R_2 + j\omega R_1 R_2 C_1]}.$$

Now let's verify that this transfer function will generate the family of frequency response curves depicted in Fig. 15.32(b). First note that when $\alpha = 0.5$ the magnitude of $H(j\omega)$ is unity for all frequencies, i.e.

$$|H(j\omega)| = \frac{|R_1 + 0.5R_2 + j\omega R_1 R_2 C_1|}{|R_1 + 0.5R_2 + j\omega R_1 R_2 C_1|} = 1.$$

When $\omega = 0$ we have

$$|H(j0)| = \frac{R_1 + \alpha R_2}{R_1 + (1-\alpha)R_2}.$$

Observe that $|H(j0)|$ at $\alpha = 1$ is the reciprocal of $|H(j0)|$ at $\alpha = 0$, that is

$$|H(j0)|_{\alpha=1} = \frac{R_1 + R_2}{R_1} = \frac{1}{|H(j0)|_{\alpha=0}}.$$

With a little thought the reader can see that the reciprocal relationship holds for all frequencies, not just $\omega = 0$. For example $\alpha = 0.4$ and $\alpha = 0.6$ are symmetric with $\alpha = 0.5$ and

$$H(j\omega)_{\alpha=0.4} = \frac{-(R_1 + 0.4R_2) + j\omega R_1 R_2 C_1}{(R_1 + 0.6R_2) + j\omega R_1 R_2 C_1}$$

while

$$H(j\omega)_{\alpha=0.6} = \frac{-(R_1 + 0.6R_2) + j\omega R_1 R_2 C_1}{(R_1 + 0.4R_2) + j\omega R_1 R_2 C_1}.$$

Hence

$$H(j\omega)_{\alpha=0.4} = \frac{1}{H(j\omega)_{\alpha=0.6}}.$$

It follows that depending on the value of α the volume control circuit can either amplify or attenuate the incoming signal.

The numerical values of R_1, R_2, and C_1 are based on two design decisions. The first design choice is the passband amplification or attenuation in the bass range (as $\omega \to 0$). The second design choice is the frequency at which this passband amplification or attenuation is changed by 3 dB. The component values which satisfy the design decisions are calculated with α equal to either 1 or 0.

As we have already observed, the maximum gain will be $(R_1 + R_2)/R_1$ and the maximum attenuation will be $R_1/(R_1 + R_2)$. If we assume $(R_1 + R_2)/R_1 \gg 1$ then the gain (or attenuation) will differ by 3 dB from its maximum value when $\omega = 1/R_2C_1$. This can be seen by noting that

$$\left|H\left(j\frac{1}{R_2C_1}\right)\right|_{\alpha=1} = \frac{|R_1 + R_2 + jR_1|}{|R_1 + jR_1|}$$

$$= \frac{\left|\dfrac{R_1 + R_2}{R_1} + j1\right|}{|1 + j1|} \approx \frac{1}{\sqrt{2}}\left(\frac{R_1 + R_2}{R_1}\right)$$

and

$$\left|H\left(j\frac{1}{R_2C_1}\right)\right|_{\alpha=0} = \frac{|R_1 + jR_1|}{|R_1 + R_2 + jR_1|}$$

$$= \frac{|1 + j1|}{\left|\dfrac{R_1 + R_2}{R_1} + j1\right|} \approx \sqrt{2}\left(\frac{R_1}{R_1 + R_2}\right).$$

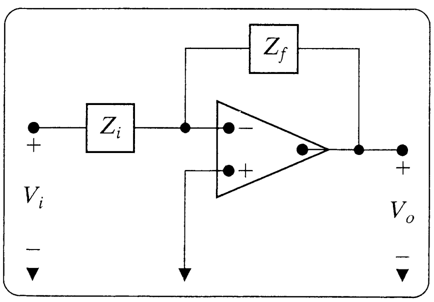
NOTE ◈ *Assess your understanding of this Practical Perspective by trying Chapter Problems 15.54 and 15.55.*

- A cascade of second-order low-pass op amp filters (Fig. 15.21) with 1 Ω resistors and capacitor values chosen to produce each factor in the Butterworth polynomial will produce an even-order Butterworth low-pass filter. Adding a prototype low-pass op amp filter will produce an odd-order Butterworth low-pass filter. (See page 725.)
- A cascade of second-order high-pass op amp filters (Fig. 15.25) with 1 F capacitors and resistor values chosen to produce each factor in the Butterworth polynomial will produce an even-order Butterworth high-pass filter. Adding a prototype high-pass op amp filter will produce an odd-order Butterworth high-pass filter. (See page 732.)
- For both high- and low-pass Butterworth filters, frequency and magnitude scaling can be used to shift the cutoff frequency from 1 rad/s and to include realistic component values in the design. Cascading an inverting amplifier will produce a nonunity passband gain. (See page 726.)
- Butterworth low-pass and high-pass filters can be cascaded to produce Butterworth bandpass filters of any order n. Butterworth low-pass and high-pass filters can be combined in parallel with a summing amplifier to produce a Butterworth bandreject filter of any order n. (See page 733.)
- If a high-Q, or narrowband, bandpass, or bandreject filter is needed, the cascade or parallel combination will not work. Instead, the circuits shown in Figs. 15.26 and 15.29 are used with the appropriate design equations. Typically, capacitor values are chosen from those commercially available, and the design equations are used to specify the resistor values. (See page 733.)

PROBLEMS

15.1 Find the transfer function V_o/V_i for the circuit shown in Fig. P15.1 if Z_f is the equivalent impedance of the feedback circuit, Z_i is the equivalent impedance of the input circuit, and the operational amplifier is ideal.

Figure P15.1



15.2

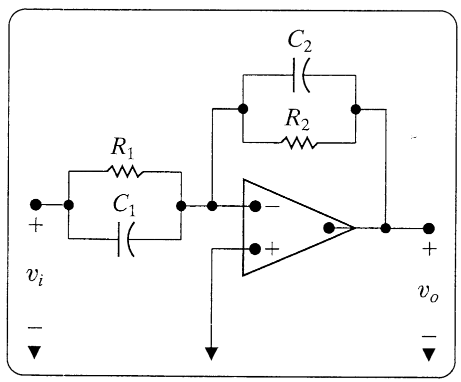
a) Use the results of Problem 15.1 to find the transfer function of the circuit shown in Fig. P15.2.

b) What is the gain of the circuit as $\omega \to 0$?

c) What is the gain of the circuit as $\omega \to \infty$?

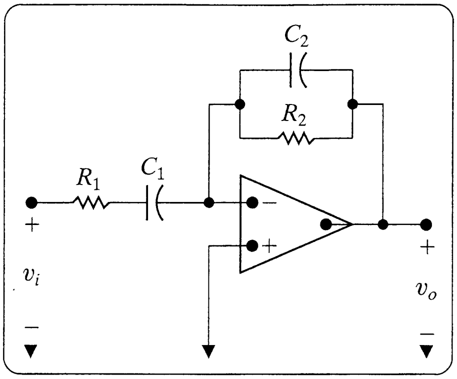
d) Do your answers to (b) and (c) make sense in terms of known circuit behavior?

Figure P15.2



15.3 Repeat Problem 15.2, using the circuit shown in Fig. P15.3.

Figure P15.3



15.4 ❖ a) Using the circuit in Fig. 15.1, design a low-pass filter with a passband gain of 10 dB and a cutoff frequency of 1 kHz. Assume a 750 nF capacitor is available.

b) Draw the circuit diagram and label all components.

15.5 ❖ a) Use the circuit in Fig. 15.4 to design a high-pass filter with a cutoff frequency of 8 kHz and a passband gain of 14 dB. Use a 3.9 nF capacitor in the design.

b) Draw the circuit diagram of the filter and label all the components.

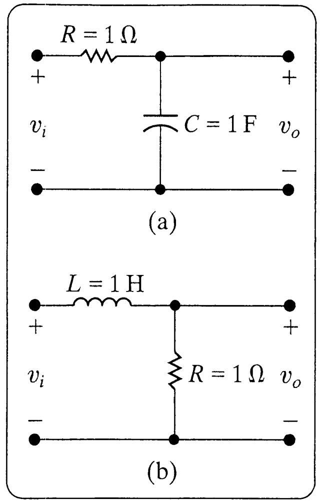
15.6 The voltage transfer function of either low-pass prototype filter shown in Fig. P15.6 is

$$H(s) = \frac{1}{s+1}.$$

Show that if either circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H'(s) = \frac{1}{(s/k_f)+1}.$$

Figure P15.6



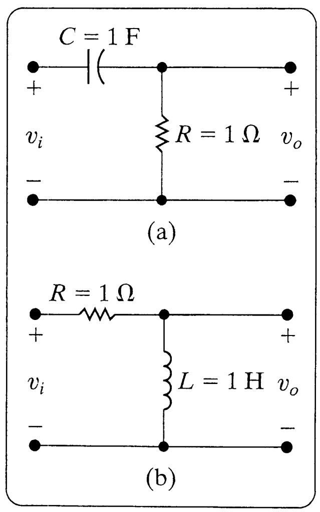
15.7 The voltage transfer function for either high-pass prototype filter shown in Fig. P15.7 is

$$H(s) = \frac{s}{s+1}.$$

Show that if either circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H'(s) = \frac{(s/k_f)}{(s/k_f)+1}.$$

Figure P15.7



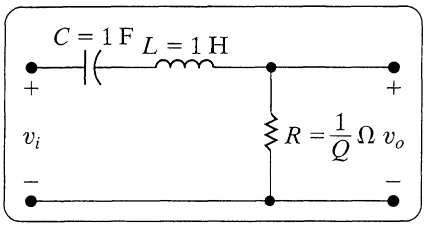
15.8 The voltage transfer function of the prototype bandpass filter shown in Fig. P15.8 is

$$H(s) = \frac{\left(\frac{1}{Q}\right)s}{s^2 + \left(\frac{1}{Q}\right)s + 1}.$$

Show that if the circuit is scaled in both magnitude and frequency, the scaled transfer function is

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left(\frac{s}{k_f}\right)^2 + \left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right) + 1}.$$

Figure P15.8



15.9 a) Specify the component values for the prototype passive bandpass filter described in Problem 15.8 if the quality factor of the filter is 20.

b) Specify the component values for the bandpass filter described in Problem 15.8 if the quality factor is 20; the center, or resonant, frequency is 40 krad/s; and the impedance at resonance is 5 kΩ.

c) Draw a circuit diagram of the scaled filter and label all the components.

15.10 An alternative to the prototype bandpass filter illustrated in Fig. P15.8 is to make $\omega_o = 1$ rad/s, $R = 1\ \Omega$, and $L = Q$ henrys.

a) What is the value of C in the prototype filter circuit?

b) What is the transfer function of the prototype filter?

c) Use the alternative prototype circuit just described to design a passive bandpass filter that has a quality factor of 16, a center frequency of 25 krad/s, and an impedance of 10 kΩ at resonance.

d) Draw a diagram of the scaled filter and label all the components.

e) Use the results obtained in Problem 15.8 to write the transfer function of the scaled circuit.

15.11 ❖ The passive bandpass filter illustrated in Fig. 14.22 has two prototype circuits. In the first prototype circuit, $\omega_o = 1$ rad/s, $C = 1$ F, $L = 1$ H, and $R = Q$ ohms. In the second prototype circuit, $\omega_o = 1$ rad/s, $R = 1\ \Omega$, $C = Q$ farads, and $L = (1/Q)$ henrys.

a) Use one of these prototype circuits (your choice) to design a passive bandpass filter that has a quality factor of 25 and a center frequency of 50 krad/s. The resistor R is 40 kΩ.

b) Draw a circuit diagram of the scaled filter and label all components.

15.12 The transfer function for the bandreject filter shown in Fig. 14.28(a) is

$$H(s) = \frac{s^2 + \left(\frac{1}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)s + \left(\frac{1}{LC}\right)}.$$

Show that if the circuit is scaled in both magnitude and frequency, the transfer function of the scaled circuit is equal to the transfer function of the unscaled circuit with s replaced by (s/k_f), where k_f is the frequency scale factor.

15.13 Show that the observation made in Problem 15.12 with respect to the transfer function for the circuit in Fig. 14.28(a) also applies to the bandreject filter circuit (lower one) in Fig. 14.31.

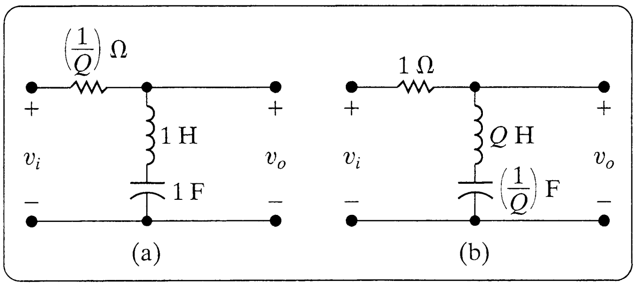
15.14 The passive bandreject filter illustrated in Fig. 14.28(a) has the two prototype circuits shown in Fig. P15.14.

a) Show that for both circuits, the transfer function is

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}.$$

b) Write the transfer function for a bandreject filter that has a center frequency of 10 krad/s and a quality factor of 8.

Figure P15.14

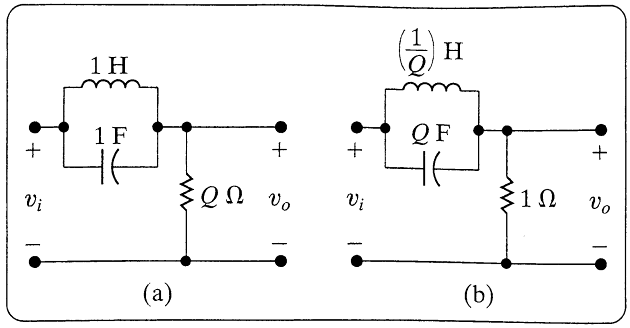


15.15 The two prototype versions of the passive bandreject filter shown in Fig. 14.31 (lower circuit) are shown in Fig. P15.15(a) and (b).

Show that the transfer function for either version is

$$H(s) = \frac{s^2 + 1}{s^2 + \left(\frac{1}{Q}\right)s + 1}.$$

Figure P15.15



15.16 Scale the bandpass filter in Problem 14.15 so that the center frequency is 200 kHz and the quality factor is still 8, using a 2.5 nF capacitor. Determine the values of the resistor, the inductor, and the two cutoff frequencies of the scaled filter.

15.17 Scale the bandreject filter in Problem 14.24 to get a center frequency of 50 krad/s, using a 200 μH inductor. Determine the values of the resistor, the capacitor, and the bandwidth of the scaled filter.

15.18 The circuit in Fig. P13.26 is scaled so that the 1 Ω resistor is replaced by a 1 kΩ resistor and the 1 F capacitor is replaced by a 200 nF capacitor.

a) What is the scaled value of L?

b) What is the expression for i_o in the scaled circuit?

15.19 Scale the circuit in Problem 13.29 so that the 50 Ω resistor is increased to 5 kΩ and the frequency of the voltage response is increased by a factor of 5000. Find $v_o(t)$.

15.20 a) Show that if the low-pass filter circuit illustrated in Fig. 15.1 is scaled in both magnitude and frequency, the transfer function of the scaled circuit is the same as Eq. 15.1 with s replaced by s/k_f, where k_f is the frequency scale factor.

b) In the prototype version of the low-pass filter circuit in Fig. 15.1, $\omega_c = 1$ rad/s, $C = 1$ F, $R_2 = 1\,\Omega$, and $R_1 = 1/K$ ohms. What is the transfer function of the prototype circuit?

c) Using the result obtained in (a), derive the transfer function of the scaled filter.

15.21 a) Show that if the high-pass filter illustrated in Fig. 15.4 is scaled in both magnitude and frequency, the transfer function is the same as Eq. 15.4 with s replaced by s/k_f, where k_f is the frequency scale factor.

b) In the prototype version of the high-pass filter circuit in Fig. 15.4, $\omega_c = 1$ rad/s, $R_1 = 1\ \Omega$, $C = 1$ F, and $R_2 = K$ ohms. What is the transfer function of the prototype circuit?

c) Using the result in (a), derive the transfer function of the scaled filter.

15.22 ❖ P a) Using 0.1 μF capacitors, design an active broadband first-order passband filter that has a lower cutoff frequency of 1000 Hz, an upper cutoff frequency of 5000 Hz, and a passband gain of 0 dB. Use prototype versions of the low-pass and high-pass filters in the design process (see Problems 15.20 and 15.21).

b) Write the transfer function for the scaled filter.

c) Use the transfer function derived in part (b) to find $H(j\omega_o)$, where ω_o is the center frequency of the filter.

d) What is the passband gain (in decibels) of the filter at ω_o?

e) Using a computer program of your choice, make a Bode magnitude plot of the filter.

15.23 ❖ P a) Using 10 nF capacitors, design an active broadband first-order bandreject filter with a lower cutoff frequency of 400 Hz, an upper cutoff frequency of 4000 Hz, and a passband gain of 0 dB. Use the prototype filter circuits introduced in Problems 15.20 and 15.21 in the design process.

b) Draw the circuit diagram of the filter and label all the components.

c) What is the transfer function of the scaled filter?

d) Evaluate the transfer function derived in (c) at the center frequency of the filter.

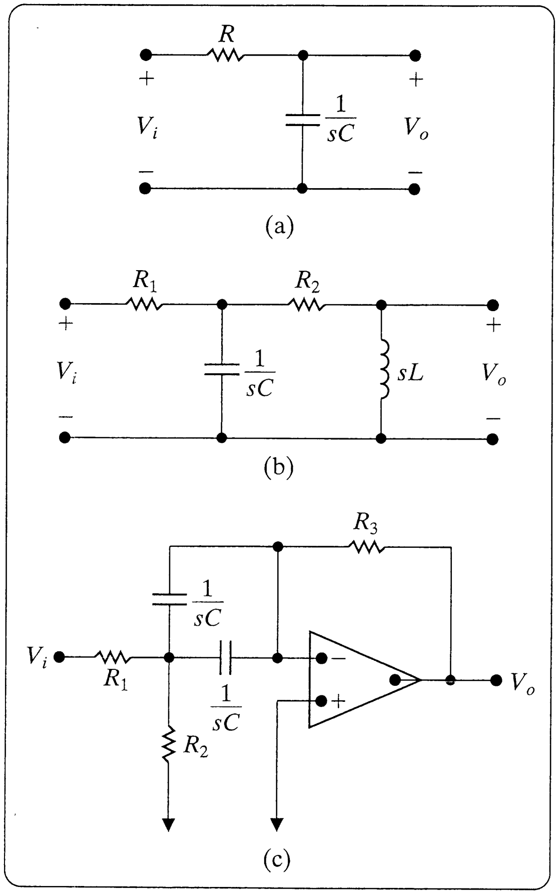
e) What is the gain (in decibels) at the center frequency?

f) Using a computer program of your choice, make a Bode magnitude plot of the filter transfer function.

15.24 For circuits consisting of resistors, capacitors, inductors, and op amps, $|H(j\omega)|^2$ involves only even powers of ω. To illustrate this, compute $|H(j\omega)|^2$ for the three circuits in Fig. P15.24 when

$$H(s) = \frac{V_o}{V_i}.$$

Figure P15.24



15.25 Design a unity-gain bandpass filter, using a cascade connection, to give a center frequency of 200 Hz and a bandwidth of 1000 Hz. Use 5 μF capacitors. Specify f_{c1}, f_{c2}, R_L, and R_H.

15.26 Design a parallel bandreject filter with a center frequency of 1000 rad/s, a bandwidth of 4000 rad/s, and a passband gain of 6. Use 0.2 μF capacitors, and specify all resistor values.

15.27 The purpose of this problem is to illustrate the advantage of an nth-order low-pass Butterworth filter over the cascade of n identical low-pass sections by calculating the slope (in decibels per decade) of each magnitude plot at the corner frequency ω_c. To facilitate the calculation, let y represent the magnitude of the plot (in decibels), and let $x = \log_{10}\omega$. Then calculate dy/dx at ω_c for each plot.

a) Show that at the corner frequency ($\omega_c = 1$ rad/s) of an nth-order low-pass prototype Butterworth filter,

$$\frac{dy}{dx} = -10n \text{ dB/dec.}$$

b) Show that for a cascade of n identical low-pass prototype sections, the slope at ω_c is

$$\frac{dy}{dx} = \frac{-20n(2^{1/n} - 1)}{2^{1/n}} \text{ dB/dec.}$$

c) Compute dy/dx for each type of filter for $n = 1, 2, 3, 4$, and ∞.

d) Discuss the significance of the results obtained in part (c).

15.28 a) Determine the order of a low-pass Butterworth filter that has a cutoff frequency of 2000 Hz and a gain of at least −30 dB at 7000 Hz.

b) What is the actual gain, in decibels, at 7000 Hz?

15.29 The circuit in Fig. 15.21 has the transfer function given by Eq. 15.34. Show that if the circuit in Fig. 15.21 is scaled in both magnitude and frequency, the transfer function of the scaled circuit is

$$H'(s) = \frac{\dfrac{1}{R^2C_1C_2}}{\left(\dfrac{s}{k_f}\right)^2 + \dfrac{2}{RC_1}\left(\dfrac{s}{k_f}\right) + \dfrac{1}{R^2C_1C_2}}.$$

15.30 a) Write the transfer function for the prototype low-pass Butterworth filter obtained in Problem 15.28(a).

b) Write the transfer function for the scaled filter in (a) (see Problem 15.29).

c) Check the expression derived in part (b) by using it to calculate the gain (in decibels) at 7000 Hz. Compare your result with that found in Problem 15.28(b).

15.31 ❖ a) Using 1 kΩ resistors and ideal op amps, design a circuit that will implement the low-pass Butterworth filter specified in Problem 15.28. The gain in the passband is one.

b) Construct the circuit diagram and label all component values.

15.32 ❖ a) Using 10 nF capacitors and ideal op amps, design a high-pass unity-gain Butterworth filter with a cutoff frequency of 2 kHz and a gain of at least −48 dB at 500 Hz.

b) Draw a circuit diagram of the filter and label all component values.

15.33 Verify the entries in Table 15.1 for $n = 5$ and $n = 6$.

15.34 The circuit in Fig. 15.25 has the transfer function given by Eq. 15.47. Show that if the circuit is scaled in both magnitude and frequency, the transfer function of the scaled circuit is

$$H'(s) = \frac{\left(\frac{s}{k_f}\right)^2}{\left(\frac{s}{k_f}\right)^2 + \frac{2}{R_2C}\left(\frac{s}{k_f}\right) + \frac{1}{R_1R_2C^2}}.$$

Hence the transfer function of a scaled circuit is obtained from the transfer function of an unscaled circuit by simply replacing s in the unscaled transfer function by s/k_f, where k_f is the frequency scaling factor.

15.35 ❖

a) Using 1 kΩ resistors and ideal op amps, design a low-pass unity-gain Butterworth filter that has a cutoff frequency of 8 kHz and is down at least 48 dB at 32 kHz.

b) Draw a circuit diagram of the filter and label all the components.

15.36 The high-pass filter designed in Problem 15.32 is cascaded with the low-pass filter designed in Problem 15.35.

a) Describe the type of filter formed by this interconnection.

b) Specify the cutoff frequencies, the midfrequency, and the quality factor of the filter.

c) Use the results of Problems 15.28 and 15.33 to derive the scaled transfer function of the filter.

d) Check the derivation of (c) by using it to calculate $H(j\omega_o)$, where ω_o is the midfrequency of the filter.

