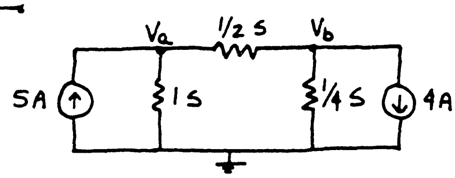
Chapter 4

Exercises

Ex. 4-1



KCL at V_a:

$$-5 + V_a + 1/2(V_a - V_b) = 0$$

$$\underline{3V_a - V_b = 10} \qquad (1)$$

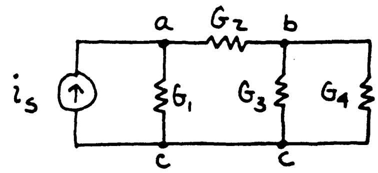
KCL at V_b: $1/2(V_b - V_a) + 1/4\, V_b + 4 = 0$

$$\underline{-2V_a + 3V_b = -16} \qquad (2)$$

Solving (1) & (2) simultaneously yields

$$\underline{V_a = 2\text{v}, \quad V_b = -4\text{v}}$$

Ex. 4-2



(1) KCL at a: $V_a(G_1+G_2) - V_b G_2 - V_c G_1 = i_s$

(2) KCL at b: $-V_a G_2 + V_b(G_2+G_3+G_4) - V_c(G_3+G_4) = 0$

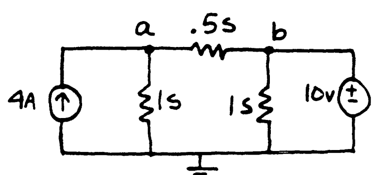
(3) KCL at c: $-V_a G_1 - (G_3+G_4)V_b + (G_1+G_3+G_4)V_c = -i_s$

To get (1) $\Rightarrow$ (1) = $-(2) - (3)$

" " (2) $\Rightarrow$ (2) = $-(1) - (3)$

" " (3) $\Rightarrow$ (3) = $-(1) - (2)$

Ex. 4-3

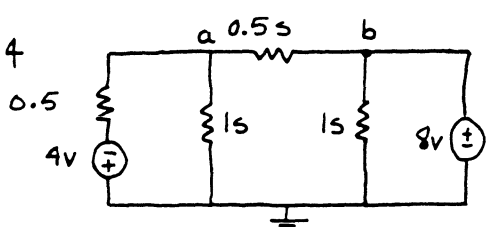


KCL at a:

$$-4 + V_a + .5(V_a - V_b) = 0$$

$$\Rightarrow V_a = \frac{.5V_b + 4}{1.5} = \frac{.5(10) + 4}{1.5} = \underline{6\text{V}}$$

Ex. 4-4



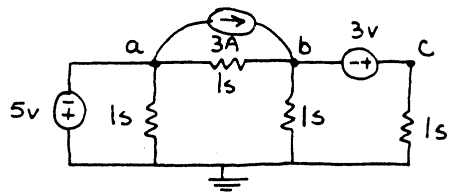
$V_b = 8\text{v}$

KCL at a: $0.5(V_a + 4) + V_a + 0.5(V_a - V_b) = 0$

$$2V_a + 2 - (.5)(8) = 0$$

$$\Rightarrow \underline{V_a = 2/2 = 1\text{v}}$$

Ex. 4-5



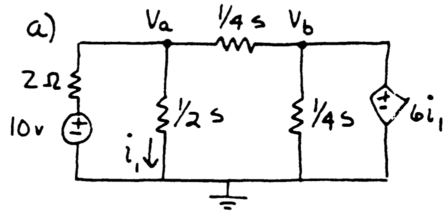
$\underline{V_a = -5v}$

KCL at b : $-3 + (V_b - V_a) + V_b + (V_b + 3) = 0$

with $V_a = -5 \Rightarrow \underline{V_b = -5/3\,v}$

also $V_c - V_b = 3 \Rightarrow \underline{V_c = 3 + V_b = 4/3\,v}$

Ex. 4-6 a)



$V_b = 6 i_1$ (1)

$V_a = 2 i_1$ (2)

KCL at V_a : $1/2 (V_a - 10) + i_1 + 1/4 (V_a - V_b) = 0$

$3V_a - V_b + 4i_1 - 20 = 0$ (3)

Solving (1), (2) & (3) simultaneously yields $\underline{V_b = 30v}$

b) for element as current source, $i_s = 3V_a$

$V_b = 6 i_1$ (1)

$V_a = 2 i_1$ (2)

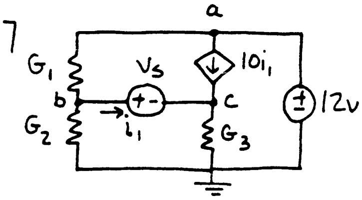
KCL at V_a : $1/2 (V_a - 10) + i_1 + i_s = 0$

$1/2 (V_a - 10) + i_1 + 3V_a = 0$

$4V_a + 4i_1 - 20 = 0$ (3)

Solving (1), (2) & (3) simultaneously yields $\underline{V_b = 15/4\,v}$

Ex. 4-7



KCL at b : $G_1 (V_b - 12) + i_1 + G_2 V_b$ (1)

KCL at c : $-i_1 - 10 i_1 + G_3 V_c = 0$

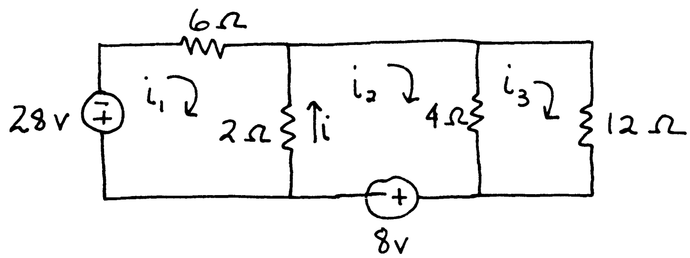
$\Rightarrow i_1 = \dfrac{G_3 V_c}{11}$ (2)

also $V_b - V_c = V_s \Rightarrow V_b = V_c + V_s$ (3)

(2) into (1) yields $(G_1 + G_2) V_b + \dfrac{G_3 V_c}{11} = 12 G_1$ (4)

(3) into (4) and solving for $V_c \Rightarrow \underline{V_c = \dfrac{12 G_1 - (G_1 + G_2) V_s}{G_1 + G_2 + G_3/11}}$

Ex. 4-8



KVL loop 1: $28 + 6i_1 + 2(i_1 - i_2) = 0$ (1)

KVL loop 2: $2(i_2 - i_1) + 4(i_2 - i_3) + 8 = 0$ (2)

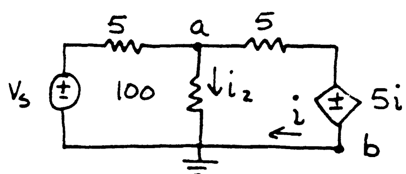
KVL loop 3: $12i_3 + 4(i_3 - i_2) = 0$ (3)

solving (1) - (3) simultaneously yields

$$i_1 = -\frac{13}{3}\,A\ ,\quad i_2 = -\frac{10}{3}\,A\ ,\quad i_3 = -\frac{5}{6}\,A$$

$$\Rightarrow\ i = i_2 - i_1 = -\frac{10}{3} - \left(-\frac{13}{3}\right) = \underline{1\,A}$$

Ex. 4-9



$V_{ab} = 5i + 5i = 10i$

$\therefore\ P_{motor} = (V_{ab})\,i = 10i^2 = 150$

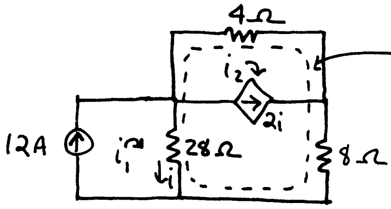
$\Rightarrow\ i = \sqrt{15}\ A$

$\therefore\ V_{ab} = (10)(\sqrt{15})\ v$

$$\Rightarrow\ i_2 = \frac{V_{ab}}{100} = \frac{\sqrt{15}}{10}\ A$$

KCL at a : $(V_{ab} - V_s)/5 + i_2 + i = 0$ $\Rightarrow\ \underline{V_s = \frac{31}{2}\sqrt{15} = 60.03\,v}$

Ex 4-10



supermesh

$i_1 = 12\,A$ (1)

KVL supermesh:

$4i_2 + 8i_3 + 28(i_3 - 12) = 0$ (2)

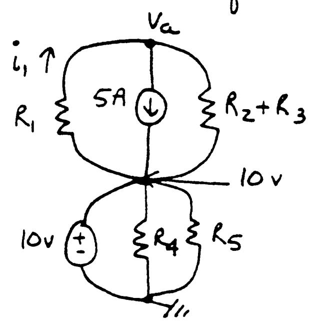
also: $2i = i_3 - i_2$ (3)

solving (1) $\rightarrow$ (3) yields $i_3 = 9A$

$\therefore\ \underline{i = 12 - 9 = 3A}$

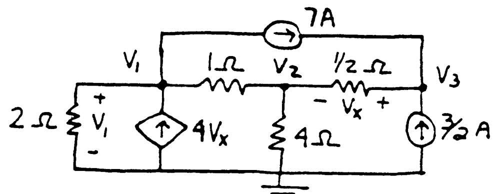
Ex. 4-11 (a) Nodal analysis since the other node is known ($= V_s$) thus only need one node eqn. at a.

(b) Nodal analysis since when the circuit is redrawn (shown below), only one node eqn. at V_a is required. Mesh analysis would require 4 mesh currents $\Rightarrow$ 4 unknowns.



Problems

P. 4-1



KCL at V_1 : $V_1/2 - 4V_x + 7 + (V_1 - V_2) = 0 \Rightarrow \underline{3V_1 - 2V_2 - 8V_x + 14 = 0}$ (1)

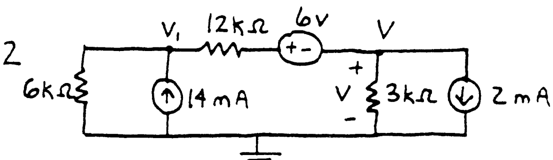
KCL at V_2 : $(V_2 - V_1) + V_2/4 - 2V_x = 0 \Rightarrow \underline{-4V_1 + 5V_2 - 8V_x = 0}$ (2)

KCL at V_3 : $-7 + 2V_x - 3/2 = 0 \Rightarrow \underline{4V_x - 17 = 0}$ (3)

Solving (1), (2) and (3) simultaneously yields

$\underline{V_1 = 24\text{ v}}$

P. 4-2



KCL at V_1 : $V_1/6 - 14 + (V_1 - 6 - V)/12 = 0$

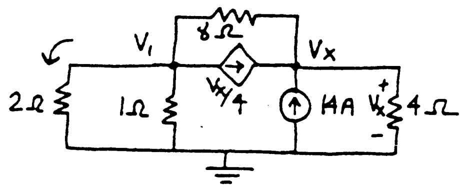
$\Rightarrow \underline{3V_1 - V - 174 = 0}$ (1)

KCL at V : $-(V_1 - 6 - V)/12 + V/3 + 2 = 0$

$\Rightarrow \underline{-V_1 + 5V + 30 = 0}$ (2)

solving (1) & (2) simultaneously yields $\underline{V = 6\text{ volts}}$

P. 4-3



KCL at V_1 : $V_1/2 + V_1/1 + (V_1 - V_x)/8 + V_x/4 = 0$

$$\Rightarrow \underline{13V_1 + V_x = 0} \qquad (1)$$

KCL at V_x : $(V_x - V_1)/8 - V_x/4 - 14 + V_x/4 = 0$

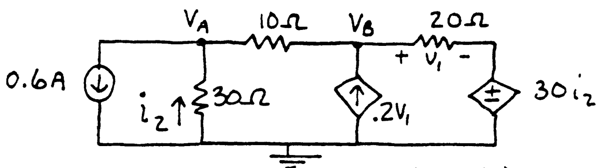
$$\Rightarrow \underline{-V_1 + V_x - 112 = 0} \qquad (2)$$

solving (1) & (2) simultaneously yields $\underline{V_x = 104\text{ v}}$

and $V_1 = -8\text{v}$

$$\therefore P_{2\Omega} = \frac{V_1^2}{R} = \frac{(8)^2}{2} = \underline{32\text{ W}}$$

P. 4-4



KCL at V_A : $\underline{6 - i_2 + \frac{(V_A - V_B)}{10} = 0}$ (1)

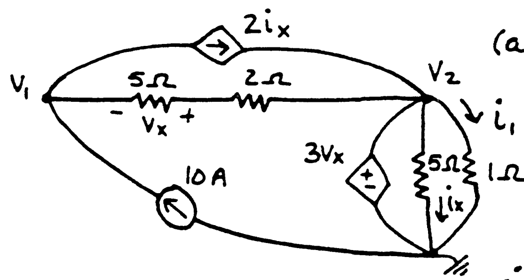
KCL at V_B : $\underline{\frac{(V_B - V_A)}{10} - 0.2V_1 + V_1/20 = 0}$ (2)

also $\underline{V_B - 30i_2 = V_1}$ (3) and $V_A = -30i_2$ (4)

solving (1) through (4) simultaneously yields

$\underline{V_1 = 72/19\text{ V}}$, $\underline{i_2 = 3/95\text{ A}}$

P. 4-5



(a) KCL at V_1 : $10 - 2i_x + V_x/5 = 0$

$$\Rightarrow \underline{V_x - 10i_x + 50 = 0} \qquad (1)$$

also $\underline{3V_x = 5i_x}$ (2)

$\therefore$ solving for i_x & V_x

$\underline{i_x = 6A}$, $\underline{V_x = 10v}$

P. 4-5 (continued)

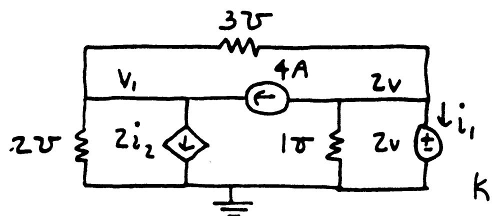
(b) $i_1 = \frac{V_2}{1} = 5 i_x = 30\,A$

$\therefore\ P_{1\Omega} = i_1^2 R = (30)^2(1) = \underline{900W}$

from KVL: $V_1 = 3V_x - V_x - 2(V_x/5) = 16v$ ∴ source is supplying power

$\therefore\ P_{\text{absorbed of source}} = -V_1 i = -(16)(10) = \underline{-160W}$

P. 4-6



KCL at V_1: $2V_1 + 2i_2 - 4 + i_2 = 0$

$\underline{2V_1 + 4i_2 - 4 = 0}$ (1)

KCL at 2v: $-i_2 + 4 + 2 + i_1 = 0$

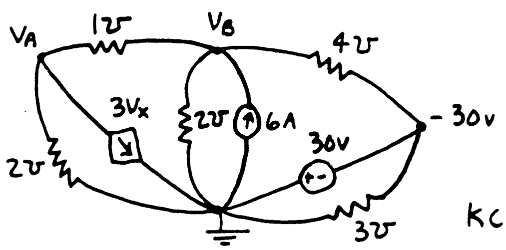
$\underline{-i_2 + 6 + i_1 = 0}$ (2)

also $\underline{3(V_1 - 2) = i_2}$ (3)

solving (1), (2) & (3) simultaneously yields

$\underline{i_1 = -6A}$

P. 4-7



KCL at V_A: $2V_A + 3V_x + V_x = 0$

$\underline{V_A + 2V_x = 0}$ (1)

KCL at V_B: $-V_x + 2V_B - 6 + 4(V_B + 30) = 0$

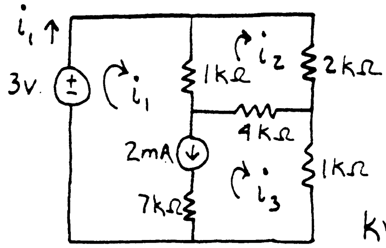
$\underline{-V_x + 6V_B + 114 = 0}$ (2)

also $\underline{V_A - V_B = V_x}$ (3)

solving (1), (2) & (3) simultaneously yields

$\underline{V_x = 6v}$

P. 4-8



KVL around mesh i_1 & i_3 combined

$$-3+(i_1-i_2)+4(i_3-i_2)+i_3=0$$

$$\underline{3-i_1+5i_2-5i_3=0} \qquad (1)$$

KVL around mesh i_2

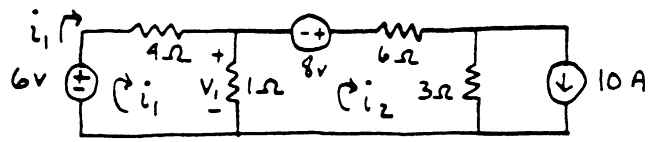
$$i_2-i_1+2i_2+4(i_2-i_3)=0$$

$$\underline{i_1-7i_2+4i_3=0} \qquad (2)$$

also $\underline{i_1-i_3=2}$ (3)

solving (1), (2) & (3) simultaneously yields $\underline{i_1 = 3\text{mA}}$

P. 4-9



KVL mesh i_1 : $-6+4i_1+(i_1-i_2)=0 \Rightarrow \underline{-6+5i_1-i_2=0}$ (1)

KVL mesh i_2 : $(i_2-i_1)-8+6i_2+3(i_2-10)=0$

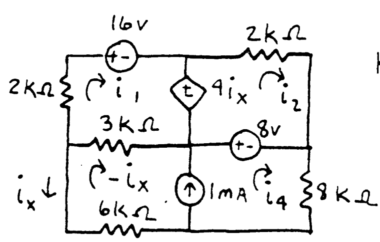
$$\Rightarrow \underline{-38-i_1+10i_2=0} \qquad (2)$$

solving (1) & (2) simultaneously yields $\underline{i_1 = 2A}$

and $i_2 = 4A$

$\therefore \underline{V_1 = 1(i_1-i_2) = -2v}$

P. 4-10



KVL mesh i_1 : $2i_1+16+4i_x+3(i_1+i_x)=0$

$$\underline{5i_1+7i_x+16=0} \qquad (1)$$

KVL mesh i_2 : $\underline{-4i_x+2i_2-8=0}$ (2)

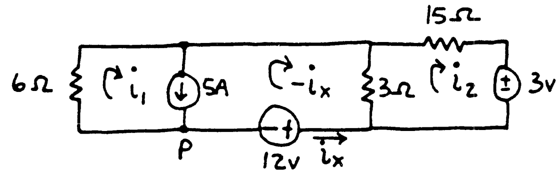
KVL outer loop : $\underline{2i_1+16+2i_2+8i_4-6i_x=0}$ (3)

also KCL bottom node : $\underline{i_4+i_x-1=0}$ (4)

solving (1), (2), (3) & (4) simultaneously $\Rightarrow$ $\underline{i_x = 2\text{mA}}$, $i_2 = 8\text{mA}$, $i_4 = -1\text{mA}$

$\therefore i_{\text{thru } 8v} = i_2 - i_4 = 9\text{mA}$ $\therefore P_{\text{supplied by } 8v} = vi = (8)(9) = \underline{72\text{ mW}}$

P. 4-11



KVL mesh i_1 & $-i_x$: $\underline{6i_1 + 3(-i_x - i_2) + 12 = 0}$ (1)

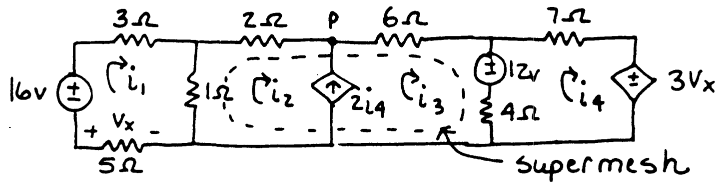
KVL mesh i_2: $3(i_2 + i_x) + 15i_2 + 3 = 0$

$\underline{18i_2 + 3i_x + 3 = 0}$ (2)

from KCL at P: $\underline{i_1 + i_x = 5}$ (3)

solving (1), (2) & (3) simultaneously yields $\underline{i_x = 5\text{A}}$

P. 4-12



KVL mesh i_1: $-16 + 3i_1 + 1(i_1 - i_2) + 5i_1 = 0$

$\Rightarrow \underline{-9i_1 + i_2 = -16}$ (1)

KVL supermesh: $1(i_2 - i_1) + 2i_2 + 6i_3 + 12 + 4(i_3 - i_4) = 0$

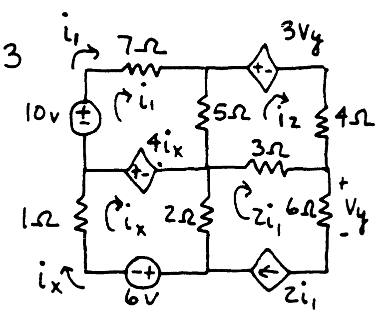
$\Rightarrow \underline{i_1 - 3i_2 - 10i_3 + 4i_4 = 12}$ (2)

KVL mesh i_4: $7i_4 + 3V_x + 4(i_4 - i_3) - 12 = 0$ also $V_x = -5i_1$

$\Rightarrow \underline{15i_1 + 4i_3 - 11i_4 = -12}$ (3)

KCL at P: $\underline{i_2 + 2i_4 - i_3 = 0}$ (4)

P. 4-13



mesh i_1: $-10 + 7i_1 + 5(i_1 - i_2) - 4i_x = 0$

$\Rightarrow \underline{12i_1 - 5i_2 - 4i_x = 10}$ (1)

mesh i_2: $3V_y + 4i_2 + 3(i_2 - 2i_1) + 5(i_2 - i_1) = 0$

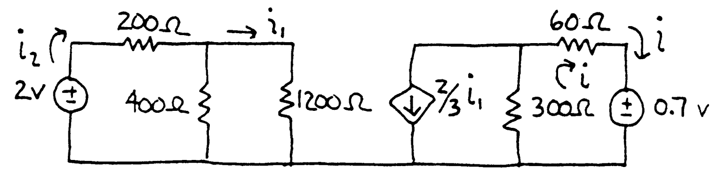
with $V_y = 12i_1$

$\Rightarrow \underline{25i_1 + 12i_2 = 0}$ (2)

mesh i_x: $i_x + 4i_x + 2(i_x - 2i_1) + 6 = 0$

$\Rightarrow \underline{4i_1 - 7i_x = 6}$ (3)

P. 4-14



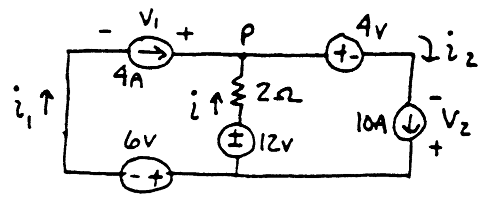
from voltage divider $1200 i_1 = 2\left(\frac{400 \| 1200}{400\|1200 + 200}\right) = 2\left(\frac{3}{5}\right) = 1.2$

$\therefore\ i_1 = .001\text{ A}$

KVL mesh i : $60i + .7 + 300(i + 2/3\, i_1) = 0$

$\Rightarrow\ \underline{i = -.0025\text{ A}}$

P. 4-15



KCL at P: $i = i_2 - i_1$

$i = 10 - 4 = 6\text{ A}$

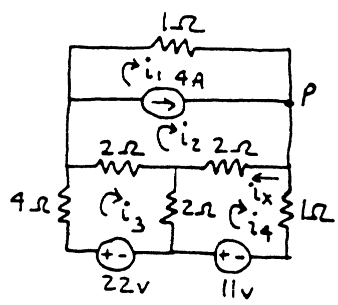
KVL i_1 : $-V_1 - 2i + 12 + 6 = 0 \Rightarrow V_1 = 18 - 2(6) = 6\text{v}$

KVL i_2 : $4 - V_2 - 12 + 2i = 0 \Rightarrow V_2 = -8 + 2(6) = 4\text{v}$

$\therefore$
$P_{6v} = vi_1 = (6)(4) = \underline{24\text{ W}}$
$P_{4A} = -V_1 i_1 = -(6)(4) = \underline{-24\text{ W}}$
$P_{2\Omega} = i^2 R = (6)^2(2) = \underline{72\text{ W}}$
$P_{12v} = -vi = -(12)(6) = \underline{-72\text{ W}}$
$P_{4v} = vi_2 = (4)(10) = \underline{40\text{ W}}$
$P_{10A} = -V_2 i_2 = -(4)(10) = \underline{-40\text{ W}}$

Power absorbed

P. 4-16



use mesh analysis

KVL i_3 : $4i_3 + 2(i_3 - i_2) + 2(i_3 - i_4) - 22 = 0$

$\underline{2i_2 - 8i_3 + 2i_4 + 22 = 0}$ (1)

KVL i_4 : $2(i_4 - i_2) + i_4 - 11 + 2(i_4 - i_3) = 0$

$\underline{2i_2 + 2i_3 - 5i_4 + 11 = 0}$ (2)

KVL i_1 & i_2 : $i_1 + 2(i_2 - i_4) + 2(i_2 - i_3) = 0$

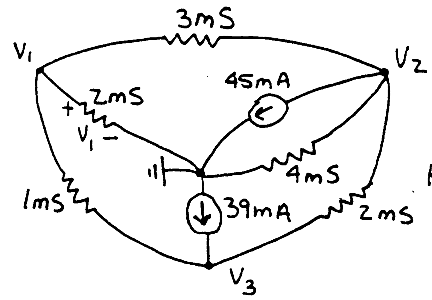
$\Rightarrow\ \underline{i_1 + 4i_2 - 2i_3 - 2i_4 = 0}$ (3)

P. 4-16 continued also $\underline{i_1 - i_2 + 4 = 0}$ (4)

and $\underline{i_2 - i_4 - i_x = 0}$ (5)

Solving (1) → (5) simultaneously yields $\underline{i_x = -1\text{A}}$

P. 4-17



KCL at V_1: $(V_1 - V_3) + 2V_1 + 3(V_1 - V_2) = 0$

$\hookrightarrow \underline{6V_1 - 3V_2 - V_3 = 0}$ (1)

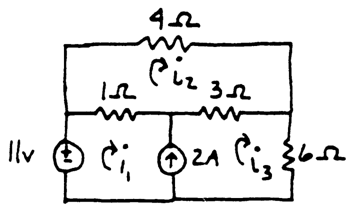
KCL at V_2: $3(V_2 - V_1) + 45 + 4V_2 + 2(V_2 - V_3) = 0$

$\hookrightarrow \underline{3V_1 - 9V_2 + 2V_3 - 45 = 0}$ (2)