15.37 ❖

a) Use 20 nF capacitors in the circuit in Fig. 15.26 to design a bandpass filter with a quality factor of 16, a center frequency of 6.4 kHz, and a passband gain of 20 dB.

b) Draw the circuit diagram of the filter and label all the components.

15.38 Show that if $\omega_o = 1$ rad/s and $C = 1$ F in the circuit in Fig. 15.26, the prototype values of R_1, R_2, and R_3 are

$$R_1 = \frac{Q}{K},$$

$$R_2 = \frac{Q}{2Q^2 - K},$$

$$R_3 = 2Q.$$

15.39 ❖

a) Design a broadband Butterworth bandpass filter with a lower cutoff frequency of 500 Hz and an upper cutoff frequency of 4500 Hz. The passband gain of the filter is 20 dB. The gain should be down at least 20 dB at 200 Hz and 11.25 kHz. Use 15 nF capacitors in the high-pass circuit and 10 kΩ resistors in the low-pass circuit.

b) Draw a circuit diagram of the filter and label all the components.

15.40

a) Derive the expression for the scaled transfer function for the filter designed in Problem 15.39.

b) Using the expression derived in (a), find the gain (in decibels) at 200 Hz and 1500 Hz.

c) Do the values obtained in part (b) satisfy the filtering specifications given in Problem 15.39?

15.41 Derive the prototype transfer function for a sixth-order high-pass Butterworth filter by first writing the transfer function for a sixth-order prototype low-pass Butterworth filter and then replacing s by $1/s$ in the low-pass expression.

15.42 The sixth-order Butterworth filter in Problem 15.41 is used in a system where the cutoff frequency is 25 krad/s.

a) What is the scaled transfer function for the filter?

b) Test your expression by finding the gain (in decibels) at the cutoff frequency.

15.43 ❖ The purpose of this problem is to guide you through the analysis necessary to establish a design procedure for determining the circuit components in a filter circuit. The circuit to be analyzed is shown in Fig. P15.43.

a) Analyze the circuit qualitatively and convince yourself that the circuit is a low-pass filter with a passband gain of R_2/R_1.

b) Support your qualitative analysis by deriving the transfer function V_o/V_i. (*Hint:* In deriving the transfer function, represent the resistors with their equivalent conductances, that is, $G_1 = 1/R_1$, and so forth.) To make the transfer function useful in terms of the entries in Table 15.1, put it in the form

$$H(s) = \frac{-Kb_o}{s^2 + b_1 s + b_o}.$$

c) Now observe that we have five circuit components—R_1, R_2, R_3, C_1, and C_2—and three transfer function constraints—K, b_1, and b_o. At first glance, it appears we have two free choices among the five components. However, when we investigate the relationships between the circuit components and the transfer function constraints, we see that if C_2 is chosen, there is an upper limit on C_1 in order for $R_2(G_2)$ to be realizable. With this in mind, show that if $C_2 = 1$ F, the three conductances are given by the expressions

$$G_1 = KG_2;$$

$$G_3 = \left(\frac{b_o}{G_2}\right) C_1;$$

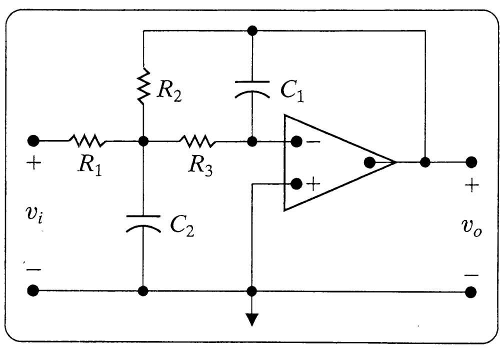
$$G_2 = \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}.$$

For G_2 to be realizable,

$$C_1 \leq \frac{b_1^2}{4b_o(1+K)}.$$

c) Based on the results obtained in (c), outline the design procedure for selecting the circuit components once K, b_o, and b_1 are known.

Figure P15.43



15.44 ❖ Assume the circuit analyzed in Problem 15.43 is part of a third-order low-pass Butterworth filter having a passband gain of 4.

a) If $C_2 = 1$ F in the prototype second-order section, what is the upper limit on C_1?

b) If the limiting value of C_1 is chosen, what are the prototype values of R_1, R_2, and R_3?

c) If the corner frequency of the filter is 2.5 kHz and C_2 is chosen to be 10 nF, calculate the scaled values of C_1, R_1, R_2, and R_3.

d) Specify the scaled values of the resistors and the capacitor in the first-order section of the filter.

e) Construct a circuit diagram of the filter and label all the component values on the diagram.

15.45 ❖ Interchange the Rs and Cs in the circuit in Fig. P15.43; that is, replace R_1 with C_1, R_2 with C_2, R_3 with C_3, C_1 with R_1, and C_2 with R_2.

a) Describe the type of filter implemented as a result of the interchange.

b) Confirm the filter type described in (a) by deriving the transfer function V_o/V_i. Write the transfer function in a form that makes it compatible with Table 15.1.

c) Set $C_2 = C_3 = 1$ F and derive the expressions for C_1, R_1, and R_2 in terms of K, b_1, and b_o. (See Problem 15.43 for the definition of b_1 and b_o.)

d) Assume the filter described in (a) is used in the same type of third-order Butterworth filter that has a passband gain of 8. With $C_2 = C_3 = 1$ F, calculate the prototype values of C_1, R_1, and R_2 in the second-order section of the filter.

15.46 ❖ a) Use the circuits analyzed in Problems 15.43 and 15.45 to implement a broadband bandreject filter having a passband gain of 0 dB, a lower corner frequency of 400 Hz, an upper corner frequency of 6400 Hz, and an attenuation of at least 30 dB at both 1000 Hz and 2560 Hz. Use 10 nF capacitors whenever possible.

b) Draw a circuit diagram of the filter and label all the components.

15.47 a) Derive the transfer function for the bandreject filter described in Problem 15.46.

b) Use the transfer function derived in part (a) to find the attenuation (in decibels) at the center frequency of the filter.

15.48 ❖ The purpose of this problem is to develop the design equations for the circuit in Fig. P15.48 on page 753. (See Problem 15.43 for suggestions on the development of design equations.)

a) Based on a qualitative analysis, describe the type of filter implemented by the circuit.

b) Verify the conclusion reached in (a) by deriving the transfer function V_o/V_i. Write the transfer function in a form that makes it compatible with the entries in Table 15.1.

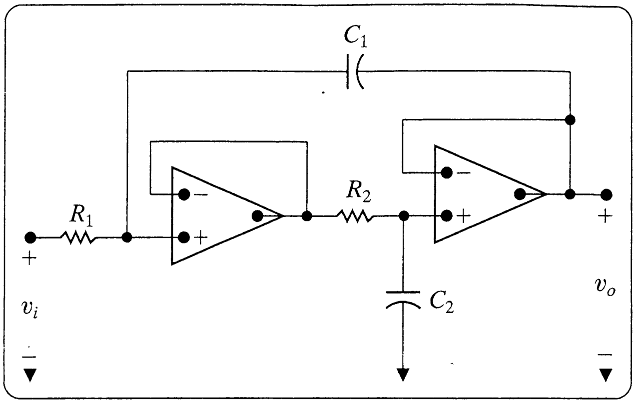
c) How many free choices are there in the selection of the circuit components?

d) Derive the expressions for the conductances $G_1 = 1/R_1$ and $G_2 = 1/R_2$ in terms of C_1, C_2, and the coefficients b_o and b_1. (See Problem 15.43 for the definition of b_o and b_1.)

e) Are there any restrictions on C_1 or C_2?

f) Assume the circuit in Fig. P15.48 is used to design a fourth-order low-pass unity-gain Butterworth filter. Specify the prototype values of R_1 and R_2 in each second-order section if 1 F capacitors are used in the prototype circuit.

Figure P15.48



15.49 ❖ The fourth-order low-pass unity-gain Butterworth filter in Problem 15.48 is used in a system where the cutoff frequency is 3 kHz. The filter has 4.7 nF capacitors.

a) Specify the numerical values of R_1 and R_2 in each section of the filter.

b) Draw a circuit diagram of the filter and label all the components.

15.50 ❖ Interchange the Rs and Cs in the circuit in Fig. P15.48, that is, replace R_1 with C_1, R_2 with C_2, and vice versa.

a) Analyze the circuit qualitatively and predict the type of filter implemented by the circuit.

b) Verify the conclusion reached in (a) by deriving the transfer function V_o/V_i. Write the transfer function in a form that makes it compatible with the entries in Table 15.1.

c) How many free choices are there in the selection of the circuit components?

d) Find R_1 and R_2 as functions of b_o, b_1, C_1, and C_2.

e) Are there any restrictions on C_1 and C_2?

f) Assume the circuit is used in a third-order Butterworth filter of the type found in (a). Specify the prototype values of R_1 and R_2 in the second-order section of the filter if $C_1 = C_2 = 1$ F.

15.51 ❖ a) The circuit in Problem 15.50 is used in a third-order high-pass Butterworth filter that has a cutoff frequency of 5 kHz. Specify the values of R_1 and R_2 if 75 nF capacitors are available to construct the filter.

b) Specify the values of resistance and capacitance in the first-order section of the filter.

c) Draw the circuit diagram and label all the components.

d) Give the numerical expression for the scaled transfer function of the filter.

e) Use the scaled transfer function derived in (d) to find the gain in dB at the cutoff frequency.

15.52 a) Show that the transfer function for a prototype bandreject filter is

$$H(s) = \frac{s^2 + 1}{s^2 + (1/Q)s + 1}.$$

b) Use the result found in (a) to find the transfer function of the filter designed in Example 15.13

15.53 ❖ a) Using the circuit shown in Fig. 15.29, design a narrow-band bandreject filter having a center frequency of 1 kHz and a quality factor of 20. Base the design on $C = 15$ nF.

b) Draw the circuit diagram of the filter and label all component values on the diagram.

c) What is the scaled transfer function of the filter?

15.54 ◆ ❖ Using the circuit in Fig. 15.32(a) design a volume control circuit to give a maximum gain of 20 dB and a gain of 17 dB at a frequency of 40 Hz. Use an 11.1 kΩ resistor and a 100 kΩ potentiometer. Test your design by calculating the maximum gain at $\omega = 0$ and the gain at $\omega = 1/R_2C_1$ using the selected values of R_1, R_2, and C_1.

15.55 ◆ ❖ Use the circuit in Fig. 15.32(a) to design a bass volume control circuit that has a maximum gain of 13.98 dB that drops off 3 dB at 50 Hz.

15.56 ◆ Plot the maximum gain in decibels versus α when $\omega = 0$ for the circuit designed in Problem 15.54. Let α vary from 0 to 1 in increments of 0.1.

15.57 ◆

a) Show that the circuits in Fig. P15.57(a) and (b) are equivalent.

b) Show that the points labeled x and y in Fig. P15.57(b) are always at the same potential.

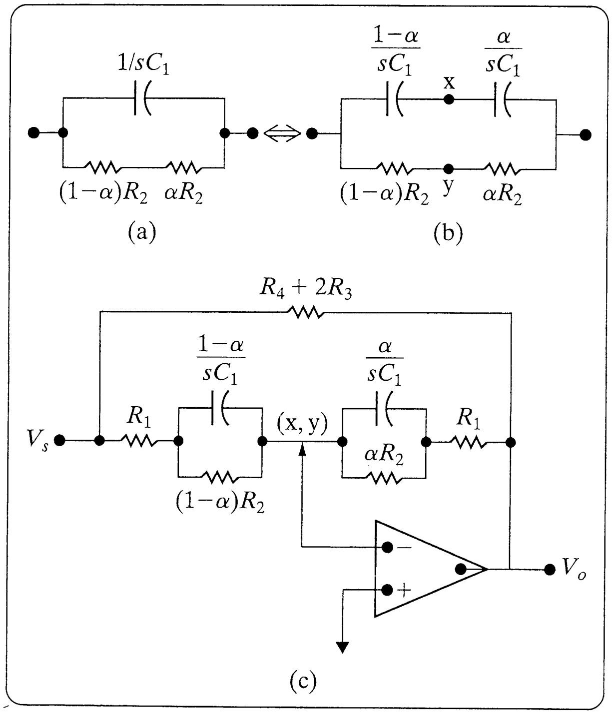
c) Using the information in (a) and (b), show that the circuit in Fig. 15.33 can be drawn as shown in Fig. P15.57(c).

d) Show that the circuit in Fig. P15.57(c) is in the form of the circuit in Fig. 15.2, where

$$Z_i = \frac{R_1 + (1-\alpha)R_2 + R_1R_2C_1s}{1 + R_2C_1s},$$

$$Z_f = \frac{R_1 + \alpha R_2 + R_1R_2C_1s}{1 + R_1R_2C_1s}.$$

Figure P15.57



15.58 ◆ An engineering project manager has received a proposal from a subordinate who claims the circuit shown in Fig. P15.58 could be used as a treble volume control circuit if $R_4 = R_1 + R_3 + 2R_5$. The subordinate further claims that the voltage transfer function for the circuit is

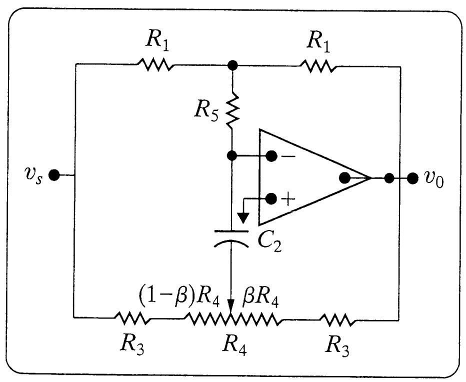
$$H(s) = \frac{V_o}{V_s}$$

$$= \frac{-\{(2R_3 + R_4) + [(1-\beta)R_4 + R_o](\beta R_4 + R_3)C_2s\}}{\{(2R_3 + R_4) + [(1-\beta)R_4 + R_3](\beta R_4 + R_o)C_2s\}}$$

where $R_o = R_1 + R_3 + 2R_5$. Fortunately the project engineer has an electrical engineering undergraduate student as an intern and therefore asks the student to check the subordinate's claim.

The student is asked to check the behavior of the transfer function as $\omega \to 0$; as $\omega \to \infty$; and the behavior when $\omega = \infty$ and β varies between 0 and 1. Based on your testing of the transfer function do you think the circuit could be used as a treble volume control? Explain.

Figure P15.58



15.59 ◆ In the circuit of Fig. P15.58 the component values are $R_1 = R_2 = 20$ kΩ, $R_3 = 5.9$ kΩ, $R_4 = 500$ kΩ, and $C_2 = 2.7$ nF.

a) Calculate the maximum boost in decibels.

b) Calculate the maximum cut in decibels.

c) Is R_4 significantly greater than R_o?

d) When $\beta = 1$, what is the boost in decibels when $\omega = 1/R_3C_2$?

e) When $\beta = 0$, what is the cut in decibels when $\omega = 1/R_3C_2$?

f) Based on the results obtained in (d) and (e), what is the significance of the frequency $1/R_3C_2$ when $R_4 \gg R_0$?

15.60 ◆ Using the component values given in Problem 15.59, plot the maximum gain in decibels versus β when $\omega = 0$. Let β vary from 0 to 1 in increments of 0.1.

APPENDIX H Answers to Selected Problems

Chapter 1

1.1 3.93 MB

1.3 3.5 s

1.6 $362\,\mu\text{m}^2$

1.9 $4 \sin 5000t$ mC

1.12 a) 300 W, A to B

b) 500 W, B to A

c) 200 W, B to A

d) 400 W, A to B

1.17 a) 42.21 mW

b) $12.14\,\mu$J

c) $140.625\,\mu$J

1.24 a) 8.45 s

b) −15.40 W (15.40 W delivered)

c) 31.55 s

d) 15.40 W (15.40 W extracted)

e) $w(0) = 0$ J; $w(10) = 112.5$ J; $w(20) = 200$ J; $w(30) = 112.5$ J; $w(40) = 0$ J

1.26 1740 W

Chapter 2

2.2 $8\ \text{k}\Omega$

2.4 $20\ \Omega$

2.14 a) 0.4 A

b) 1.6 A

c) 120 V

d) $P_{40\Omega} = 160$ W; $P_{300\Omega} = 48$ W; $P_{75\Omega} = 192$ W

e) 400 W

2.16 a) $i_g = 6$ A; $i_o = 2$ A

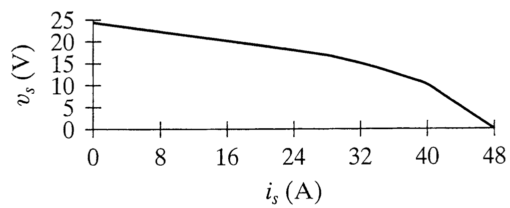
b) 240 V

c) $P_{\text{dev}} = P_{\text{abs}} = 1440$ W

2.21 a) −30 V source; 20 Ω resistor

b) 10 W

2.23 a)



b) 24 V source; 0.25 Ω resistor

c) 19.2 A

d) 96 A

e) 48 A

f) A linear model cannot predict the nonlinear behavior of the voltage source.

2.26 a) $P_{4\Omega} = 900$ W; $P_{20\Omega} = 1620$ W; $P_{5\Omega} = 180$ W; $P_{22.5\Omega} = 360$ W; $P_{15\Omega} = 1500$ W

b) 4560 W

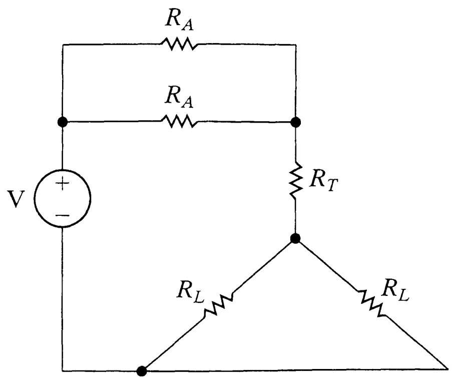
c) $P_{\text{supp}} = 4560\text{ W} = P_{\text{dis}}$

2.29 a) 5.55 V

b) $P_{\text{supp}} = 44.82\text{ mW} = P_{\text{dis}}$

2.34 $i = 385$ mA, so a warning sign should be posted and precautions taken.

2.35



2.36 a) $P_{\text{arm}} = 59.17$ W; $P_{\text{leg}} = 29.59$ W; $P_{\text{trunk}} = 7.40$ W

b) $t_{\text{arm}} = 1414.23$ s; $t_{\text{leg}} = 7071.13$ s; $t_{\text{trunk}} = 70{,}677.37$ s

c) All values are much greater than a few minutes.

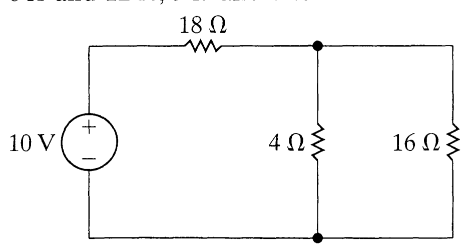
2.37 a) 40 V

b) No, 12 V/800 Ω = 15 mA will only cause a shock.

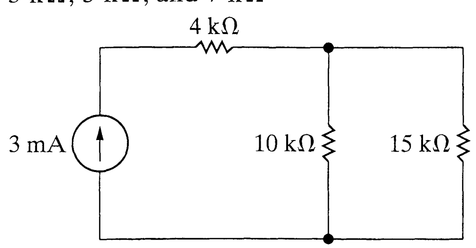
2.38 3000 V

Chapter 3

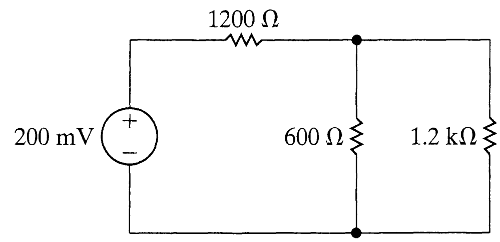
3.1 a) 6 Ω and 12 Ω; 9 Ω and 7 Ω



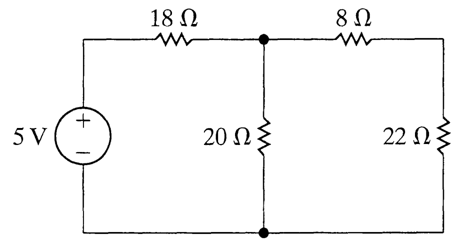
b) 3 kΩ, 5 kΩ, and 7 kΩ



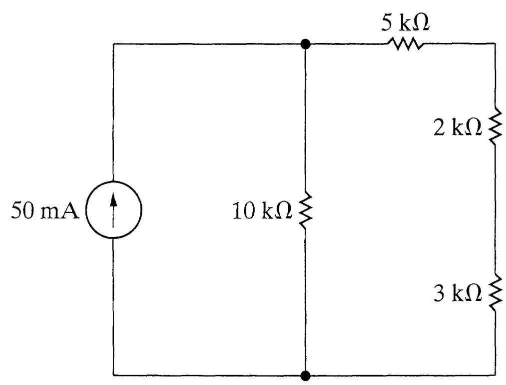
c) 300 Ω, 400 Ω, and 500 Ω



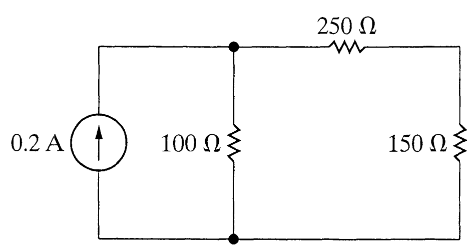
3.2 a) 10 Ω and 40 Ω; 100 Ω and 25 Ω



b) 9 kΩ, 18 kΩ, and 6 kΩ



c) 600 Ω, 200 Ω, and 300 Ω



3.5 a) $21.2\,\Omega$

b) $10\,k\Omega$

c) $1600\,\Omega$

3.6 a) $30\,\Omega$

b) $5\,k\Omega$

c) $80\,\Omega$

3.13 a) 10 V

b) $P_{R_1} = 800$ mW; $P_{R_2} = 200$ mW

c) $R_1 = 1600\,\Omega$; $R_2 = 400\,\Omega$ (minimum values)

3.14 $26.67\,\Omega$

3.21 $R_1 = 800\,\Omega$; $R_2 = 1.6\,k\Omega$; $R_3 = 16\,k\Omega$; $R_4 = 16\,k\Omega$

3.22 a) 15 V, positive at the top

b) 20 mA, right to left

c) 1.5 mA, top to bottom

d) 18 V, positive at the top

3.23 a) 30 mV, positive on the left

b) 4.5 V, positive at the top

c) 0.4 A, bottom to top

3.31 a) $i_m = \dfrac{(10/99)}{10 + (10/99)} i_{\text{meas}} = \dfrac{1}{100} i_{\text{meas}}$

b) $\dfrac{1}{100{,}000}$

c) Yes

3.34 a) $49{,}980\,\Omega$

b) $4980\,\Omega$

c) $230\,\Omega$

d) $5\,\Omega$

3.49 a) $2000\,\Omega$

b) 8.4 mA

c) $800\,\Omega$; 28.8 mW

d) $500\,\Omega$; 2.88 mW

3.53 $v_1 = 23.2$ V; $v_2 = 21.0$ V

3.54 a) Equivalent wye is $5\,\Omega, 20\,\Omega, 4\,\Omega$; $R_{\text{ab}} = 33\,\Omega$.

b) Equivalent delta is $100\,\Omega, 80\,\Omega, 20\,\Omega$; $R_{\text{ab}} = 33\,\Omega$.

c) Convert R_4–R_5–R_6 delta to wye; convert R_3–R_4–R_6 wye to delta.

3.55 $90\,\Omega$

3.71 $R_1 = 1.0372\,\Omega,\ R_2 = 1.1435\,\Omega,\ R_3 = 1.2\,\Omega,\ R_4 = 1.1435\,\Omega,\ R_5 = 1.0372\,\Omega,\ R_a = 0.0259\,\Omega,\ R_b = 0.0068\,\Omega,\ R_c = 0.0068\,\Omega,\ R_d = 0.0259\,\Omega$

3.72 $P_{\text{diss}} = 624$ W$= P_{\text{del}}$

3.73 a) $R_1 = 0.4269\,\Omega,\ R_2 = 0.4617\,\Omega,\ R_3 = 0.48\,\Omega,\ R_4 = 0.4617\,\Omega,\ R_5 = 0.4269\,\Omega,\ R_a = 0.0085\,\Omega,\ R_b = 0.0022\,\Omega,\ R_c = 0.0022\,\Omega,\ R_d = 0.0085\,\Omega$

b) $i_1 = 26.51$ A, $i_1^2 R_1 = 300$ W or 200 W/m; $i_2 = 25.49$ A, $i_2^2 R_2 = 300$ W or 200 W/m; $i_b = 52$ A, $i_b^2 R_b = 6$ W or 200 W/m; $P_{\text{del}} = 1548$ W $= P_{\text{diss}}$

Chapter 4

4.1 a) 11

b) 9

c) 9

d) 7

e) 6

f) 4

g) 6

4.2 a) 2

b) 5

c) 7

d) 1; 4; 7

4.3 a) 7

b) 3

c) 4

d) top mesh; leftmost mesh

4.4 a) 5

b) 3

c) $-i_g + i_{R_1} + i_{R_2} = 0$; $-i_{R_1} + i_{R_4} + i_{R_3} = 0$; $i_{R_5} - i_{R_2} - i_{R_3} = 0$

d) 2

e) $R_1 i_{R_1} + R_3 i_{R_3} - R_2 i_{R_2} = 0$; $R_3 i_{R_3} + R_5 i_{R_5} - R_4 i_{R_4} = 0$

4.6 -5 V

4.9 $v_1 = 120$ V, $v_2 = 96$ V

4.10 a) $i_a = 8$ A, $i_b = 2$ A, $i_c = 6$ A, $i_d = 2$ A, $i_e = 4$ A

b) 360 W

4.19 750 W delivered

4.20 a) $P_{25A} = -8800$ W; $P_{dev} = 8800$ W

b) $P_{84i_\Delta} = 628.32$ W, $P_{40\Omega} = 3097.6$ W, $P_{160\Omega} = 774.4$ W, $P_{10\Omega} = 1960$ W, $P_{20\Omega} = 2247.2$ W, $P_{8\Omega} = 92.48$ W, $P_{diss} = 8800$ W

4.21 10 V

4.26 $v_1 = -37.5$ V, $P_{25V} = 31.25$ W delivered

4.27 25 V

4.31 a) $i_a = 5.6$ A, $i_b = 3.2$ A, $i_c = -2.4$ A

b) $i_a = -8.8$ A, $i_b = -1.6$ A, $i_c = 7.2$ A

4.32 a) $P_{230V} = -1012$ W, $P_{460V} = -16{,}928$ W, $P_{dev} = 17{,}940$ W

b) $P_{abs} = 17{,}940$ W

4.33 259.2 W

4.34 2016 W delivered

4.37 a) -312 W delivered (312 W absorbed)

b) 21,000 W

c) $P_{dev} = 21{,}000$ W $= P_{diss}$

4.38 a) 2 mA

b) 304 mW delivered

c) 0.9 mW delivered

4.43 740 W

4.46 a) $i_a = -4.2$ A, $i_b = 7.4$ A, $i_c = 4.68$ A, $i_d = 11.6$ A, $i_e = 2.72$ A

b) $P_{dev} = 1329.632$ W $= P_{abs}$

4.52 a) Three unknown node voltages, but only two unknown mesh currents, so choose mesh current method.

b) 3.6 W

c) No, since it is straightforward to calculate the voltage drop across the current source from mesh currents.

d) 52.8 W delivered

4.54 a) Node-voltage method because constraint equations are easier to formulate.

b) 480 mW absorbed

4.55 a) -1 mA

b) -1 mA

4.58 a) −0.85 A

b) −0.85 A

4.59 $V_{Th} = 60$ V, $R_{Th} = 10\,\Omega$

4.62 $I_N = 1$ mA, top to bottom; $R_N = 3.75$ kΩ

4.63 a) 51.3 V

b) −5%

4.65 $v_{Th} = 160$ V, negative at the top; $R_{Th} = 56.4$ kΩ

4.72 1.43 μA

4.75 2.5 Ω and 22.5 Ω

4.76 a) 6 Ω

b) 24 W

4.87 a) 50 V

b) 250 W

4.88 30 V

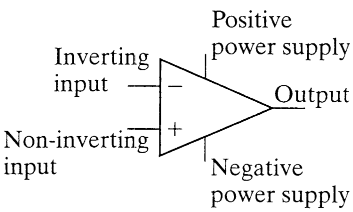
4.101 $v_1 = 39.583$ V, $v_2 = 102.5$ V

4.102 $v_1 = 37.5$ V, $v_2 = 105$ V

4.103 $v_1 = 52.083$ V, $v_2 = 117.5$ V

Chapter 5

5.1 a)



b) Input resistance is infinite, so $i_n = 0$ A

c) Gain in the linear region is infinite, so $v_p - v_n = 0$

d) −10 V

5.2 a) −15 V (saturated)

b) −10 V

c) −4 V

d) 7 V

e) $-1.08\text{ V} \le v_a \le 4.92\text{ V}$

5.3 −1 mA

5.6 a) Use a single 20 kΩ resistor in the forward path and 6 series-connected 20 kΩ resistors in the feedback path, OR, use 6 parallel-connected 20 kΩ resistors in the forward path and a single 20 kΩ resistor in the feedback path.

b) ±18 V

5.7 a) Non-inverting amplifier

b) 9 V

5.16 a) Inverting summing amplifier

b) −4 V

c) $-2.5\text{ V} \le v_b \le -1.3\text{ V}$

5.17 a) 14 V

b) $3.818\text{ V} \le v_a \le 9.273\text{ V}$

5.19 $R_a = 24$ kΩ, $R_b = 12$ kΩ, $R_c = 8$ kΩ, $R_d = 6$ kΩ

5.22 a) Non-inverting amplifier

b) $2v_s$

c) $-6\text{ V} \le v_s \le 4\text{ V}$

5.23 a) 10.54 V

b) $-4.554\text{ V} \le v_g \le 4.554\text{ V}$

c) 181.76 kΩ

5.29 20 kΩ

5.30 a) 4.2 V

b) $-771\text{ mV} \le v_c \le 1371\text{ mV}$

5.31 a) −1.53 V

b) 33.5 kΩ

c) 80 kΩ

5.37 a) 24.98

b) −0.04

c) 624.5

5.38 $19.93\text{ k}\Omega \le R_x \le 20.07\text{ k}\Omega$

5.39 a) −19.9915 V

b) 403.2 μV

c) 5002.02 Ω

d) −20; 0 V; 5000 Ω

5.48 a) 2 kΩ

b) 12 mΩ

Chapter 6

6.1

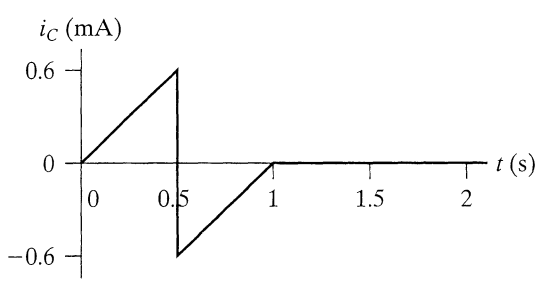
a)

$i = 0$	$t < 0$
$i = 50t$ A	$0 \le t \le 5$ ms
$i = 0.5 - 50t$ A	$5 \text{ ms} \le t \le 10$ ms
$i = 0$	$10 \text{ ms} \le t \le \infty$

b)

$v = 0$	$t < 0$
$v = 1$ V	$0 \le t \le 5$ ms
$v = -1$ V	$5 \text{ ms} \le t \le 10$ ms
$v = 0$	$10 \text{ ms} \le t \le \infty$
$p = 0$	$t < 0$
$p = 50t$ W	$0 \le t \le 5$ ms
$p = 50t - 0.5$ W	$5 \text{ ms} \le t \le 10$ ms
$p = 0$	$10 \text{ ms} \le t \le \infty$
$w = 0$	$t < 0$
$w = 25t^2$ J	$0 \le t \le 5$ ms
$w = 25t^2 - 0.5t + 0.0025$ J	$5 \text{ ms} \le t \le 10$ ms
$w = 0$	$10 \text{ ms} \le t \le \infty$

6.14



6.15

a) $8 \times 10^5 t + 12$ V

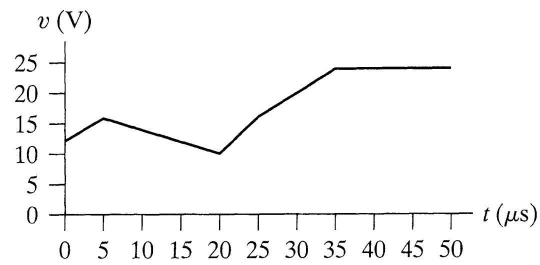
b) $-4 \times 10^5 t + 18$ V

c) $12 \times 10^5 t - 14$ V

d) $8 \times 10^5 t - 4$ V

e) 24 V

f)



6.21 15 H

6.22

a) $6e^{-t}$ A

b) $4e^{-t} - 2$ A

c) $2e^{-t} + 2$ A

d) 36 J

e) 54 J

f) 18 J

g) 54 J − 36 J = 18 J (checks)

6.26 2 μF with 25 V positive at the top

6.27

a) $10e^{-t}$ V

b) $6.67e^{-t} - 2.67$ V

c) $3.33e^{-t} + 2.67$ V

d) 100 μJ

e) 132 μJ

f) 32 μJ

g) 100 μJ + 32 μJ = 132 μJ (checks)

6.34

a) $16\frac{di_2}{dt} + 32i_2 = 2\frac{di_g}{dt}$

b) $-16e^{-t} + 32e^{-2t} + 32e^{-t} - 32e^{-2t} = 16e^{-t}$

c) $34e^{-t} - 4e^{-2t}$ V, $\quad t \ge 0$

d) 30 V; yes

6.38

a) 50 mH; 2.4

b) 0.2×10^{-6} Wb/A

6.39 0.8 nWb/A; 1.2 nWb/A

6.47

a) 2721.6 mJ

b) 2721.6 mJ

c) 518.4 mJ

d) 518.4 mJ

6.48 a) -4.5 A
b) no

6.49 $v = \frac{1}{3}v_s(t) + v(0)$

6.51 The finger causes no change in the output voltage.

Chapter 7

7.1 a) $i_1(0^-) = 5$ mA, $i_2(0^-) = 15$ mA
b) $i_1(0^+) = 5$ mA, $i_2(0^+) = -5$ mA
c) $5e^{-20,000t}$ mA
d) $-5e^{-20,000t}$ mA
e) The current in a resistor can change instantaneously.

7.2 a) 4 A
b) 80 ms
c) $i = 4e^{-12.5t}$ A, $t \geq 0$; $v_1 = -180e^{-12.5t}$ V, $t \geq 0^+$; $v_2 = -200e^{-12.5t}$ V, $t \geq 0^+$
d) 56.89%

7.3 a) 0
b) 160 mA
c) 65 mA
d) 160 mA
e) 225 mA
f) 0
g) $160e^{-200t}$ mA, $t \geq 0$
h) 0
i) -3.2 V
j) 0
k) $-3.2e^{-200t}$ V, $t \geq 0^+$
l) $225 - 160e^{-200t}$ mA, $t \geq 0^+$

7.21 a) $i = 1.6e^{-50t}$ mA, $t \geq 0^+$, $v_1 = 32e^{-50t} + 8$ V, $t \geq 0$, $v_2 = -8e^{-50t} + 8$ V, $t \geq 0$
b) 800 μJ
c) $w_{\text{trapped}} = 160\ \mu$J, $w_{\text{diss}} = 640\ \mu$J

7.22 a) $9.9e^{-1000t}$ mA
b) 42.14%

7.33 a) $i_L(t) = -2 - 3e^{-5000t}$ A, $t \geq 0$; $v_o(t) = 48 - 48e^{-5000t}$ V, $t \geq 0$
b) $v_L(0^+) = 60$ V, $v_o(0^+) = 0$ V

7.34 a) $5 + 15e^{-1000t}$ A, $t \geq 0$
b) $50 - 450e^{-1000t}$ V, $t \geq 0^+$

7.35 $i_o(t) = 2.5 + 7.5e^{-1250t}$ A, $t \geq 0$; $v(t) = -150e^{-1250t}$ V, $t \geq 0$

7.47 $-60 + 90e^{-2000t}$ V, $t \geq 0$

7.48 a) $60 - 60e^{-100t}$ V, $t \geq 0$
b) $1 - 0.6e^{-100t}$ mA, $t \geq 0^+$
c) $1 + 2.4e^{-100t}$ mA, $t \geq 0^+$
d) $4 - 2.4e^{-100t}$ mA, $t \geq 0^+$
e) 3.4 mA

7.61 a) 50 V
b) -24 V
c) 0.1 μs
d) -18.5 A
e) $-24 + 74e^{-10^7 t}$ V, $t \geq 0$
f) $-18.5e^{-10^7 t}$ A, $t \geq 0^+$

7.62 a) 90 V
b) -60 V
c) 1000 μs
d) 916.3 μs

7.63 a) -13 mA
b) -12 mA
c) 80 μs
d) $-12 - e^{-12,500t}$ mA, $t \geq 0$

7.64 a) $6 - 6e^{-10t}$ A, $t \geq 0$
b) $100e^{-10t}$ V, $t \geq 0$
c) $7 - 7e^{-10t}$ A, $t \geq 0$
d) $-1 + e^{-10t}$ A, $t \geq 0$
e) Yes, check initial conditions and final values.

7.66
a) $4 - 4e^{-20t}$ A, $t \geq 0$
b) $80e^{-20t}$ V, $t \geq 0^+$
c) $2.4 - 2.4e^{-20t}$ A, $t \geq 0$
d) $1.6 - 1.6e^{-20t}$ A, $t \geq 0$
e) Yes, check initial conditions and final values.

7.69 $-100 + 130e^{-200t}$ V, $t \geq 0$

7.70
a) 7.5 A
b) 6.14 A
c) 226.48 mA
d) −220.73 V
e) −110.4 V

7.84
a) 2.25
b) 272.1 μs

7.85 83.09 ms

7.89 80 ms

7.90 $v_o = 8 - 1600t$ V, $0 \leq t \leq t_{sat}$;
$v_2 = 11e^{-200t} - 15$ V, $0 \leq t \leq t_{sat}$;
$v_f = 23 - 1600t - 11e^{-200t}$ V, $0 \leq t \leq t_{sat}$

7.103
a) 1.091 MΩ
b) 0.29 s

7.104
a) 8.55 flashes/min
b) 559.3 kΩ

7.105
a) 24.3 flashes/min
b) 99.06 mA
c) $43.39 per year

Chapter 8

8.1
a) −100 rad/s, −400 rad/s
b) overdamped
c) 1562.5 Ω
d) $-160 + j120$ rad/s, $-160 - j120$ rad/s
e) 1250 Ω

8.2
a) $R = 8$ kΩ, $L = 40$ H, $\alpha = 625$ rad/s, $\omega_0 = 500$ rad/s
b) $i_R(t) = 4e^{-1000t} - e^{-250t}$ mA, $t \geq 0^+$;
$i_C(t) = 0.2e^{-250t} - 3.2e^{-1000t}$ mA, $t \geq 0^+$;
$i_L(t) = 0.8e^{-250t} - 0.8e^{-1000t}$ mA, $t \geq 0^+$

8.7
a) 40 mH
b) 625 Ω
c) 100 V
d) 40 mA
e) $e^{-20,000t}(40\cos 15,000t + 220\sin 15,000t)$ mA, $t \geq 0$

8.8
a) $\alpha = 500$ rad/s, $\omega_0 = 400$ rad/s, $L = 1.5625$ H, $C = 4\ \mu$F, $A_1 = -31$ mA, $A_2 = 76$ mA
b) $38.75e^{-200t} - 23.75e^{-800t}$ V, $t \geq 0$
c) $155e^{-200t} - 95e^{-800t}$ mA, $t \geq 0^+$
d) $-124e^{-200t} + 19e^{-800t}$ mA, $t \geq 0$

8.9
a) $R = 1$ kΩ, $C = 1\ \mu$F, $D_1 = 6000$ V/s, $D_2 = 8$ V
b) $(-3000t + 2)e^{-500t}$ mA, $t \geq 0^+$

8.16 $5e^{-2000t} + 10e^{-8000t}$ V, $t \geq 0$

8.17 $15e^{-2000t}\cos 3122.5t + 4.8e^{-2000t}\sin 3122.5t$ V, $t \geq 0$

8.18 $15e^{-4000t}$ V, $t \geq 0$

8.24 $60 - 120e^{-5000t} + 15e^{-20,000t}$ mA, $t \geq 0$

8.25 $60 - 105e^{-8000t}\cos 6000t - 90e^{-8000t}\sin 6000t$ mA, $t \geq 0$

8.26 $60 - 750,000te^{-10,000t} - 105e^{-10,000t}$ mA, $t \geq 0$

8.40 $-2.4e^{-4t}\cos 3t - 3.2e^{-4t}\sin 3t$ A, $t \geq 0^+$

8.42
a) $10,000te^{-10^5 t} + 0.1e^{-10^5 t}$ A, $t \geq 0$
b) $25 \times 10^5 te^{-10^5 t} + 50e^{-10^5 t}$ V, $t \geq 0$

8.46
a) 24 V
b) −53,248 V/s
c) $-40 - 25.6e^{-160t} + 89.6e^{-640t}$ V, $t \geq 0^+$

8.51
a) $v_o = 10t^2$ V, $0 \leq t \leq 0.5^-$ s;
$v_o = -5t^2 + 15t - 3.75$ V, 0.5^+ s $\leq t \leq t_{sat}$;
$v_{o1} = -1.6t$ V, $0 \leq t \leq 0.5^-$ s;
$v_{o1} = 0.8t - 1.2$ V, 0.5^+ s $\leq t \leq t_{sat}$
b) 3.5 s

8.52 $v_o = 10 - 20e^{-t} + 10e^{-2t}$ V, $0 \leq t \leq 0.5$ s; $v_o = -5 + 19.42e^{-(t-0.5)} - 12.87e^{-2(t-0.5)}$ V, $0.5 \leq t \leq \infty$; $v_{o1} = -0.8 + 0.8e^{-2t}$ V, $0 \leq t \leq 0.5$ s; $v_{o1} = 0.4 - 0.91e^{-2(t-0.5)}$ V, $0.5 \leq t \leq \infty$

8.60 a) 55.23 μs

b) 262.42 V

c) $t_{\max} = 53.63\ \mu$s, $v(t_{\max}) = 262.15$ V

8.61 a) 40 mJ

b) −27,808.04 V

c) 568.15 V

Chapter 9

9.1 a) 600 Hz

b) 1.67 ms

c) 10 V

d) 6 V

e) −53.13° or −0.9273 rad

f) 662.64 μs

g) 245.97 μs

9.2 $\dfrac{V_m}{2}$

9.3 a) 40 V

b) 50 Hz

c) 314.159 rad/s

d) 1.05 rad

e) 60°

f) 20 ms

g) 6.67 ms

h) $40 \cos 100\pi t$ V

i) 8.33 ms

j) 16.67 ms

9.9 a) $-195.72e^{-1066.67t} + 200\cos(800t - 11.87°)$ mA

b) $-195.72e^{-1066.67t}$ mA, $200\cos(800t - 11.87°)$ mA

c) 28.39 mA

d) 0.2 A, 800 rad/s, −11.87°

e) 36.87°

9.12 a) 1000 Hz

b) 0°

c) −90°

d) 8 Ω

e) 1.27 mH

f) $j8\ \Omega$

9.13 a) 314,159.27 rad/s

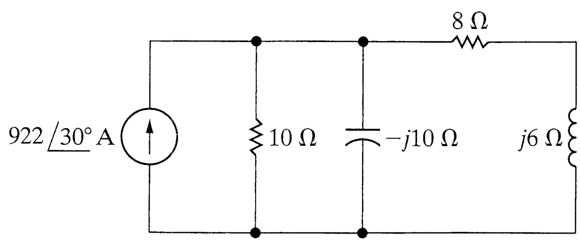
b) 90°

c) −15.92 Ω

d) 0.2 μF

e) $-j15.92\ \Omega$

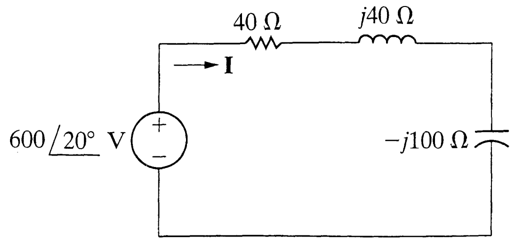
9.14 a)



b) $5000.25\ \angle 17.47°$ V

c) $5000.25\cos(2 \times 10^4 t + 17.47°)$ V

9.15 a)



b) $8.32 \angle 76.31^\circ$ A

c) $8.32 \cos(8000t + 76.31^\circ)$ A

9.16 $17.68 \cos(50t - 135^\circ)$ mA

9.17 a) $200 \angle 36.87^\circ$ mS

b) 160 mS

c) 120 mS

d) 10 A

9.18 $42.43 \cos(2000t + 45^\circ)$ V

9.19 a) $50 \angle 45^\circ$ Ω

b) 50 μs

9.20 $42.43 \cos(50{,}000t + 45^\circ)$ V

9.35 0.667 Ω

9.39 $10 \cos 200t$ mV

9.42 $227.68 \angle -18.43^\circ$ V, positive at the top; $3.6 + j10.8$ Ω

9.43 $2.2 \angle 0^\circ$ A, flowing top to bottom; $30 - j40$ Ω

9.51 $188.43 \angle -42.88^\circ$ V

9.54 $36 \cos 2000t$ V

9.56 $j80$ V $= 80 \angle 90^\circ$ V

9.59 $4 - j2$ A $= 4.47 \angle -26.57^\circ$ A

9.66 a) $i_g = 5 \cos(5000t - 36.87^\circ)$ A, $i_L = \cos(5000t - 180^\circ)$ A

b) 0.5

c) 9.0 mJ, 12 mJ

9.67 $850 + j850$ V, positive at the top; $85 + j85$ Ω

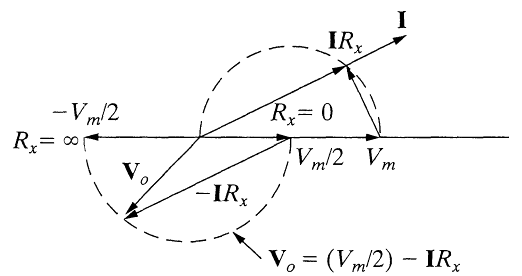
9.71 $512 \angle 60^\circ$ kΩ

9.75 a) $247.11 \angle 1.68^\circ$ V

b) −32 Ω, $241.13 \angle 1.90^\circ$ V

c) −26.90 Ω

9.76 As R_x varies from 0 to ∞, the amplitude of v_o remains constant and its phase angle changes from 0 to -180°.



9.85 a) $\mathbf{I}_1 = 24 \angle 0^\circ$ A, $\mathbf{I}_2 = 2.04 \angle 0^\circ$ A, $\mathbf{I}_3 = 21.96 \angle 0^\circ$ A, $\mathbf{I}_4 = 19.40 \angle 0^\circ$ A, $\mathbf{I}_5 = 4.6 \angle 0^\circ$ A, $\mathbf{I}_6 = 2.55 \angle 0^\circ$ A

b) $0.42 \angle 0^\circ$ A

9.86 a) 0 A

b) 0.436 A

c) When the two loads are equal, more current is drawn from the primary.