KCL at V_3: $(V_3 - V_1) - 39 + 2(V_3 - V_2) = 0 \Rightarrow \underline{V_1 + 2V_2 - 3V_3 + 39 = 0}$ (3)

Solving (1), (2) & (3) simultaneously yields $\underline{V_1 = 1\text{v}}$

P. 4-18



KVL mesh i_1 & i_3

$-11 + (i_1 - i_2) + 3(i_3 - i_2) + 6i_3 = 0$

$\hookrightarrow \underline{i_1 - 4i_2 + 9i_3 - 11 = 0}$ (1)

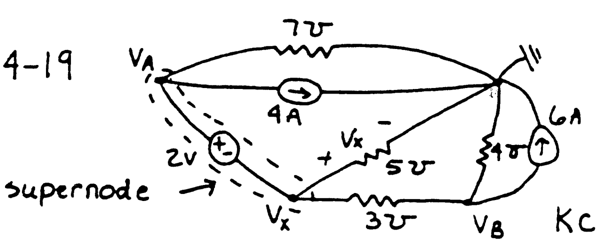
KVL mesh i_2: $4i_2 + 3(i_2 - i_3) + (i_2 - i_1) = 0$

$\hookrightarrow \underline{i_1 - 8i_2 + 3i_3 = 0}$ (2)

KCL at middle node: $\underline{i_1 + 2 - i_3 = 0}$ (3)

Solving (1), (2) & (3) simultaneously yields $\underline{i_1 = -\tfrac{1}{2}\text{A}}$

P. 4-19



KCL supernode:

$7V_A + 5V_x + 3(V_x - V_B) + 4 = 0$

$\hookrightarrow \underline{7V_A - 3V_B + 8V_x + 4 = 0}$ (1)

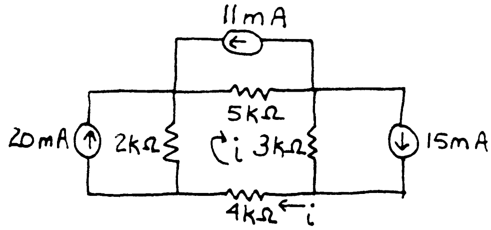
KCL at V_B: $3(V_B - V_x) + 4V_B + 6 = 0$

$\hookrightarrow \underline{7V_B - 3V_x + 6 = 0}$ (2)

also $\underline{V_A - V_x = 2}$ (3)

Solving (1), (2) & (3) simultaneously yields $\underline{V_x = -1.5\text{v}}$

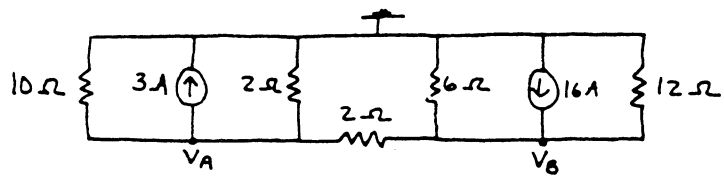
P. 4-20



KVL mesh i : $2(i-20)+5(i+11)+3(i-15)+4i=0$

$14i - 140 = 0 \quad \Rightarrow \quad \underline{i = 10\text{mA}}$

P. 4-21



KCL at V_A : $V_A/10 + 3 + V_A/2 + 1/2(V_A - V_B) = 0$

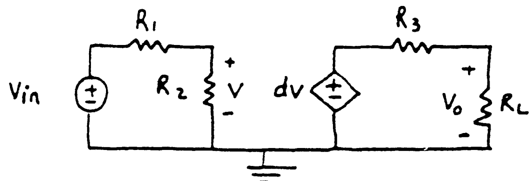
$\underline{11V_A - 5V_B + 30 = 0}$ (1)

KCL at V_B : $V_B/12 - 16 + V_B/6 + 1/2(V_B - V_A) = 0$

$\underline{2V_A - 3V_B + 64 = 0}$ (2)

Solving (1) & (2) simultaneously yields $\underline{V_A = 10\text{v},\ V_B = 28\text{v}}$

P. 4-22



$d = 10$

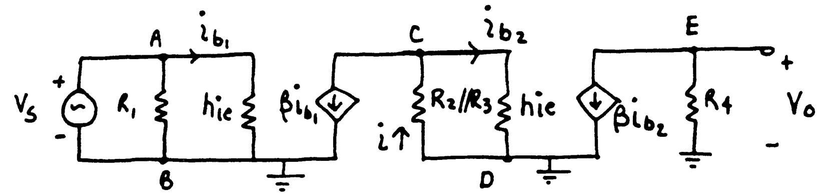
from voltage divider $V = V_{in} \dfrac{R_2}{R_1+R_2}$

from Voltage divider $V_o = dV \dfrac{R_L}{R_L+R_3}$

$= (10)\, V_{in} \dfrac{R_2}{R_1+R_2} \cdot \dfrac{R_L}{R_L+R_3}$

$$\therefore \quad \underline{\frac{V_o}{V_{in}} = \frac{10 R_L R_2}{(R_L+R_3)(R_1+R_2)}}$$

P. 4-23



KVL around ABA: $hie\, i_{b_1} - V_s = 0 \quad \Rightarrow \quad \underline{V_s = 1.3\, i_{b_1}}$ (1)

KCL at E : $V_o/R_4 + \beta i_{b_2} = 0 \quad \Rightarrow \quad \underline{V_o = -(2)(50)\, i_{b_2} = -100\, i_{b_2}}$ (2)

KCL at C : $i = \beta i_{b_1} + i_{b_2} = \underline{50 i_{b_1} + i_{b_2}}$ (3)

KVL around CDC: $hie\, i_{b_2} + i(R_2//R_3) = 0$

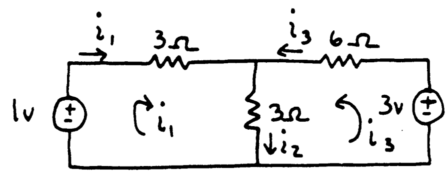
$\quad 1.3\, i_{b_2} + (50 i_{b_1} + i_{b_2})(1.985) = 0$

$\quad \underline{i_{b_2} = -30.213\, i_{b_1}}$ (4)

(4) into (2) yields $\quad i_{b_1} = \dfrac{V_o}{3021.31}$

$\therefore$ from (1) get $\quad V_s = \dfrac{1.3}{3021.31} V_o \quad \Rightarrow \quad \underline{\dfrac{V_o}{V_s} = 2324.08}$

P. 4-24



(a)

KVL mesh i_1 : $-1 + 3i_1 + 3(i_1 + i_3) = 0 \quad \Rightarrow \quad \underline{6i_1 + 3i_3 = 1}$ (1)

KVL mesh i_3 : $6i_3 + 3(i_3 + i_1) - 3 = 0 \quad \Rightarrow \quad \underline{i_1 + 3i_3 = 1}$ (2)

from KCL : $\underline{i_1 + i_3 = i_2}$ (3)

solving (1), (2) & (3) simultaneously yields $\underline{i_1 = 0}$, $\underline{i_2 = 1/3\,A}$, $\underline{i_3 = 1/3\,A}$

(b) calculate power absorbed

$P_{1v} = v i_1 = \underline{0}$

$P_{R_1} = i_1^2 R_1 = \underline{0}$

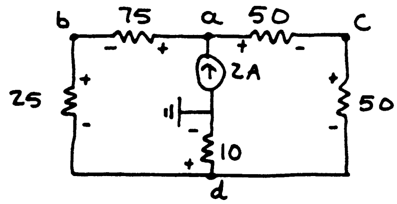
$P_{R_2} = i_2^2 R_2 = (1/3)^2(3) = \underline{1/3\,W}$

$P_{R_3} = i_3^2 R_3 = (1/3)^2(6) = \underline{2/3\,W}$

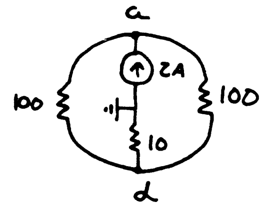
$P_{3v} = -v i_3 = -(3)(1/3) = \underline{-1\,W}$

See that $\sum P = 0$

P. 4-25 simplify ckt.



$\Rightarrow$



by symmetry 1A flows from a to b & from a to c

$\therefore$ $\underline{V_{ab} = 75v}$ and $\underline{V_{ac} = 50v}$

$\underline{V_{bd} = 25v}$ and $\underline{V_{cd} = 50v}$

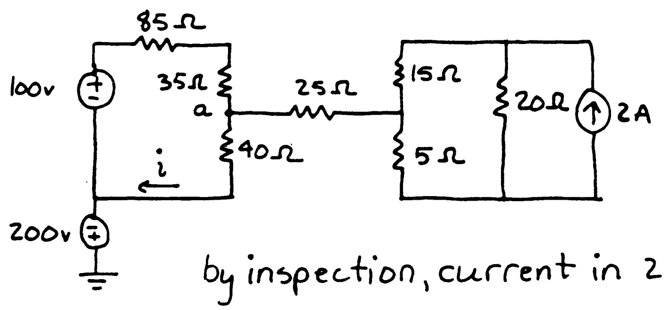
with respect to ground

$\underline{V_d = (10)(2A) = 20v}$

from KVL: $\underline{V_b = V_d + V_{bd} = 45v}$

$\underline{V_c = V_d + V_{cd} = 70v}$ and $\underline{V_A = V_b + V_{ab} = 120v}$

P. 4-26



by inspection, current in 25Ω = 0

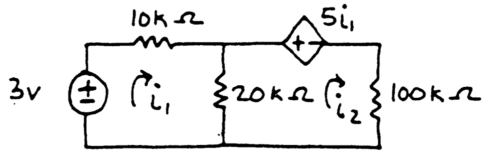
$\therefore$ $i = \frac{100v}{85+35+40} = \frac{100}{160} = 5/8 \text{ A}$

from KVL: $V_A = -200 + 40i$

$= -200 + 40(5/8)$

$\underline{V_A = -175v}$

P. 4-27



KVL i_1 ↻ : $-3 + 10i_1 + 20(i_1 - i_2) = 0 \Rightarrow \underline{30i_1 - 20i_2 = 3}$ (1)

KVL i_2 ↻ : $5i_1 + 100i_2 + 20(i_2 - i_1) = 0 \Rightarrow \underline{i_1 = 8i_2}$ (2)

solving (1) & (2) simultaneously $\Rightarrow i_1 = \frac{6}{55}$ mA , $i_2 = \frac{3}{220}$ mA

$\therefore P_{\text{deliv. to } 5i_1 \text{ and } 100k\Omega} = (5i_1)(i_2) + 100(i_2)^2$

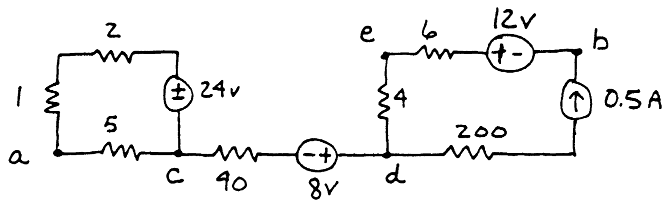
$= 5(6/55)(3/220) + 100(3/220)^2 = 0.026$ mA

$= 2.6 \times 10^{-5}$ W

$\therefore$ Energy in 24 hr. $= Pt = (2.6\times10^{-5})(24\text{hr})(3600\text{s/hr})$

$= \underline{2.25 \text{ J}}$

P. 4-28



KVL around acdeba ↻: $V_{ab} = V_{ac} + V_{cd} + V_{de} + V_{eb}$

by inspection current between c-d is zero $\therefore V_{cd} = -8$v

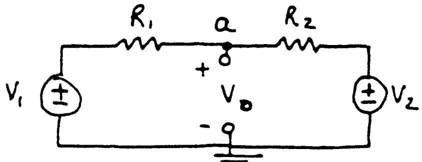
from voltage divider $V_{ac} = 24\left(\frac{5}{5+1+2}\right) = 15$v

and by inspection $V_{de} = (4)(-.5) = -2$v

$V_{eb} = 6(-.5) + 12 = 9$v

$\therefore \underline{V_{ab} = 15 - 8 - 2 + 9 = 14\text{V}}$

P. 4-29



KCL at a : $\frac{V_o - V_1}{R_1} + \frac{V_o - V_2}{R_2} = 0$

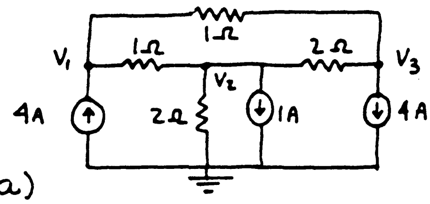
$\Rightarrow V_o = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2$

now want $\frac{R_2}{R_1+R_2} = 2/3$ and $\frac{R_1}{R_1+R_2} = 1/3$

solving the two above simultaneously yields

$\underline{R_1 = 1\Omega \text{ and } R_2 = 2\Omega}$

P. 4-30



(a)

KCL at v_1 : $-4 + (v_1 - v_3)/1 + (v_1 - v_2)/1 = 0$

$\Rightarrow \underline{2v_1 - v_2 - v_3 = 4}$ (1)

KCL at v_2 : $(v_2 - v_1)/1 + v_2/2 + 1 + (v_2 - v_3)/2 = 0$

$\Rightarrow \underline{-2v_1 + 4v_2 - v_3 + 2 = 0}$ (2)

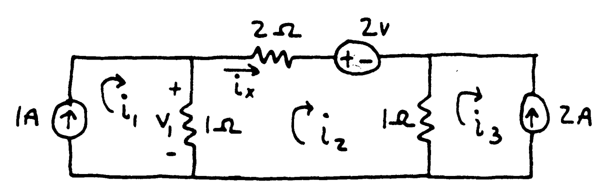
KCL at v_3 : $(v_3 - v_1)/1 + (v_3 - v_2)/2 + 4 = 0$

$\Rightarrow \underline{-2v_1 - v_2 + 3v_3 + 8 = 0}$ (3)

Solving (1), (2) & (3) simultaneously $\Rightarrow$ $\boxed{\begin{array}{l} v_1 = -1\text{v} \\ v_2 = -2\text{v} \\ v_3 = -4\text{v} \end{array}}$

(b) $P_{absorbed} = vi = -v_2(1) = -(2)(1) = \underline{-2W}$

P. 4-31



$i_1 = 1A$

$i_3 = -2A$

KVL mesh i_2 : $1(i_2 - i_1) + 2i_2 + 2 + 1(i_2 - i_3) = 0$

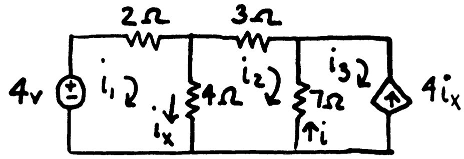
$\Rightarrow \underline{i_2 = -3/4\,A = i_x}$

now $v_1 = 1(i_1 - i_2) = 1 + 3/4 = \underline{1.75\text{ v}}$

P4-32 KCL : $-1 + \frac{v_a}{1} + \frac{v_a}{3} + \frac{v_a - 6}{2} = 0$

$\Rightarrow \underline{v_a = 24/11 = 2.18\text{v}}$

P 4-33



mesh 1 : $6i_1 - 4i_2 = 4$ (1)

mesh 2 : $-4i_1 + 14i_2 - 7[4(i_2-i_1)] = 0$ (2)

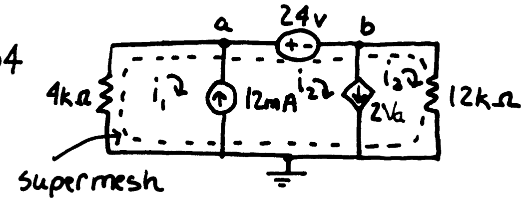
where $4i_x = 4(i_1-i_2)$

solving (1) & (2) yields : $i_1 = -14/3\,A$, $i_2 = -8\,A$

now note $i_3 = 4(i_2-i_1) = 4(-8+14/3) = -40/3\,A$

$\Rightarrow i = i_3 - i_2 = \underline{-16/3 = -5.33\,A}$

P 4-34



Supermesh: $4i_1 + 24 + 12i_3 = 0$ (1)

also : $i_2 - i_1 = 12$ (2)

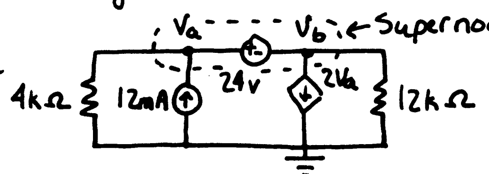
and : $-i_3 + i_2 = 2V_a$ (3)

and : $V_a = -4i_1$ (4)

a) mesh

solving (1) – (4) simultaneously yields $\underline{V_a = 6V}$

b) nodal



inside supernode

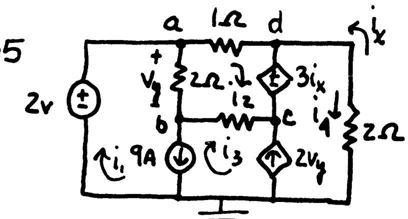
$V_b = V_a - 24$ (1)

KCL at supernode : $\frac{V_a}{4} - 12 + 2V_a + \frac{V_b}{12} = 0$ (2)

plugging (1) into (2) and multiplying by 12 yields

$27V_a - 144 + V_a - 24 = 0 \Rightarrow \underline{V_a = 6V}$

P 4-35



a) mesh analysis

combine loop i_1, 3 & 4 into supermesh

$\Rightarrow -2 + (i_1-i_2)2 + (i_3-i_2) - 3i_x + 2i_4 = 0$ (1)

loop i_2 : $-2i_1 + 4i_2 - i_3 + 3i_x = 0$ (2)

also : $i_1 - i_3 = 9$ (3)

$i_4 - i_3 = 2V_y$ (4)

$2(i_1-i_2) = V_y$ (5)

and $V_c = -2i_x - 3i_x$ (6)

$i_4 = -i_x$ (7)

solving (1) – (7) simultaneously yields

$i_x = -1A$, $\underline{V_c = 5V}$

P 4-35 (continued)

b) <u>nodal analysis</u>

$V_a = 2\text{v}$

KCL at b: $9 + \frac{V_b - V_c}{1} + \frac{V_b - 2}{2} = 0 \Rightarrow 3V_b - 2V_c = -16$ (1)

form supernode around nodes c & d, then KCL yields

$$\frac{V_d - 2}{1} + \frac{V_d}{2} - 2V_y + \frac{V_c - V_b}{1} = 0$$

with $V_y = 2 - V_b$ & multiplying above thru by 2 yields

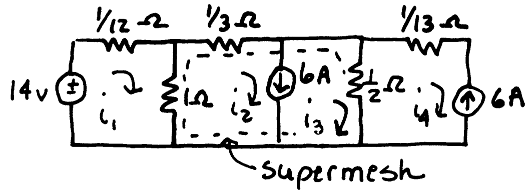
$$2V_b + 2V_c + 3V_d = 12 \qquad (2)$$

also: $V_d - V_c = 3i_x = 3(-V_d/2) \Rightarrow -V_c + 2.5V_d = 0$ (3)

Solving (1) – (3) yields

$\underline{V_c = 5\text{v}}$ and $V_d = 2\text{v} \Rightarrow \underline{i_x = -\frac{V_d}{2} = -1\text{A}}$

P 4-36



mesh (1): $-14 + \frac{1}{12}i_1 + 1(i_1 - i_2) = 0$ (1)

supermesh:

$1(i_2 - i_1) + \frac{1}{3}i_2 + \frac{1}{3}i_3 + \frac{1}{2}(i_3 - i_4) = 0$ (2)

also $i_2 - i_3 = 6$ (3) and $i_4 = -6$ (4)

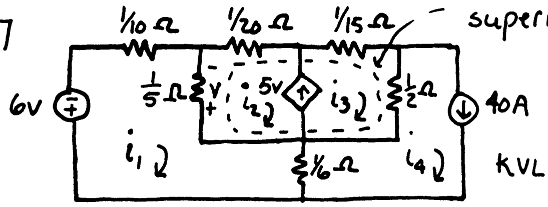
plugging (3) and (4) into (2) yields

$$(i_2 - i_1) + \frac{1}{3}i_2 + \frac{1}{3}(i_2 - 6) + \frac{1}{2}[(i_2 - 6) + 6] = 0 \qquad (5)$$

Solving (1) and (5) simultaneously yields

$\underline{i_1 = 22.89\text{A}}$, $\underline{i_2 = 10.79\text{ A}}$

P 4-37



KVL mesh1 :

$$6 + \frac{i_1}{10} + \frac{1}{5}(i_1 - i_2) + \frac{1}{6}(i_1 - 40) = 0 \qquad (1)$$

KVL supermesh:

$$\frac{1}{5}(i_2 - i_1) + \frac{1}{20}i_2 + \frac{1}{15}i_3 + \frac{1}{2}(i_3 - i_4) = 0 \qquad (2)$$

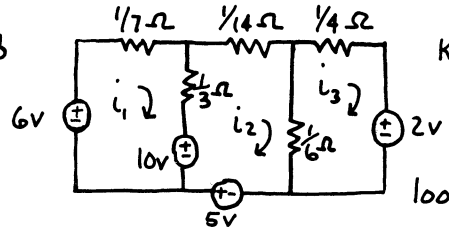
also note : $i_3 - i_2 = 5v = 5\frac{(i_2 - i_1)}{5} \Rightarrow i_3 = 2i_2 - i_1 \qquad (3)$

and note : $i_4 = 40 \qquad (4)$

Solving (1)-(4) simultaneously leads to

$i_1 = 10\text{A}$, $i_2 = 20\text{A}$, $\underline{i_3 = 30\text{A}}$

P 4-38



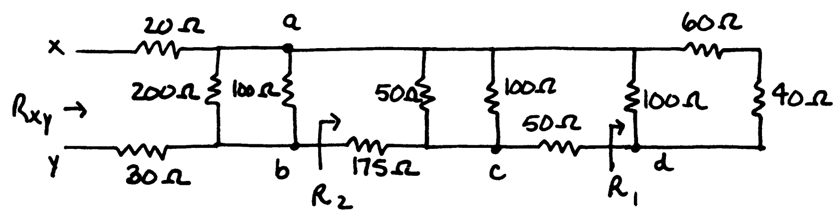
KVL loop1 : $\left(\frac{1}{7} + \frac{1}{3}\right)i_1 - \frac{1}{3}i_2 + 10 - 6 = 0 \qquad (1)$

loop 2 : $-\frac{1}{3}i_1 + \left(\frac{1}{3} + \frac{1}{14} + \frac{1}{6}\right)i_2 - \frac{1}{6}i_3 - 10 - 5 = 0 \qquad (2)$

loop 3 : $-\frac{1}{6}i_2 + \left(\frac{1}{6} + \frac{1}{4}\right)i_3 + 2 = 0 \qquad (3)$

Solving (1)-(3) simultaneously yields $\underline{i_1 = 21\text{A}}$, $\underline{i_2 = 42\text{A}}$, $\underline{i_3 = 12\text{A}}$

P 4-39

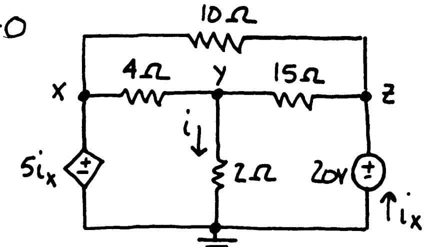


$$R_1 = \frac{1}{\frac{1}{100} + \frac{1}{60+40}} = 50\Omega$$

$$R_2 = 175 + \frac{1}{\frac{1}{50} + \frac{1}{100} + \frac{1}{50+R_1}} = 175 + \frac{1}{\frac{1}{50} + \frac{1}{100} + \frac{1}{100}} = 200\Omega$$

$$R_{xy} = 20 + 30 + \frac{1}{\frac{1}{200} + \frac{1}{100} + \frac{1}{R_2}} = 20 + 30 + \frac{1}{\frac{1}{200} + \frac{1}{100} + \frac{1}{200}} = \underline{100\Omega}$$

P 4-40



by inspection : $V_x = 5i$, $V_z = 20v$, $i = \frac{V_y}{2}$

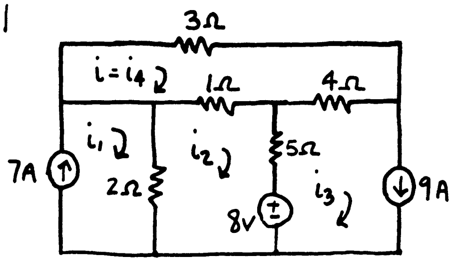
KCL at y : $\frac{V_y - 5i_x}{4} + \frac{V_y}{2} + \frac{V_y - 20}{15} = 0$ (1)

KCL at z : $\frac{20 - 5i_x}{10} + \frac{20 - V_y}{15} - i_x = 0$ (2)

Solving (1) & (2) simultaneously yields $V_y \doteq 4.713\,v$

$\Rightarrow i = \frac{V_y}{2} = \frac{4.713}{2} = \underline{2.356\ A}$

P 4-41



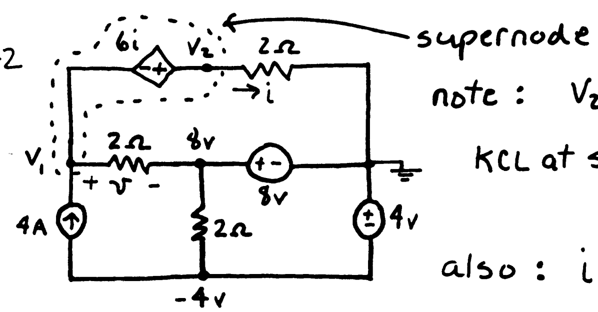
$i_1 = 7A$, $i_3 = 9A$

mesh i_2 : $1(i_2 - i_4) + 5(i_2 - 9) + 8 + 2(i_2 - 7) = 0$ (1)

mesh i_4 : $3i_4 + 4(i_4 - 9) + 1(i_4 - i_2) = 0$ (2)

solving (1) & (2) simultaneously yields : $\underline{i = i_4 = 5.38\,A}$

P 4-42



note : $V_2 = V_1 + 6i$ (1)

KCL at supernode :

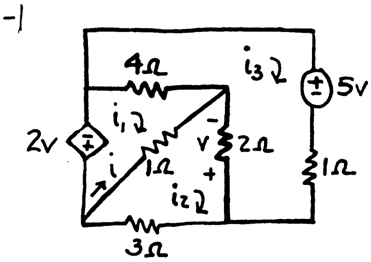
$$-4 + \frac{V_1 - 8}{2} + \frac{V_1 + 6i}{2} = 0 \qquad (2)$$

also : $i = \frac{V_2}{2}$ (3)

solving (1) - (3) simultaneously yields : $\underline{v = 24v}$

Advanced Problems

AP 4-1



$V = 2(i_3 - i_2)$ (1)

loop 1 : $2v + 4(i_1 - i_3) + 1(i_1 - i_2)$ (2)

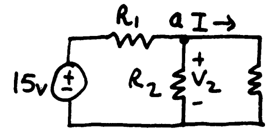
plugging (1) into (2) yields :

$4(i_3 - i_2) + 4(i_1 - i_3) + i_1 - i_2 = 0 \Rightarrow i_1 = i_2$

now $\underline{i = i_2 - i_1 = i_2 - i_2 = 0}$

AP 4-2

a) model



need to keep V_2 across R_2 as $4.8 \le V_2 \le 5.4$

I = .3 or .1A depending on whether activated or not

↑ display is active ↑ not active

KCL at a : $\frac{V_2 - 15}{R_1} + \frac{V_2}{R_2} + I = 0$

$\Rightarrow V_2' = 4.8v\ (I' = .3A)$; $V_2'' = 5.4v\ (I'' = .1A)$

assumed that maximum I results in minimum V_2 and visa-versa

now plug in V_2' ; V_2'' into KCL eqn. to generate 2 eqns and then solve for R_1 ; R_2 $\Rightarrow$ $\underline{R_1 = 7.89\Omega}$, $\underline{R_2 = 4.83\Omega}$

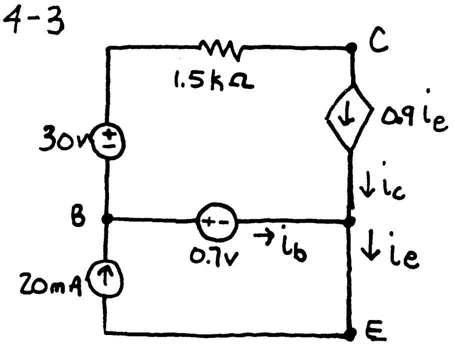
b) $I_{R_1 max} = \frac{15 - 4.8}{7.89} = 1.292A \Rightarrow P_{R_1 max} = (1.292)^2(7.89) = 13.17\ W$

$I_{R_2 max} = \frac{5.4}{4.83} = 1.118A \Rightarrow P_{R_2 max} = \frac{(5.4)^2}{4.83} = 6.03\ W$

$I_{15v\,max} = 1.292\ A$

c) No; if the supply voltage (15v) were to rise or drop, the voltage at the display at the display would drop below 4.8v or rise above 5.4v. The power dissipated in the resistors is excessive. Most of the power from the supply is dissipated in the resistors, not the display.