Chapter 10

10.1 a) $P = 409.58$ W (abs); $Q = 286.79$ VAR (abs)

b) $P = 103.53$ W (abs); $Q = -386.37$ VAR (del)

c) $P = -1000$ W (del); $Q = -1732.05$ VAR (del)

d) $P = -250$ W (del); $Q = 433.01$ VAR (abs)

13.49 a) $\frac{200}{s+200}$ pole at −200 rad/s

b) $\frac{s}{s+200}$ zero at 0, pole at −200 rad/s

c) $\frac{s}{s+8000}$ zero at 0, pole at −8000 rad/s

d) $\frac{8000}{s+8000}$ pole at −8000 rad/s

e) $\frac{100}{s+500}$ pole at −500 rad/s

13.57 $1-e^{-t}$ V, $0 \leq t \leq 1$; $(e-1)e^{-t}$ V, $1 \leq t \leq \infty$

13.58 e^{-t} V, $0 \leq t \leq 1$; $(1-e)e^{-t}$ V, $1 \leq t \leq \infty$

13.76 $16.97\cos(3t+8.13^\circ)$ V

13.77 a) $\frac{-10^4 s}{(s+400)(s+1000)}$

b) $13.13\cos(400t-156.8^\circ)$ V

13.83 a) 80 V

b) 20 V

c) 0 V

d) $32\delta(t)\ \mu$A

e) 16 V

f) 4 V

g) 20 V

13.84 a) 0.8 A

b) 0.6 A

c) 0.2 A

d) −0.6 A

e) $0.6e^{-2\times10^6 t}u(t)$ A

f) $-0.6e^{-2\times10^6 t}u(t)$ A

g) $-1.6\times10^{-3}\delta(t)-7200e^{-2\times10^6 t}u(t)$ V

13.89 a) $i_2(0^-)=i_2(0^+)=0$ A;
$i_L(0^-)=i_L(0^+)=35.36$ A

b) $V_0 =$

$$\frac{1440\pi(122.92\sqrt{2}s-3000\pi\sqrt{2})}{(s+1475\pi)(s^2+14{,}400\pi^2)}+\frac{300\sqrt{2}}{s+1475\pi};$$

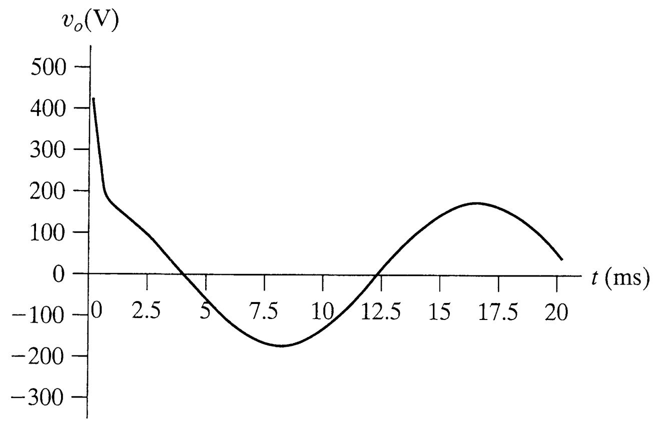
$v_0(t) =$

$252.89e^{-1475\pi t}+172.62\cos(120\pi t+6.85^\circ)$ V;

$v_0(0^+)=424.26$ V

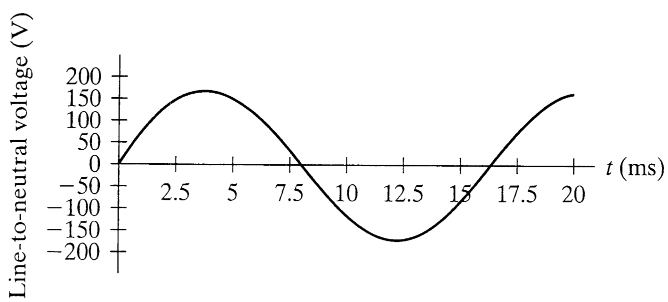
c) $\mathbf{V}_0 = 122.06\ \angle 6.85^\circ$ V (rms)

d)



13.90 a) $-20.58e^{-1475\pi t}+172.62\cos(120\pi t-83.15^\circ)$ V

b)



c) Voltage spikes in Problem 13.89 but not here

Chapter 14

14.1 a) 2021.27 Hz

b) $H(j\omega_c)=0.71\ \angle -45^\circ$;
$H(j0.2\omega_c)=0.98\ \angle -11.31^\circ$;
$H(j5\omega_c)=0.2\ \angle -78.69^\circ$

c) $v_o(\omega_c)=7.07\cos(12{,}700t-45^\circ)$ V;
$v_o(0.2\omega_c)=9.81\cos(2540t-11.31^\circ)$ V;
$v_o(5\omega_c)=1.96\cos(63{,}500t-78.69^\circ)$ V

14.2 a) $31.42\ \Omega$

b) 895.77 Hz

14.9 a) $5.31\ \text{k}\Omega$

b) 333.86 Hz

14.10 a) $125\ \Omega$

b) $3\ \text{k}\Omega$

14.15 a) 100 krad/s

b) 15.92 kHz

c) 8

d) 93.95 krad/s

e) 14.96 kHz

f) 106.45 krad/s

g) 16.94 kHz

h) 12.5 krad/s or 1.99 kHz

14.16 a) $R = 5\ \text{k}\Omega,\ L = 50$ mH

b) $f_{c1} = 2.88$ kHz, $f_{c2} = 3.52$ kHz

c) 636.62 Hz

14.24 a) 1 Mrad/s

b) 159.15 kHz

c) 15

d) 967.22 krad/s

e) 153.94 kHz

f) 1.03 Mrad/s

g) 164.55 kHz

h) 10.61 kHz

14.25 a) $R = 397.89\ \Omega,\ L = 3.17$ mH

b) $f_{c1} = 3.62$ kHz, $f_{c2} = 4.42$ kHz

c) 800 Hz

14.31 a) $L = 0.39$ H, $C = 0.1\ \mu$F

b) $0.948|V_{\text{peak}}|$

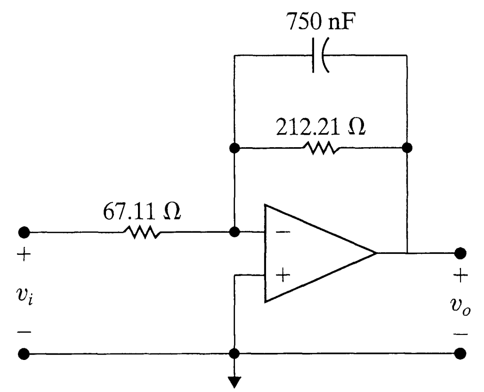
c) $0.344|V_{\text{peak}}|$

14.32 $L = 0.225$ H, $C = 0.057\ \mu$F, $0.344|V_{\text{peak}}|$

Chapter 15

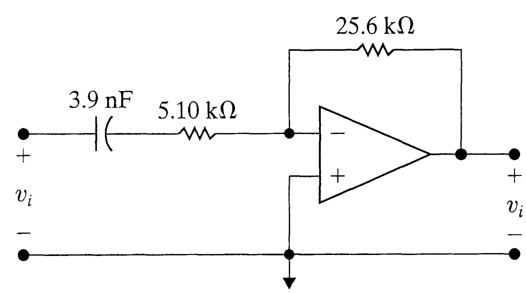
15.4 a) $R_1 = 67.11\ \Omega,\quad R_2 = 212.21\ \Omega$

b)



15.5 a) $R_1 = 5.10\ \text{k}\Omega,\quad R_2 = 25.6\ \text{k}\Omega$

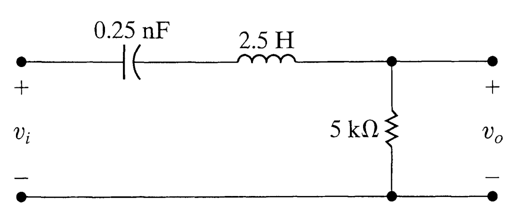
b)



15.9 a) $R = 0.05\ \Omega,\quad L = 1$ H $\quad C = 1$ F

b) $R = 5\ \text{k}\Omega,\quad L = 2.5$ H $\quad C = 0.25$ nF

c)

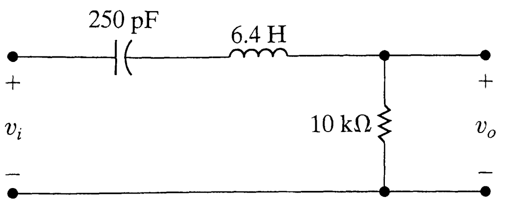


15.10 a) $1/Q$

b) $\dfrac{s/Q}{s^2 + s/Q + 1}$

c) $R = 10\ \text{k}\Omega, \quad L = 6.4\ \text{H}, \quad C = 250\ \text{pF}$

d)



e) $\dfrac{1562.5s}{s^2 + 1562.5s + 625 \times 10^6}$

15.25 $f_{c1} = 38.52$ Hz, $f_{c2} = 1038.52$ Hz, $R_L = 30.65\ \Omega$, $R_H = 826.43\ \Omega$

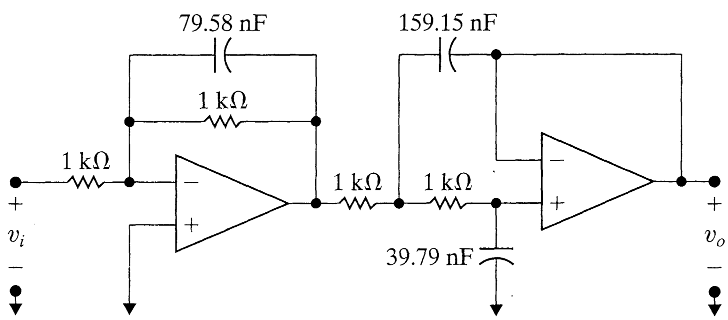
15.26 $R_L = 21.18\ \text{k}\Omega,\ R_H = 1.18\ \text{k}\Omega,\ \dfrac{R_f}{R_i} = 6$

15.28 a) 3

b) −32.65 dB

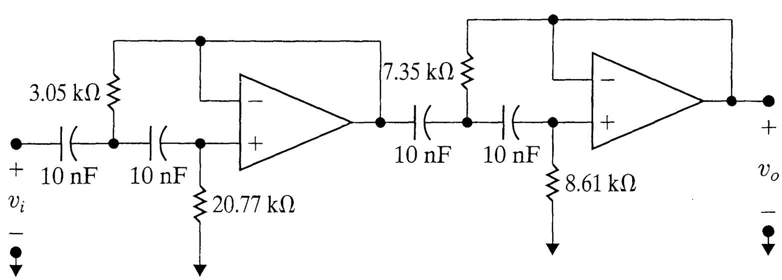
15.31 a) First-order circuit: $R = 1\ \text{k}\Omega$, $C = 79.58$ nF
Second-order circuit: $R = 1\ \text{k}\Omega$,
$C_1 = 159.15$ nF, $C_2 = 39.79$ nF

b)



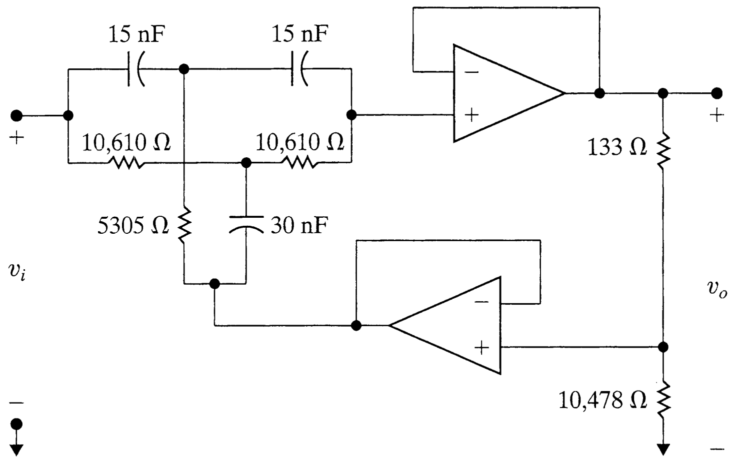
15.32 a) First section: $R_1 = 3.05\ \text{k}\Omega$, $R_2 = 20.77\ \text{k}\Omega$
Second section: $R_1 = 7.35\ \text{k}\Omega$, $R_2 = 8.61\ \text{k}\Omega$

b)



15.53 a) $R = 10{,}610\ \Omega;\quad \sigma R = 10{,}478\ \Omega;$
$(1-\sigma)R = 133\ \Omega$

b)



c) $$\frac{s^2 + 4 \times 10^6\pi^2}{s^2 + 100\pi s + 4 \times 10^6\pi^2}$$

15.54 $C = 39.79$ nF, $|H(j\omega)|_{\max} = 20.01$ dB,
$|H(j/R_2C_1)| = 17.04$ dB

15.55 Choose $R_1 = 100\ \text{k}\Omega$, then $R_2 = 400\ \text{k}\Omega$,
$C_1 = 7.96$ nF.

Chapter 16

16.1 a) $\omega_{oa} = 31{,}415.93$ rad/s, $\omega_{ob} = 3978.87$ rad/s

b) $f_{oa} = 5$ kHz, $f_{ob} = 25$ kHz

c) $a_{va} = 0$, $a_{vb} = 25$ V

d) $a_{va} = 0;\quad a_{ka} = 0$ for k even;

$a_{ka} = \frac{-80}{\pi k}\sin\frac{\pi k}{2}$ for k odd;

$b_{ka} = 0$ for k even; $b_{ka} = \frac{240}{\pi k}$ for k odd;

$a_{ab} = 25$; $a_{kb} = \frac{200}{\pi k}\sin\frac{\pi k}{4}$ for all k;

$b_{kb} = 0$ for all k

e) $$v_a(t) = \frac{80}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} -\frac{1}{n}\sin\frac{\pi n}{2}\cos n\omega_o t + \frac{3}{n}\sin n\omega_o t \ \text{V};$$

$$v_b(t) = 25 + \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi}{4}\cos n\omega_o t \ \text{V}$$

APPENDIX B Complex Numbers

Complex numbers were invented to permit the extraction of the square roots of negative numbers. Complex numbers simplify the solution of problems that would otherwise be very difficult. The equation $x^2 + 8x + 41 = 0$, for example, has no solution in a number system that excludes complex numbers. These numbers, and the ability to manipulate them algebraically, are extremely useful in circuit analysis.

B.1 ◆ Notation

There are two ways to designate a complex number: with the cartesian, or rectangular, form or with the polar, or trigonometric, form. In the **rectangular form**, a complex number is written in terms of its real and imaginary components; hence

$$n = a + jb, \tag{B.1}$$

where a is the real component, b is the imaginary component, and j is by definition $\sqrt{-1}$.[1]

In the **polar form**, a complex number is written in terms of its magnitude (or modulus) and angle (or argument); hence

$$n = ce^{j\theta}, \tag{B.2}$$

where c is the magnitude, θ is the angle, e is the base of the natural logarithm, and, as before, $j = \sqrt{-1}$. In the literature, the symbol $\angle\theta^\circ$ is frequently used in place of $e^{j\theta}$; that is, the polar form is written

$$n = c \angle\theta^\circ. \tag{B.3}$$

Although Eq. B.3 is more convenient in printing text material, Eq. B.2 is of primary importance in mathematical operations because the rules for manipulating an exponential quantity are well known. For example, because $(y^x)^n = y^{xn}$, then $(e^{j\theta})^n = e^{jn\theta}$; because $y^{-x} = 1/y^x$, then $e^{-j\theta} = 1/e^{j\theta}$; and so forth.

Because there are two ways of expressing the same complex number, we need to relate one form to the other. The transition from the polar to the rectangular form makes use of Euler's identity:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta. \tag{B.4}$$

[1] You may be more familiar with the notation $i = \sqrt{-1}$. In electrical engineering, i is used as the symbol for current, and hence in electrical engineering literature, j is used to denote $\sqrt{-1}$.

A complex number in polar form can be put in rectangular form by writing

$$\begin{aligned} ce^{j\theta} &= c(\cos\theta + j\sin\theta) \\ &= c\cos\theta + jc\sin\theta \\ &= a + jb. \end{aligned} \tag{B.5}$$

The transition from rectangular to polar form makes use of the geometry of the right triangle, namely,

$$\begin{aligned} a + jb &= (\sqrt{a^2 + b^2})e^{j\theta} \\ &= ce^{j\theta}, \end{aligned} \tag{B.6}$$

where

$$\tan\theta = b/a. \tag{B.7}$$

It is not obvious from Eq. B.7 in which quadrant the angle θ lies. The ambiguity can be resolved by a graphical representation of the complex number.

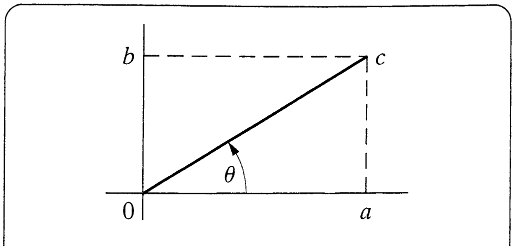


Figure B.1 The graphical representation of $a + jb$ when a and b are both positive.

B.2 ◆ The Graphical Representation of a Complex Number

A complex number is represented graphically on a complex-number plane, which uses the horizontal axis for plotting the real component and the vertical axis for plotting the imaginary component. The angle of the complex number is measured counterclockwise from the positive real axis. The graphical plot of the complex number $n = a + jb = c\ \angle\theta^\circ$, if we assume that a and b are both positive, is shown in Fig. B.1.

This plot makes very clear the relationship between the rectangular and polar forms. Any point in the complex-number plane is uniquely defined by giving either its distance from each axis (that is, a and b) or its radial distance from the origin (c) and the angle of the radial measurement θ.

It follows from Fig. B.1 that θ is in the first quadrant when a and b are both positive, in the second quadrant when a is negative and b is positive, in the third quadrant when a and b are both negative, and in the fourth quadrant when a is positive and b is negative. These observations are illustrated in Fig. B.2, where we have plotted $4 + j3$, $-4 + j3$, $-4 - j3$, and $4 - j3$.

Note that we can also specify θ as a clockwise angle from the positive real axis. Thus in Fig. B.2(c) we could also designate $-4 - j3$ as $5\ \angle -143.13^\circ$. In Fig. B.2(d) we observe that $5\ \angle 323.13^\circ = 5\ \angle -36.87^\circ$. It is customary to express θ in terms of negative values when θ lies in the third or fourth quadrant.

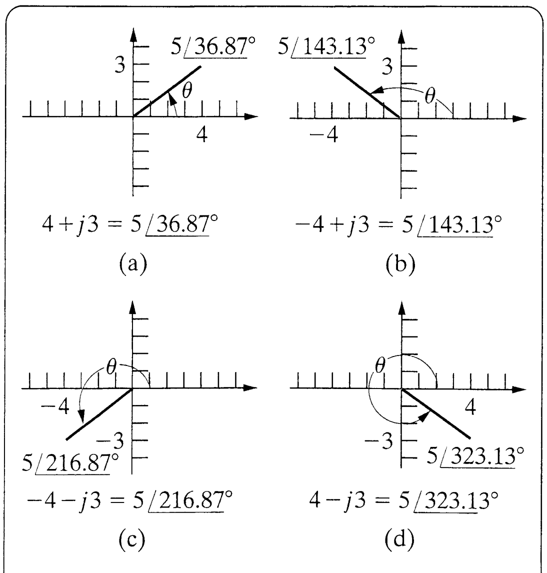


Figure B.2 The graphical representation of four complex numbers.

The graphical interpretation of a complex number also shows the relationship between a complex number and its conjugate. The **conjugate of a complex number** is formed by reversing the sign of its imaginary component. Thus the conjugate of $a+jb$ is $a-jb$, and the conjugate of $-a+jb$ is $-a-jb$. When we write a complex number in polar form, we form its conjugate simply by reversing the sign of the angle θ. Therefore the conjugate of $c \angle\theta^\circ$ is $c \angle -\theta^\circ$. The conjugate of a complex number is designated with an asterisk. In other words, n^* is understood to be the conjugate of n. Figure B.3 shows two complex numbers and their conjugates plotted on the complex-number plane.

Note that conjugation simply reflects the complex numbers about the real axis.