AP 4-3

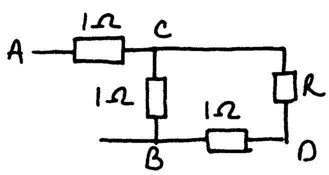


$i_e = 20\text{mA}$

$i_c = .9 i_e = 18\text{mA}$

$i_b = i_e - i_c = 2\text{mA}$

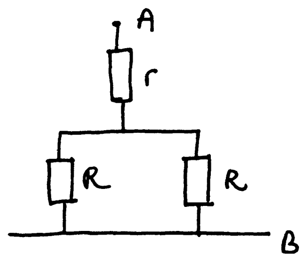
AP 4-4 The resistance R looking to the right from A to B is the same as the resistance looking to the right from C to D. So the right half of the network can be represented by the figure below:



Thus $R = 1 + \dfrac{1(R+1)}{1+R+1} = \sqrt{3}\ \Omega$

With the left half of the network included, the resistance between A and B is $\sqrt{3}/2$ or $.866\ \Omega$.

AP 4-5 Notice that if R is the resistance between A and B, the network can be drawn like:

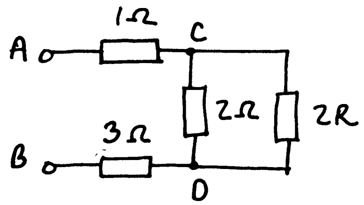


so $R = r + R/2$

or $\underline{R = 2r}$

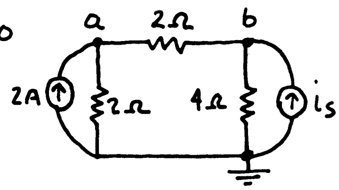
AP 4-6 The portion of the network to the right of C and D is the same as the original network with every resistance doubled. So the original network can be represented by the figure below. If R is the resistance between A and B, then have

$$R = 4 + \frac{4R}{2+2R} \quad \text{and} \quad R = \frac{5+\sqrt{41}}{2} = 5.70\ \Omega$$



Design Problems

DP 4-1 simplify to



KCL at node a : $v_a\left(\frac{1}{2}+\frac{1}{2}\right) - \frac{1}{2}v_b - 2 = 0$ (1)

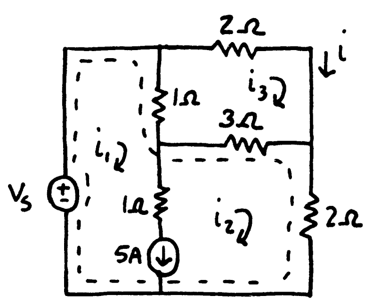
at node b : $-\frac{1}{2}v_a + \left(\frac{1}{2}+\frac{1}{4}\right)v_b - i_s = 0$ (2)

now $v_{ba} = 3 = v_b - v_a \Rightarrow v_b = 3 + v_a$ (3)

plugging (3) into (1) yields : $\underline{v_a = 7\text{v}}$ & $\underline{v_b = 10\text{v}}$

thus from (2) get : $\underline{i_s = 4\text{A}}$

DP 4-2



$i_3 = i = 3\text{A}$

supermesh : $-V_s + 1(i_1 - i_3) + 3(i_2 - i_3) + 2i_2 = 0$

$\hookrightarrow -V_s + i_1 + 5i_2 = 12$ (1)

mesh 3 : $1(i_3 - i_1) + 2i_3 + 3(i_3 - i_2) = 0$

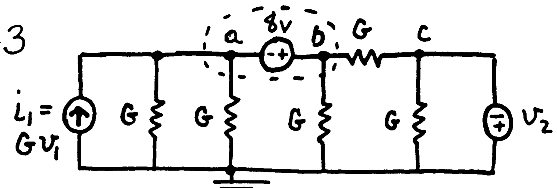
$\hookrightarrow i_1 + 3i_2 = 18$ (2)

also $i_1 - i_2 = 5$ (3)

combining (2) & (3) yields $i_2 = 3.25\text{A},\ i_1 = 8.25\text{A}$

from (1) $\underline{V_s = 12.5\text{v}}$

DP 4-3



$G = .5\,S$
$i_1 = Gv_1$
$v_c = -v_2$

supernode : $v_a(G+G) + v_b(G+G) - Gv_c = i_1$ (1)

also : $v_b - v_a = 8$ (2)

combining (1) & (2) yields $v_a + (8+v_a) + .5v_2 = .5v_1$

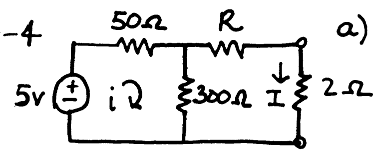
$2v_a = .5v_1 - .5v_2 - 8$

$\Rightarrow v_a = \dfrac{.5v_1 - .5v_2 - 8}{2} = 0$

$\therefore\ .5v_1 - .5v_2 = 8$

so let $\underline{v_2 = 2v}$ and $\underline{v_1 = 18v}$ one solution

DP 4-4



desire
$50mA \le I \le 75mA$

a) KVL left mesh: $-5 + 50i + 300(i - I) = 0$ (1)

right mesh: $(R+2)I + 300(I - i) = 0$ (2)

solving (1) & (2) for I $\Rightarrow$ $I = \dfrac{150}{1570 + 35R}$ (3)

so if $R = 100$, then $\underline{I = 29.59\,mA}$ $\Rightarrow$ lamp will not light

b) from (3) note that as $R\downarrow$ $I\uparrow$, so try $R = 50\Omega \Rightarrow I = 45mA$ (won't light)

try $R = 25\Omega \Rightarrow I = 61mA \Rightarrow$ will light

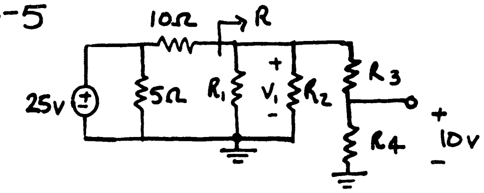
now check if $R \pm 10\%$ will light and not burn out

$-10\% \to 22.5\Omega \to I = 63.63\,mA$

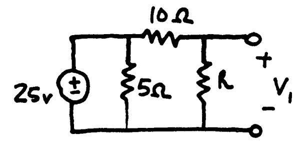
$+10\% \to 27.5\Omega \to I = 59.23\,mA$

lamp will stay on

DP 4-5



$R = R_1 // R_2 // (R_3 + R_4)$



$$V_1 = 25 \frac{R}{10+R}$$

using voltage divider

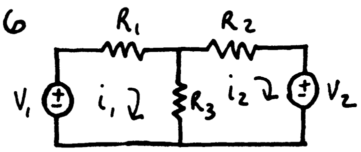
$$10 = \frac{R_4}{R_3+R_4} V_1 = \frac{R_4}{R_3+R_4} \frac{(R_1 // R_2 // (R_3+R_4))}{10 + (R_1 // R_2 // (R_3+R_4))} 25$$

choose $R_1 = R_2 = 25\Omega$ $\longrightarrow$ $10 = \frac{R_4}{20} \frac{(12.5 // 20)}{10 + (12.5 // 20)} 25 \Rightarrow \underline{R_4 = 18.4\Omega}$

one solution

& $R_3 + R_4 = 20 \Rightarrow \underline{R_3 = 1.6\Omega}$

DP 4-6



mesh i_1 : $(R_1+R_3)i_1 - R_3 i_2 - V_1 = 0$ (1)

mesh i_2 : $-R_3 i_1 + (R_2+R_3)i_2 + V_2 = 0$ (2)

from (1) & (2) get: $i_1 = \dfrac{\begin{bmatrix} V_1 & -R_3 \\ -V_2 & (R_2+R_3) \end{bmatrix}}{\Delta}$ $\quad i_2 = \dfrac{\begin{bmatrix} (R_1+R_3) & V_1 \\ -R_3 & -V_2 \end{bmatrix}}{\Delta}$

where $\Delta = (R_1+R_3)(R_2+R_3) - R_3^2$

now if $R_1 = R_2 = R_3 = 1K$ where K represents 1000

then $\Delta = 4 - 1 = 3K^2$

so have $i_1 = \dfrac{[2V_1 - V_2]K}{3K^2}$, $i_2 = \dfrac{[-2V_2 + V_1]K}{3K^2}$

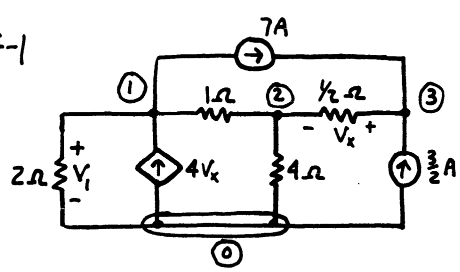
$\Rightarrow i = i_1 - i_2 = \dfrac{V_1 + V_2}{3K}$

if $V_1 = V_2 = 1v \Rightarrow i = 2/3\,mA$ okay

if $V_1 = V_2 = 2v \Rightarrow i = 4/3\,mA$ okay

Spice Problems

SP 4-1

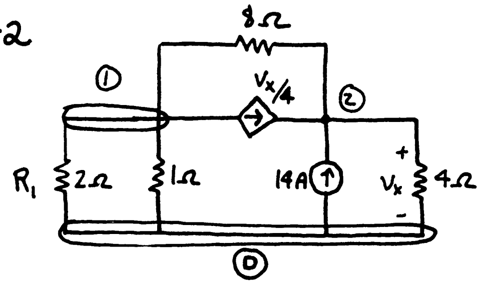


ans. $\underline{V_1 = 24v}$

input file :

```
R1   1 0 2
R2   1 2 1
R3   2 0 4
R4   2 3 .5
G1   0 1 3 2 4
I1   1 3 7
I2   0 3
.print dc v(1)
.END
```

SP 4-2



Ans. $V_x = V_2 = 104\,V$

$V_1 = -8\,V$

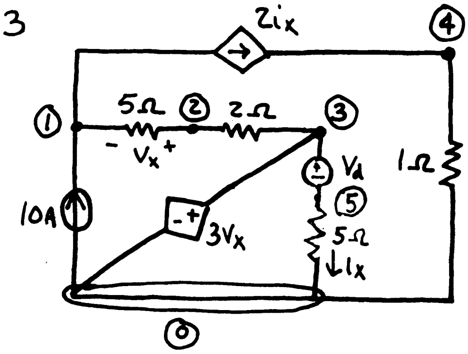
$\Rightarrow P_{2\Omega} = \frac{(8)^2}{2} = \underline{32\,W}$

input file :

```
R1   1 0 2
R2   1 0 1
R3   1 2 8
R4   2 0 4
G1   1 2 2 0 .25
I1   0 2 14
.DC  I1 14 14 1
.PRINT DC V(1) V(2) I(R1)
.END
```

Also I(R1) = −4 A

SP 4-3



Ans. $i_x = i_{R_3} = \underline{6\,A}$

$V(2,1) = V_x = V_2 - V_1 = \underline{10\,V}$

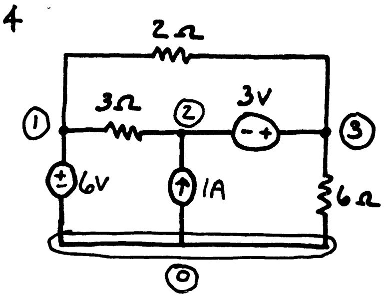
$P_{1\Omega} = 1^2\, 2i_x = \underline{12\,W}$

$P_{10A} = -(10A)\,V_1 = \underline{-160\,W}$

input file

```
R1  1 2 5
R2  2 3 2
R3  5 0 5
R4  4 0 1
I1  0 1 10
VDummy 3 5 DC 0 ; Note the dummy source was
F1  1 4 VDummy 2 ; necessary because the format
E1  3 0 2 1 3    ; of F (cccs) requires the
.DC I1 10 10 1   ; current source to be dependent
.PRINT DC I(R3) V(2,1) V(1)  ; on a voltage source
.END
```

SP 4-4



ans. $V_1 = 6\,V$

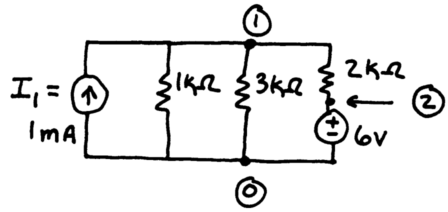
$V_2 = 4\,V$

$V_3 = 7\,V$

input file

```
V1  1 0 DC 6
I1  0 2 1
V2  3 2 3
R1  1 2 3
R2  1 3 2
R3  3 0 6
.END
```

SP 4-7



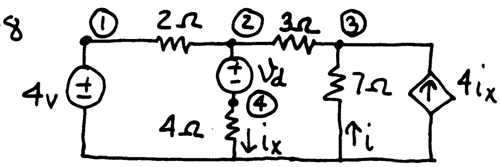
input file

```
*uses .OP Default
I1  0 1  DC  1m
V1  2 0  DC  6
R1  1 0  1K
R2  1 2  2K
R3  1 0  3K
.PRINT DC V(1) V(2)
.END
```

Output: V(1) = 2.1818 v
V(2) = 6.00 v

current through voltage source V(1)
I(V1) = -1.909×10^{-3} A

SP 4-8



$V_d = 0$ measures i_x for CCCS

input file:

```
V1  1 0  DC 4
VD  2 4  DC 0
F1  0 3  VD 4
R1  1 2  2
R2  4 0  4
R3  2 3  3
R4  0 3  7     ; I flows 0 to 3
.DC V1 4 4 1
.PRINT DC V(1) I(R4)
. END
```

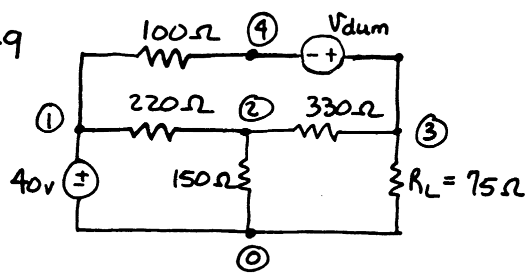
output:

V(1) = 4.00 v

I (R4) = 8.00 A

then i = 8.0 A

SP 4-9

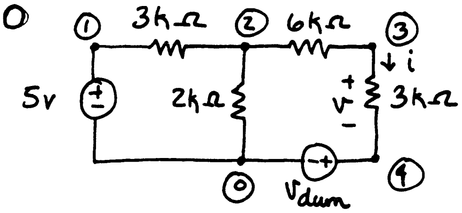


input file

```
V1    1 0  DC 40
R1    1 2  220
R2    2 0  150
R3    2 3  330
R4    3 0  75
R5    1 4  100
VDUM  3 4
. DC  40 40 1
.PRINT DC V(1)
.END
```

output: V(3) = 17.0569 v

SP 4-10



input file

```
R3   2  3  6000
R1   1  2  3k
R2   3  4  3K
R4   2  0  2K
V1   1  0  DC 5
VDum 4  0  0
.PRINT DC V(3)
.END
```

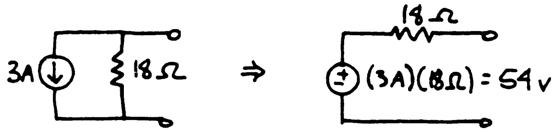
output : $V(3) = 0.5882$ v

$I(V_{DUM}) = 1.961E-4$ A

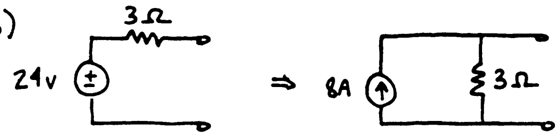
Chapter 5

Exercises

Ex. 5-1 (a)

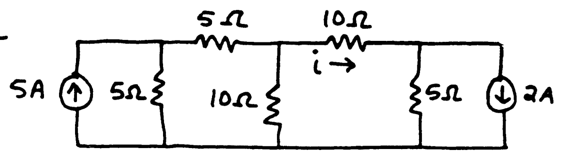


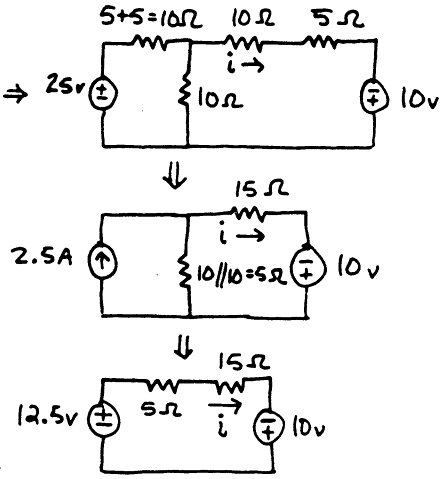
(b)



where $8A = 24v/3\Omega$

Ex. 5-2

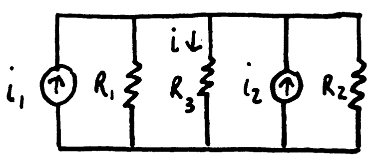




∴ from KVL

$$i = \frac{10v + 12.5v}{5\Omega + 15\Omega} = \underline{1.125\ A}$$

Ex 5-3 Using the method of voltage source transform, we change the two voltage sources into two current sources

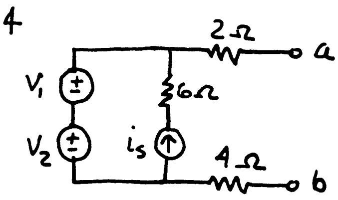


where $i_1 = \frac{V_1}{R_1} = -\frac{1}{2}e^{-t}\ (A)$

$i_2 = \frac{V_2}{R_2} = \frac{1}{2}e^{-2t}\ (A)$

because $R_1 = R_2 = R_3 = 2\Omega \Rightarrow i = \frac{1}{2+2+2}(i_1 + i_2) = \underline{\frac{1}{6}(e^{-2t} - e^{-t})\ A}$

Ex 5-4

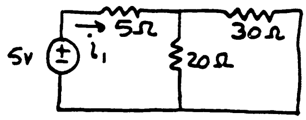


terminals a-b are open

⇒ no current in the 2Ω & 4Ω resistors

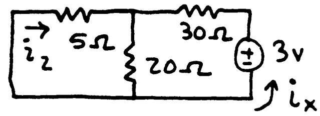
∴ $\underline{V_{ab} = V_1 + V_2}$

Ex. 5-5 consider 5v source only (short 3v source)



$$i_1 = \frac{5v}{5 + 20/\!/30} = \frac{5}{5+12} = \underline{0.294\ A}$$

consider 3v source (short 5v source)

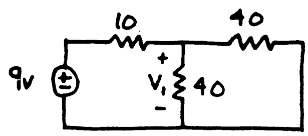


$$i_x = \frac{3v}{30 + 20/\!/5} = \frac{3}{30+4} = 0.088\ A$$

∴ from current divider $i_2 = -i_x\left(\frac{20}{20+5}\right) = \underline{-0.071\ A}$

∴ $\underline{i = i_1 + i_2 = 0.294 - 0.071 = 0.223\ A}$

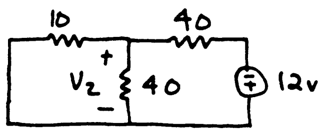
Ex. 5-6 consider 9v source only (short 12v source)



voltage divider ⇒

$$V_1 = 9v\left(\frac{40/\!/40}{10 + 40/\!/40}\right) = 9\left(\frac{20}{10+20}\right) = \underline{6v}$$

consider 12v source (short 9v source)

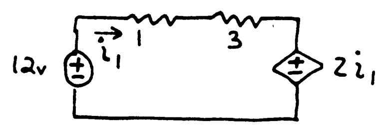


Voltage divider ⇒

$$V_2 = -12v\left(\frac{40/\!/10}{40 + 40/\!/10}\right) = \underline{-2v}$$

∴ $\underline{V = V_1 + V_2 = 6 - 2 = 4v}$

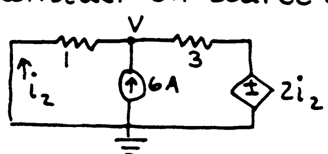
Ex. 5-7 consider 12v source only (open 6A source)



KVL: $-12 + i_1(1+3) + 2i_1 = 0$

⇒ $\underline{i_1 = 2A}$

consider 6A source only (short 12v source)



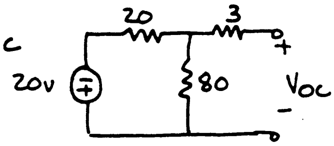
KCL at V: $-i_2 - 6 + (V - 2i_2)/3 = 0$

also $V = -(1)i_2$

∴ solve for $\underline{i_2 = -3A}$

∴ $\underline{i = i_1 + i_2 = 2 - 3 = -1A}$

Ex. 5-8 find V_{oc}

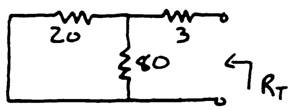


no current flows through 3 Ω

∴ from voltage divider

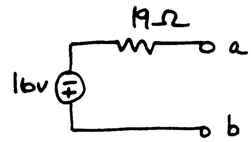
$$V_{oc} = -20v\left(\frac{80\Omega}{80\Omega + 20\Omega}\right) = \underline{-16v}$$

to find R_T, kill 20v source

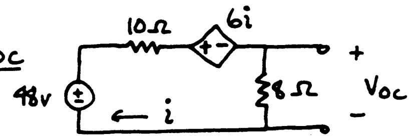


$R_T = 20 /\!/ 80 + 3 = \underline{19\Omega}$

∴ Thev. equiv. ckt ⇒



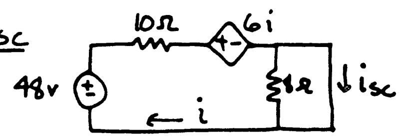
Ex. 5-9 <u>find V_{oc}</u>



KVL ↻ : $-48 + 10i + 6i + 8i = 0 \Rightarrow i = 2A$

∴ $\underline{V_{oc} = 8i = 16v}$

<u>find i_{sc}</u>

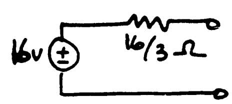


since no current flows through 8Ω ⇒ $i_{sc} = i$

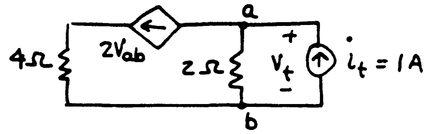
KVL ↻ : $-48 + 10i_{sc} + 6i_{sc} = 0 \Rightarrow \underline{i_{sc} = 3A}$