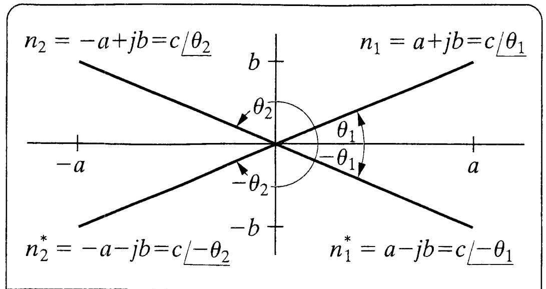


Figure B.3 The complex numbers n_1 and n_2 amd their conjugates n_1^* and n_2^*

B.3 ◆ Arithmetic Operations

Addition (Subtraction)

To add or subtract complex numbers, we must express the numbers in rectangular form. Addition involves adding the real parts of the complex numbers to form the real part of the sum, and the imaginary parts to form the imaginary part of the sum. Thus, if we are given

$$n_1 = 8 + j16$$

and

$$n_2 = 12 - j3,$$

then

$$n_1 + n_2 = (8 + 12) + j(16 - 3) = 20 + j13.$$

Subtraction follows the same rule. Thus

$$n_2 - n_1 = (12 - 8) + j(-3 - 16) = 4 - j19.$$

If the numbers to be added or subtracted are given in polar form, they are first converted to rectangular form. For example, if

$$n_1 = 10 \angle 53.13^\circ$$

and

$$n_2 = 5 \angle -135^\circ,$$

then

$$\begin{aligned} n_1 + n_2 &= 6 + j8 - 3.535 - j3.535 \\ &= (6 - 3.535) + j(8 - 3.535) \\ &= 2.465 + j4.465 = 5.10 \angle 61.10^\circ, \end{aligned}$$

and

$$n_1 - n_2 = 6 + j8 - (-3.535 - j3.535)$$
$$= 9.535 + j11.535$$
$$= 14.966 \angle 50.42^\circ.$$

Multiplication (Division)

Multiplication or division of complex numbers can be carried out with the numbers written in either rectangular or polar form. However, in most cases, the polar form is more convenient. As an example, let's find the product $n_1 n_2$ when $n_1 = 8 + j10$ and $n_2 = 5 - j4$. Using the rectangular form, we have

$$n_1 n_2 = (8 + j10)(5 - j4) = 40 - j32 + j50 + 40$$
$$= 80 + j18$$
$$= 82 \angle 12.68^\circ.$$

If we use the polar form, the multiplication $n_1 n_2$ becomes

$$n_1 n_2 = (12.81 \angle 51.34^\circ)(6.40 \angle -38.66^\circ)$$
$$= 82 \angle 12.68^\circ$$
$$= 80 + j18.$$

The first step in dividing two complex numbers in rectangular form is to multiply the numerator and denominator by the conjugate of the denominator. This reduces the denominator to a real number. We then divide the real number into the new numerator. As an example, let's find the value of n_1/n_2, where $n_1 = 6 + j3$ and $n_2 = 3 - j1$. We have

$$\frac{n_1}{n_2} = \frac{6 + j3}{3 - j1} = \frac{(6 + j3)(3 + j1)}{(3 - j1)(3 + j1)}$$
$$= \frac{18 + j6 + j9 - 3}{9 + 1}$$
$$= \frac{15 + j15}{10} = 1.5 + j1.5$$
$$= 2.12 \angle 45^\circ.$$

In polar form, the division of n_1 by n_2 is

$$\frac{n_1}{n_2} = \frac{6.71 \angle 26.57^\circ}{3.16 \angle -18.43^\circ} = 2.12 \angle 45^\circ$$
$$= 1.5 + j1.5.$$

B.4 ◆ Useful Identities

In working with complex numbers and quantities, the following identities are very useful:

$$\pm j^2 = \mp 1, \tag{B.8}$$

$$(-j)(j) = 1, \tag{B.9}$$

$$j = \frac{1}{-j}, \tag{B.10}$$

$$e^{\pm j\pi} = -1, \tag{B.11}$$

$$e^{\pm j\pi/2} = \pm j. \tag{B.12}$$

Given that $n = a + jb = c\,\angle\theta^\circ$, it follows that

$$nn^* = a^2 + b^2 = c^2, \tag{B.13}$$

$$n + n^* = 2a, \tag{B.14}$$

$$n - n^* = j2b, \tag{B.15}$$

$$n/n^* = 1\,\angle 2\theta^\circ. \tag{B.16}$$

B.5 ◆ The Integer Power of a Complex Number

To raise a complex number to an integer power k, it is easier to first write the complex number in polar form. Thus

$$\begin{aligned} n^k &= (a + jb)^k \\ &= (ce^{j\theta})^k = c^k e^{jk\theta} \\ &= c^k(\cos k\theta + j\sin k\theta). \end{aligned}$$

For example,

$$\begin{aligned} (2e^{j12^\circ})^5 &= 2^5 e^{j60^\circ} = 32e^{j60^\circ} \\ &= 16 + j27.71, \end{aligned}$$

and

$$\begin{aligned} (3 + j4)^4 &= (5e^{j53.13^\circ})^4 = 5^4 e^{j212.52^\circ} \\ &= 625e^{j212.52^\circ} \\ &= -527 - j336. \end{aligned}$$

B.6 ◆ The Roots of a Complex Number

To find the kth root of a complex number, we must recognize that we are solving the equation

$$x^k - ce^{j\theta} = 0, \tag{B.17}$$

which is an equation of the kth degree and therefore has k roots.

To find the k roots, we first note that

$$ce^{j\theta} = ce^{j(\theta+2\pi)} = ce^{j(\theta+4\pi)} = \cdots. \tag{B.18}$$

It follows from Eqs. B.17 and B.18 that

$$x_1 = (ce^{j\theta})^{1/k} = c^{1/k}e^{j\theta/k}, \tag{B.19}$$

$$x_2 = [ce^{j(\theta+2\pi)}]^{1/k} = c^{1/k}e^{j(\theta+2\pi)/k}, \tag{B.20}$$

$$x_3 = [ce^{j(\theta+4\pi)}]^{1/k} = c^{1/k}e^{j(\theta+4\pi)/k}, \tag{B.21}$$

$$\vdots$$

We continue the process outlined by Eqs. B.19, B.20, and B.21 until the roots start repeating. This will happen when the multiple of π is equal to $2k$. For example, let's find the four roots of $81e^{j60^\circ}$. We have

$$x_1 = 81^{1/4}e^{j60/4} = 3e^{j15^\circ},$$

$$x_2 = 81^{1/4}e^{j(60+360)/4} = 3e^{j105^\circ},$$

$$x_3 = 81^{1/4}e^{j(60+720)/4} = 3e^{j195^\circ},$$

$$x_4 = 81^{1/4}e^{j(60+1080)/4} = 3e^{j285^\circ},$$

$$x_5 = 81^{1/4}e^{j(60+1440)/4} = 3e^{j375^\circ} = 3e^{j15^\circ}.$$

Here, x_5 is the same as x_1, so the roots have started to repeat. Therefore we know the four roots of $81e^{j60^\circ}$ are the values given by x_1, x_2, x_3, and x_4.

It is worth noting that the roots of a complex number lie on a circle in the complex-number plane. The radius of the circle is $c^{1/k}$. The roots are uniformly distributed around the circle, the angle between adjacent roots being equal to $2\pi/k$ radians, or $360/k$ degrees. The four roots of $81e^{j60^\circ}$ are shown plotted in Fig. B.4.

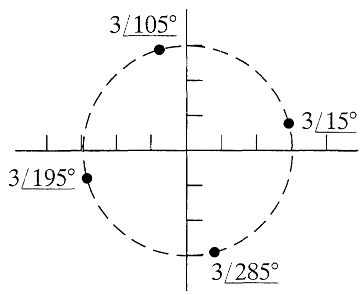


Figure B.4 The four roots of $81e^{j60^\circ}$.

APPENDIX C More on Magnetically Coupled Coils and Ideal Transformers

C.1 ◆ Equivalent Circuits for Magnetically Coupled Coils

At times, it is convenient to model magnetically coupled coils with an equivalent circuit that does not involve magnetic coupling. Consider the two magnetically coupled coils shown in Fig. C.1. The resistances R_1 and R_2 represent the winding resistance of each coil. The goal is to replace the magnetically coupled coils inside the shaded area with a set of inductors that are not magnetically coupled. Before deriving the equivalent circuits, we must point out an important restriction: The voltage between terminals b and d must be zero. In other words, if terminals b and d can be shorted together without disturbing the voltages and currents in the original circuit, the equivalent circuits derived in the material that follows can be used to model the coils. This restriction is imposed because, while the equivalent circuits we develop both have four terminals, two of those four terminals are shorted together. Thus, the same requirement is placed on the original circuits.

We begin developing the circuit models by writing the two equations that relate the terminal voltages v_1 and v_2 to the terminal currents i_1 and i_2. For the given references and polarity dots,

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \tag{C.1}$$

and

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}. \tag{C.2}$$

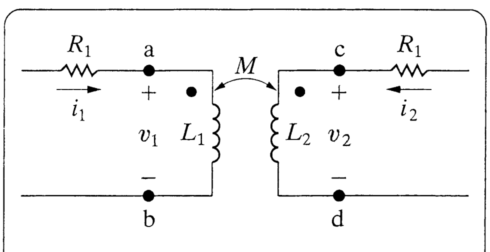


Figure C.1 The circuit used to develop an equivalent circuit for magnetically coupled coils.

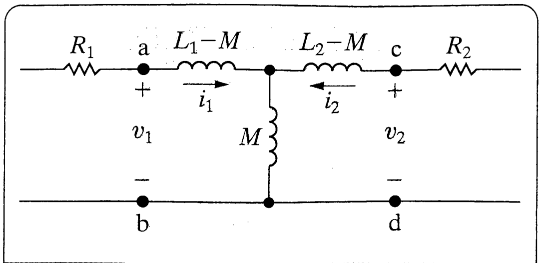


Figure C.2 The T-equivalent circuit for the magnetically coupled coils of Fig. C.1.

The T-Equivalent Circuit

To arrive at an equivalent circuit for these two magnetically coupled coils, we seek an arrangement of inductors that can be described by a set of equations equivalent to Eqs. C.1 and C.2. The key to finding the arrangement is to regard Eqs. C.1 and C.2 as mesh-current equations with i_1 and i_2 as the mesh variables. Then we need one mesh with a total inductance of L_1 H and a second mesh with a total inductance of L_2 H. Furthermore, the two meshes must have a common inductance of M H. The T-arrangement of coils shown in Fig. C.2 satisfies these requirements.

You should verify that the equations relating v_1 and v_2 to i_1 and i_2 reduce to Eqs. C.1 and C.2. Note the absence of magnetic coupling between the inductors and the zero voltage between b and d.

The π-Equivalent Circuit

We can derive a π-equivalent circuit for the magnetically coupled coils shown in Fig. C.1. This derivation is based on solving Eqs. C.1 and C.2 for the derivatives di_1/dt and di_2/dt and then regarding the resulting expressions as a pair of node-voltage equations. Using Cramer's method for solving simultaneous equations, we obtain expressions for di_1/dt and di_2/dt:

$$\frac{di_1}{dt} = \frac{\begin{vmatrix} v_1 & M \\ v_2 & L_2 \end{vmatrix}}{\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}} = \frac{L_2}{L_1L_2 - M^2}v_1 - \frac{M}{L_1L_2 - M^2}v_2; \tag{C.3}$$

$$\frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & v_1 \\ M & v_2 \end{vmatrix}}{L_1L_2 - M^2} = \frac{-M}{L_1L_2 - M^2}v_1 + \frac{L_1}{L_1L_2 - M^2}v_2. \tag{C.4}$$

Now we solve for i_1 and i_2 by multiplying both sides of Eqs. C.3 and C.4 by dt and then integrating:

$$i_1 = i_1(0) + \frac{L_2}{L_1L_2 - M^2}\int_0^t v_1 d\tau - \frac{M}{L_1L_2 - M^2}\int_0^t v_2 d\tau \tag{C.5}$$

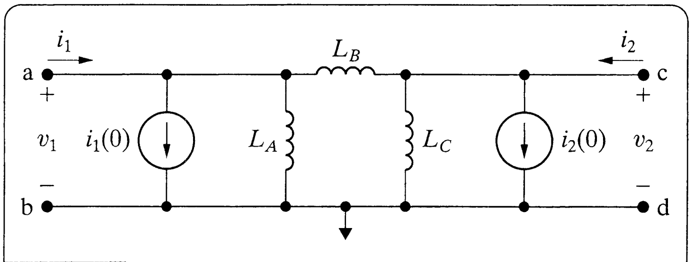


Figure C.3 The circuit used to derive the π-equivalent circuit for magnetically coupled coils.

and

$$i_2 = i_2(0) - \frac{M}{L_1L_2 - M^2}\int_0^t v_1 d\tau + \frac{L_1}{L_1L_2 - M^2}\int_0^t v_2 d\tau \quad \text{(C.6)}$$

If we regard v_1 and v_2 as node voltages, Eqs. C.5 and C.6 describe a circuit of the form shown in Fig. C.3.

All that remains to be done in deriving the π-equivalent circuit is to find L_A, L_B, and L_C as functions of L_1, L_2, and M. We easily do so by writing the equations for i_1 and i_2 in Fig. C.3 and then comparing them with Eqs. C.5 and C.6. Thus

$$\begin{aligned} i_1 &= i_1(0) + \frac{1}{L_A}\int_0^t v_1 d\tau + \frac{1}{L_B}\int_0^t (v_1 - v_2) d\tau \\ &= i_1(0) + \left(\frac{1}{L_A} + \frac{1}{L_B}\right)\int_0^t v_1 d\tau - \frac{1}{L_B}\int_0^t v_2 d\tau \end{aligned} \quad \text{(C.7)}$$

and

$$\begin{aligned} i_2 &= i_2(0) + \frac{1}{L_C}\int_0^t v_2 d\tau + \frac{1}{L_B}\int_0^t (v_2 - v_1) d\tau \\ &= i_2(0) - \frac{1}{L_B}\int_0^t v_1 d\tau + \left(\frac{1}{L_B} + \frac{1}{L_C}\right)\int_0^t v_2 d\tau. \end{aligned} \quad \text{(C.8)}$$

Then

$$\frac{1}{L_B} = \frac{M}{L_1L_2 - M^2}, \quad \text{(C.9)}$$

$$\frac{1}{L_A} = \frac{L_2 - M}{L_1L_2 - M^2}, \quad \text{(C.10)}$$

$$\frac{1}{L_C} = \frac{L_1 - M}{L_1L_2 - M^2}. \quad \text{(C.11)}$$

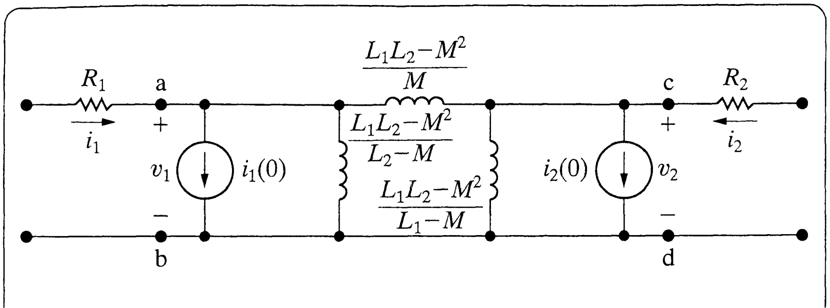


Figure C.4 The π-equivalent circuit for the magnetically coupled coils of Fig. C.1.

When we incorporate Eqs. C.9–C.11 into the circuit shown in Fig. C.3, the π-equivalent circuit for the magnetically coupled coils shown in Fig. C.1 is as shown in Fig. C.4.

Note that the initial values of i_1 and i_2 are explicit in the π-equivalent circuit but implicit in the T-equivalent circuit. We are focusing on the sinusoidal steady-state behavior of circuits containing mutual inductance, so we can assume that the initial values of i_1 and i_2 are zero. We can thus eliminate the current sources in the π-equivalent circuit, and the circuit shown in Fig. C.4 simplifies to the one shown in Fig. C.5.

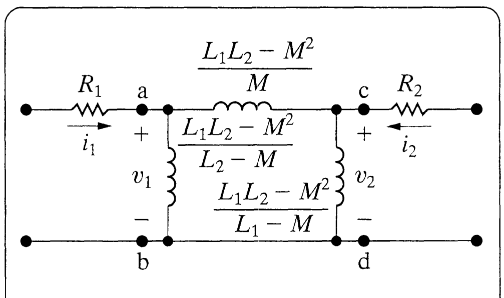


Figure C.5 The π-equivalent circuit used for sinusoidal steady-state analysis.

The mutual inductance carries its own algebraic sign in the T- and π-equivalent circuits. In other words, if the magnetic polarity of the coupled coils is reversed from that given in Fig. C.1, the algebraic sign of M reverses. A reversal in magnetic polarity requires moving one polarity dot without changing the reference polarities of the terminal currents and voltages.

Example C.1 illustrates the application of the T-equivalent circuit.

EXAMPLE ◆ C.1

a) Use the T-equivalent circuit for the magnetically coupled coils shown in Fig. C.6 to find the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$. The source frequency is 400 rad/s.

b) Repeat (a), but with the polarity dot on the secondary winding moved to the lower terminal.

SOLUTION

a) For the polarity dots shown in Fig. C.6, M carries a value of +3 H in the T-equivalent circuit. Therefore the three inductances in the equivalent circuit are

$$L_1 - M = 9 - 3 = 6 \text{ H};$$

$$L_2 - M = 4 - 3 = 1 \text{ H};$$

$$M = 3 \text{ H}.$$

Figure C.7 shows the T-equivalent circuit, and Fig. C.8 shows the frequency-domain equivalent circuit at a frequency of 400 rad/s.

Figure C.9 shows the frequency-domain circuit for the original system.

Here the magnetically coupled coils are modeled by the circuit shown in Fig. C.8. To find the phasor currents $\mathbf{I}_1$ and $\mathbf{I}_2$, we first find the node voltage across the 1200 Ω inductive reactance. If we use the lower node as the reference, the single node-voltage equation is

$$\frac{\mathbf{V} - 300}{700 + j2500} + \frac{\mathbf{V}}{j1200} + \frac{\mathbf{V}}{900 - j2100} = 0.$$

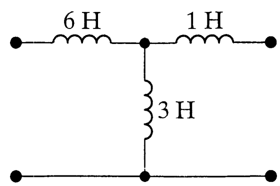


Figure C.7 The T-equivalent circuit for the magnetically coupled coils in Example C.1.

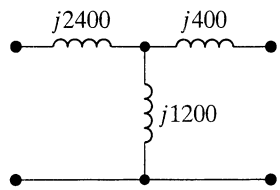


Figure C.8 The frequency-domain model of the equivalent circuit at 400 rad/s.

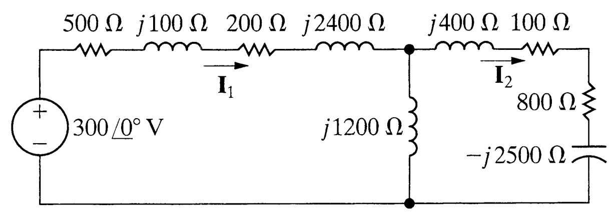


Figure C.9 The circuit of Fig. C.6, with the magnetically coupled coils replaced by their T-equivalent circuit.

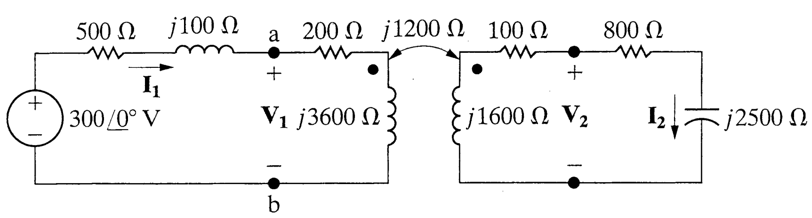


Figure C.6 The frequency-domain equivalent circuit for Example C.1.

Continued ◆

Solving for **V** yields

$$\mathbf{V} = 136 - j8 = 136.24 \angle -3.37^\circ \text{ V (rms)}.$$

Then

$$\mathbf{I}_1 = \frac{300 - (136 - j8)}{700 + j2500} = 63.25 \angle -71.57^\circ \text{ mA (rms)}$$

and

$$\mathbf{I}_2 = \frac{136 - j8}{900 - j2100} = 59.63 \angle 63.43^\circ \text{ mA (rms)}.$$

b) When the polarity dot is moved to the lower terminal of the secondary coil, M carries a value of -3 H in the T-equivalent circuit. Before carrying out the solution with the new T-equivalent circuit, we note that reversing the algebraic sign of M has no effect on the solution for $\mathbf{I}_1$ and shifts $\mathbf{I}_2$ by 180°. Therefore we anticipate that

$$\mathbf{I}_1 = 63.25 \angle -71.57^\circ \text{ mA (rms)}$$

and

$$\mathbf{I}_2 = 59.63 \angle -116.57^\circ \text{ mA (rms)}.$$

We now proceed to find these solutions by using the new T-equivalent circuit. With $M = -3$ H, the three inductances in the equivalent circuit are

$$L_1 - M = 9 - (-3) = 12 \text{ H};$$

$$L_2 - M = 4 - (-3) = 7 \text{ H};$$

$$M = -3 \text{ H}.$$

At an operating frequency of 400 rad/s, the frequency-domain equivalent circuit requires two inductors and a capacitor, as shown in Fig. C.10.

The resulting frequency-domain circuit for the original system appears in Fig. C.11.

As before, we first find the node voltage across the center branch, which in this case is a capacitive reactance of $-j1200\ \Omega$. If we use the lower node as reference, the node-voltage equation is

$$\frac{\mathbf{V} - 300}{700 + j4900} + \frac{\mathbf{V}}{-j1200} + \frac{\mathbf{V}}{900 + j300} = 0.$$

Solving for **V** gives

$$\mathbf{V} = -8 - j56$$
$$= 56.57 \angle -98.13^\circ \text{ V (rms)}.$$

Then

$$\mathbf{I}_1 = \frac{300 - (-8 - j56)}{700 + j4900}$$
$$= 63.25 \angle -71.57^\circ \text{ mA (rms)}$$

and

$$\mathbf{I}_2 = \frac{-8 - j56}{900 + j300}$$
$$= 59.63 \angle -116.57^\circ \text{ mA (rms)}.$$

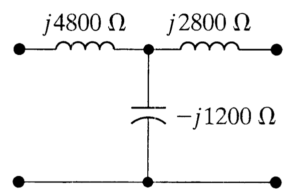


Figure C.10 The frequency-domain equivalent circuit for M = −3 H and ω = 400 rad/s.

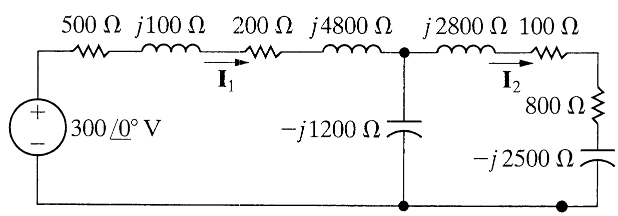


Figure C.11 The frequency-domain equivalent circuit for Example C.1(b).

C.2 ◆ The Need for Ideal Transformers in the Equivalent Circuits

The inductors in the T- and π-equivalent circuits of magnetically coupled coils can have negative values. For example, if $L_1 = 3$ mH, $L_2 = 12$ mH, and $M = 5$ mH, the T-equivalent circuit requires an inductor of -2 mH, and the π-equivalent circuit requires an inductor of -5.5 mH. These negative inductance values are not troublesome when you are using the equivalent circuits in computations. However, if you are to build the equivalent circuits with circuit components, the negative inductors can be bothersome. The reason is that whenever the frequency of the sinusoidal source changes, you must change the capacitor used to simulate the negative reactance. For example, at a frequency of 50 krad/s, a -2 mH inductor has an impedance of $-j100\ \Omega$. This impedance can be modeled with a capacitor having a capacitance of 0.2 μF. If the frequency changes to 25 krad/s, the -2 mH inductor impedance changes to $-j50\ \Omega$. At 25 krad/s, this requires a capacitor with a capacitance of 0.8 μF. Obviously, in a situation where the frequency is varied continuously, the use of a capacitor to simulate negative inductance is practically worthless.

You can circumvent the problem of dealing with negative inductances by introducing an ideal transformer into the equivalent circuit. This doesn't completely solve the modeling problem, because ideal transformers can only be approximated. However, in some situations the approximation is good enough to warrant a discussion of using an ideal transformer in the T- and π-equivalent circuits of magnetically coupled coils.

An ideal transformer can be used in two different ways in either the T-equivalent or the π-equivalent circuit. Figure C.12 shows the two arrangements for each type of equivalent circuit.

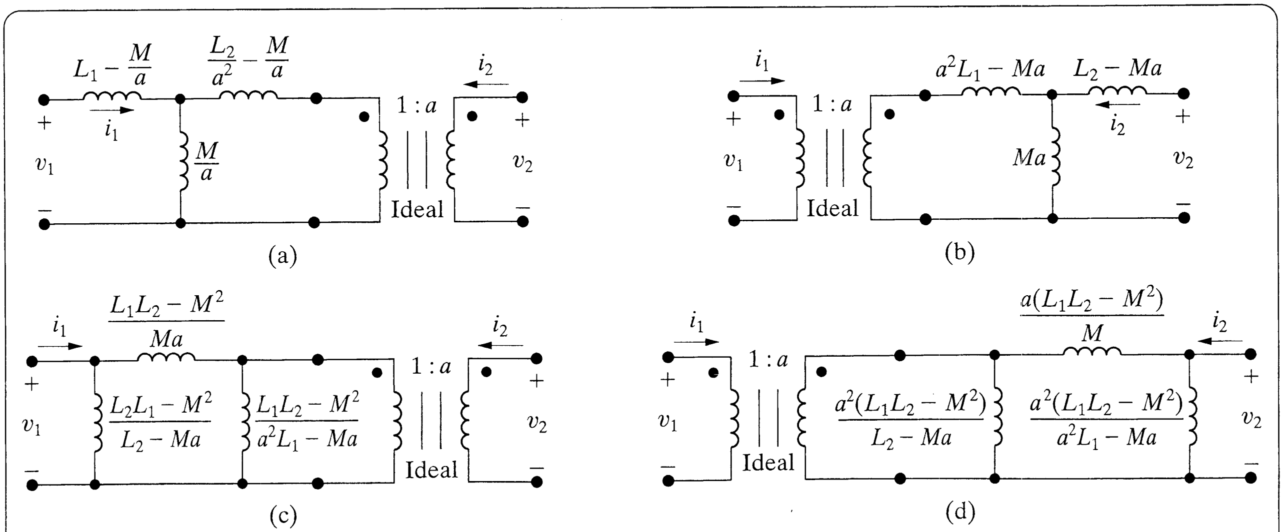


Figure C.12 The four ways of using an ideal transformer in the T- and π-equivalent circuit for magnetically coupled coils.

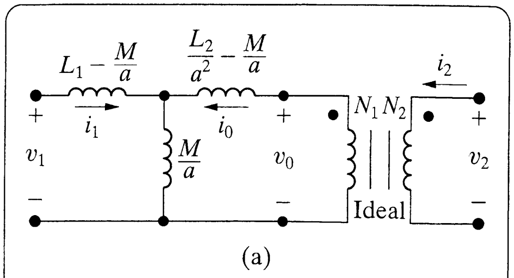


Figure C.13 The circuit of Fig. C.12(a) with i_0 and v_0 defined.

Verifying any of the equivalent circuits in Fig. C.12 requires showing only that, for any circuit, the equations relating v_1 and v_2 to di_1/dt and di_2/dt are identical to Eqs. C.1 and C.2. Here, we validate the circuit shown in Fig. C.12(a); we leave it to you to verify the circuits in Figs. C.12(b), (c), and (d). To aid the discussion, we redrew the circuit shown in Fig. C.12(a) as Fig. C.13, adding the variables i_0 and v_0.

From this circuit,

$$v_1 = \left(L_1 - \frac{M}{a}\right)\frac{di_1}{dt} + \frac{M}{a}\frac{d}{dt}(i_1 + i_0) \tag{C.12}$$

and

$$v_0 = \left(\frac{L_2}{a^2} - \frac{M}{a}\right)\frac{di_0}{dt} + \frac{M}{a}\frac{d}{dt}(i_0 + i_1). \tag{C.13}$$

The ideal transformer imposes constraints on v_0 and i_0:

$$v_0 = \frac{v_2}{a}; \tag{C.14}$$

$$i_0 = ai_2. \tag{C.15}$$

Substituting Eqs. C.14 and C.15 into Eqs. C.12 and C.13 gives

$$v_1 = L_1\frac{di_1}{dt} + \frac{M}{a}\frac{d}{dt}(ai_2) \tag{C.16}$$

and

$$\frac{v_2}{a} = \frac{L_2}{a^2}\frac{d}{dt}(ai_2) + \frac{M}{a}\frac{di_1}{dt}. \tag{C.17}$$

From Eqs. C.16 and C.17,

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \tag{C.18}$$

and

$$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}. \tag{C.19}$$

Equations C.18 and C.19 are identical to Eqs. C.1 and C.2; thus, insofar as terminal behavior is concerned, the circuit shown in Fig. C.13 is equivalent to the magnetically coupled coils shown inside the box in Fig. C.1.

In showing that the circuit in Fig. C.13 is equivalent to the magnetically coupled coils in Fig. C.1, we placed no restrictions on the turns ratio a. Therefore, an infinite number of equivalent circuits are possible. Furthermore, we can always find a turns ratio to make all the inductances positive. Three values of a are of particular interest:

$$a = \frac{M}{L_1}, \quad \text{(C.20)}$$

$$a = \frac{L_2}{M}, \quad \text{(C.21)}$$

and

$$a = \sqrt{\frac{L_2}{L_1}}. \quad \text{(C.22)}$$

The value of a given by Eq. C.20 eliminates the inductances $L_1 - M/a$ and $a^2L_1 - aM$ from the T-equivalent circuits and the inductances $(L_1L_2 - M^2)/(a^2L_1 - aM)$ and $a^2(L_1L_2 - M^2)/(a^2L_1 - aM)$ from the π-equivalent circuits. The value of a given by Eq. C.21 eliminates the inductances $(L_2/a^2) - (M/a)$ and $L_2 - aM$ from the T-equivalent circuits and the inductances $(L_1L_2 - M^2)/\ (L_2 - aM)$ and $a^2(L_1L_2 - M^2)/(L_2 - aM)$ from the π-equivalent circuits.

Also note that when $a = M/L_1$, the circuits in Figs. C.12(a) and (c) become identical, and when $a = L_2/M$, the circuits in Figs. C.12(b) and (d) become identical. Figures C.14 and C.15 summarize these observations.

In deriving the expressions for the inductances there, we used the relationship $M = k\sqrt{L_1L_2}$. Expressing the inductances as functions of the self-inductances L_1 and L_2 and the coefficient of coupling k allows the values of a given by Eqs. C.20 and C.21 not only to reduce the number of inductances needed in the equivalent circuit, but also to guarantee that all the inductances will be positive. We leave to you to investigate the consequences of choosing the value of a given by Eq. C.22.

The values of a given by Eqs. C.20–C.22 can be determined experimentally. The ratio M/L_1 is obtained by driving the coil designated as having N_1 turns by a sinusoidal voltage source. The source frequency is set high enough that $\omega L_1 \gg R_1$, and the N_2 coil is left open. Figure C.16 shows this arrangement.

With the N_2 coil open,

$$\mathbf{V}_2 = j\omega M\mathbf{I}_1. \quad \text{(C.23)}$$

Now, as $j\omega L_1 \gg R_1$, the current I_1 is

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{j\omega L_1}. \quad \text{(C.24)}$$

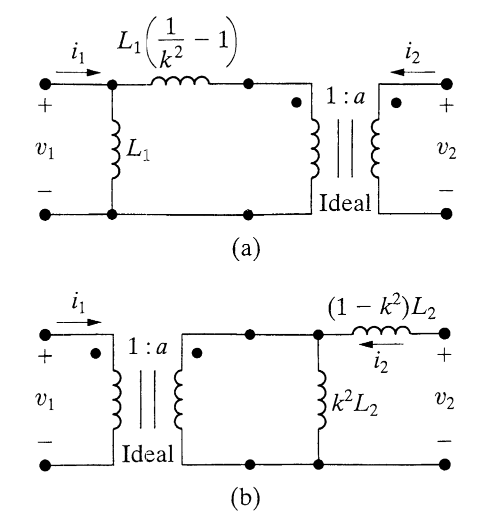


Figure C.14 Two equivalent circuits when $a = M/L_1$.

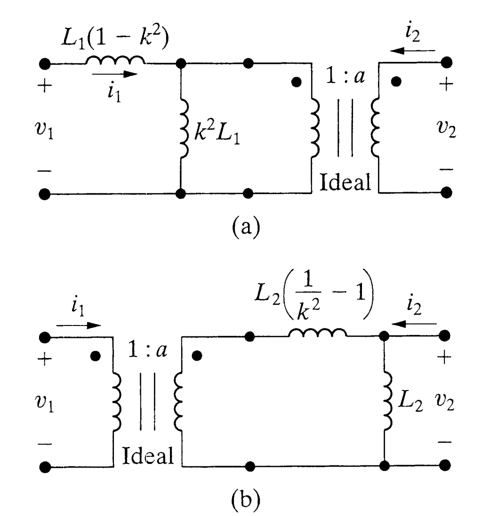


Figure C.15 Two equivalent circuits when $a = L_2/M$.

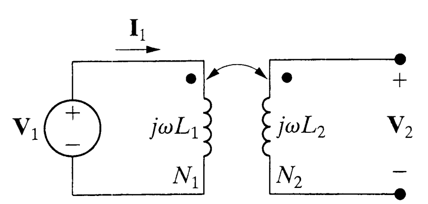


Figure C.16 Experimental determination of the ratio M/L_1.

Substituting Eq. C.24 into Eq. C.23 yields

$$\left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_2=0} = \frac{M}{L_1}, \tag{C.25}$$

in which the ratio M/L_1 is the terminal voltage ratio corresponding to coil 2 being open; that is, $\mathbf{I}_2 = 0$.

We obtain the ratio L_2/M by reversing the procedure; that is, coil 2 is energized and coil 1 is left open. Then

$$\frac{L_2}{M} = \left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_1=0}. \tag{C.26}$$

Finally, we observe that the value of a given by Eq. C.22 is the geometric mean of these two voltage ratios; thus

$$\sqrt{\left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_2=0}\left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_1=0}} = \sqrt{\frac{M}{L_1}\frac{L_2}{M}} = \sqrt{\frac{L_2}{L_1}}. \tag{C.27}$$

For coils wound on nonmagnetic cores, the voltage ratio is not the same as the turns ratio, as it very nearly is for coils wound on ferromagnetic cores. Because the self-inductances vary as the square of the number of turns, Eq. C.27 reveals that the turns ratio is approximately equal to the geometric mean of the two voltage ratios, or

$$\sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1} = \sqrt{\left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_2=0}\left(\frac{\mathbf{V}_2}{\mathbf{V}_1}\right)_{I_1=0}}. \tag{C.28}$$

APPENDIX D The Decibel

Telephone engineers who were concerned with the power loss across the cascaded circuits used to transmit telephone signals introduced the decibel. Figure D.1 defines the problem.

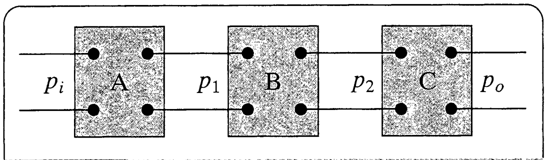


Figure D.1 Three cascaded circuits.

There, p_i is the power input to the system, p_1 is the power output of circuit A, p_2 is the power output of circuit B, and p_o is the power output of the system. The power gain of each circuit is the ratio of the power out to the power in. Thus

$$\sigma_A = \frac{p_1}{p_i}, \quad \sigma_B = \frac{p_2}{p_1}, \quad \text{and} \quad \sigma_C = \frac{p_o}{p_2}.$$

The overall power gain of the system is simply the product of the individual gains, or

$$\frac{p_o}{p_i} = \frac{p_1}{p_i}\frac{p_2}{p_1}\frac{p_o}{p_2} = \sigma_A\sigma_B\sigma_C.$$

The multiplication of power ratios is converted to addition by means of the logarithm; that is,

$$\log_{10}\frac{p_o}{p_i} = \log_{10}\sigma_A + \log_{10}\sigma_B + \log_{10}\sigma_C.$$

This log ratio of the powers was named the **bel**, in honor of Alexander Graham Bell. Thus we calculate the overall power gain, in bels, simply by summing the power gains, also in bels, of each segment of the transmission system. In practice, the bel is an inconveniently large quantity. One-tenth of a bel is a more useful measure of power gain; hence the **decibel**. The number of decibels equals 10 times the number of bels, so

$$\text{Number of decibels} = 10\log_{10}\frac{p_o}{p_i}.$$

When we use the decibel as a measure of power ratios, in some situations the resistance seen looking into the circuit equals the resistance loading the circuit, as illustrated in Fig. D.2.

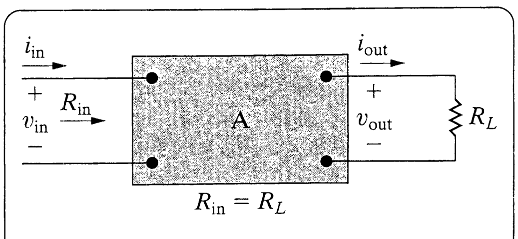


Figure D.2 A circuit in which the input resistance equals the load resistance.

When the input resistance equals the load resistance, we can convert the power ratio to either a voltage ratio or a current ratio:

$$\frac{p_o}{p_i} = \frac{v_{\text{out}}^2/R_L}{v_{\text{in}}^2/R_{\text{in}}} = \left(\frac{v_{\text{out}}}{v_{\text{in}}}\right)^2$$

or

$$\frac{p_o}{p_i} = \frac{i_{\text{out}}^2 R_L}{i_{\text{in}}^2 R_L} = \left(\frac{i_{\text{out}}}{i_{\text{in}}}\right)^2$$

These equations show that the number of decibels becomes

$$\text{Number of decibels} = 20 \log_{10} \frac{v_{\text{out}}}{v_{\text{in}}}$$

$$= 20 \log_{10} \frac{i_{\text{out}}}{i_{\text{in}}}. \tag{D.1}$$

The definition of the decibel used in Bode diagrams (see Appendix E) is borrowed from the results expressed by Eq. D.1, since these results apply to any transfer function involving a voltage ratio, a current ratio, a voltage-to-current ratio, or a current-to-voltage ratio. You should keep the original definition of the decibel firmly in mind because it is of fundamental importance in many engineering applications.

When you are working with transfer function amplitudes expressed in decibels, having a table that translates the decibel value to the actual value of the output/input ratio is helpful. Table D.1 gives some useful pairs. The ratio corresponding to a negative decibel value is the reciprocal of the positive ratio. For example, −3 dB corresponds to an output/input ratio of 1/1.41, or 0.707. Interestingly, −3 dB corresponds to the half-power frequencies of the filter circuits discussed in Chapters 14 and 15.

TABLE D.1 Some dB-Ratio Pairs

dB	RATIO	dB	RATIO
0	1.00	30	31.62
3	1.41	40	100.00
6	2.00	60	10^3
10	3.16	80	10^4
15	5.62	100	10^5
20	10.00	120	10^6

The decibel is also used as a unit of power when it expresses the ratio of a known power to a reference power. Usually the reference power is 1 mW and the power unit is written dBm, which stands for "decibels relative to one milliwatt." For example, a power of 20 mW corresponds to ±13 dBm.

AC voltmeters commonly provide dBm readings that assume not only a 1 mW reference power but also a 600 Ω reference resistance (a value commonly used in telephone systems). Since a power of 1 mW in 600 Ω corresponds to 0.7746 V (rms), that voltage is read as 0 dBm on the meter. For analog meters, there usually is exactly a 10 dB difference between adjacent ranges. Although the scales may be marked 0.1, 0.3, 1, 3, 10, and so on, in fact 3.16 V on the 3 V scale lines up with 1 V on the 1 V scale.

Some voltmeters provide a switch to choose a reference resistance (50, 135, 600, or 900 Ω) or to select dBm or dBV (decibels relative to one volt).

APPENDIX E Bode Diagrams

As we have seen, the frequency response plot is a very important tool for analyzing a circuit's behavior. Up to this point, however, we have shown qualitative sketches of the frequency response without discussing how to create such diagrams. The most efficient method for generating and plotting the amplitude and phase data is to use a digital computer; we can rely on it to give us accurate numerical plots of $|H(j\omega)|$ and $\theta(j\omega)$ versus ω. However, in some situations, preliminary sketches using Bode diagrams can help ensure the intelligent use of the computer.

A Bode diagram, or plot, is a graphical technique that gives a feel for the frequency response of a circuit. These diagrams are named in recognition of the pioneering work done by H. W. Bode.[1] They are most useful for circuits in which the poles and zeros of $H(s)$ are reasonably well separated.

Like the qualitative frequency response plots seen thus far, a Bode diagram consists of two separate plots: One shows how the amplitude of $H(j\omega)$ varies with frequency, and the other shows how the phase angle of $H(j\omega)$ varies with frequency. In Bode diagrams, the plots are made on semilog graph paper for greater accuracy in representing the wide range of frequency values. In both the amplitude and phase plots, the frequency is plotted on the horizontal log scale, and the amplitude and phase angle are plotted on the linear vertical scale.

E.1 ◆ Real, First-Order Poles and Zeros

To simplify the development of Bode diagrams, we begin by considering only cases where all the poles and zeros of $H(s)$ are real and first order. Later we will present cases with complex and repeated poles and zeros. For our purposes, having a specific expression for $H(s)$ is helpful. Hence we base the discussion on

$$H(s) = \frac{K(s + z_1)}{s(s + p_1)}, \tag{E.1}$$

from which

$$H(j\omega) = \frac{K(j\omega + z_1)}{j\omega(j\omega + p_1)}. \tag{E.2}$$

The first step in making Bode diagrams is to put the expression for $H(j\omega)$ in a **standard form**, which we derive simply by dividing out the

[1] See H. W. Bode, *Network Analysis and Feedback Design* (New York: Van Nostrand, 1945).

poles and zeros:

$$H(j\omega) = \frac{Kz_1(1 + j\omega/z_1)}{p_1(j\omega)(1 + j\omega/p_1)}. \tag{E.3}$$

Next we let K_o represent the constant quantity Kz_1/p_1, and at the same time we express $H(j\omega)$ in polar form:

$$H(j\omega) = \frac{K_o|1 + j\omega/z_1|\underline{/\psi_1}}{|\omega|\ \underline{/90^\circ}\ |1 + j\omega/p_1|\underline{/\beta_1}}$$

$$= \frac{K_o|1 + j\omega/z_1|}{|\omega||1 + j\omega/p_1|}\underline{/(\psi_1 - 90^\circ - \beta_1)}. \tag{E.4}$$

From Eq. E.4,

$$|H(j\omega)| = \frac{K_o|1 + j\omega/z_1|}{\omega|1 + j\omega/p_1|}, \tag{E.5}$$

$$\theta(\omega) = \psi_1 - 90^\circ - \beta_1. \tag{E.6}$$

By definition, the phase angles ψ_1 and β_1 are

$$\psi_1 = \tan^{-1}\omega/z_1; \tag{E.7}$$

$$\beta_1 = \tan^{-1}\omega/p_1. \tag{E.8}$$

The Bode diagrams consist of plotting Eq. E.5 (amplitude) and Eq. E.6 (phase) as functions of ω.

E.2 ◆ Straight-Line Amplitude Plots

The amplitude plot involves the multiplication and division of factors associated with the poles and zeros of $H(s)$. We reduce this multiplication and division to addition and subtraction by expressing the amplitude of $H(j\omega)$ in terms of a logarithmic value: the decibel (dB).[2] The amplitude of $H(j\omega)$ in decibels is

$$A_{dB} = 20\log_{10}|H(j\omega)|. \tag{E.9}$$

To give you a feel for the unit of decibels, Table E.1 provides a translation between the actual value of several amplitudes and their values in decibels. Expressing Eq. E.5 in terms of decibels gives

$$A_{dB} = 20\log_{10}\frac{K_o|1 + j\omega/z_1|}{\omega|1 + j\omega/p_1|}$$

$$= 20\log_{10}K_o + 20\log_{10}|1 + j\omega/z_1|$$

$$- 20\log_{10}\omega - 20\log_{10}|1 + j\omega/p_1|. \tag{E.10}$$

TABLE E.1 Actual Amplitudes and Their Decibel Values

A_{dB}	A	A_{dB}	A
0	1.00	30	31.62
3	1.41	40	100.00
6	2.00	60	10^3
10	3.16	80	10^4
15	5.62	100	10^5
20	10.00	120	10^6

[2] See Appendix D for more information regarding the decibel.

The key to plotting Eq. E.10 is to plot each term in the equation separately and then combine the separate plots graphically. The individual factors are easy to plot because they can be approximated in all cases by straight lines.

The plot of $20 \log_{10} K_o$ is a horizontal straight line because K_o is not a function of frequency. The value of this term is positive for $K_o > 1$, zero for $K_o = 1$, and negative for $K_o < 1$.

Two straight lines approximate the plot of $20 \log_{10} |1 + j\omega/z_1|$. For small values of ω, the magnitude $|1 + j\omega/z_1|$ is approximately 1, and therefore

$$20 \log_{10} |1 + j\omega/z_1| \to 0 \quad \text{as } \omega \to 0. \tag{E.11}$$

For large values of ω, the magnitude $|1 + j\omega/z_1|$ is approximately ω/z_1, and therefore

$$20 \log_{10} |1 + j\omega/z_1| \to 20 \log_{10}(\omega/z_1) \quad \text{as } \omega \to \infty. \tag{E.12}$$

On a log frequency scale, $20 \log_{10}(\omega/z_1)$ is a straight line with a slope of 20 dB/decade (a decade is a 10-to-1 change in frequency). This straight line intersects the 0 dB axis at $\omega = z_1$. This value of ω is called the **corner frequency**. Thus, on the basis of Eqs. E.11 and E.12, two straight lines can approximate the amplitude plot of a first-order zero, as shown in Fig. E.1.

The plot of $-20 \log_{10} \omega$ is a straight line having a slope of -20 dB/decade that intersects the 0 dB axis at $\omega = 1$. Two straight lines approximate the plot of $-20 \log_{10} |1 + j\omega/p_1|$. Here the two straight lines intersect on the 0 dB axis at $\omega = p_1$. For large values of ω, the straight line $20 \log_{10}(\omega/p_1)$ has a slope of -20 dB/decade. Figure E.2 shows the straight-line approximation of the amplitude plot of a first-order pole.

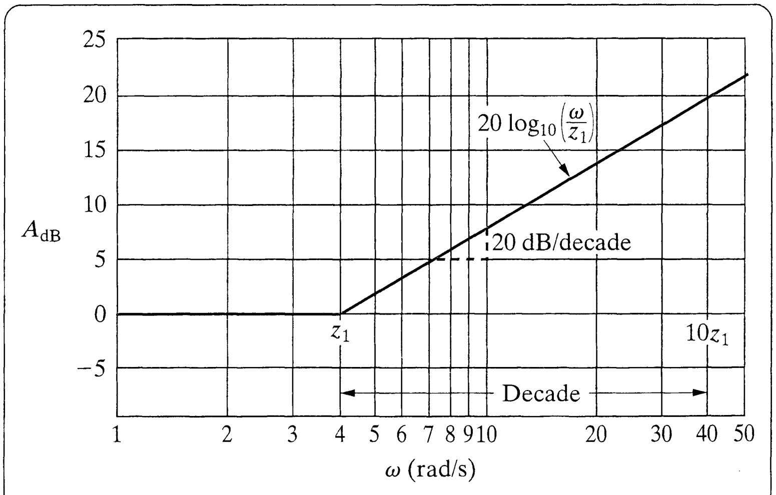


Figure E.1 A straight-line approximation of the amplitude plot of a first-order zero.

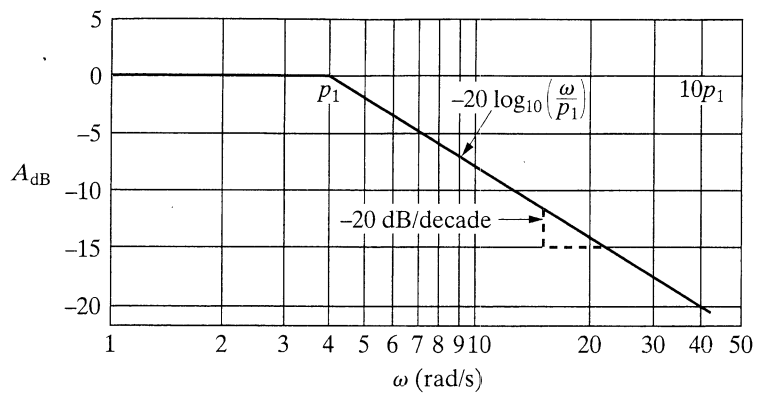


Figure E.2 A straight-line approximation of the amplitude plot of a first-order pole.