∴ $R_T = V_{oc}/i_{sc} = \underline{16/3\ \Omega}$

Thev. equiv. ckt ⇒



Ex. 5-10 no independent sources ∴ $V_{oc} = i_{sc} = 0$ ⇒ apply 1A test source

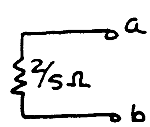


$V_{ab} = V_t$

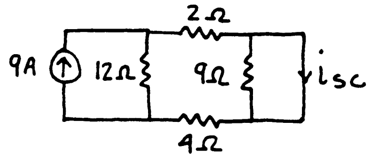
KCL at a : $2V_t + V_t/2 - 1 = 0 \Rightarrow V_t = 2/5\ v$

$$\therefore R_T = \frac{V_t}{i_t} = 2/5\ \Omega$$

Thev. equiv. ckt ⇒



Ex. 5-11 find i_{sc}



no current flows through 9Ω

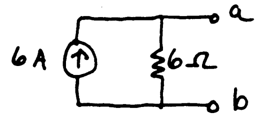
$\therefore$ from current divider

$$i_{sc} = 9\left(\frac{12}{12+6}\right) = \underline{6A}$$

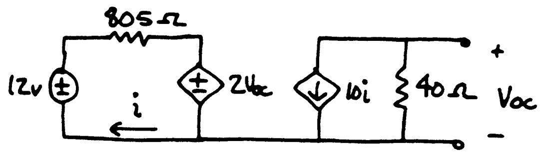
open 9A source to find R_T

$$\therefore R_T = 9 // (2+4+12) = \frac{(9)(18)}{9+18} = \underline{6\Omega}$$

$\therefore$ Norton equiv. ckt $\Rightarrow$



Ex. 5-12 find V_{oc}

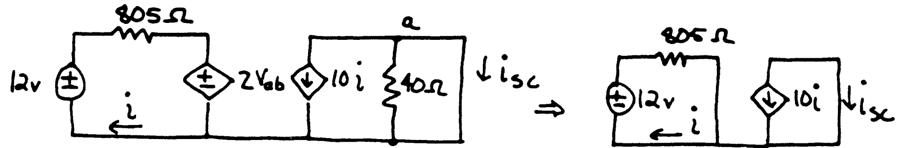


KVL around left mesh: $-12 + 805i + 2V_{oc} = 0$ (1)

from right loop: $V_{oc} = -10i(40) = -40i$ (2)

solving (1) and (2) simultaneously $\Rightarrow$ $\underline{V_{oc} = -960v}$

find i_{sc}



$V_{ab} = 0$ here

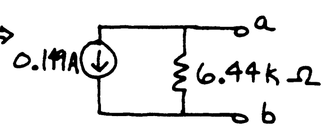
$i_{sc} = -10i$

from left loop: $i = 12v/805\Omega = -i_{sc}/10$

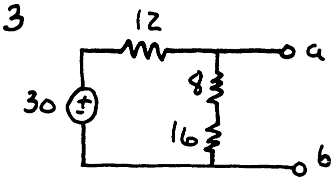
$$\therefore \underline{i_{sc} = -0.149A}$$

$$\therefore R_T = V_{oc}/i_{sc} = \frac{-960}{-0.149} = 6.44 \times 10^3 \Omega = \underline{6.44k\Omega}$$

Norton equiv. ckt $\Rightarrow$



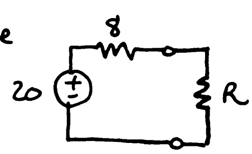
Ex 5-13



$$R_T = \frac{12 \times 24}{12+24} = \frac{12 \times 24}{36} = 8\Omega$$

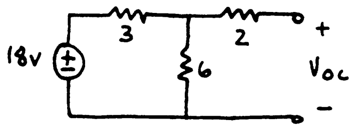
$$V_{oc} = \frac{24}{12+24} 30 = 20v$$

so have



$$i = \frac{20}{8+R} \text{ A}$$

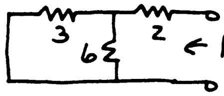
Ex. 5-14 find V_{oc}



from voltage divider

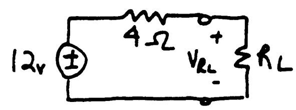
$$V_{oc} = 18v\left(\frac{6}{6+3}\right) = \underline{12v}$$

find R_T (short 18v source)



$$\leftarrow R_T = 3 /\!/ 6 + 2 = \underline{4\Omega}$$

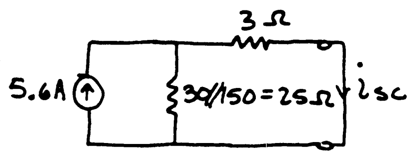
∴ Thev. equiv. ckt ⇒



for max power to R_L ⇒ $R_L = R_T = 4\Omega$

$$\therefore \underset{\text{to } R_L}{P_{max}} = \frac{(V_{R_L})^2}{R_L} = \frac{(6)^2}{4} = \underline{9W}$$

Ex. 5-15 find i_{sc}



from current divider

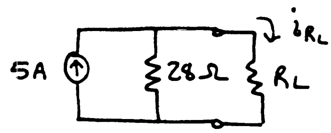
$$i_{sc} = 5.6A\left(\frac{25}{25+3}\right)$$

$$\underline{i_{sc} = 5A}$$

find R_T (open 5.6A source)

$$\leftarrow R_T = 25 + 3 = \underline{28\Omega}$$

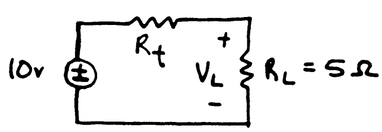
∴ Norton equiv. ckt ⇒



for max power $R_L = R_T = 28\Omega$

$$\therefore P_{L\,max} = (i_{R_L})^2 R_L = (5/2)^2(28) = \underline{175W}$$

Ex. 5-16



$$P_{L\,max} = \frac{V_{L\,max}^2}{R_L} = \frac{\left[10v\left(\frac{5}{5+R_t}\right)\right]^2}{R_L}$$

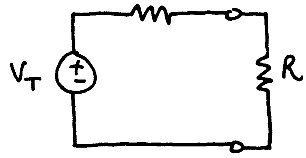
now for V_L to be maximized R_t must be minimized

∴ choose $R_t = 1\Omega$

$$\therefore P_{L\,max} = \frac{\left[10\left(\frac{5}{6}\right)\right]^2}{5} = \underline{13.9\,W}$$

Ex 5-17

$20\,\Omega = R$ at P_{max}

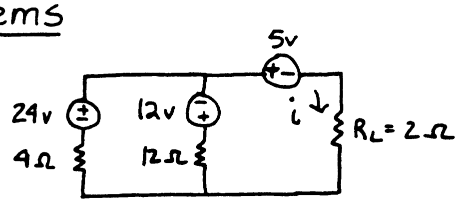


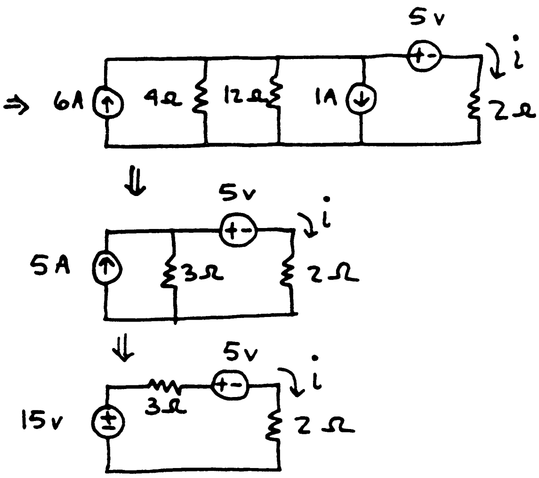
$$P_{max} = 5 = \left(\frac{V_T}{40}\right)^2 20 = \frac{V_T^2}{80}$$

$$\Rightarrow \underline{V_T = \sqrt{400} = 20\text{ V}}$$

Problems

P. 5-1

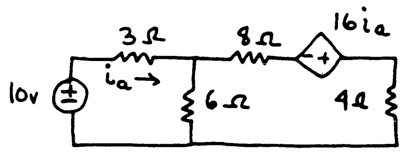


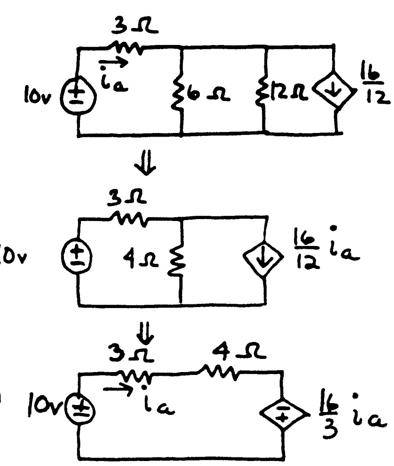


$$i = \frac{15v - 5v}{3\Omega + 2\Omega} = 2A$$

$$P_{R_L} = i^2 R_L = (2)^2(2) = 8W$$

P. 5-2

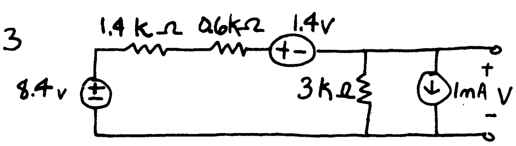


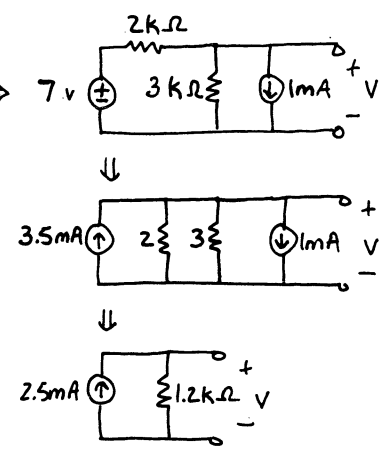


KVL ↻ : $-10 + 3i_a + 4i_a + \frac{16}{3} i_a = 0$

$\Rightarrow i_a = 6A$

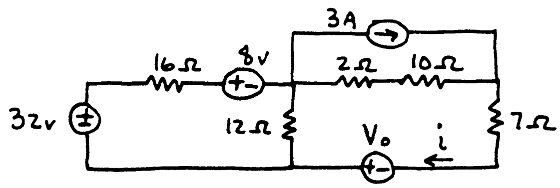
P. 5-3

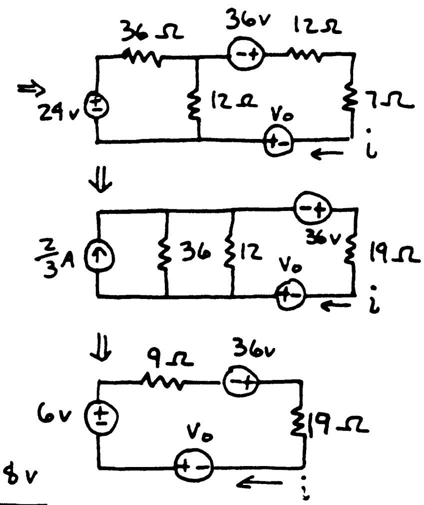




$V = (1.2k\Omega)(2.5mA) = 3$ volts

P. 5-4

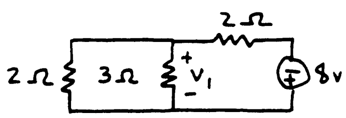




KVL ↻ : $-6 + i(9+19) - 36 - V_o = 0$

$\Rightarrow V_o = -42 + 28(5/2) = 28\text{v}$

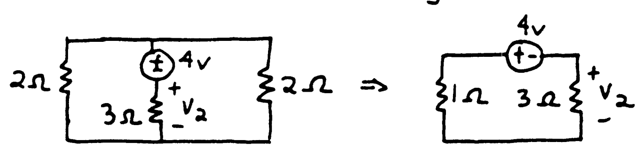
P. 5-5 consider 8v source only (short 4v source)



from voltage divider

$V_1 = -8\text{v}\left[\dfrac{2\|3}{2\|3+2}\right] = (-8)\left(\dfrac{1.2}{3.2}\right) = \underline{-3\text{v}}$

consider 4v source only (short 8v source)

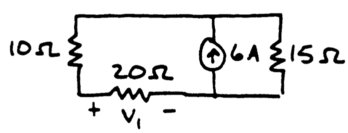


Voltage divider

$V_2 = -4\text{v}\left(\dfrac{3}{3+1}\right) = \underline{-3\text{v}}$

$\therefore\ V = V_1 + V_2 = -3 - 3 = -6$ volts

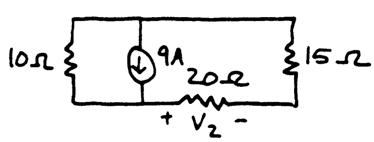
P. 5-6 consider 6A source only (open 9A source)



from current divider

$V_1/20 = 6\left[\dfrac{15}{15+30}\right] \Rightarrow \underline{V_1 = 40\text{v}}$

consider 9A source only (open 6A source)

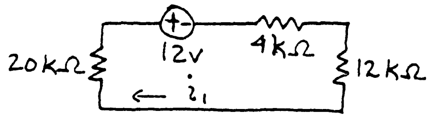


current divider :

$V_2/20 = 9\left[\dfrac{10}{10+35}\right] \Rightarrow \underline{V_2 = 40\text{v}}$

$\therefore\ V = V_1 + V_2 = 40 + 40 = 80\text{v}$

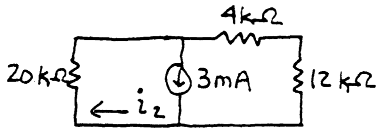
P. 5-7 consider 12v source only (open both current sources)



KVL: $20i_1 + 12 + 4i_1 + 12i_1 = 0$

$\Rightarrow \underline{i_1 = -1/3\,\text{mA}}$

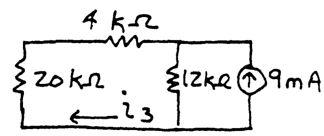
consider 12mA source only (short 12V and open 6mA sources)



from current divider

$i_2 = 3\left[\frac{16}{16+20}\right] = \underline{\frac{4}{3}\,\text{mA}}$

consider 9mA source only (short 12v and open 12mA sources)

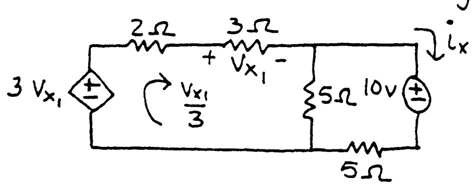


from current divider

$i_3 = -9\left[\frac{12}{24+12}\right] = \underline{-3\,\text{mA}}$

$\therefore \underline{i = i_1 + i_2 + i_3 = -1/3 + 4/3 - 3 = -2\,\text{mA}}$

P. 5-8 consider 10V source only (open 4A source)



KVL 1st mesh:

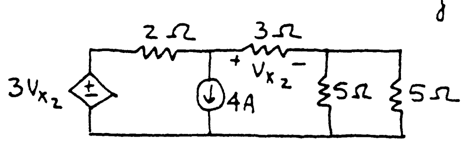
$-3V_{x_1} + 5\left(\frac{V_{x1}}{3}\right) + 5\left(\frac{V_{x1}}{3} - i_x\right) = 0$

$\underline{V_{x_1} = 15 i_x}$ (1)

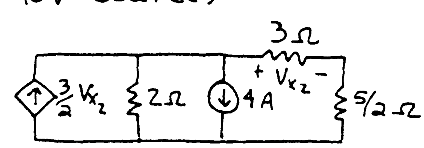
KVL 2nd mesh: $\underline{5(i_x - V_{x1}/3) + 10 + 5i_x = 0}$ (2)

Solving (1) and (2) simultaneously $\Rightarrow \underline{V_{x_1} = 10\,\text{v}}$

Consider 4A source only (short 10v source)



$\Rightarrow$

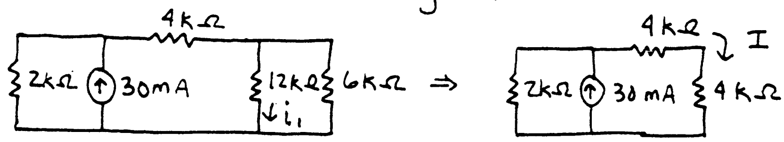


using current divider

$\frac{V_{x2}}{3} = \left(3/2\,V_{x_2} - 4\right)\left(\frac{2}{2+3+5/2}\right) \Rightarrow \underline{V_{x_2} = 16\,\text{v}}$

$\therefore \underline{V_x = V_{x_1} + V_{x2} = 10 + 16 = 26\,\text{V}}$

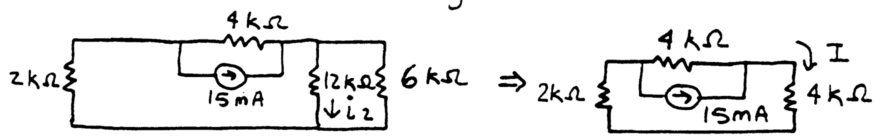
P. 5-9 consider 30mA source only (open 15mA and short 60v sources)



current divider ⇒ $I = 30\left(\frac{2}{2+8}\right) = 6\text{ mA}$

$\therefore\ i_1 = I\left(\frac{6}{6+12}\right) = \underline{2\text{ mA}}$

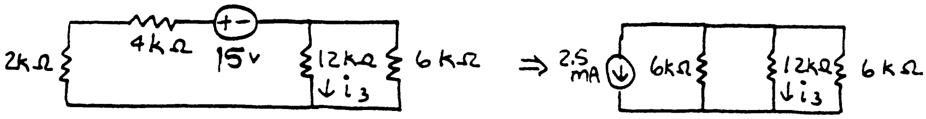
consider 15mA source only (open 30mA source and short 60v source)



current divider ⇒ $I = 15\left(\frac{4}{4+6}\right) = 6\text{mA}$

$\therefore\ i_2 = I\left(\frac{6}{6+12}\right) = \underline{2\text{mA}}$

consider 15v source only (open both current sources)

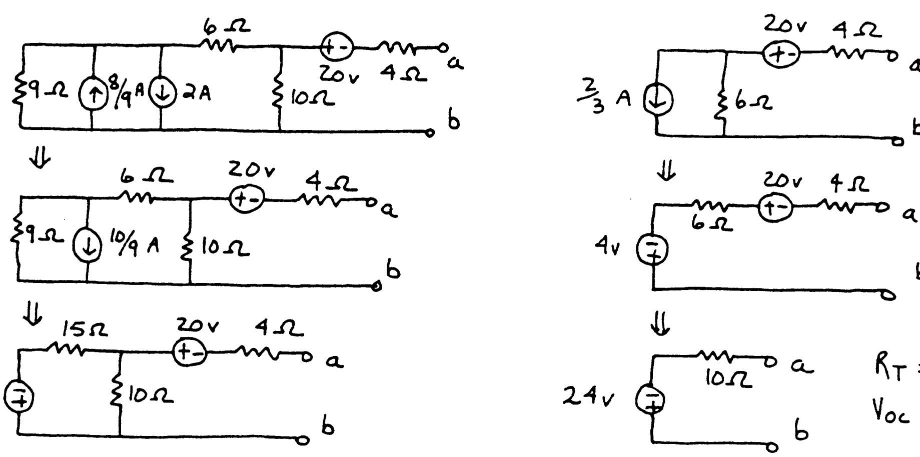


from current divider

$i_3 = -2.5\left(\frac{6/\!/6}{6/\!/6+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-.5\text{mA}}$

$\therefore\ \underline{i = i_1 + i_2 + i_3 = 2 + 2 - .5 = 3.5\text{mA}}$

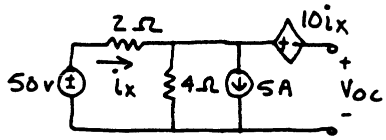
P. 5-10 use source transformations



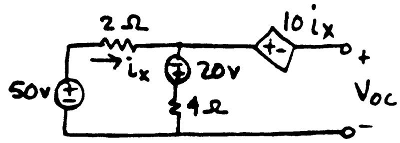
$R_T = 10\Omega$

$V_{oc} = -24\text{v}$

P. 5-11 find V_{oc}



⇒

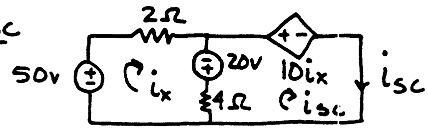


KVL around 1st mesh ↻ : $-50 + 2i_x - 20 + 4i_x = 0 \Rightarrow i_x = 70/6\ A$

KVL around 2nd mesh ↻ : $-4i_x + 20 + 10i_x + V_{oc} = 0$

$\Rightarrow \underline{V_{oc} = -90v}$

find i_{sc}



KVL i_x mesh ↻ : $-50 + 2i_x - 20 + 4(i_x - i_{sc}) = 0$

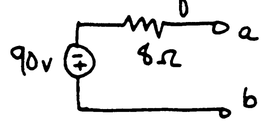
↳ $\underline{6i_x - 4i_{sc} - 70 = 0}$ (1)

KVL i_{sc} mesh ↻ : $4(i_{sc} - i_x) + 20 + 10i_x = 0$

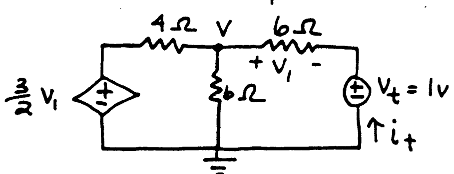
↳ $\underline{6i_x + 4i_{sc} + 20 = 0}$ (2)

solving (1) and (2) simultaneously $\Rightarrow \underline{i_{sc} = -45/4\ A}$

$\therefore R_T = \dfrac{V_{oc}}{i_{sc}} = \underline{8\Omega}$ Thev equiv. ckt:



P. 5-12 since no independent sources $V_{oc} = i_{sc} = 0$ ∴ apply test source



$V = V_1 + V_t$

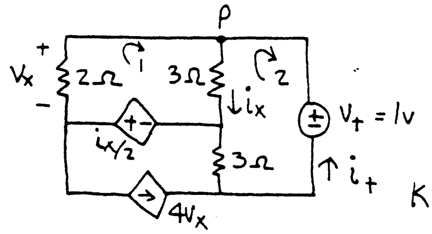
KCL at V : $(V - 3/2\,V_1)/4 + V/6 + V_1/6 = 0$ & with $V = V_1 + 1$

↳ $V_1 = -2A$

now $i_T = -\dfrac{V_1}{6} = \underline{1/3\,A}$

$\therefore R_T = \dfrac{V_t}{i_t} = \dfrac{1}{1/3} = \underline{3\Omega}$

P. 5-13 Since only have dependent sources, $V_{oc} = i_{sc} = 0$, ∴ use test source



KVL ↻ mesh 1 : $-V_x + 3i_x - \frac{1}{2}i_x = 0$

$\Rightarrow \underline{V_x = 5/2\, i_x}$ (1)

KVL ↻ mesh 2 : $1 + 3(-i_t + 4V_x) - 3i_x = 0$

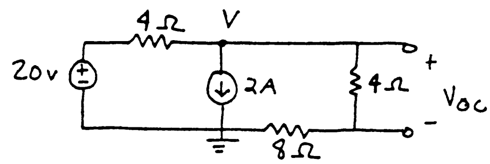
$\Rightarrow \underline{12V_x - 3i_t - 3i_x + 1 = 0}$ (2)

KCL at P : $\underline{V_x/2 + i_x - i_T = 0}$ (3)

solving (1), (2) & (3) simultaneously yields $\underline{i_T = -1/9\ A}$

$\therefore R_T = \frac{V_T}{i_T} = \frac{1}{-1/9} = \underline{-9\Omega}$

P. 5-14 find V_{oc}



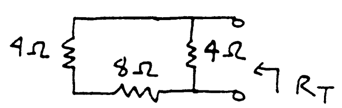
KCL at V : $(V-20)/4 + 2 + V_{oc}/4 = 0$

$\Rightarrow \underline{V + V_{oc} - 12 = 0}$ (1)

voltage divider : $V_{oc} = V\left(\frac{4}{4+8}\right) = \underline{V\left(\frac{1}{3}\right)}$ (2)

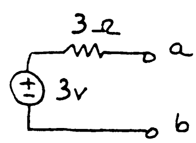
solving (1) & (2) simultaneously $\Rightarrow \underline{V_{oc} = 3V}$

find R_T (kill all sources)

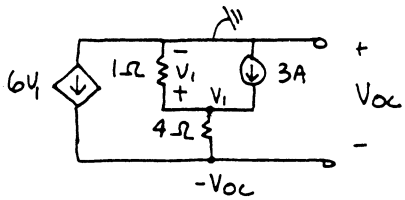


$\underline{R_T = 12 \| 4 = 3\Omega}$

Theu. equiv. ckt ⇒



P. 5-15 find V_{oc}



KCL at V_1 : $V_1 - 3 + (V_1 + V_{oc})/4 = 0$

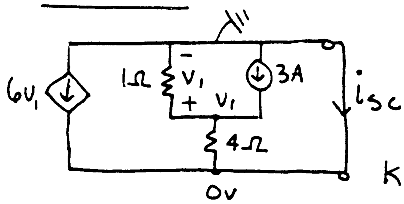
$\rightarrow$ $\underline{5V_1 + V_{oc} - 12 = 0}$ (1)

KCL at $-V_{oc}$: $-6V_1 + (-V_{oc} - V_1)/4 = 0$

$\rightarrow$ $\underline{-25V_1 = V_{oc}}$ (2)

solving (1) & (2) simultaneously $\Rightarrow$ $\underline{V_{oc} = 15\text{v}}$

find i_{sc}



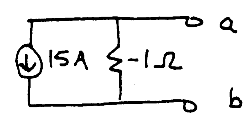
KCL at V_1 : $V_1 - 3 + (V_1)/4 = 0$

$\Rightarrow$ $\underline{V_1 = 12/5\text{ v}}$

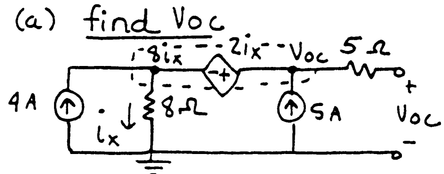
KCL at 0v : $-6V_1 - V_1/4 - i_{sc} = 0$

$\Rightarrow$ $\underline{i_{sc} = -15\text{A}}$

$\therefore$ $R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{15}{-15} = \underline{-1\Omega}$ $\Rightarrow$



P. 5-16 (a) find V_{oc}

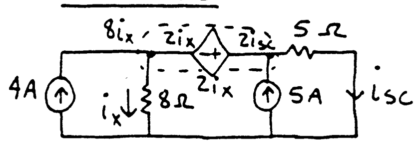


KCL at supernode : $-4 + i_x - 5 = 0$

$\Rightarrow$ $\underline{i_x = 9\text{A}}$

$V_{oc} = 8i_x + 2i_x = 10i_x = \underline{90\text{v}}$

find i_{sc}



KCL at supernode : $-4 + i_x - 5 + i_{sc} = 0$

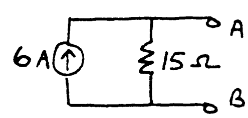
$\rightarrow$ $\underline{i_x + i_{sc} = 9}$ (1)

$\underline{5i_{sc} = 10i_x}$ (2)

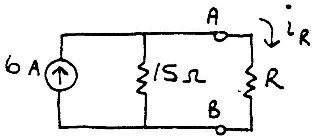
solving (1) & (2) simultaneously $\Rightarrow$ $\underline{i_{sc} = 6\text{ A}}$

$\therefore$ $R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{90}{6} = \underline{15\Omega}$

Norton equiv. ckt $\Rightarrow$



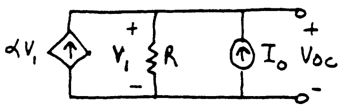
P. 5-16 continued

(b) 

for max power to $R \Rightarrow R = R_T = 15\,\Omega$

$\therefore P_{max} = i_R^2 R = \left(\frac{15}{3}A\right)^2 (15\,\Omega) = \underline{135 \text{ W}}$

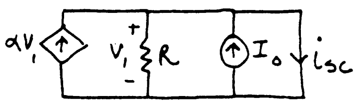
P. 5-17 <u>find V_{oc}</u>



KCL at top: $-\alpha V_1 + V_1/R - I_o = 0$, but $V_1 = V_{oc}$

$\Rightarrow \underline{V_{oc} = \dfrac{R I_o}{1 - \alpha R}}$

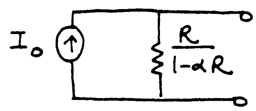
<u>find i_{sc}</u>



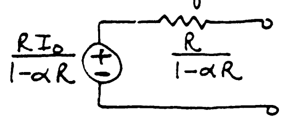
$V_1 = 0$ $\therefore \underline{i_{sc} = I_o}$

$\therefore \underline{R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{R}{1 - \alpha R}}$

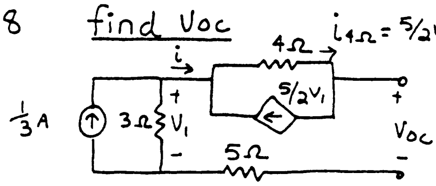
<u>Norton equiv.</u>



<u>Thev. equiv.</u>



P. 5-18 <u>find V_{oc}</u>



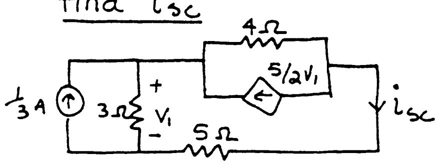
by inspection $i = 0$

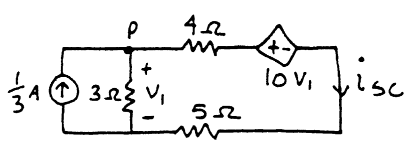
from left mesh: $V_1 = 3(1/3) = 1\text{V}$

from KVL: $-V_1 + 4 i_{4\Omega} + V_{oc} = 0$

$\Rightarrow \underline{V_{oc} = V_1 - 4(5/2 V_1) = -9\text{V}}$

<u>find i_{sc}</u>



$\Rightarrow$ 

from KVL: $-V_1 + 4 i_{sc} + 10 V_1 + 5 i_{sc} = 0$

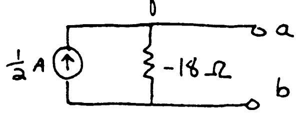
$\hookrightarrow \underline{9 V_1 + 9 i_{sc} = 0}$ (1)

from KCL at P: $\underline{-1/3 + V_1/3 + i_{sc} = 0}$ (2)

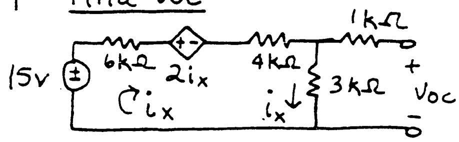
(1) & (2) yields $\underline{i_{sc} = 1/2 \text{ A}}$

$\therefore R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{-9}{1/2} = \underline{-18\,\Omega}$

Norton equiv. ckt:



P. 5-19 find V_{oc}

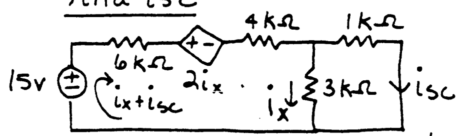


KVL $\curvearrowright i_x$: $-15 + i_x(6+4) + 2i_x + 3i_x = 0$

$\Rightarrow i_x = 1mA$

$\therefore \underline{V_{oc} = 3i_x = 3v}$

find i_{sc}



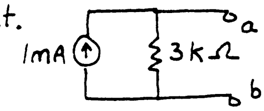
KVL $\curvearrowright i_x + i_{sc}$: $-15 + 2i_x + 10(i_x + i_{sc}) + 3i_x = 0$

$\underline{-15 + 15i_x + 10i_{sc} = 0}$ (1)

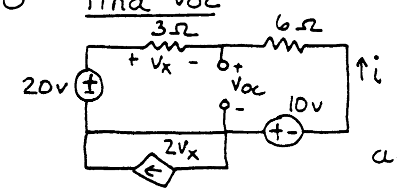
KVL $\curvearrowright i_{sc}$: $\underline{-3i_x + i_{sc} = 0}$ (2)

solving (1) & (2) simultaneously yields : $\underline{i_{sc} = 1mA}$

$\therefore \underline{R_T = \frac{V_{oc}}{i_{sc}} = \frac{3}{1} = 3\,k\Omega}$ Norton equiv. ckt.



P. 5-20 find V_{oc}



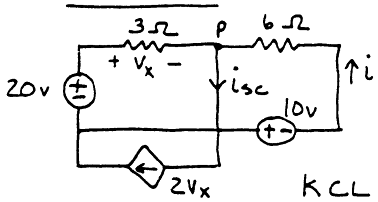
KVL left mesh $\curvearrowright$: $\underline{-20 + V_x + V_{oc} = 0}$ (1)

KVL right mesh $\curvearrowright$: $\underline{-V_{oc} - 6i - 10 = 0}$ (2)

also $\underline{V_x = -3i}$ (3)

solving (1), (2) & (3) simultaneously yields $\Rightarrow \underline{V_{oc} = 10v}$

find i_{sc}



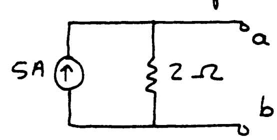
KVL left mesh : $-20 + V_x = 0 \Rightarrow \underline{V_x = 20v}$ (1)

KVL right mesh : $\underline{6i + 10 = 0}$ (2)

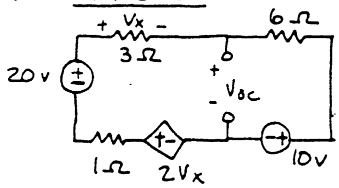
KCL at P : $\underline{-V_x/3 - i + i_{sc} = 0}$ (3)

(1), (2) & (3) yield: $\underline{i_{sc} = 5A}$

$\therefore R_T = \frac{V_{oc}}{i_{sc}} = \frac{10}{5} = \underline{2\,\Omega}$ Norton equiv. ckt.



P. 5-21 find V_{oc}



KVL ↻ entire loop : $-20 + V_x + 6i + 10 - 2V_x + i = 0$

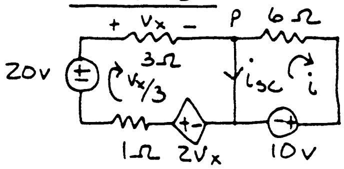
$\hookrightarrow$ $7i - V_x - 10 = 0$ (1)

KVL ↻ right loop : $-V_{oc} + 6i + 10 = 0$ (2)

also $V_x = 3i$ (3)

solving (1), (2) & (3) yields $\Rightarrow$ $V_{oc} = 25\text{v}$

find i_{sc}



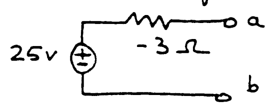
KVL ↻ $V_x/3$ mesh : $-20 + V_x - 2V_x + V_x/3 = 0$

$\Rightarrow$ $V_x = -30\text{v}$

KVL ↻ i mesh : $6i + 10 = 0$ $\Rightarrow$ $i = -5/3\text{ A}$

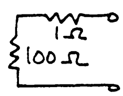
KCL at P : $-V_x/3 + i_{sc} + i = 0$ $\Rightarrow$ $i_{sc} = V_x/3 - i = -25/3\text{ A}$

$\therefore$ $R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{25}{-25/3} = -3\Omega$ Thev. equiv. ckt



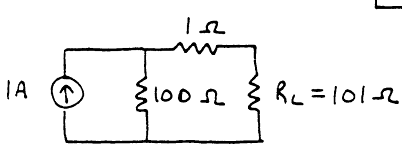
P. 5-22 (a) for max power need $R_L = R_T$ $\therefore$ find R_T

kill current source

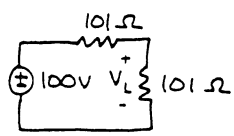


$\leftarrow$ $R_T = 101\Omega = R_L$

(b)

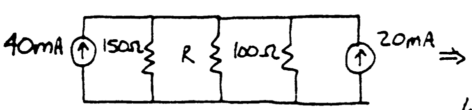


$\Rightarrow$

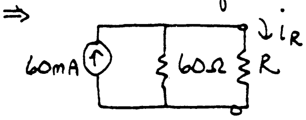


$P_{max} = V_L^2/101 = (50)^2/101 = 24.75\text{W}$

P. 5-23 (a) use source transformations to reduce ckt.



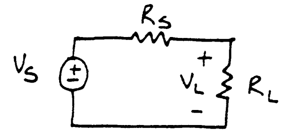
$\Rightarrow$ Norton equiv. where $R_T = 60\Omega$



$\therefore$ want $R_L = 60\Omega$

(b) $P_{max} = i_R^2 (R) = (30)^2 (60) = 54{,}000\mu\text{W} = 54\text{mW}$

P. 5-24



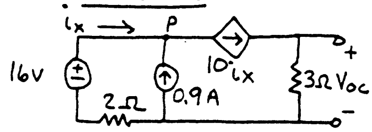
$$V_L = V_S\left[\frac{R_L}{R_S+R_L}\right]$$

$$\therefore\ P_L = \frac{V_L^2}{R_L} = \frac{V_S^2 R_L}{(R_S+R_L)^2}$$

By inspection,
P_L is max when you vary R_S to get the smallest denominator. $\therefore$ set $R_S = 0$

P. 5-25 find $R_T = V_{oc}/i_{sc}$

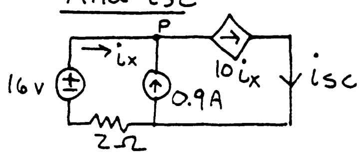
find V_{oc}



KCL at P: $-i_x - .9 + 10 i_x = 0 \Rightarrow i_x = 0.1A$

$\therefore$ $V_{oc} = 3(10 i_x) = 3V$

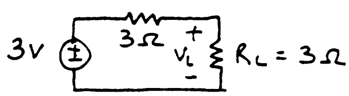
find i_{sc}



KCL at P: $-i_x - .9 + 10 i_x = 0 \Rightarrow i_x = 0.1A$

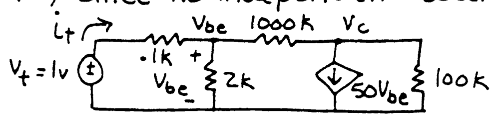
$\therefore$ $i_{sc} = 10 i_x = 1A$

$\therefore$ $R_T = V_{oc}/i_{sc} = 3\Omega = R_L$ for max power



$$P_{L_{max}} = \frac{V_L^2}{R_L} = \frac{(1.5)^2}{3} = 0.75\ W$$

P. 5-26 (a) since no independent sources apply test source



KCL at V_{be}: $-i_t + \frac{1}{2}V_{be} + (V_{be} - V_c)/1000 = 0$

(1) $-1000 i_t + 501 V_{be} - V_c = 0$

KCL at V_c: $(V_c - V_{be})/1000 + 50V_{be} + V_c/100 = 0$

$11V_c + 50000 V_{be} = 0$ (2)

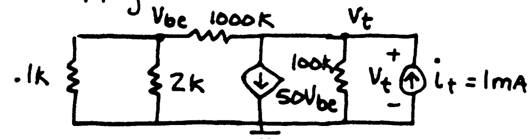
also $(1 - V_{be})/.1 = i_t$ (3)

Solving (1), (2) & (3) simultaneously yields $i_t = 3.35mA$

$$\therefore\ R_{IN} = \frac{V_t}{i_t} = \frac{1v}{3.35mA} = .299\ k\Omega = 299\ \Omega$$

P. 5-26 continued

(b) apply test source



KCL at Vbe : $V_{be}/.1 + V_{be}/2 + (V_{be} - V_t)/1000 = 0$

$\hookrightarrow$ $\underline{10501 V_{be} = V_t}$ (1)

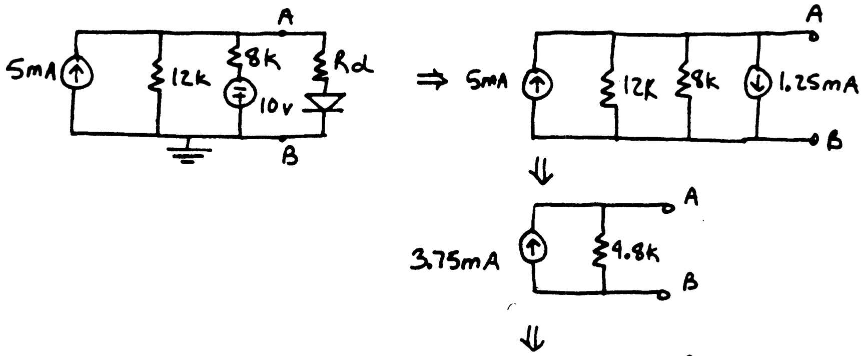
KCL at V_t : $(V_t - V_{be})/1000 + 50 V_{be} + V_t/100 - 1 = 0$

$\hookrightarrow$ $\underline{11 V_t + 49999 V_{be} - 1000 = 0}$ (2)

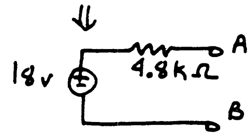
solving (1) & (2) yields : $\underline{V_t = 63.5\text{ v}}$

$\therefore$ $\underline{R_{out} = V_t / i_t = 63.5\text{v}/1\text{mA} = 63.5\text{ k}\Omega}$

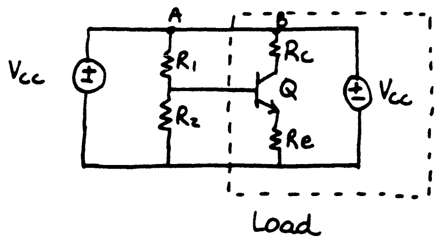
P. 5-27 redraw the ckt.



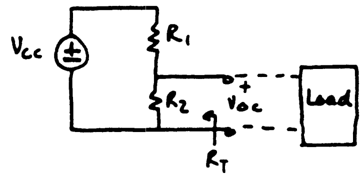
Thev. equiv. ckt. →



P. 5-28 redraw ckt as:



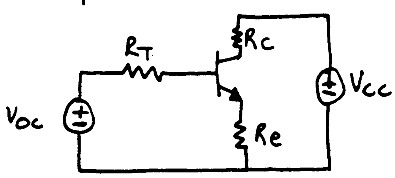
since points A & B are at same potential virtually no current exists between A-B ∴ open ckt.



find R_T : kill Vcc source $\Rightarrow R_T = R_1 // R_2 = R_1R_2/R_1+R_2$

find Voc : voltage divider $V_{oc} = V_{cc}\left(\frac{R_2}{R_1+R_2}\right)$

∴ can replace above ckt as:

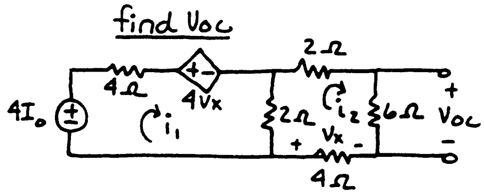


where $V_{oc} = V_{cc}\left(\frac{R_2}{R_1+R_2}\right)$

$R_T = \frac{R_2R_1}{R_2+R_1}$

P. 5-29 (a) for max power $R_L = R_T$

find Voc



KVL i_1: $-4I_o + 4i_1 + 4V_x + 2(i_1 - i_2) = 0$

$\underline{6i_1 - 2i_2 + 4V_x - 4I_o = 0}$ (1)

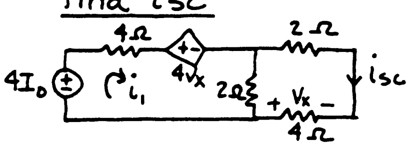
KVL i_2: $2i_2 + 6i_2 - V_x + 2(i_2 - i_1) = 0$

$\underline{-2i_1 + 10i_2 - V_x = 0}$ (2)

also $\underline{V_x = -4i_2}$ (3) and $\underline{V_{oc} = 6i_2}$ (4)

solving (1), (2), (3) & (4) yields $\underline{V_{oc} = I_o}$

find isc



KVL i_1: $-4I_o + 4i_1 + 4V_x + 2(i_1 - i_{sc}) = 0$

$\underline{6i_1 - 2i_{sc} + 4V_x - 4I_o = 0}$ (1)

KVL i_{sc}: $2i_{sc} + 4i_{sc} + 2(i_{sc} - i_1) = 0$

$\underline{-2i_1 + 8i_{sc} = 0}$ (2)

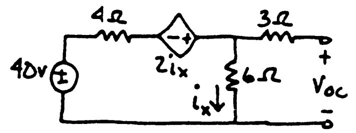
also $\underline{V_x = -4i_{sc}}$ (3)

Solving (1), (2) & (3) yields $\underline{i_{sc} = 2/3 I_o}$ ∴ $\underline{R_T = \frac{V_{oc}}{i_{sc}} = 3/2\,\Omega = R_L}$

(b) $P_{L\max} = 54\ \text{W} = \frac{(V_{oc}/2)^2}{R_L} = I_o^2/6$

$\Rightarrow \underline{I_o = 18\text{A}}$

P. 5-30 (a) find V_{oc} (transform current source)

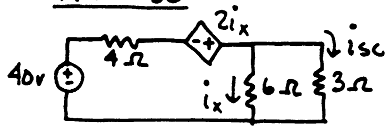


KVL: $-40 + 4i_x - 2i_x + 6i_x = 0$

$\Rightarrow i_x = 5A$

$\therefore$ $\underline{V_{oc} = 6i_x = 30v}$

find i_{sc}

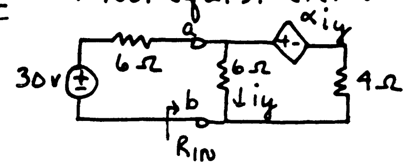


KVL: $-40 + 4(i_x + i_{sc}) - 2i_x + 6i_x = 0$

$\underline{-40 + 8i_x + 4i_{sc} = 0}$ (1)

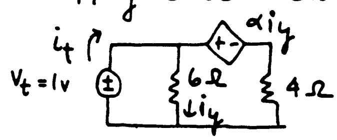
also $\underline{6i_x = 3i_{sc}}$ (2) Solving (1) & (2) yields $\underline{i_{sc} = 5A}$

$\therefore$ $R_T = \frac{V_{oc}}{i_{sc}} = \frac{30}{5} = \underline{6\,\Omega}$ Thev. equiv. ckt :



(b) for max power $R_{IN} = R_T = 6\,\Omega$

apply a test source since no independent sources present



$R_{IN} = \frac{V_t}{i_t}$

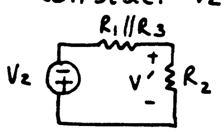
$i_y = 1/6$

KVL: $\alpha(1/6) + 4(i_t - 1/6) - 1 = 0$

$\underline{\alpha + 24i_t - 10 = 0}$ (1)

from (1) and $R_{IN} = 1/i_t = 6$ yields $\underline{\alpha = 6}$

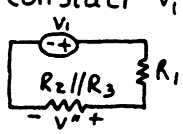
P. 5-31 consider V_2 source only



voltage divider: $V' = -V_2\left[\frac{R_2}{R_2 + R_1/\!/R_3}\right]$

$V' = -V_2\left[\frac{R_2(R_1+R_3)}{R_1R_2 + R_1R_3 + R_2R_3}\right]$

consider V_1 source only



voltage divider $V'' = V_1\left[\frac{R_2/\!/R_3}{R_2/\!/R_3 + R_1}\right]$

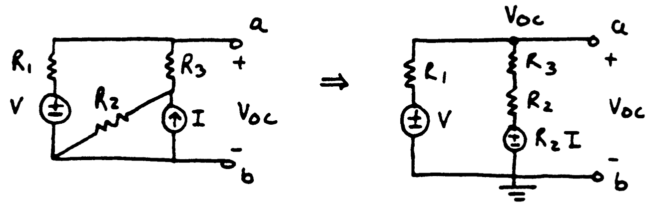
$V'' = V_1\left[\frac{R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}\right]$

consider i_1 source only

$V''' = 0$ since no current flows through R_2, R_3 and R_1.

$\therefore$ $V = V' + V'' + V''' = \frac{V_1R_2R_3 - V_2(R_2(R_1+R_3))}{R_1R_2 + R_1R_3 + R_2R_3}$

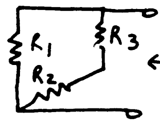
P. 5-32 find V_{oc}



KCL at V_{oc} : $(V_{oc}-V)/R_1 + (V_{oc} - R_2 I)/R_2+R_3 = 0$

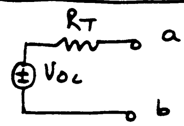
$$\Rightarrow \quad V_{oc} = \frac{V(R_2+R_3) + R_1 R_2 I}{R_1 + R_2 + R_3}$$

find R_T (kill sources)



$\leftarrow$ $R_T = R_1 /\!/ (R_2+R_3) = \dfrac{R_1(R_2+R_3)}{R_1+R_2+R_3}$

Thev. equiv. ckt. $\Rightarrow$

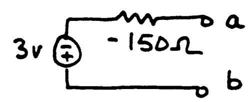


P. 5-33 From the graph, when $V_{ab} = v = 0 \Rightarrow i = i_{sc} = 20\text{mA}$

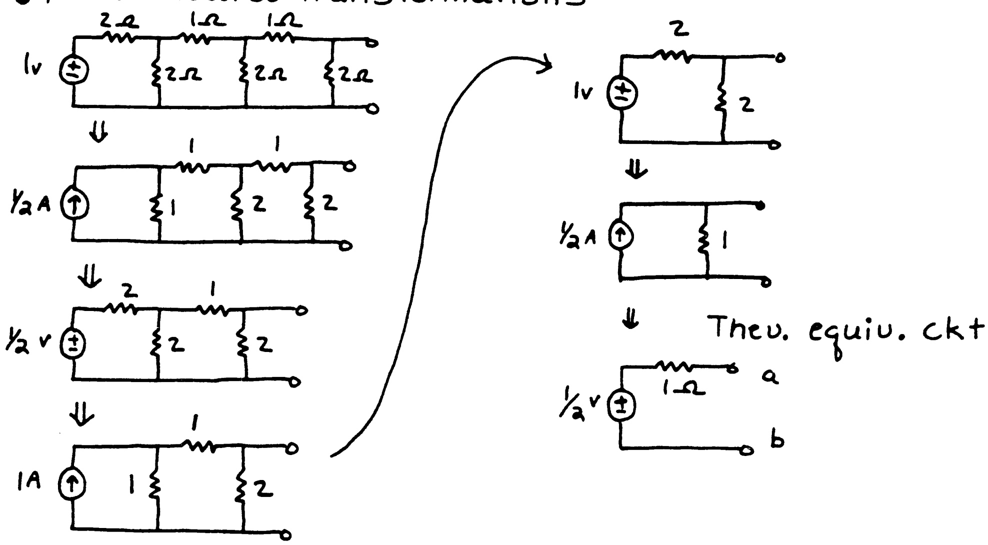
when $i = 0 \Rightarrow v = V_{oc} = -3v$

$\therefore \; R_T = \dfrac{V_{oc}}{i_{sc}} = \dfrac{-3v}{20mA} = -.15k\Omega = -150\Omega$

Thev. equiv. ckt $\Rightarrow$

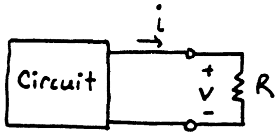


P. 5-34 use source transformations



Thev. equiv. ckt

P. 5-35

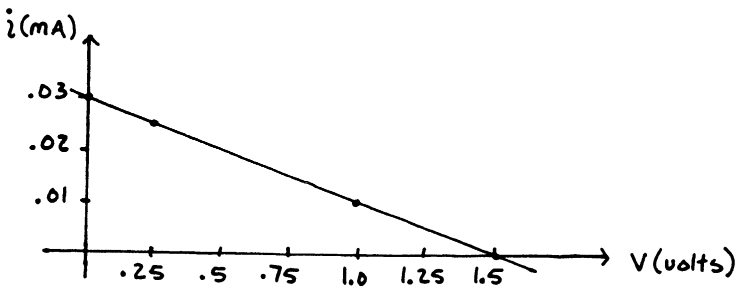


with R absent (open ckt): $V = 1.5\text{v} \Rightarrow i = V/\infty = 0$

with $R = 10\,k\Omega$: $V = 0.25\,\text{v} \Rightarrow i = V/R = 0.025\,\text{mA}$

with $R = 100\,k\Omega$: $V = 1.0\,\text{v} \Rightarrow i = 0.01\,\text{mA}$

<u>plot i vs v</u>



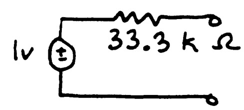
since i vs v is linear $\Rightarrow$ <u>circuit is linear</u>

$V_{oc} = 1.0\,\text{v}$ (when $i = 0$, terminals opened)

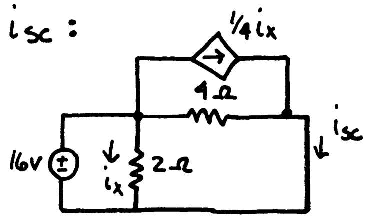
$i_{sc} = .03\,\text{mA}$ (when $v = 0$, terminals shorted)

$$\therefore\ R_T = \frac{V_{oc}}{i_{sc}} = \frac{1.0\,\text{v}}{.03\,\text{mA}} = \underline{33.3\,k\Omega}$$

Thev. equiv. ckt $\Rightarrow$



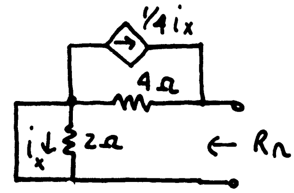
P 5-36 for i_{sc}:



$$i_x = \frac{16}{2} = 8\text{ A}$$

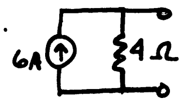
$$\therefore\ i_{sc} = \frac{16}{4} + \frac{8}{4} = \underline{6\text{ A}}$$

to find R_n, turn off 16V source

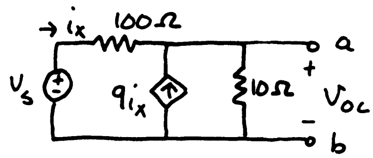


note: $i_x = 0 \Rightarrow R_{eq} = R_n = \underline{4\ \Omega}$

Norton equiv.