Figure E.3 shows a plot of Eq. E.10 for $K_o = \sqrt{10}$, $z_1 = 0.1$ rad/s, and $p_1 = 5$ rad/s. Each term in Eq. E.10 is labeled on Fig. E.3, so you can verify that the individual terms sum to create the resultant plot, labeled $20 \log_{10} |H(j\omega)|$.

Example E.1 illustrates the construction of a straight-line amplitude plot for a transfer function characterized by first-order poles and zeros.

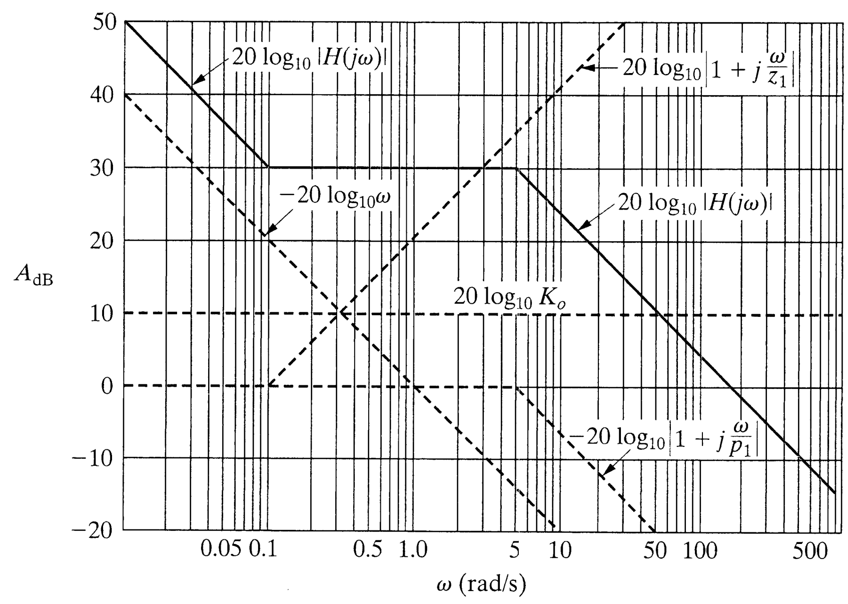


Figure E.3 A straight-line approximation of the amplitude plot for Eq. E.10.

EXAMPLE ◆ E.1

For the circuit in Fig. E.4:

a) Compute the transfer function, $H(s)$.

b) Construct a straight-line approximation of the Bode amplitude plot.

c) Calculate $20\log_{10}|H(j\omega)|$ at $\omega = 50$ rad/s and $\omega = 1000$ rad/s.

d) Plot the values computed in (c) on the straight-line graph; and

e) Suppose that $v_i(t) = 5\cos(500t + 15°)$ V, and then use the Bode plot you constructed to predict the amplitude of $v_o(t)$ in the steady state.

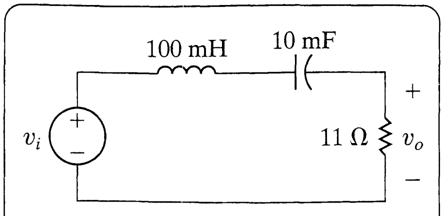


Figure E.4 The circuit for Example E.1.

SOLUTION

a) Transforming the circuit in Fig. E.4 into the s-domain and then using s-domain voltage division gives

$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + \frac{1}{LC}}.$$

Substituting the numerical values from the circuit, we get

$$H(s) = \frac{110s}{s^2 + 110s + 1000} = \frac{110s}{(s+10)(s+100)}.$$

b) We begin by writing $H(j\omega)$ in standard form:

$$H(j\omega) = \frac{0.11 j\omega}{[1 + j(\omega/10)][1 + j(\omega/100)]}.$$

The expression for the amplitude of $H(j\omega)$ in decibels is

$$\begin{aligned} A_{dB} &= 20\log_{10}|H(j\omega)| \\ &= 20\log_{10} 0.11 + 20\log_{10}|j\omega| \\ &\quad - 20\log_{10}\left|1 + j\frac{\omega}{10}\right| - 20\log_{10}\left|1 + j\frac{\omega}{100}\right|. \end{aligned}$$

Figure E.5 shows the straight-line plot. Each term contributing to the overall amplitude is identified.

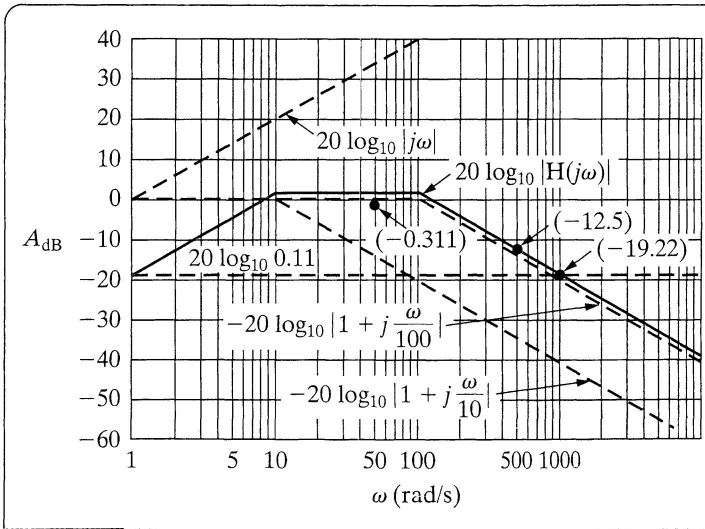


Figure E.5 The straight-line amplitude plot for the transfer function of the circuit in Fig. E.4.

Continued ◆

c) We have

$$H(j50) = \frac{0.11(j50)}{(1 + j5)(1 + j0.5)}$$
$$= 0.9648 \angle{-15.25^\circ},$$
$$20\log_{10}|H(j50)| = 20\log_{10} 0.9648$$
$$= -0.311 \text{ dB};$$
$$H(j1000) = \frac{0.11(j1000)}{(1 + j100)(1 + j10)}$$
$$= 0.1094 \angle{-83.72^\circ};$$
$$20\log_{10} 0.1094 = -19.22 \text{ dB}.$$

d) See Fig. E.5.

e) As we can see from the Bode plot in Fig. E.5, the value of A_{dB} at $\omega = 500$ rad/s is approximately -12.5 dB. Therefore,

$$|A| = 10^{(-12.5/20)} = 0.24$$

and

$$V_{mo} = |A|V_{mi} = (0.24)(5) = 1.19 \text{ V}.$$

We can compute the actual value of $|H(j\omega)|$ by substituting $\omega = 500$ into the equation for $|H(j\omega)|$:

$$H(j500) = \frac{0.11(j500)}{(1 + j50)(1 + j5)} = 0.22 \angle{-77.54^\circ}.$$

Thus, the actual output voltage magnitude for the specified signal source at a frequency of 500 rad/s is

$$V_{mo} = |A|V_{mi} = (0.22)(5) = 1.1 \text{ V}.$$

E.3 ◆ More Accurate Amplitude Plots

We can make the straight-line plots for first-order poles and zeros more accurate by correcting the amplitude values at the corner frequency, one half the corner frequency, and twice the corner frequency. At the corner frequency, the actual value in decibels is

$$A_{\text{dB}_c} = \pm 20\log_{10}|1 + j1|$$
$$= \pm 20\log_{10}\sqrt{2}$$
$$\approx \pm 3 \text{ dB}. \quad \text{(E.13)}$$

The actual value at one half the corner frequency is

$$A_{\text{dB}_{c/2}} = \pm 20\log_{10}\left|1 + j\frac{1}{2}\right|$$
$$= \pm 20\log_{10}\sqrt{5/4}$$
$$\approx \pm 1 \text{ dB}. \quad \text{(E.14)}$$

At twice the corner frequency, the actual value in decibels is

$$\begin{aligned} A_{\mathrm{dB}_{2c}} &= \pm 20 \log_{10} |1 + j2| \\ &= \pm 20 \log_{10} \sqrt{5} \\ &\approx \pm 7 \text{ dB}. \end{aligned} \qquad \text{(E.15)}$$

In Eqs. E.13–E.15, the plus sign applies to a first-order zero, and the minus sign applies to a first-order pole. The straight-line approximation of the amplitude plot gives 0 dB at the corner and one half the corner frequencies, and ±6 dB at twice the corner frequency. Hence the corrections are ±3 dB at the corner frequency and ±1 dB at both one half the corner frequency and twice the corner frequency. Figure E.6 summarizes these corrections.

A 2-to-1 change in frequency is called an **octave**. A slope of 20 dB/decade is equivalent to 6.02 dB/octave, which for graphical purposes is equivalent to 6 dB/octave. Thus the corrections enumerated correspond to one octave below and one octave above the corner frequency.

If the poles and zeros of $H(s)$ are well separated, inserting these corrections into the overall amplitude plot and achieving a reasonably accurate curve is relatively easy. However, if the poles and zeros are close together, the overlapping corrections are difficult to evaluate, and you're better off using the straight-line plot as a first estimate of the amplitude characteristic. Then use a computer to refine the calculations in the frequency range of interest.

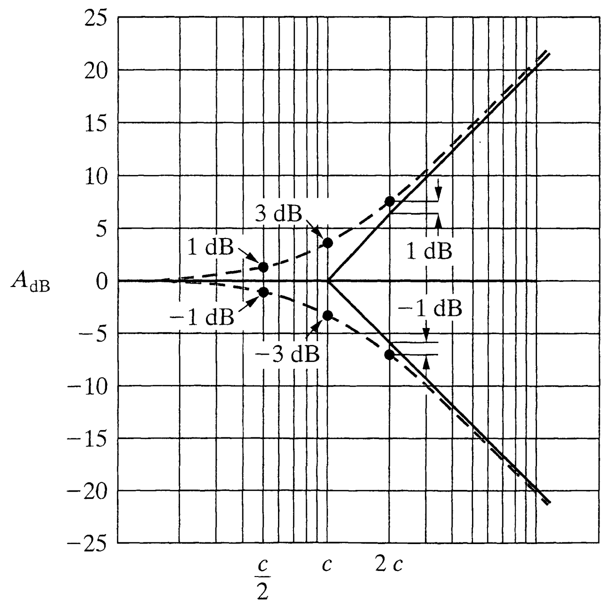


Figure E.6 Corrected amplitude plots for a first-order zero and pole.

E.4 ◆ Straight-Line Phase Angle Plots

We can also make phase angle plots by using straight-line approximations. The phase angle associated with the constant K_o is zero, and the phase angle associated with a first-order zero or pole at the origin is a constant $\pm 90°$. For a first-order zero or pole not at the origin, the straight-line approximations are as follows:

- For frequencies less than one tenth the corner frequency, the phase angle is assumed to be zero.
- For frequencies greater than 10 times the corner frequency, the phase angle is assumed to be $\pm 90°$.
- Between one tenth the corner frequency and 10 times the corner frequency, the phase angle plot is a straight line that goes through $0°$ at one-tenth the corner frequency, $\pm 45°$ at the corner frequency, and $\pm 90°$ at 10 times the corner frequency.

In all these cases, the plus sign applies to the first-order zero and the minus sign to the first-order pole. Figure E.7 depicts the straight-line approximation for a first-order zero and pole. The dashed curves show the exact variation of the phase angle as the frequency varies. Note how closely the straight-line plot approximates the actual variation in phase angle. The maximum deviation between the straight-line plot and the actual plot is approximately $6°$.

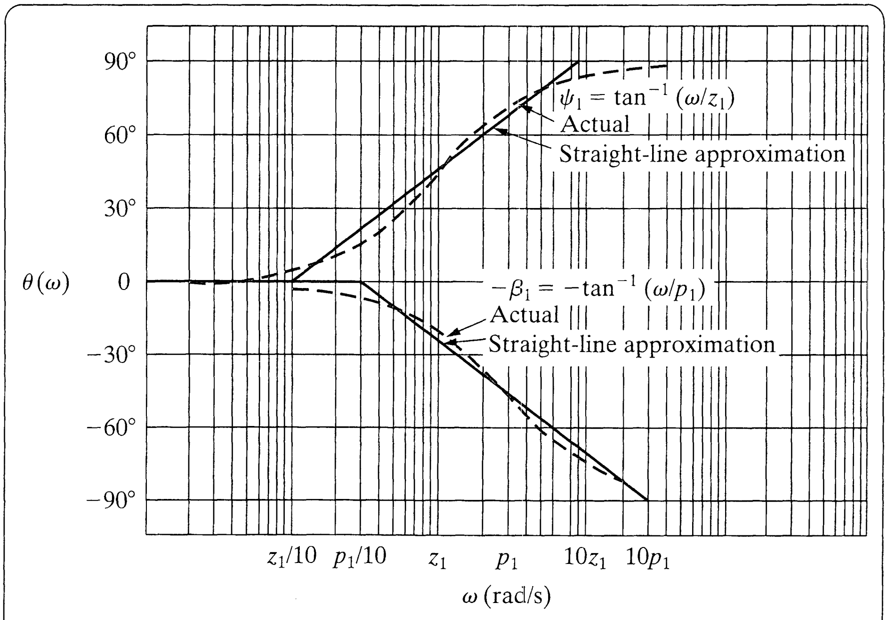


Figure E.7 Phase angle plots for a first-order zero and pole.

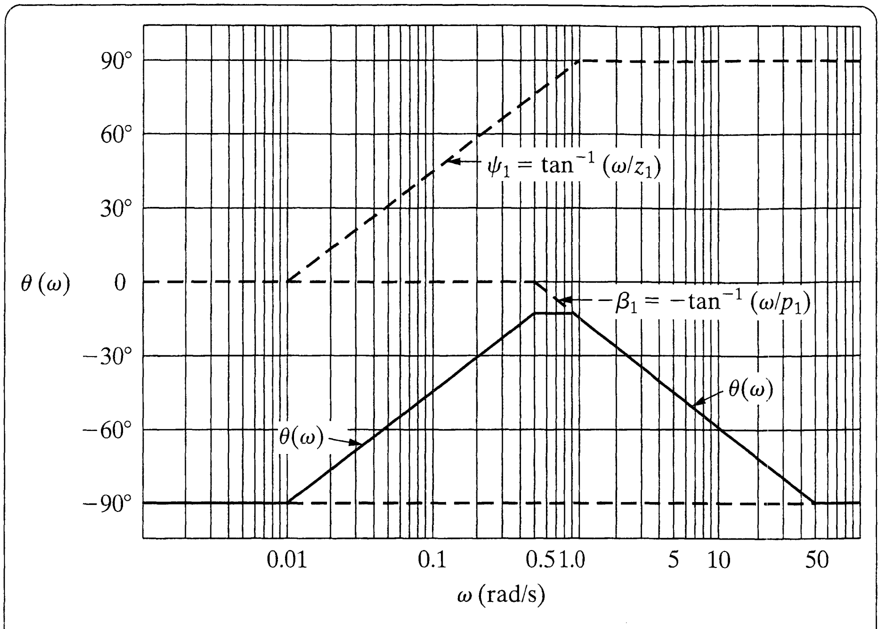


Figure E.8 A straight-line approximation of the phase angle plot for Eq. B.1.

Figure E.8 depicts the straight-line approximation of the phase angle of the transfer function given by Eq. B.1. Equation B.6 gives the equation for the phase angle; the plot corresponds to $z_1 = 0.1$ rad/s, and $p_1 = 5$ rad/s.

An illustration of a phase angle plot using a straight-line approximation is given in Example E.2.

EXAMPLE ◆ E.2

a) Make a straight-line phase angle plot for the transfer function in Example E.1.

b) Compute the phase angle $\theta(\omega)$ at $\omega = 50, 500,$ and 1000 rad/s.

c) Plot the values of (b) on the diagram of (a).

d) Using the results from Example E.1(e) and (b) of this example, compute the steady-state output voltage if the source voltage is given by $v_i(t) = 10\cos(500t - 25°)$ V.

SOLUTION

a) From Example E.1,

$$H(j\omega) = \frac{0.11(j\omega)}{[1 + j(\omega/10)][1 + j(\omega/100)]}$$

$$= \frac{0.11|j\omega|}{|1 + j(\omega/10)||1 + j(\omega/100)|}\angle(\psi_1 - \beta_1 - \beta_2).$$

Continued ◆

Therefore,

$$\theta(\omega) = \psi_1 - \beta_1 - \beta_2,$$

where $\psi_1 = 90°$, $\beta_1 = \tan^{-1}(\omega/10)$, and $\beta_2 = \tan^{-1}(\omega/100)$. Figure E.9 depicts the straight-line approximation of $\theta(\omega)$.

b) We have

$$H(j50) = 0.96 \angle -15.25°,$$

$$H(j500) = 0.22 \angle -77.54°,$$

$$H(j1000) = 0.11 \angle -83.72°.$$

Thus,

$$\theta(j50) = -15.25°,$$

$$\theta(j500) = -77.54°,$$

and

$$\theta(j1000) = -83.72°.$$

c) See Fig. E.9.

d) We have

$$V_{mo} = |H(j500)|V_{mi} = (0.22)(10) = 2.2 \text{ V},$$

and

$$\theta_o = \theta(\omega) + \theta_i = -77.54° - 25° = -102.54°.$$

Thus,

$$v_o(t) = 2.2\cos(500t - 102.54°) \text{ V}.$$

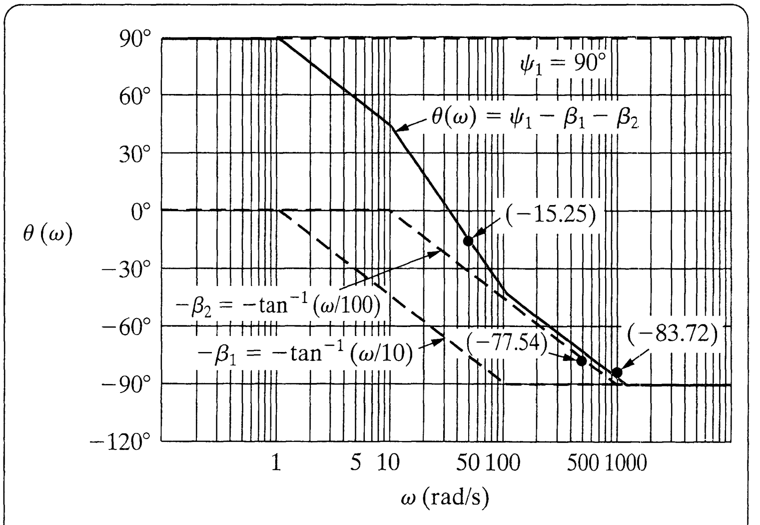


Figure E.9 A straight-line approximation of $\theta(\omega)$ for Example E.2.

E.5 ◆ Bode Diagrams: Complex Poles and Zeros

Complex poles and zeros in the expression for $H(s)$ require special attention when you make amplitude and phase angle plots. Let's focus on the contribution that a pair of complex poles makes to the amplitude and phase angle plots. Once you understand the rules for handling complex poles, their application to a pair of complex zeros becomes apparent.

The complex poles and zeros of $H(s)$ always appear in conjugate pairs. The first step in making either an amplitude or a phase angle plot of a transfer function that contains complex poles is to combine the conjugate pair into a single quadratic term. Thus, for

$$H(s) = \frac{K}{(s + \alpha - j\beta)(s + \alpha + j\beta)}, \quad \text{(E.16)}$$

we first rewrite the product $(s + \alpha - j\beta)(s + \alpha + j\beta)$ as

$$(s + \alpha)^2 + \beta^2 = s^2 + 2\alpha s + \alpha^2 + \beta^2. \quad \text{(E.17)}$$

When making Bode diagrams, we write the quadratic term in a more convenient form:

$$s^2 + 2\alpha s + \alpha^2 + \beta^2 = s^2 + 2\zeta\omega_n s + \omega_n^2. \quad \text{(E.18)}$$

A direct comparison of the two forms shows that

$$\omega_n^2 = \alpha^2 + \beta^2 \quad \text{(E.19)}$$

and

$$\zeta\omega_n = \alpha. \quad \text{(E.20)}$$

The term ω_n is the corner frequency of the quadratic factor, and ζ is the damping coefficient of the quadratic term. The critical value of ζ is 1. If $\zeta < 1$, the roots of the quadratic factor are complex, and we use Eq. E.18 to represent the complex poles. If $\zeta \geq 1$, we factor the quadratic factor into $(s + p_1)(s + p_2)$ and then plot amplitude and phase in accordance with the discussion previously. Assuming that $\zeta < 1$, we rewrite Eq. E.16 as

$$H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad \text{(E.21)}$$

We then write Eq. E.21 in standard form by dividing through by the poles and zeros. For the quadratic term, we divide through by ω_n, so

$$H(s) = \frac{K}{\omega_n^2} \frac{1}{1 + (s/\omega_n)^2 + 2\zeta(s/\omega_n)}, \tag{E.22}$$

from which

$$H(j\omega) = \frac{K_o}{1 - (\omega^2/\omega_n^2) + j(2\zeta\omega/\omega_n)}, \tag{E.23}$$

where

$$K_o = \frac{K}{\omega_n^2}.$$

Before discussing the amplitude and phase angle diagrams associated with Eq. E.23, for convenience we replace the ratio ω/ω_n by a new variable, u. Then

$$H(j\omega) = \frac{K_o}{1 - u^2 + j2\zeta u}. \tag{E.24}$$

Now we write $H(j\omega)$ in polar form:

$$H(j\omega) = \frac{K_o}{|(1 - u^2) + j2\zeta u| \underline{/\beta_1}}, \tag{E.25}$$

from which

$$\begin{aligned} A_{\text{dB}} &= 20\log_{10}|H(j\omega)| \\ &= 20\log_{10} K_o - 20\log_{10}|(1 - u^2) + j2\zeta u|, \end{aligned} \tag{E.26}$$

and

$$\theta(\omega) = -\beta_1 = -\tan^{-1}\frac{2\zeta u}{1 - u^2}. \tag{E.27}$$

E.6 ◆ Amplitude Plots

The quadratic factor contributes to the amplitude of $H(j\omega)$ by means of the term $-20\log_{10}|1-u^2+j2\zeta u|$. Because $u=\omega/\omega_n$, $u\to 0$ as $\omega\to 0$, and $u\to\infty$ as $\omega\to\infty$. To see how the term behaves as ω ranges from 0 to ∞, we note that

$$-20\log_{10}|(1-u^2)+j2\zeta u| = -20\log_{10}\sqrt{(1-u^2)^2+4\zeta^2u^2}$$

$$= -10\log_{10}[u^4+2u^2(2\zeta^2-1)+1], \quad \text{(E.28)}$$

as $u\to 0$,

$$-10\log_{10}[u^4+2u^2(2\zeta^2-1)+1]\to 0, \quad \text{(E.29)}$$

and as $u\to\infty$,

$$-10\log_{10}[u^4+2u^2(2\zeta^2-1)+1]\to -40\log_{10}u. \quad \text{(E.30)}$$

From Eqs. E.29 and E.30, we conclude that the approximate amplitude plot consists of two straight lines. For $\omega<\omega_n$, the straight line lies along the 0 dB axis, and for $\omega>\omega_n$, the straight line has a slope of −40 dB/decade. These two straight lines join on the 0 dB axis at $u=1$ or $\omega=\omega_n$. Figure E.10 shows the straight-line approximation for a quadratic factor with $\zeta<1$.