P 5-37 for V_{oc}:



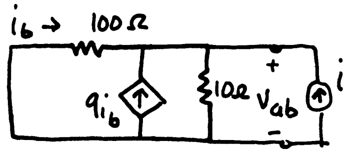
$$i_x = \frac{V_s - V_{oc}}{100}$$

KCL at terminal a:

$$\frac{1}{100}(V_{oc} - V_s) - 9\left[\frac{1}{100}(V_s - V_{oc})\right] + \frac{1}{10}V_{oc} = 0$$

$$\Rightarrow \underline{V_{oc} = \frac{1}{2}V_s}$$

use current source at a-b to find R_T:

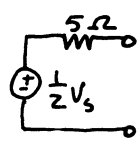


$$i_b = -\frac{V_{ab}}{100}$$

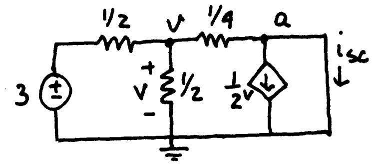
KCL: $\frac{1}{100}V_{ab} - 9\left[\frac{1}{100}(-V_{ab})\right] + \frac{1}{10}V_{ab} - i = 0$

$$\Rightarrow i = \frac{1}{5}V_{ab} \qquad \therefore\ R_T = \frac{V_{ab}}{i} = 5\ \Omega$$

so Thev. equiv.



P 5-38 find i_{sc}:

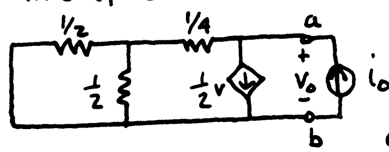


KCL at V: $\frac{V-3}{1/2} + \frac{V}{1/2} + \frac{V}{1/4} = 0$

$\Rightarrow V = 3/4\,V$

KCL at a: $i_{sc} = \frac{V}{1/4} - \frac{V}{2} = \underline{\frac{21}{8}\text{ A}}$

to find R_T set source $= 0$

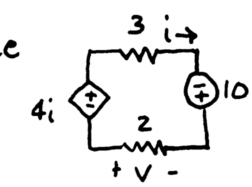


at V: $\frac{V}{1/2} + \frac{V}{1/2} + \frac{V-V_o}{1/4} = 0 \quad \Rightarrow V = \frac{V_o}{2}$

at a: $i_o = \frac{1}{2}V + \frac{V_o - V}{1/4} = \frac{9}{4}V_o$

$\therefore R_T = \frac{V_o}{i_o} = \underline{\frac{4}{9}\,\Omega}$

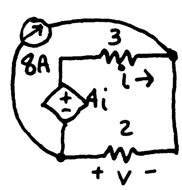
P 5-39 1) 10v source



KVL: $-4i + 3i - 10 + 2i = 0 \quad \Rightarrow i = 10\text{ A}$

so $\underline{V = -2(10) = -20\text{ V}}$

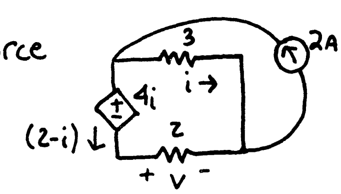
2) 8A source



KVL: $3i - 4i + 2(8+i) = 0 \quad \Rightarrow i = -16\text{ A}$

so $\underline{V = -2(8+i) = 16\text{ v}}$

3) 2A source

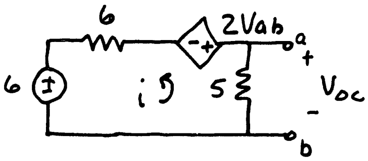


KVL: $3i - 2(2-i) - 4i = 0 \quad \Rightarrow i = 4\text{A}$

so $\underline{V = 2(2-i) = -4\text{v}}$

$\therefore \underline{V_T = -20 + 16 - 4 = -8\text{v}}$

P 5-40 find V_{oc} :

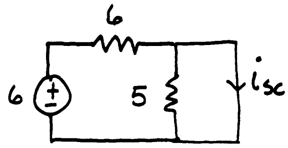


KVL : $6 + 11i + 2V_{ab} = 0$

$\hookrightarrow 6 + 11i + 2(5(-i)) = 0 \Rightarrow i = -6A$

$\therefore V_{oc} = -i(5) = \underline{30V}$

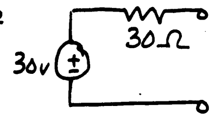
find R_T :



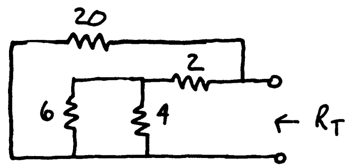
$-6 + 6i_{sc} = 0$ so $i_{sc} = 1A$

$\therefore R_T = V_{oc}/i_{sc} = \underline{30\,\Omega}$

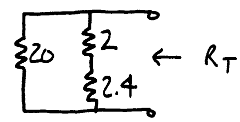
so have



P 5-41 find R_T :

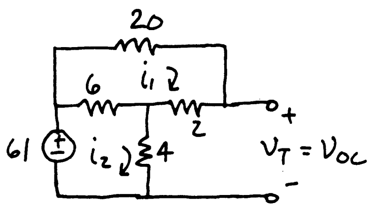


$\Rightarrow$



$$R_T = \frac{20(2+2.4)}{20+2+2.4} = \underline{3.61\,\Omega}$$

find V_T :



$V_T = 2i_1 + 4i_2$

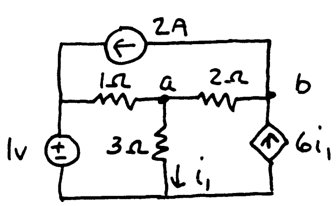
mesh i_1 : $28i_1 - 6i_2 = 0$ (1)

mesh i_2 : $-6i_1 + 10i_2 - 61 = 0$ (2)

solving (1) & (2) yields : $i_1 = 1.5A$, $i_2 = 7A$

$\therefore \underline{V_T = 3 + 28 = 31V}$

P 5-42



KCL at b : $i + 6i_1 - 2 = 0$

$\Rightarrow i_1 = 1/3 - 1/6\, i$ (1)

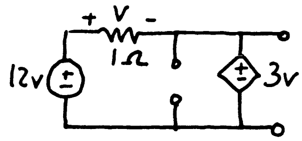
KVL around left lower mesh:

$1(i_1 + i) + 3i_1 - 1 = 0$ (2)

plugging (1) into (2) $\Rightarrow \underline{i = -1A}$

P 5-43 first find v_{oc} by superposition

12v source

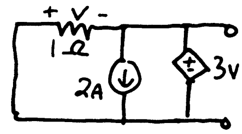


KVL: $v + 3v - 12 = 0$

$\Rightarrow v = 3$ volts

$\therefore v_{oc} = 3v = 9$ volts

i source



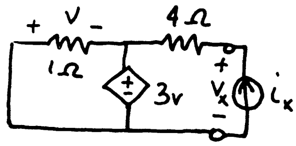
KVL: $v + 3v = 0$

$\Rightarrow v = 0$

$\therefore v_{oc} = 0$

so total $\underline{v_{oc} = 9 + 0 = 9 \text{ volts}}$

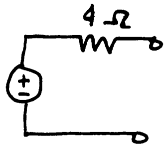
find R_T



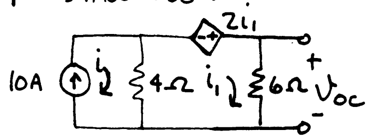
KVL: $4i_x + 3v = v_x$ also $v = -3v \Rightarrow v = 0$

so $v_x = 4i_x \Rightarrow R_T = v_x / i_x = 4\Omega$

Thev. equiv.



P 5-44 find v_{oc}:

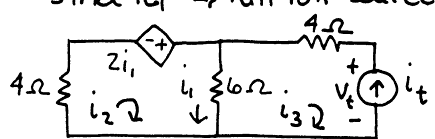


$i = 10A$; KVL 2nd mesh: $4i_1 - 2i_1 + 6i_1 - 4i = 0$

$\Rightarrow i_1 = 5A$

so $v_{oc} = 6i_1 = \underline{30v}$

find $R_T \Rightarrow$ kill 10A source & drive terminals a-b with current source



$i_3 = -i_t$

KVL i_2: $4i_2 + 2i_1 + 6(i_1 + i_t) = 0$ (1)

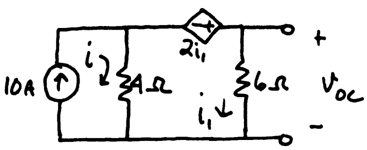
also: $i_2 = i_1 - i_t$ (2)

(1) & (2) yields $i_t = -6i_1$

now $v_t = 6i_1 + 4i_t = -i_t + 4i_t = 3i_t$

$\therefore R_T = \dfrac{v_t}{i_t} = \underline{3\Omega}$

P 5-45 find v_{oc}

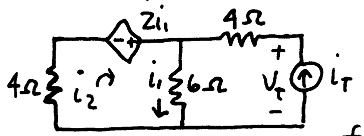


KVL: $4i_1 - 2i_1 + 6i_1 - 4i = 0$ & $i = 10A$

$\Rightarrow i_1 = 40/8 = 5A$

$\therefore \underline{v_{oc} = 6i_1 = 30v}$

find R_T ⇒ kill 10A source & drive terminals a-b



KVL left loop: $4i_2 + 2i_1 + 6(i_1 + i_t) = 0$ (1)

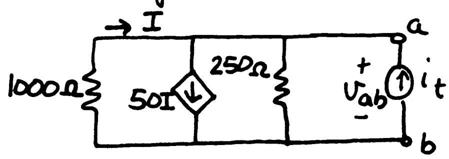
also: $i_2 = i_1 - i_t$ (2)

from (1) & (2) ⇒ $i_1 = -\frac{1}{6} i_t$

now $v_t = 6i_1 + 4i_t = -i_t + 4i_t = 3i_t$ ⇒ $R_T = \frac{v_t}{i_t} = \underline{3\Omega}$

Advanced Problems

AP 5-1 apply test source to output



KCL: $-I + 50I + \frac{v_{ab}}{250} - i_t = 0$ (1)

also: $I = \frac{v_{ab}}{1000}$ (2)

(1) & (2) yield $i_t = \frac{53}{1000} v_{ab}$ or $R_{out} = \frac{v_{ab}}{i_t} = \frac{1000}{53} = \underline{18.9\Omega}$

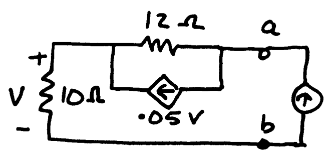
AP 5-2 all 24A flows thru 10Ω so $v = 10(24) = 240v$

current is $.05v = 12A$

So the voltage across 12Ω ⇒ $v_{12} = 12(12) = 144v$

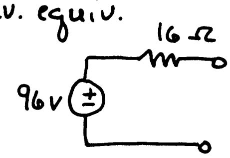
then $\underline{V_T = 240 - 144 = 96V}$

to find R_T apply 1A at output:



V = 10 volts

Thev. equiv.

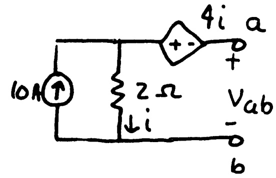


so current source = .5A

∴ have .5A thru 12Ω ∴ $V_{12\Omega} = 6v$

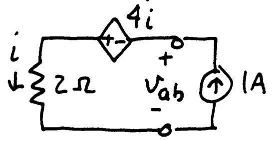
$\Rightarrow v_{ab} = 6 + 10 = 16v \Rightarrow \underline{R_T = 16\Omega}$

AP 5-3 1) disconnect R_L
open circuit a-b



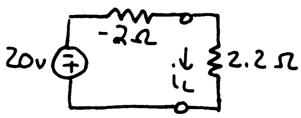
KVL: $-V_{ab} - 4i + 2i = 0$, $i = 10A$
$\Rightarrow V_T = V_{ab} = -2i = \underline{-20v}$

2) set independent source = 0 and place 1A source at a-b



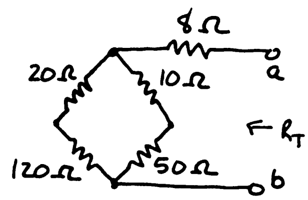
KVL: $-V_{ab} - 4i + 2i = 0$, $i = 1A$
$\Rightarrow V_{ab} = -2A$
$\therefore R_T = V_{ab}/1A = \underline{-2\Omega}$

3) Connect $R_L = 2.2\Omega$



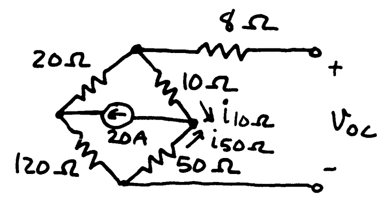
$$i_L = \frac{-20}{-2+R_L} = \frac{-20}{0.2} = \underline{-100A}$$

AP 5-4 $P_{max} = V_T^2/4R_T$
find $R_T \Rightarrow$ kill i source



$R_T = 8 + (20+120)//(10+50)$
$= 50\Omega$

find V_{oc}:

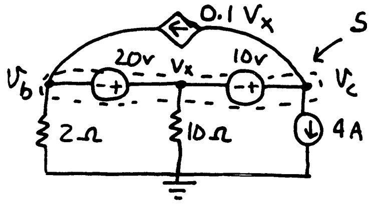


$$i_{10\Omega} = \frac{120+50}{120+50+20+10} 20A$$
$= 17A$

$\therefore V_{10\Omega} = 10(17) = 170v$
$V_{50\Omega} = 50(17-20) = -150v$
$\Rightarrow V_{oc} = V_{10\Omega} + V_{50\Omega} = 170 - 150 = \underline{20v}$

$$\therefore P_{max} = \frac{20^2}{4(50)} = \underline{2W}$$

AP 5-5



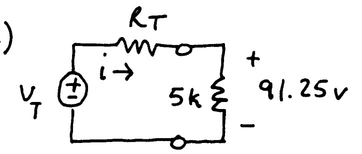
Supernode KCL: $\frac{V_b}{2} + \frac{V_x}{10} + 4 = 0$ (1)

also $V_b = V_x - 20$ (2)

(1) & (2) yields $\underline{V_x = 10v}$

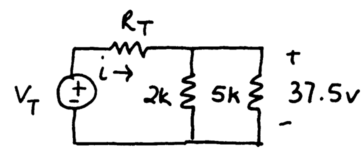
AP 5-6 When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current and dissipate no power. Then Box A will warm up and Box B will cool off.

AP 5-7 a)



$$i = \frac{91.25}{5\times10^3} \Rightarrow V_T = \left(\frac{91.25}{5\times10^3}\right)R_T + 91.25 \quad ①$$

b)



$$i = \frac{37.5}{10/7\times10^3} \Rightarrow V_T = \frac{37.5}{10/7\times10^3} R_T + 37.5 \quad ②$$

solving ① & ② to get $\underline{R_T = 6.72\,k\Omega}$ & $\underline{V_T = 213.9\ V}$

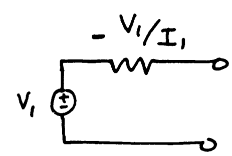
AP 5-8 When $\underline{0 < V < V_p}$ it works as a pure resistor

so $R = V_P/I_P$ $\quad V_{oc} = 0$

when $\underline{V_p < V < V_m}$ it is linear but shows negative resistance characteristic

$$\Rightarrow V_{oc} = V_{oc}\big|_{I=0} = V_1$$

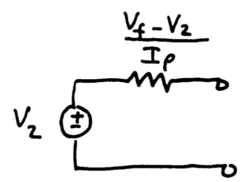
$$R = \frac{V_{oc}}{I_{sc}} = -\frac{V_1}{I_1}$$



when $\underline{V_m < V < V_f}$ it is linear

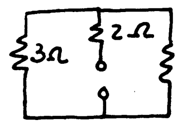
$$\text{so } V_{oc} = V\big|_{I=0} = V_2$$

$$R = \frac{V_{oc}}{I_{sc}} = \frac{V_f - V_2}{I_p}$$



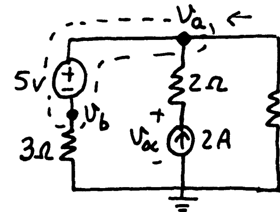
AP 5-9 We are asked to vary $R = R_t$, not R_{load}.
Thus $R_t = R_{load}$ is not the optimal solution.
Clearly when $\underline{R=0}$, maximum current and voltage (hence maximum power) are delivered to the load.

AP 5-10 find R_T → kill sources



$R_T = 2 + 3//3$
$= \underline{3.5\,\Omega}$

find V_{oc} :



← supernode KCL: $\frac{v_b}{3} - 2 + \frac{v_a}{3} = 0$ (1)

also: $v_a - v_b = 5$ (2)

solving (1) & (2) yields: $v_a = 5.5\text{v}$

now $V_{oc} = v_a + 2(2) = 5.5 + 4 = \underline{9.5\text{v}}$

$\therefore\ P_{max} = V_{oc}^2/4R_T = (9.5)^2/4(3.5) = \underline{6.45\text{ W}}$

AP 5-11 (a) $v_o = -g R_L v$ and $v = \frac{R_2}{R_1+R_2} v_i$ $\Rightarrow$ $\frac{v_o}{v_i} = -g \frac{R_L R_2}{R_1+R_2}$

(b) so have $\frac{v_o}{v_i} = -g \frac{(5\times10^3)(10^3)}{1.1\times10^3} = -170$

$\Rightarrow \underline{g = 0.0374\ S}$

Design Problems

DP 5-1 find Thev. equiv. with R_L disconnected

$R_T = 6 + \frac{5(20)}{25} = 10\,\Omega$

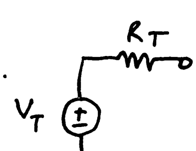
$V_T = v_{oc} = \frac{20}{25}(100) = 80\text{v}$

now $P = i^2 R_L = \left(\frac{V_T}{R_L+R_T}\right)^2 R_L = \frac{80^2 R_L}{(R_L+10)^2} = 150$

$\Rightarrow R_L^2 + \left(20 - \frac{6400}{150}\right) R_L + 100 = 0 \Rightarrow R_L = 11.335 \pm 5.333$

$\therefore\ \underline{R_L = 6.00\,\Omega \text{ or } 16.67\,\Omega}$

DP 5-2 Thev. equiv.



$I = 10^{-2}\text{ A}$

$V_T = -30(500)I = -150\text{v}$

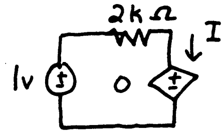
deactivate source $\Rightarrow I = 0$ $\therefore\ R_T = 500\,\Omega$

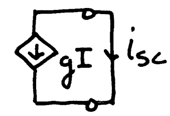
so set $R_L = 500\,\Omega$ then $P_{max} = V_T^2/4R_L = \frac{(150)^2}{4(500)} = \underline{11.25\text{W}}$

DP 5-3 at terminal a-b : $R_o = R_T = V_{oc}/i_{sc}$

Short circuit output $\Rightarrow V_{ab} = 0$

then $d V_{ab} = 0$

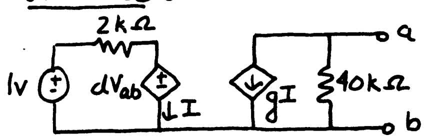




$\therefore I = \dfrac{1}{2\times10^3} = .5\,mA$

so $-g\dfrac{(10)^{-3}}{2} = i_{sc}$

find V_{oc} :



right mesh : $V_{ab} = (-gI)(40{,}000)$ (1)

left mesh : $2000\,I + .0004\,V_{ab} = 1$ (2)

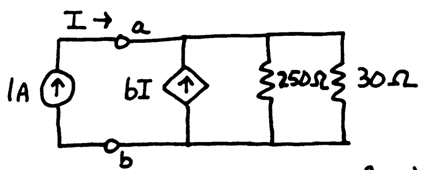
from (1) and (2) get $I = 1/(2000 - 16g)$

now $R_o = 50{,}000 = V_{oc}/i_{sc} = \dfrac{V_{oc}}{-g\,10^{-3}/2}$ (3)

also $V_{oc} = -g(40{,}000)I = \dfrac{-g(40{,}000)}{(2000-16g)}$ (4)

solving (3) & (4) yields $\underline{g = 25}$

DP 5-4



let I = 1A , then total current to R_p is

$I_{R_p} = 1 + b1$ where $R_p = \dfrac{250(30)}{250+30} = 26.79\,\Omega$

$\therefore V_{ab} = (1+b1)(26.79)$

now $R_{in} = 2705 = (1+b1)(26.79)/1A \Rightarrow 1+b1 = 100.99 \simeq 101$

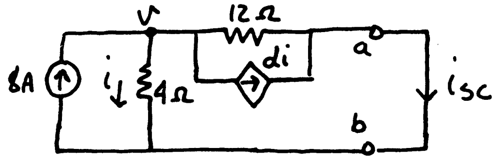
$\therefore \underline{b = 100}$

DP 5-5 desire $R_T = 64\,\Omega$

find V_{oc} : with open circuit $i = 8A$, so current thru 12Ω is $i_{12} = d8$

$\therefore V_{oc} = 8\times4 + 8d\times12 = 32(1+3d)$

find i_{sc} :



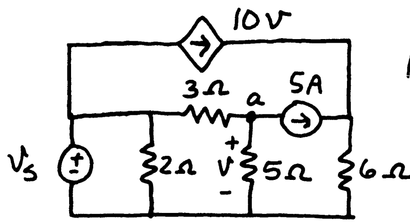
KCL at node v : $v/4 + v/12 + d(v/4) = 8 \Rightarrow v = \dfrac{96}{4+3d}$ (1)

also : $i_{sc} = d\,v/4 + v/12$ (2)

plugging (1) into (2) yields $\Rightarrow i_{sc} = \dfrac{8(3d+1)}{4+3d}$

$\therefore R_T = 64 = \dfrac{V_{oc}}{i_{sc}} = 4(4+3d)$ so $\underline{d = 4}$ thus $\underline{V_T = V_{oc} = 416\,V}$

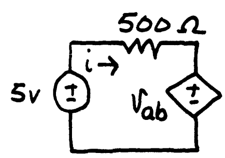
DP5-6

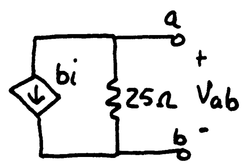


KCL at node a: $v\left(\frac{1}{5}+\frac{1}{3}\right)+5-\frac{v_s}{3}=0$

so if $v=0$ then $\underline{v_s=15\,\text{v}}$

DP 5-7





<u>find i_{sc}</u>: $v_{ab}=0$ $\therefore\ i=5/500=10\,\text{mA}$

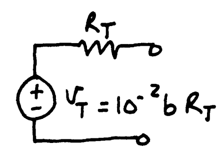
$\therefore\ i_{sc}=-bi=-10^{-2}b$

<u>find v_{oc}</u>: KVL left loop: $-5+500i+v_{ab}=0$ (1)

also $v_{ab}=-25(bi)$ (2)

$\Rightarrow v_{ab}=\dfrac{5}{(1-20/b)}$

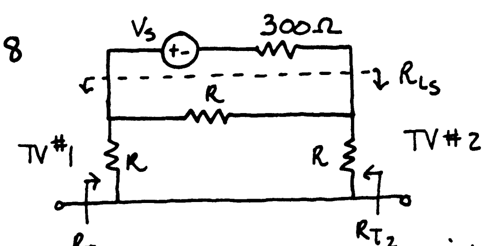
$\therefore\ R_T=\dfrac{v_{ab}}{i_{sc}}=\dfrac{-500}{(b-20)}$



need $60<R_T<70\,\Omega$

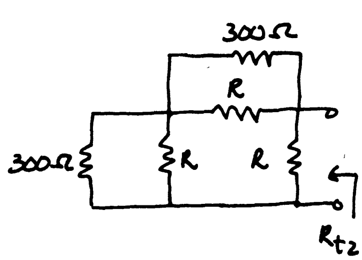
so if $b=12.5$ $\Rightarrow$ $\underline{R_T=66.66\,\Omega}$ and $\underline{|v_T|=8.33\,\text{v}}$

DP 5-8



For maximum power to each TV, require $R_{T_1}=300\,\Omega$, $R_{T_2}=300\,\Omega$. Futhermore, for the source (antenna) to deliver maximum power, we require R_{Ls} (seen by antenna) $=300\,\Omega$

Note that calculation of R_{Ls}, R_{t_1}, R_{t_2} leads to identical circuits, hence consider R_{T_2} (kill v_s)



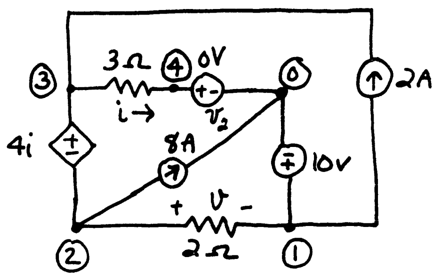
$$R_{t2}=R\,/\!/\left[(300/\!/R)+(300/\!/R)\right]$$

$$=\frac{2R\left(\frac{R\,300}{R+300}\right)}{R+R300/R+300}=300$$

$\Rightarrow \underline{R=600\,\Omega}$

Spice Problems

SP 5-2



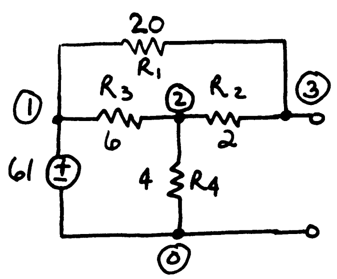
input file

```
V1  1 0 DC 10
V2  4 0
R1  3 4 3
R2  2 1 2
I1  1 3 DC 2
I2  2 0 DC 8
H1  3 2 V2 4
.DC V1 10 10 1
.PRINT DC V(2,1) I(V2)
.END
```

ans. $i = I(V2) = -2.00A$

$v = V(2,1) = -8.00V$

SP 5-5



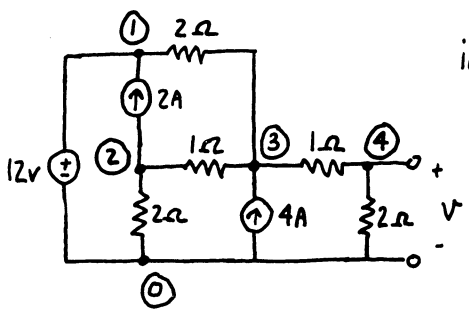
input file

```
R2  2 3 2
R4  2 0 4
R3  1 2 6
R1  1 3 20
V1  1 0 DC 61
.TF V(3) V1
.END
```

ans. $\frac{V(3)}{V1} = 5.082E-01$

so $\underline{V_T = 31V}$ ∴ R_T = output resistance = $\underline{3.62\ \Omega}$

SP 5-6

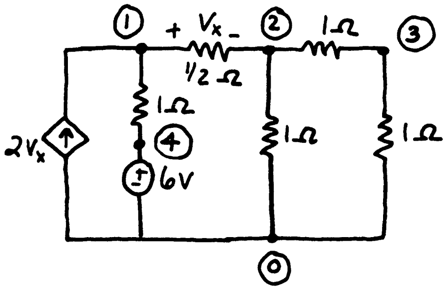


input file

```
R1  1 3 2
R2  2 0 2
R3  2 3 1
R4  3 4 1
R5  4 0 2
V1  1 0 DC 12
I1  2 1 DC 2
I2  0 3 DC 4
.DC V1 12 12 1
.PRINT DC V(4)
.END
```

ans. $\underline{V(4) = 4.9524}$

SP 5-8

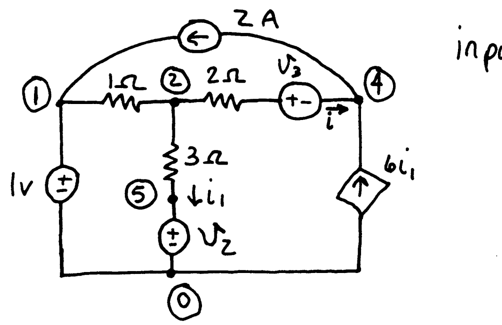


input file

```
R1   1 4 1
R2   1 2 .5
R3   2 0 1
R4   2 3 1
R5   3 0 1
V1   4 0 DC 6
G1   0 1 1 2 2
.DC  V1 6 6 1
.PRINT DC V(3)
.END
```

ans. $\underline{V(3) = 1.714\,v}$

SP 5-9

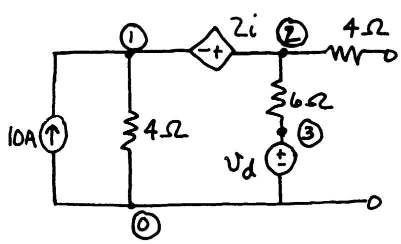


input file

```
V1   1 0 DC 1
R1   1 2 1
R2   2 5 3
R3   2 3 2
IS   4 1 DC 2
V2   5 0 0
V3   3 4 0
F1   0 4 V2 6
.DC  V1 1 1 1
.PRINT DC I(V3)
.END
```

ans. $\underline{I(V3) = -1.000\ A} = i$

SP 5-10



input file

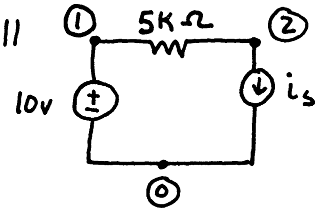
```
I1   0 1 DC 10
R1   1 0 4
H1   2 1 VD 2
R2   2 3 6
VD   3 0 DC 0
.TF  V(2) I2
.END
```

SPICE Output : V(2) = 30.00 V

output resistance = 3.00 Ω

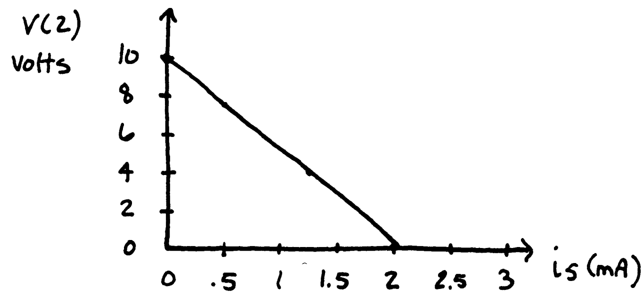
$\therefore\ \underline{V_T = 30\,v}$ & $\underline{R_T = 3 + 4 = 7\,\Omega}$

SP 5-11

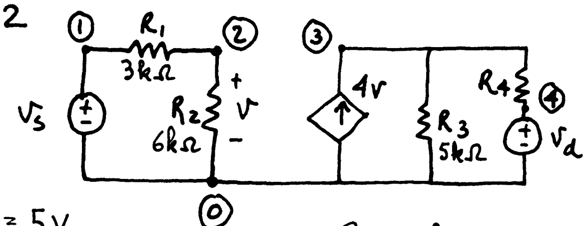


input file

```
vleft  1 0 dc 10v
R1     1 2 5k
is     2 0 dc 1mA
.dc is 0 2e-3 0.2e-3
.probe V(2)
.options nopage
.end
```



SP 5-12



$v_s = 5V$

$R_4 = 2k\Omega$

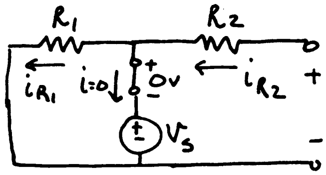
input file

```
* Amplifier
VS  1 0 DC 5
R1  1 2 3k
R2  2 0 6K
G1  0 3 2 0 4
R3  3 0 5K
R4  3 4 2K
VD  4 0 DC 0
.DC VS 5 5
.PRINT I(VD)
.END
```

Chapter 6

Exercises

Ex 6-1



$v_1 = v_s$

$i_{R_1} = \frac{v_s}{R_1}$ and $i_{R_2} = \frac{v_o - v_s}{R_2}$

now $i_{R_1} = i_{R_2}$ $\therefore$ $\frac{v_s}{R_1} = \frac{v_o - v_s}{R_2}$

or $\frac{v_o}{v_s} = 1 + \frac{R_2}{R_1}$

Ex. 6-2 referring to figure 12-10 (b)

from voltage divider : $V_1 = \frac{R_1}{R_1 + R_2} V_o \Rightarrow \frac{V_o}{V_1} = 1 + \frac{R_2}{R_1}$

$$\frac{V_o}{V_1} = 1 + \frac{40}{10} = 5$$

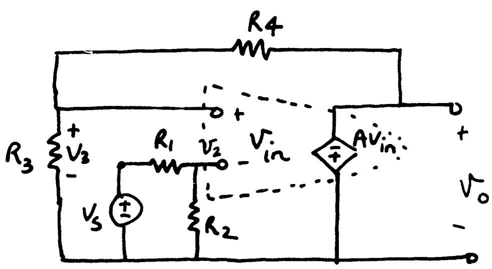
from KVL : $V_s + V_i = V_1$, $V_i = -\frac{V_o}{A}$

$\Rightarrow V_s = V_1 + \frac{V_o}{A}$

$\frac{V_s}{V_o} = \frac{V_1}{V_o} + \frac{1}{A} = \frac{1}{5} + \frac{1}{10^6} \simeq 1/5$

$\therefore \frac{V_o}{V_s} = 5$

Ex 6-3 use op amp model



$V_3 = \frac{R_3}{R_3 + R_4} v_o$

$V_2 = \frac{R_2}{R_2 + R_1} V_s$

$V_{in} = V_3 - V_2 = \frac{R_3}{R_3 + R_4} v_o - \frac{R_2}{R_2 + R_1} v_s$ (1)

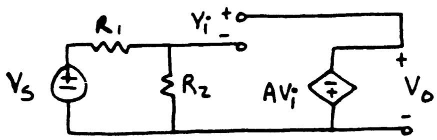
now $v_o = -A v_{in}$ or $v_{in} = -\frac{v_o}{A}$ (2)

equating (1) & (2) and solving fo v_o/v_s yields

$$\frac{v_o}{v_s} = \frac{R_2}{R_1 + R_2}\left(1 + \frac{R_4}{R_3}\right)$$

if $R_2 >> R_1$ $\Rightarrow$ $\frac{v_o}{v_s} = 1 + \frac{R_4}{R_3}$

Ex. 6-4



from voltage divider

$$V_1 = V_s\left(\frac{R_2}{R_1+R_2}\right) = V_o - V_i$$

$$\Rightarrow V_i = V_o - V_s\left(\frac{R_2}{R_1+R_2}\right)$$

also $V_o = -AV_i$

$$= -A\left(V_o - V_s\left(\frac{R_2}{R_1+R_2}\right)\right)$$

$$\Rightarrow \underline{\frac{V_o}{V_s} = \left(\frac{R_2}{R_1+R_2}\right)\frac{A}{1+A} = \frac{kA}{1+A}}$$

Ex 6-5 from example 6-2

$$k = \frac{R_1}{R_1+R_2} = \frac{50}{50+100} = \frac{1}{3}$$

$$\frac{v_o}{v_s} = \frac{-A(1-k)}{1+Ak} = \frac{-10^4(1-1/3)}{1+10^4(\frac{1}{3})} = \underline{-1.9994}$$

compared to approximation when $A >> 1$

then $\frac{v_o}{v_s} = -2.0$

Ex. 6-6 Assume $i_1 << i, i_L \Rightarrow i = i_L \quad \therefore V_o = V_s + v_i$

KVL @ rt. mesh: $V_o = -AV_i - R_o i$

$$= -AV_i - R_o(V_o/R_L)$$

$$V_o = -A(V_o - V_s) - R_o(V_o/R_L)$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{A}{1+\frac{R_o}{R_L}+A} = \frac{10^4}{1+.1+10^4} = \underline{0.99989}$$

now $R_{IN} = \frac{V_s}{i_1} = -\frac{V_s}{V_i/R_i} = -\frac{V_s}{V_i}R_i = -\frac{V_s R_i}{-\frac{V_o}{A}\left(1+\frac{R_o}{R_L}\right)}$

$$\Rightarrow R_{IN} = \left(\frac{V_s}{V_o}\right)\frac{R_i A}{1+R_o/R_L} = (.99989)^{-1}\frac{(10^5)(10^4)}{1+.1} = \underline{909\ M\Omega}$$

Ex. 6-7 w/o the buffer

$$V_o = \frac{R_L}{R_1+R_L} V_s = \frac{500}{1500} V_s = 1/3\, V_s$$

$$\Rightarrow P_1 = \frac{V_o^2}{R_L} = 1/9 \frac{V_s^2}{R_L}$$

with the buffer

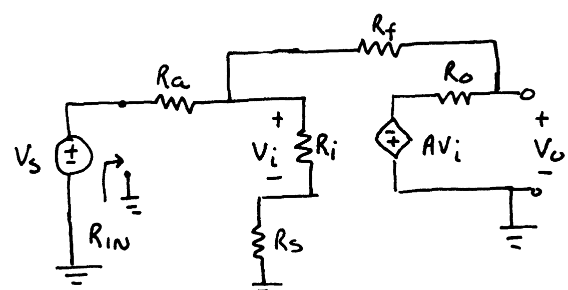
$$V_o = \frac{R_L}{R_o+R_L} V_s \qquad \Rightarrow \text{with gain} = 1.0 \text{ of buffer}$$

$$\Rightarrow P_2 = \frac{V_o^2}{R_L} = \frac{V_s^2}{R_L} \frac{R_L^2}{(R_o+R_L)^2}$$

$$= \frac{V_s^2}{R_L} \frac{(500)^2}{(510)^2} = .961 \frac{V_s^2}{R_L}$$

$$\therefore \frac{P_2}{P_1} = \frac{.961}{1/9} = 8.65$$

Ex 6-8



Answers : $A_v = \frac{V_o}{V_s} = \frac{R_o(R_i+R_s) + A R_i R_f}{(R_f+R_o)(R_i+R_s) + R_a(R_f+R_o+R_i+R_s) - A R_i R_a}$

$$R_{IN} = R_a + \frac{(R_f+R_o)(R_i+R_s)}{(R_f+R_o+R_i+R_s) - AR_i}$$

$$R_{out} = \frac{R_o[R_a(R_f+R_i+R_s) + R_f(R_i+R_s)]}{-AR_iR_a}$$

if the op amp is ideal (A very large, R_i large, R_o small)

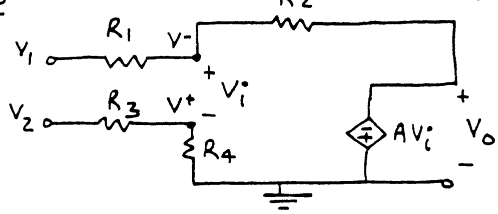
$$A_v = \frac{V_o}{V_s} \simeq \frac{AR_iR_f}{(1-A)R_iR_a} \simeq -\frac{R_f}{R_a}$$

$$R_{IN} \simeq R_a \quad \text{and} \quad R_{out} \simeq \frac{R_o(R_aR_i + R_fR_i)}{-AR_iR_a} = -\frac{R_o(R_a+R_f)}{AR_a}$$

if $R_f = 10R_a$ then $\Rightarrow R_{out} = -\frac{11 R_o}{A} \rightarrow 0$

Problems

P. 6-1



(a) Voltage divider yields

$$V^+ = \frac{R_4}{R_3+R_4} V_2 \qquad (1)$$

KCL at V^- yields

$$\frac{V^- - V_1}{R_1} = \frac{V_o - V^-}{R_2}$$

$$\Rightarrow V^- = \frac{R_1}{R_1+R_2} V_o + \frac{R_2}{R_1+R_2} V_1 \qquad (2)$$

now $V_o = -AV_i$

$$V_o = -A(V^- - V^+) = -A\left[\frac{R_1}{R_1+R_2}V_o + \frac{R_2}{R_1+R_2}V_1 - \frac{R_4}{R_3+R_4}V_2\right]$$

$$\Rightarrow V_o\left[-\frac{1}{A} - \frac{R_1}{R_1+R_2}\right] = \frac{R_2}{R_1+R_2}V_1 - \frac{R_4}{R_3+R_4}V_2$$

let $A \to \infty$ and solve for V_o

$$\therefore\ V_o = -\frac{R_2}{R_1}V_1 + \frac{1+R_2/R_1}{1+R_3/R_4}V_2$$

(b) want $V_o = 4V_2 - 11V_1$

$$\therefore\ \frac{R_2}{R_1} = 11 \quad \text{and} \quad \frac{1+R_2/R_1}{1+R_3/R_4} = 4$$

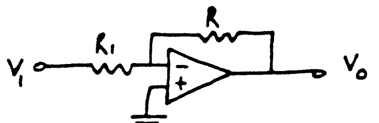
$$1+11 = 4(1+R_3/R_4)$$

$$\Rightarrow \frac{R_3}{R_4} = 2$$

let $R_1 = 10\,k\Omega$ $\Rightarrow$ $R_2 = 11R_1 = 110\,k\Omega$

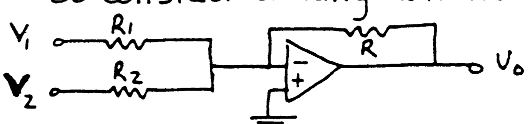
let $R_3 = 20\,k\Omega$ $\Rightarrow$ $R_4 = \frac{1}{2}R_3 = 10\,k\Omega$

P. 6-2



we know this ckt yields $V_o = -\frac{R}{R_1}V_1$

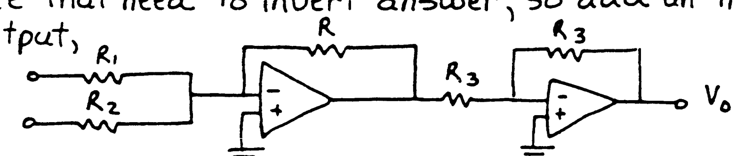
so consider adding another input lead into the neg. terminal



with $V_i = 0$, KCL at neg. input yields

$$-V_1/R_1 - V_2/R_2 = V_o/R \quad \Rightarrow V_o = -\frac{R}{R_1}V_1 - \frac{R}{R_2}V_2$$

See that need to invert answer, so add an inverter to the output,



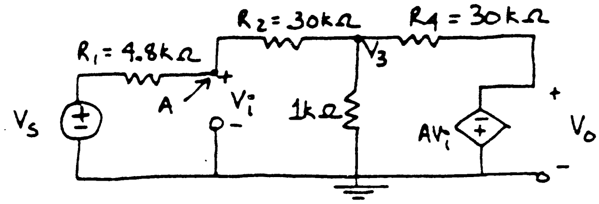
let $R_3 = 10\,k\Omega$

now $\frac{R}{R_1} = 6$ and $\frac{R}{R_2} = 2$ $\quad\therefore$ let $R = 60\,k\Omega$

$R_1 = 10\,k\Omega$

$R_2 = 30\,k\Omega$

P. 6-3



KCL at A : $(V_i - V_s)/4.8 = (V_3 - V_i)/30 \Rightarrow V_3 = 7.25V_i - 6.25V_s$

KCL at V_3 : $(V_3 - V_i)/30 + V_3 + (V_3 - V_o)/30 = 0$

$\Rightarrow 32V_3 = V_i + V_o$

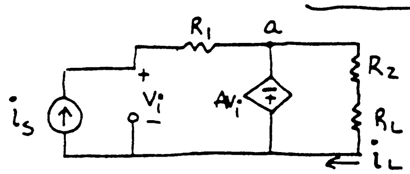
$32(7.25V_i - 6.25V_s) = V_i + V_o$

using $V_i = -\frac{V_o}{A}$

$$\Rightarrow \frac{V_o}{V_s} = -\frac{200}{1 + \frac{232}{A}}$$

for $A \to \infty$ $\quad \frac{V_o}{V_s} = -200$

P. 6-4



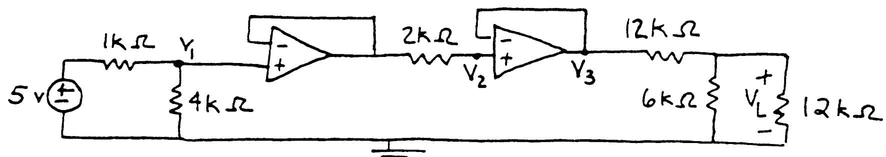
if $A \gg 1$ and $V_i \approx 0 \Rightarrow V_a = -i_s R_1$

then $i_L = \frac{V_a}{R_2 + R_L} = -\frac{R_1 i_s}{R_2 + R_L}$

$\simeq -\frac{R_1 i_s}{R_2}$ if $R_2 \gg R_L$

so $\frac{i_L}{i_s} = -\frac{R_1}{R_2} = -100$

P. 6-5



no current into the buffers

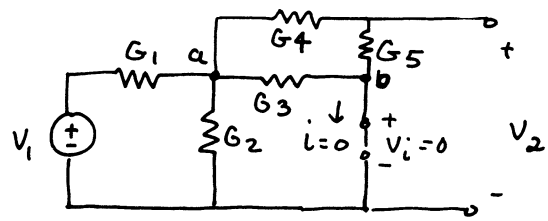
$\therefore V_1 = 5\left[\frac{4}{1+4}\right] = 4\text{ v}$

$V_2 = V_1$ (since buffer has gain = 1)

$V_2 = V_3$

$\Rightarrow V_L = V_3\left[\frac{12\|6}{12\|6 + 12}\right] = 1/4\ V_3 = 1/4\ V_1 = 1\text{ v}$

P 6-6



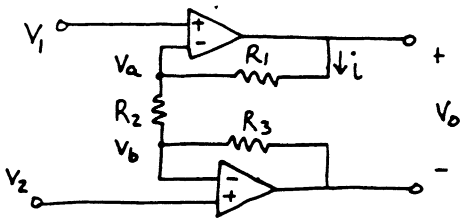
$V_b \to 0$

KCL node a : $(G_1 + G_2 + G_3 + G_4)V_a - G_1V_1 - G_4V_2 = 0$ (1)

KCL node b : $G_3V_a + G_5V_2 = 0$ (2)

from (1) & (2) $\Rightarrow$ $\dfrac{V_2}{V_1} = \dfrac{-G_1G_3}{G_5(G_1+G_2+G_3+G_4)+G_3G_4}$

P. 6-7 Assume $A = \infty$ (no current into the operational amps.)



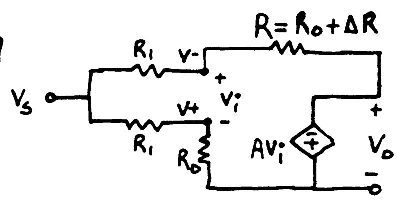
with the voltage across the input to the op amps = 0, then have $V_a = V_1$ and $V_b = V_2$

$\therefore$ from KVL : $V_o - iR_3 - iR_2 - iR_1 = 0$

$\Rightarrow V_o = i(R_1+R_2+R_3)$

also : $i = \dfrac{V_a - V_b}{R_2} = \dfrac{V_1-V_2}{R_2}$ $\Rightarrow$ $V_o = \dfrac{(R_1+R_2+R_3)(V_1-V_2)}{R_2}$

P. 6-9



from voltage divider : $V+ = \dfrac{R_o}{(R_o+R_1)}V_s$ (1)

KCL at V- : $(V- - V_s)/R_1 = (V_o - V^-)/R$

$\Rightarrow V_- = \dfrac{R}{R_1+R}V_s + \dfrac{R_1}{R_1+R}V_o$ (2)

now $V_o = -AV_i = -A(V_- - V_+)$ (3) $\Rightarrow$ plugging (1) & (2) into (3) yields

$V_o(-1/A - R/R_1+R) = \left(\dfrac{R}{R_1+R} - \dfrac{R_o}{R_o+R_1}\right)V_s$

assuming $\dfrac{R_1}{R_1+R} \gg 1/A$ $\Rightarrow -V_o\left(\dfrac{R_1}{R_1+R}\right) = \left(\dfrac{R}{R_1+R} - \dfrac{R_o}{R_o+R_1}\right)V_s$

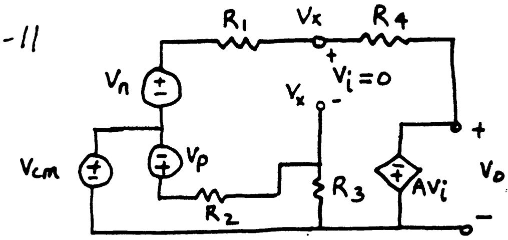
$\Rightarrow V_o = -\dfrac{V_s}{R_1}\left[R - \dfrac{R_o(R_1+R)}{R_o+R_1}\right]$

$= -\dfrac{V_s}{R_1}\left[\dfrac{(R_o+\Delta R)(R_o+R_1) - R_o(R_1+R_o+\Delta R)}{R_o+R_1}\right]$

$= -\dfrac{V_s}{R_1}\dfrac{R_1\Delta R}{(R_o+R_1)}$

$V_o = -V_s\dfrac{R_o}{R_o+R_1}\dfrac{\Delta R}{R_o}$

P. 6-11



with $V_i = 0$, from voltage divider

$$V_x = (V_{cm} + V_p)\left(\frac{R_3}{R_2+R_3}\right) \quad (1)$$

KCL at V_x (+ terminal)

$$\frac{(V_{cm}+V_n-V_x)}{R_1} = \frac{(V_x - V_o)}{R_4}$$

$$\Rightarrow V_o = V_x\left(1+\frac{R_4}{R_1}\right) - \frac{R_4}{R_1}\left(V_{cm}+V_n\right) \quad (2)$$

when $R_3/R_2 = R_4/R_1$ eqn. (1) becomes : $V_x = (V_{cm}+V_p)\left(\frac{R_4/R_1}{1+R_4/R_1}\right)$ (3)

plugging (3) into (2) yields

$$\underline{V_o = -R_4/R_1\,(V_n - V_p)}$$

P. 6-12 Select an inverting amplifier in which the gain is

$$\frac{V_o}{V_s} = -\frac{R_2}{R_1}$$ (see figure 12-12)

then $R_2 = 10R_1$

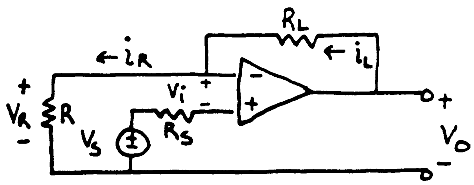
since the maximum current is 10mA through R_1

$$R_1 \geq \frac{1V}{10mA} = \underline{0.1\,k\Omega}$$

$\therefore$ if $R_1 = 0.1\,k\Omega$, then

$$\underline{R_2 = 10R_1 = 1\,k\Omega}$$

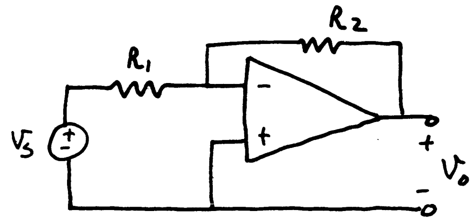
P. 6-13



Assume that $V_i = 0$ and that the current entering the op amp is negligible.

$$\therefore V_R = V_s = i_L R \Rightarrow \underline{\frac{i_L}{V_s} = \frac{1}{R}}$$

P 6-14



see table 6-3

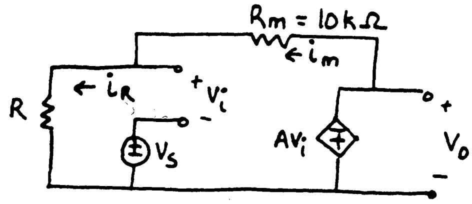
for 741 : $A = 2\times10^5$ $R_i = 2\,M\Omega$

$R_o = 75\,\Omega$

using ideal case $\frac{V_o}{V_s} = -\frac{R_2}{R_1}$

use $R_1 = 1000\,\Omega = 1k\Omega$ then $\underline{R_2 = 100k\Omega}$

P. 6-15



assume the op amp draws no current $\Rightarrow$ $i_R = i_m$

from KVL : $V_o = (R_m + R) i_R$ (1)

$V_i = i_R R - V_s$ (2)

also $V_o = -AV_i$ (3)

(2) into (3) $\Rightarrow$ $V_o = A(V_s - i_R R)$ (4)

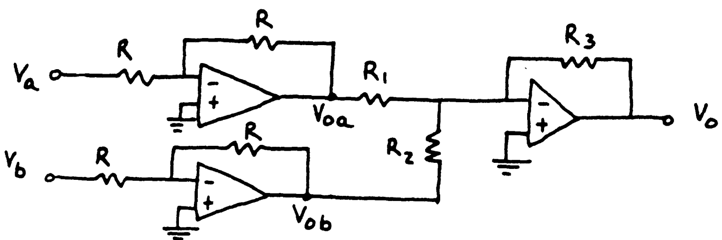
Solving for i_R in (1) and plugging into (4) yields

$$\frac{V_o}{V_s} = \frac{A}{1+kA} \quad \text{where } k = R/(R+R_m)$$

for $A \gg 1$ $\Rightarrow$ $\frac{V_o}{V_s} = \frac{1}{k} = \frac{R_m + R}{R}$

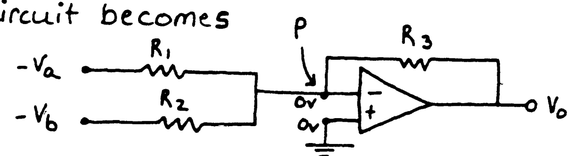
so $i_R = i_m = \frac{V_o}{R+R_m} = \frac{(R_m+R)V_s}{R(R+R_m)} = \frac{V_s}{R}$ $\Rightarrow$ $R = \frac{V_s}{i_R} = \frac{1V}{10^{-4}A} = 10k\Omega$

P. 6-16



(a) assume ideal op amps $\Rightarrow$ $V_{oa} = -V_a$ and $V_{ob} = -V_b$

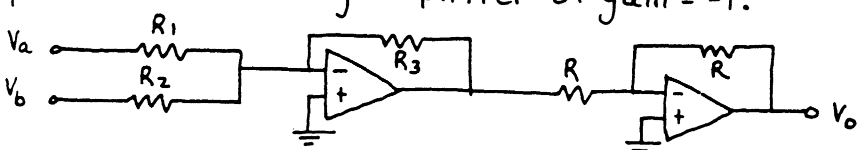
$\therefore$ the circuit becomes



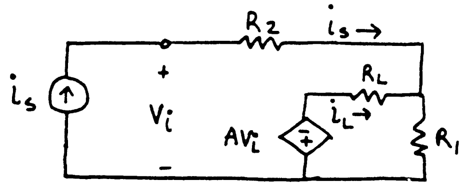
KCL at P: $V_a/R_1 + V_b/R_2 = V_o/R_3$ $\Rightarrow$ $V_o = \frac{R_3}{R_1} V_a + \frac{R_3}{R_2} V_b$

(b) the circuit functions as a weighted adder

(c) use above ckt. with input of $+V_a$ and $+V_b$ and then invert the output with an inverting amplifier of gain = -1.