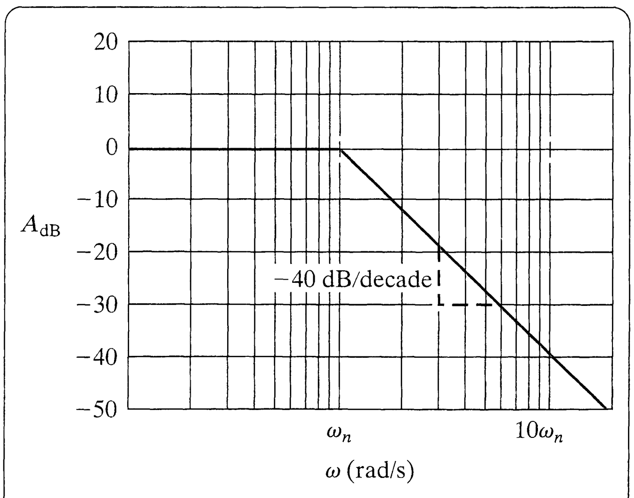


Figure E.10 The amplitude plot for a pair of complex poles.

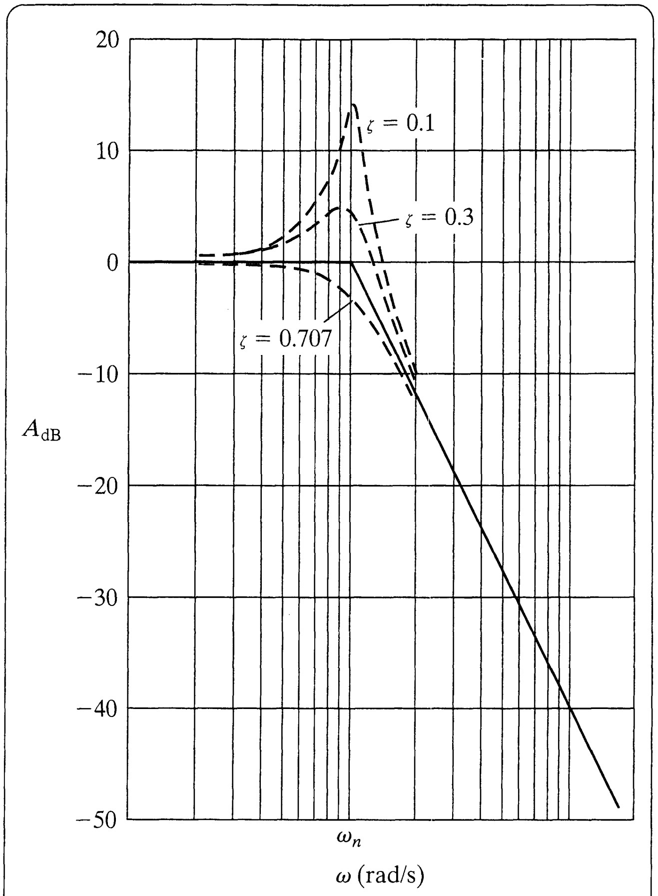


Figure E.11 The effect of ζ on the amplitude plot.

E.7 ◆ Correcting Straight-Line Amplitude Plots

Correcting the straight-line amplitude plot for a pair of complex poles is not as easy as correcting a first-order real pole, because the corrections depend on the damping coefficient ζ. Figure E.11 shows the effect of ζ on the amplitude plot. Note that as ζ becomes very small, a large peak in the amplitude occurs in the neighborhood of the corner frequency $\omega_n (u = 1)$. When $\zeta \geq 1/\sqrt{2}$, the corrected amplitude plot lies entirely below the straight-line approximation. For sketching purposes, the straight-line amplitude plot can be corrected by locating four points on the actual curve. These four points correspond to (1) one half the corner frequency, (2) the frequency at which the amplitude reaches its peak value, (3) the corner

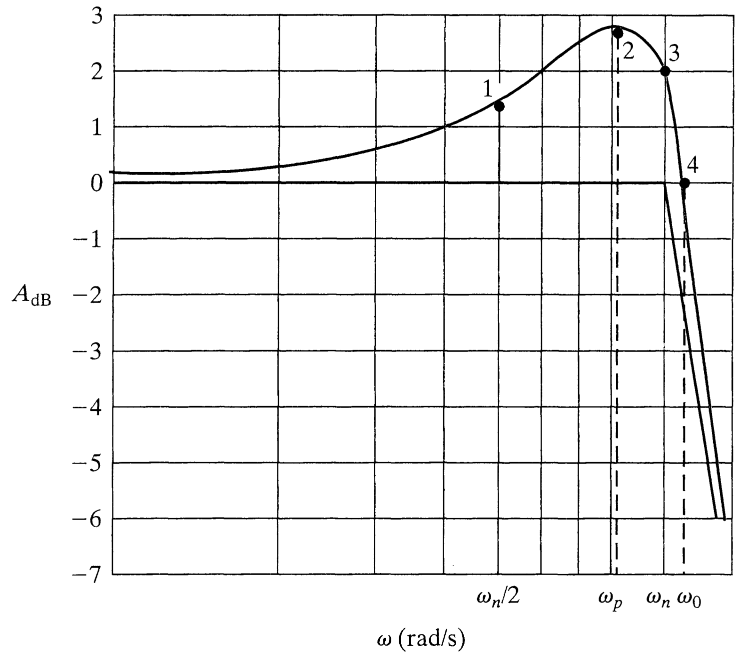


Figure E.12 Four points on the corrected amplitude plot for a pair of complex poles.

frequency, and (4) the frequency at which the amplitude is zero. Figure E.12 shows these four points.

At one half the corner frequency (point 1), the actual amplitude is

$$A_{dB}(\omega_n/2) = -10\log_{10}(\zeta^2 + 0.5625). \tag{E.31}$$

The amplitude peaks (point 2) at a frequency of

$$\omega_p = \omega_n\sqrt{1 - 2\zeta^2}, \tag{E.32}$$

and it has a peak amplitude of

$$A_{dB}(\omega_p) = -10\log_{10}[4\zeta^2(1 - \zeta^2)]. \tag{E.33}$$

At the corner frequency (point 3), the actual amplitude is

$$A_{dB}(\omega_n) = -20\log_{10} 2\zeta. \tag{E.34}$$

The corrected amplitude plot crosses the 0 dB axis (point 4) at

$$\omega_o = \omega_n\sqrt{2(1 - 2\zeta^2)} = \sqrt{2}\omega_p. \tag{E.35}$$

The derivations of Eqs. E.31, E.34, and E.35 follow from Eq. E.28. Evaluating Eq. E.28 at $u = 0.5$ and $u = 1.0$, respectively, yields Eqs. E.31 and E.34. Equation E.35 corresponds to finding the value of u that makes $u^4 + 2u^2(2\zeta^2 - 1) + 1 = 1$. The derivation of Eq. E.32 requires differentiating Eq. E.28 with respect to u and then finding the value of u where the derivative is zero. Equation E.33 is the evaluation of Eq. E.28 at the value of u found in Eq. E.32.

Example E.3 illustrates the amplitude plot for a transfer function with a pair of complex poles.

EXAMPLE ◆ E.3

Compute the transfer function for the circuit shown in Fig. E.13.

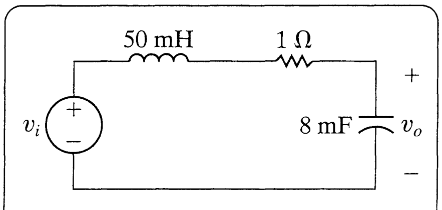


Figure E.13 The circuit for Example E.3.

a) What is the value of the corner frequency in radians per second?
b) What is the value of K_o?
c) What is the value of the damping coefficient?
d) Make a straight-line amplitude plot ranging from 10 to 500 rad/s.
e) Calculate and sketch the actual amplitude in decibels at $\omega_n/2$, ω_p, ω_n, and ω_o.
f) From the straight-line amplitude plot, describe the type of filter represented by the circuit in Fig. E.13 and estimate its cutoff frequency, ω_c.

SOLUTION

Transform the circuit in Fig. E.13 to the s-domain and then use s-domain voltage division to get

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}.$$

Substituting the component values,

$$H(s) = \frac{2500}{s^2 + 20s + 2500}.$$

a) From the expression for $H(s)$, $\omega_n^2 = 2500$; therefore, $\omega_n = 50$ rad/s.

b) By definition, K_o is $2500/\omega_n^2$, or 1.

c) The coefficient of s equals $2\zeta\omega_n$; therefore

$$\zeta = \frac{20}{2\omega_n} = 0.20.$$

Continued ◆

d) See Fig. E.14.

e) The actual amplitudes are

$$A_{\text{dB}}(\omega_n/2) = -10\log_{10}(0.6025) = 2.2 \text{ dB},$$

$$\omega_p = 50\sqrt{0.92} = 47.96 \text{ rad/s},$$

$$A_{\text{dB}}(\omega_p) = -10\log_{10}(0.16)(0.96) = 8.14 \text{ dB},$$

$$A_{\text{dB}}(\omega_n) = -20\log_{10}(0.4) = 7.96 \text{ dB},$$

$$\omega_o = \sqrt{2}\omega_p = 67.82 \text{ rad/s},$$

$$A_{\text{dB}}(\omega_o) = 0 \text{ dB}.$$

Figure E.14 shows the corrected plot.

f) It is clear from the amplitude plot in Fig. E.14 that this circuit acts as a low-pass filter. At the cutoff frequency, the magnitude of the transfer function, $|H(j\omega_c)|$, is 3 dB less than the maximum magnitude. From the corrected plot, the cutoff frequency appears to be about 55 rad/s, almost the same as that predicted by the straight-line Bode diagram.

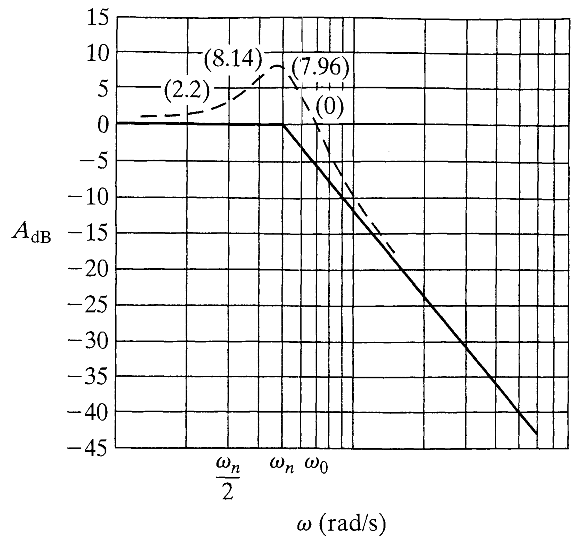


Figure E.14 The amplitude plot for Example E.3.

E.8 ◆ Phase Angle Plots

The phase angle plot for a pair of complex poles is a plot of Eq. E.27. The phase angle is zero at zero frequency and is -90° at the corner frequency. It approaches -180° as $\omega(u)$ becomes large. As in the case of the amplitude plot, ζ is important in determining the exact shape of the phase angle plot. For small values of ζ, the phase angle changes rapidly in the vicinity of the corner frequency. Figure E.15 shows the effect of ζ on the phase angle plot.

We can also make a straight-line approximation of the phase angle plot for a pair of complex poles. We do so by drawing a line tangent to the phase angle curve at the corner frequency and extending this line until it intersects with the 0° and -180° lines. The line tangent to the phase angle curve at -90° has a slope of $-2.3/\zeta$ rad/decade ($-132/\zeta$ degrees/decade), and it intersects the 0° and -180° lines at $u_1 = 4.81^{-\zeta}$ and $u_2 = 4.81^{\zeta}$, respectively. Figure E.16 depicts the straight-line approximation for $\zeta = 0.3$ and shows the actual phase angle plot. Comparing the straight-line approximation to the actual curve indicates that the approximation is reasonable in the vicinity of the corner frequency. However, in the neighborhood of u_1 and u_2, the error is quite large. In Example E.4, we summarize our discussion of Bode diagrams.

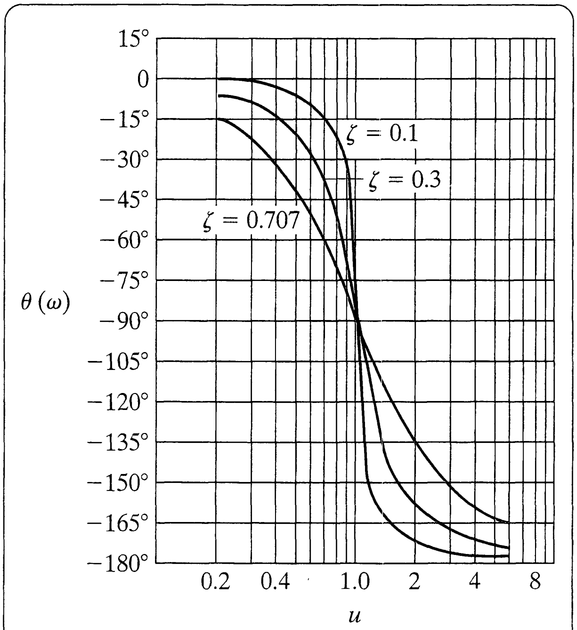


Figure E.15 The effect of ζ on the phase angle plot.

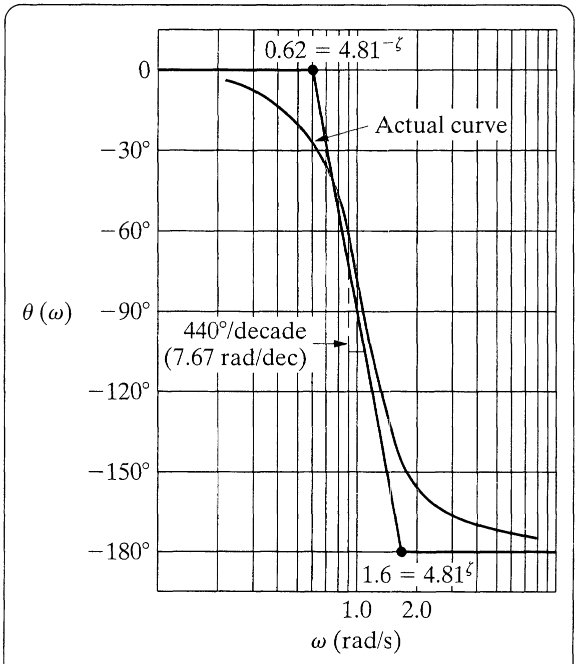


Figure E.16 A straight-line approximation of the phase angle for a pair of complex poles.

EXAMPLE E.4

a) Compute the transfer function for the circuit shown in Fig. E.17.

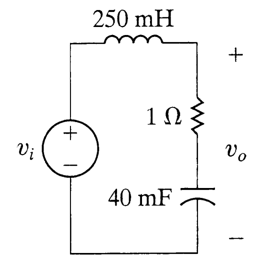


Figure E.17 The circuit for Example E.4.

b) Make a straight-line amplitude plot of $20\log_{10}|H(j\omega)|$.

c) Use the straight-line amplitude plot to determine the type of filter represented by this circuit and then estimate its cutoff frequency.

d) What is the actual cutoff frequency?

e) Make a straight-line phase angle plot of $H(j\omega)$.

f) What is the value of $\theta(\omega)$ at the cutoff frequency from (c)?

g) What is the actual value of $\theta(\omega)$ at the cutoff frequency?

SOLUTION

a) Transform the circuit in Fig. E.17 to the s-domain and then perform s-domain voltage division to get

$$H(s) = \frac{\frac{R}{L}s + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.$$

Substituting the component values from the circuit gives

$$H(s) = \frac{4(s+25)}{s^2 + 4s + 100}.$$

b) The first step in making Bode diagrams is to put $H(j\omega)$ in standard form. Because $H(s)$ contains a quadratic factor, we first check the value of ζ. We find that $\zeta = 0.2$ and $\omega_n = 10$, so

$$H(s) = \frac{s/25 + 1}{1 + (s/10)^2 + 0.4(s/10)},$$

from which

$$H(j\omega) = \frac{|1 + j\omega/25| \angle \psi_1}{|1 - (\omega/10)^2 + j0.4(\omega/10)| \angle \beta_1}.$$

Note that for the quadratic factor, $u = \omega/10$. The amplitude of $H(j\omega)$ in decibels is

$$A_{\text{dB}} = 20\log_{10}|1 + j\omega/25| - 20\log_{10}\left[\left|1 - \left(\frac{\omega}{10}\right)^2 + j0.4\left(\frac{\omega}{10}\right)\right|\right],$$

and the phase angle is

$$\theta(\omega) = \psi_1 - \beta_1,$$

where

$$\psi_1 = \tan^{-1}(\omega/25),$$

$$\beta_1 = \tan^{-1}\frac{0.4(\omega/10)}{1 - (\omega/10)^2}.$$

Figure E.18 shows the amplitude plot.

Continued ◆

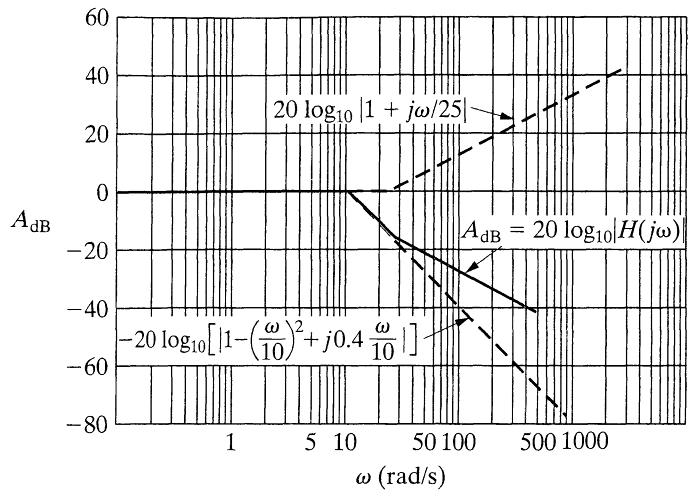


Figure E.18 The amplitude plot for Example E.4.

c) From the straight-line amplitude plot in Fig. E.18, this circuit acts as a low-pass filter. At the cutoff frequency, the amplitude of $H(j\omega)$ is 3 dB less than the amplitude in the passband. From the plot, we predict that the cutoff frequency is approximately 13 rad/s.

d) To solve for the actual cutoff frequency, replace s with $j\omega$ in $H(s)$, compute the expression for $|H(j\omega)|$, set $|H(j\omega_c)| = (1/\sqrt{2})$ $H_{\max} = 1/\sqrt{2}$, and solve for ω_c. First,

$$H(j\omega) = \frac{4(j\omega) + 100}{(j\omega)^2 + 4(j\omega) + 100}.$$

Then,

$$|H(j\omega_c)| = \frac{\sqrt{(4\omega_c)^2 + 100^2}}{\sqrt{(100 - \omega_c^2)^2 + (4\omega_c)^2}} = \frac{1}{\sqrt{2}}.$$

Solving for ω_c gives us

$$\omega_c = 16 \text{ rad/s}.$$

e) Figure E.19 shows the phase angle plot. Note that the straight-line segment of $\theta(\omega)$ between 1.0 and 2.5 rad/s does not have the same slope as the segment between 2.5 and 100 rad/s.

f) From the phase angle plot in Fig. E.19, we estimate the phase angle at the cutoff frequency of 16 rad/s to be $-65°$.

g) We can compute the exact phase angle at the cutoff frequency by substituting $s = j16$ into the transfer function $H(s)$:

$$H(j16) = \frac{4(j16 + 25)}{(j16)^2 + 4(j16) + 100}.$$

Computing the phase angle, we see

$$\theta(\omega_c) = \theta(j16) = -125.0°.$$

Note the large error in the predicted angle. In general, straight-line phase angle plots do not give satisfactory results in the frequency band where the phase angle is changing. The straight-line phase angle plot is useful only in predicting the general behavior of the phase angle, not in estimating actual phase angle values at particular frequencies.

Continued ◆

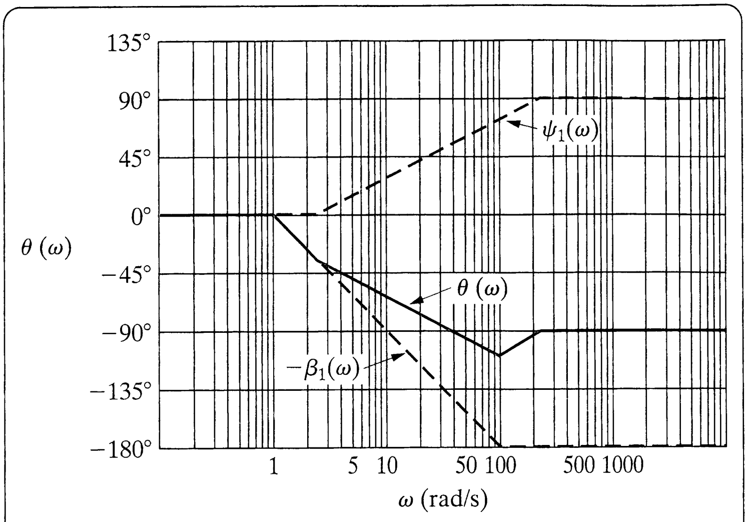


Figure E.19 The phase angle plot for Example E.4.

APPENDIX F

An Abbreviated Table of Trigonometric Identities

1. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

2. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

3. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

4. $\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

5. $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

6. $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

7. $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

8. $2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

9. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

10. $\sin 2\alpha = 2\sin\alpha\cos\alpha$

11. $\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

12. $\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$

13. $\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$

14. $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

15. $\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$

APPENDIX G

An Abbreviated Table of Integrals

1. $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$

2. $\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2-2ax+2)$

3. $\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax$

4. $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$

5. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

6. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

7. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

8. $\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$

9. $\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$

10. $\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$

11. $\int \sin ax\cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}, \quad a^2 \neq b^2$

12. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$

13. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$

14. $$\int_0^{\infty} \frac{a\,dx}{a^2 + x^2} = \begin{cases} \frac{\pi}{2}, a > 0; \\ 0, a = 0; \\ \frac{-\pi}{2}, a < 0 \end{cases}$$

15. $$\int_0^{\infty} \frac{\sin ax}{x}\,dx = \begin{cases} \frac{\pi}{2}, a > 0; \\ \frac{-\pi}{2}, a < 0 \end{cases}$$

16. $$\int x^2 \sin ax\,dx = \frac{2x}{a^2} \sin ax - \frac{a^2x^2 - 2}{a^3} \cos ax$$

17. $$\int x^2 \cos ax\,dx = \frac{2x}{a^2} \cos ax + \frac{a^2x^2 - 2}{a^3} \sin ax$$

18. $$\int e^{ax} \sin^2 bx\,dx = \frac{e^{ax}}{a^2 + 4b^2} \left[(a \sin bx - 2b \cos bx) \sin bx + \frac{2b^2}{a} \right]$$

19. $$\int e^{ax} \cos^2 bx\,dx = \frac{e^{ax}}{a^2 + 4b^2} \left[(a \cos bx + 2b \sin bx) \cos bx + \frac{2b^2}{a} \right]$$