P. 6-17



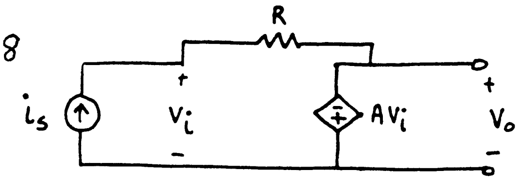
with $v_i \simeq 0$, R_1 & R_2 are in parallel

$\therefore$ from current divider

$$i_s = -\frac{R_1}{R_1+R_2}\, i_L$$

$$\Rightarrow \underline{\frac{i_L}{i_s} = \frac{-(R_1+R_2)}{R_1}}$$

P. 6-18

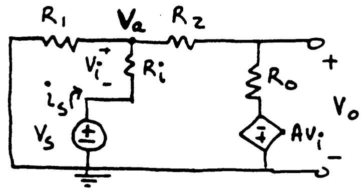


assume $v_i \simeq 0$

$\therefore$ i_s flows through R

from KVL : $-v_o - i_s R = 0$ $\quad\Rightarrow \underline{\dfrac{v_o}{i_s} = -R}$

P. 6-19 (a)



KCL at V_a: $\quad i_s + \dfrac{(-Av_i - v_a)}{R_2+R_o} - \dfrac{v_a}{R_1} = 0 \quad (1)$

also : $\quad v_a = v_s - i_s R_i \quad (2)$

$\quad v_i = -i_s R_i \quad (3)$

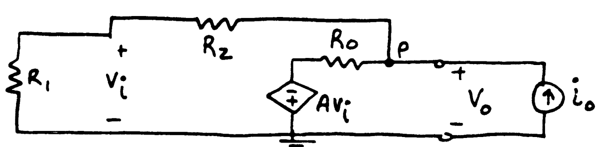
(3) and (2) into (1) yields

$$v_s\left[\frac{1}{R_1} + \frac{1}{R_2+R_o}\right] = i_s\left[1 + \frac{AR_i}{R_2+R_o} + \frac{R_i}{R_2+R_o} + \frac{R_i}{R_1}\right]$$

assuming $A \gg 1 \Rightarrow \dfrac{A}{R_2+R_o} \gg \dfrac{1}{R_1}$

$$\Rightarrow \underline{R_{in} = \frac{v_s}{i_s} = \left[\frac{AR_1}{R_1+R_2+R_o}\right] R_i}$$

(b) assume $R_i \gg R_2$ and set $v_s = 0$



KCL at P: $\quad i_o - v_o/(R_1+R_2) - (v_o + Av_i)/R_o = 0 \quad (1)$

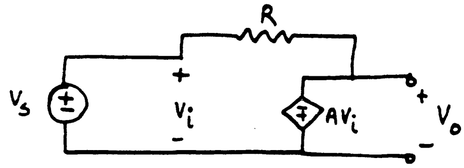
also: $v_i = v_o\left(\dfrac{R_1}{R_1+R_2}\right) \quad (2)$

(2) into (1) yields $\quad i_o = \dfrac{v_o}{R_1+R_2} + \dfrac{v_o}{R_o} - \dfrac{v_o A R_1}{(R_1+R_2)R_o}$

for $A \gg 1$: $\quad i_o \simeq \dfrac{v_o A R_1}{(R_1+R_2)R_o}$

$$\Rightarrow \underline{R_{out} = \frac{v_o}{i_o} = \frac{(R_1+R_2)R_o}{AR_1}}$$

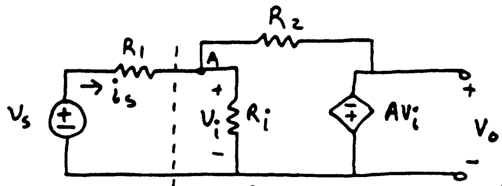
P. 6-20



$V_s = V_i$

$$\Rightarrow \frac{V_o}{V_s} = \frac{V_o}{V_i} = \frac{-AV_i}{V_i} = -A$$

P. 6-21



← find the Thev. equiv. to right of dotted line and add R_T to R_1 to get total $R_{in} = V_s/i_s$

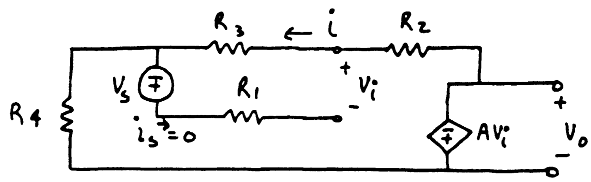
KCL at A: $i_s = V_i/R_i + (V_i + AV_i)/R_2$

$$\Rightarrow \frac{i_s}{V_i} = \frac{1}{R_i} + \frac{1}{R_2} + \frac{A}{R_2} \simeq \frac{A}{R_2} \qquad \text{for } A \gg 1$$

$$\therefore \; R_T = \frac{R_2}{A}$$

Thus, the total input impedance $\Rightarrow R_{in} = R_1 + R_2/A \simeq R_1$

P. 6-22



Since $i_s = 0$, from KVL

$-V_s - V_i + R_3 i = 0$

$\Rightarrow i = (V_s + V_i)/R_3$ (1)

also: $i = V_o/(R_4 + R_3 + R_2)$ (2)

equating (1) and (2) and using $V_i = -V_o/A$ yields

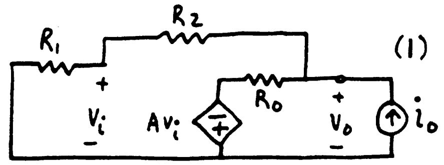
$$V_o\left[\frac{1}{AR_3} + \frac{1}{R_2 + R_3 + R_4}\right] = \frac{V_s}{R_3}$$

assume $AR_3 \gg R_2 + R_3 + R_4$

$$\Rightarrow V_o = \frac{(R_3 + R_2 + R_4)\, V_s}{R_3}$$

Note: V_o is indep. of R_1

P. 6-23 (a)



(1) KCL at output node :

$$i_o + (-Av_i - v_o)/R_o + (v_i - v_o)/R_2 = 0$$

(2) also : $v_i = \dfrac{R_1 v_o}{R_1 + R_2} = k v_o$

(2) into (1) yields $\quad i_o + (-Akv_o - v_o)/R_o + (kv_o - v_o)/R_2 = 0$

assume $Ak \gg 1$

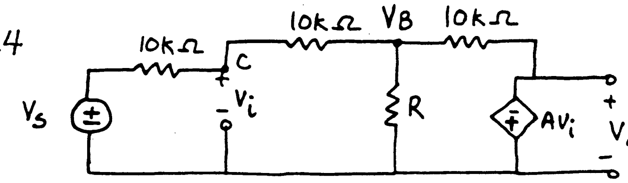
$$\Rightarrow \quad i_o - \frac{Akv_o}{R_o} + \frac{(k-1)}{R_2} v_o = 0$$

assume $\dfrac{Ak}{R_o} \gg \dfrac{(k-1)}{R_2} \quad \Rightarrow \quad \underline{R_{out} = \dfrac{v_o}{i_o} = \dfrac{R_o}{Ak}}$

(b) for $A = 10^5$, $k = 0.1$, $R_o = 1\,k\Omega$

$\Rightarrow \underline{R_{out} = 0.1\,\Omega}$

P. 6-24



KCL at c : $(v_i - v_s)/10 + (v_i - v_B)/10 = 0 \Rightarrow v_B = 2v_i - v_s$ (1)

KCL at v_B : $(v_B - v_i)/10 + v_B/R + (v_B - v_o)/10 = 0$ (2)

also : $v_i = -v_o/A$ (3)

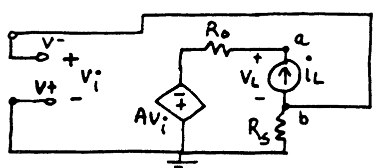
plugging (3) and (1) into (2) yields

$$\Rightarrow \quad -2(2 + 10/R)v_s = [1 + (3 + 20/R)1/A]\, v_o$$

assume $(3 + 20/R)1/A \ll 1$

$$\Rightarrow \underline{R = 10/(-v_o/v_s - 2) = 10/(100-2) = .102\,k\Omega = 102\,\Omega}$$

P. 6-25 (a)



replace R_L with current source i_L and find $R = v_L/i_L$

set $v_s = 0$

now $v^+ = 0$ and $\underline{v_i = v^- - v^+ = -i_L R_s}$

from KVL : $-v_L + i_L R_o - Av_i - v_i = 0$

$$\Rightarrow v_L = i_L R_o - (1+A)(-i_L R_s) = i_L[R_o + (1+A)R_s]$$

$\therefore \underline{R_{out} = \dfrac{v_L}{i_L} = R_o + (1+A)R_s \simeq AR_s}$ for $A \gg 1$

(b) $R_{out} = AR_s + R_o$

$= (10^5)(10^4) + 10^3$

$\underline{R_{out} \simeq 10^9\,\Omega}$

P. 6-26 (a) without amplifier: $\underline{V_o/V_s = \frac{R_L}{R_S+R_L} = 1/2}$

with amplifier: $\underline{\frac{V_o}{V_s} = 1}$ since $R_{IN} \gg 500\,\Omega$ and $R_L \gg R_o$

(b) $P_a = V_o^2/R_L = V_s^2/R_L$

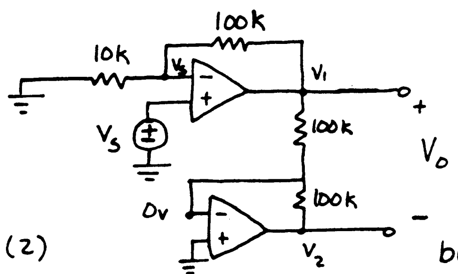
direct: $P_d = V_o^2/R_L = 1/4\, V_s^2/R_L$

$\therefore$ $\underline{\text{power gain} = \frac{P_a}{P_d} = 4}$

(c) desire $P_a/P_d = 16 = \dfrac{V_s^2/R_L}{\dfrac{\left(R_L/(R_S+R_L)\right)^2 V_s^2}{R_L}}$

$\therefore$ $\frac{R_L}{R_S+R_L} = 1/\sqrt{16}$, with $R_S = 500\,\Omega$ $\Rightarrow$ $\underline{R_L = 167\,\Omega}$

P. 6-27 assume ideal amps



KCL at V_s:

$V_s/10 + (V_s - V_1)/100 = 0$

$\Rightarrow$ $V_1 = 11\,V_s$ (1)

also: $V_o = V_1 - V_2 = 2V_1$ (2)

by inspection $V_2 = -V_1$

(1) into (2) yields

$V_o = 22\,V_s$ $\Rightarrow$ $\underline{V_o/V_s = 22}$

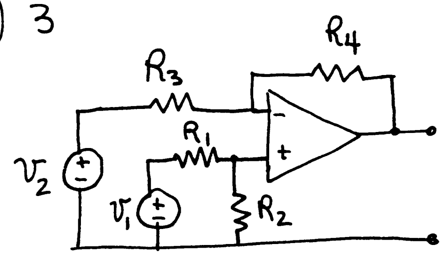
P 6-28

see Exer 6-3

$$v_o = \frac{R_2}{R_1+R_2}\left(1+\frac{R_4}{R_3}\right)v_1 - \frac{R_4}{R_3}v_2$$

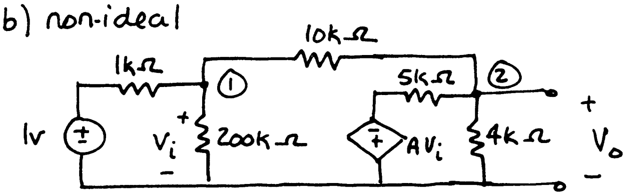
$$= \frac{1}{3}(1+4)\,6 - \left(\frac{20}{5}\right)3$$

$$= 10 - 12 = -2\,\text{V}$$



P 6-29 a) ideal $v_o = -\frac{10k\Omega}{1k\Omega}(1v) = \underline{-10v}$

b) non-ideal



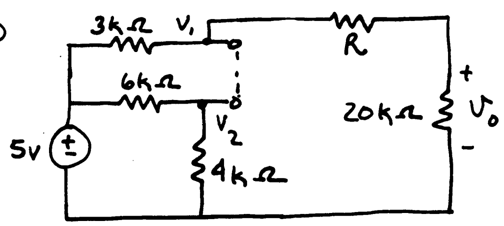
$A = 10^4$

KCL node 1 : $\frac{V_i - 1}{1} + \frac{V_i}{200} + \frac{V_i - V_o}{10} = 0$ (1)

node 2 : $\frac{V_o - V_i}{10} + \frac{V_o + AV_i}{5} + \frac{V_o}{4} = 0$ (2)

from (1) and (2) get $\underline{v_o = -9.97V}$

P 6-30



$V_2 = \frac{4}{4+6} V_s$ and $V_1 = V_2$

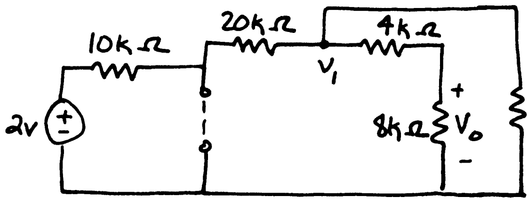
$= 2v$

KCL at V_1 : $\frac{5-2}{3k\Omega} + \frac{v_o - 2}{R} = 0$ $\Rightarrow R = 2 - v_o$ (R in kΩ)

at neg. value of $v_o = -14v$ get $R = 2-(-14) = 16k\Omega$

so saturates at $R \geq 16k\Omega$

P 6-31



KCL into '-' terminal:

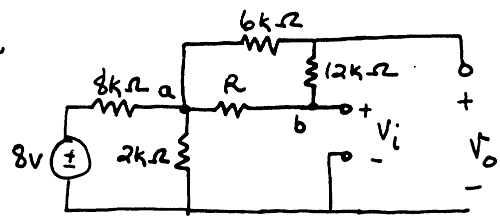
$$\frac{2}{10}+\frac{V_1}{20}=0 \Rightarrow V_1=-4v$$

KCL node V_1: $$\frac{V_1}{20}+\frac{V_1}{5}+\frac{V_1-V_o}{4}=0$$

$$\Rightarrow V_o=-8v$$

$$\therefore I_o=\frac{V_o}{8k\Omega}+\frac{V_o-V_1}{4}=\underline{-2mA}$$

P 6-32



KCL node a: $$\frac{V_a-8}{8}+\frac{V_a-V_o}{6}+\frac{V_a}{2}+\frac{V_a-V_i}{R}=0 \qquad (1)$$

KCL node b: $$\frac{V_i-V_a}{R}+\frac{V_i-V_o}{12}=0 \qquad (2)$$

now let $V_i=0$ and multiply (1) by 24 & (2) by -12

$$3(V_a-8)+4(V_a-V_o)+12V_a+\frac{24V_a}{R}=0 \qquad (3)$$

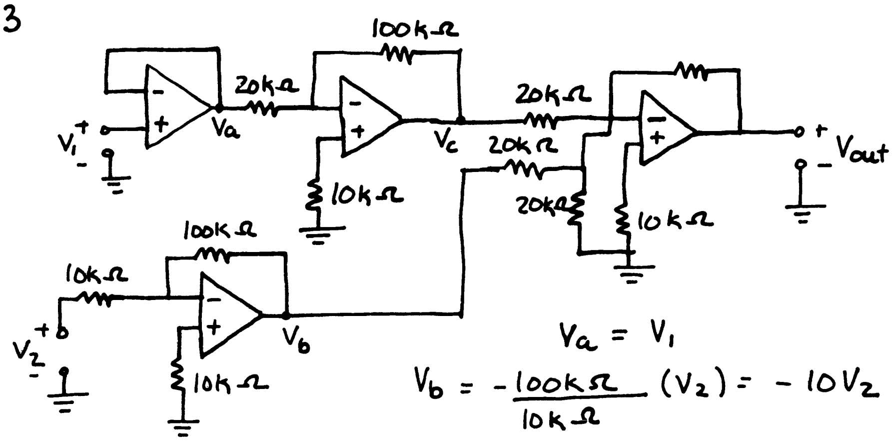
$$\frac{12V_a}{R}+V_o=0 \qquad (4)$$

solving for V_a $\Rightarrow$ $$V_a=\frac{24}{\left[19+\frac{24+48}{R}\right]}$$

want $V_o=-1.95v$ or $V_a=\dfrac{1.95R}{12}$

thus $$\frac{1.95R}{12}=\frac{24}{\left[19+72/R\right]} \Rightarrow \underline{R=4k\Omega}$$

P 6-33



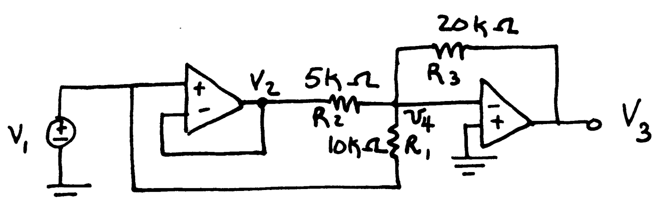
$$V_a = V_1$$

$$V_b = -\frac{100k\Omega}{10k\Omega}(V_2) = -10V_2$$

$$V_c = -\frac{100k\Omega}{20k\Omega}(V_a) = -5V_a = -5V_1$$

$$V_{out} = -80k\Omega\left(\frac{V_c}{20k\Omega} + \frac{V_b}{20k\Omega}\right) = -4V_c - 4V_b = \underline{20V_1 + 40V_2}$$

P 6-34



$V_1 = V_2 = V_s$

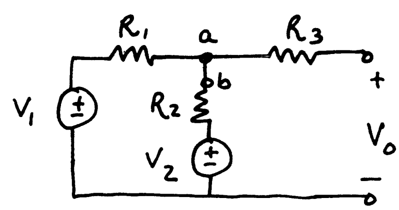
$V_3 = V_o$

$v_4 \to 0$ virtual ground

$$V_o = 20\left(\frac{V_s}{10} + \frac{V_s}{5}\right) = 2V_s + 4V_s = \underline{6V_s}$$

since KCL at v_4 is $\frac{v_o}{20} = \frac{v_s}{10} + \frac{v_s}{5}$

P6-35



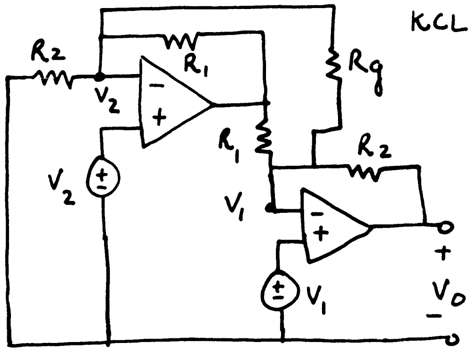
$v_b = V_2$ since no current

$V_a = v_b$ due to short so $v_a = v_b = v_2$

KCL : $\frac{V_1 - V_a}{R_1} + \frac{(v_o - v_a)}{R_3} = 0$ so $\frac{v_1 - v_2}{R_1} + \frac{v_o - v_2}{R_3} = 0$

$$\Rightarrow \underline{V_o = \left(1 + \frac{R_3}{R_1}\right)V_2 - \frac{R_3}{R_1}V_1}$$

P 6-36



KCL V_1 : $\frac{V_1-V_o}{R_2}+\frac{V_1-V_a}{R_1}+\frac{V_1-V_2}{R_g}=0$ (1)

V_2 : $\frac{V_2-V_a}{R_1}+\frac{V_2-V_1}{R_g}+\frac{V_2}{R_2}=0$ (2)

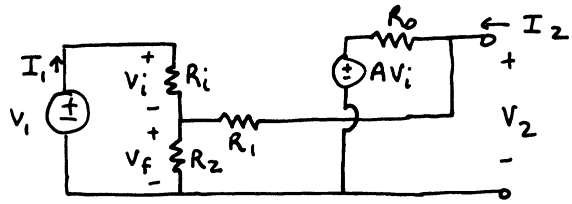
solving (1), (2) for V_o :

$$V_o=\left(1+\frac{R_2}{R_1}+\frac{2R_2}{R_g}\right)(V_1-V_2)$$

Let $R_g=200\,k\Omega$, $R_2=50\,k\Omega$ then have $10=\left(1+\frac{50}{R_1}+\frac{100}{200}\right)$

so need $\underline{R_1=5.88\,k\Omega}$

P 6-37



a) $V_i=V_1-V_f$

$$V_2=AV_1-A\left[\frac{\frac{R_2R_i}{R_2+R_i}\,V_2}{R_1+\frac{R_2R_i}{R_2+R_i}}\right]$$

Let $\beta\equiv R_2'/(R_2'+R_1)$ with $R_2'=R_2/\!/R_i$

$V_2=AV_1-A\beta V_2$ $\therefore$ $\underline{\frac{V_2}{V_1}=\frac{A}{(1+A\beta)}}$

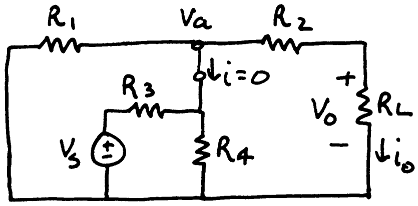
Feedback eqn: $\frac{V_2}{V_1}=\frac{A}{1+A\beta}$ $\quad \left.\begin{array}{l}V_1=V_i+V_F\\ I_1=V_i/R_i\end{array}\right\}$ $\begin{array}{l}V_i=V_2/A\\ V_F=\beta V_2\end{array}$

$\therefore$ $\frac{V_1}{I_1}=\frac{V_2\left[\frac{1}{A}+\beta\right]}{V_2/AR_i}=(1+A\beta)R_i$ $\quad$ so $\underline{R_{IN}=(1+A\beta)R_i}$

b) $R_2'=10/\!/100=9.09\,k\Omega$ $\Rightarrow$ $\beta=\frac{9.09}{9.09+10}=.476$

so $\frac{V_2}{V_1}=\frac{10^4}{1+10^4(.476)}=\underline{2.10}$; $R_{IN}=[1+10^4(.476)]100=\underline{476\,M\Omega}$

P 6-39



$V_1 = \frac{R_4}{R_3+R_4} V_s = a V_s$, $V_a = V_1 = a V_s$

$i = -\frac{V_a}{R_1}$, $i_o = \frac{V_a - V_o}{R_2}$

$\therefore -\frac{V_a}{R_1} = \frac{V_a - V_o}{R_2}$ or $-\frac{a V_s}{R_1} = \frac{a V_s}{R_2} - \frac{V_o}{R_2}$

so $V_o = a\left(1+\frac{R_2}{R_1}\right)V_s = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)V_s = \left(\frac{20}{30}\right)\left(1+\frac{98}{7}\right)(.1) = 1.0\text{ v}$

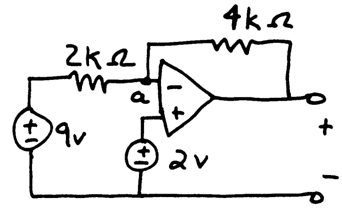
so $i_o = \frac{V_o}{R_L} = \frac{10 V_s}{2k\Omega} = \underline{0.5\text{ mA}}$

Advanced Problems

AP 6-1 1st op amp has -3v input and is inverting amplifier

$V_{o1} = -\left(\frac{6}{2}\right)(-3) = \underline{9\text{v}}$

Second op amp equiv. ckt:



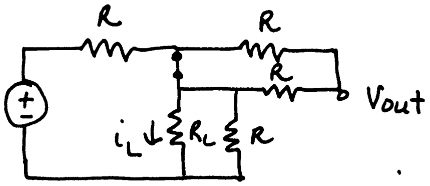
KCL node a:

$$\frac{9-2}{2} + \frac{V_o - 2}{4} = 0$$

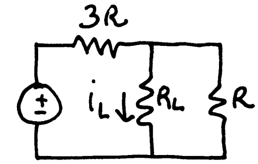
$\Rightarrow \underline{V_o = -12\text{ v}}$

AP 6-2

$v_1 = V_o$



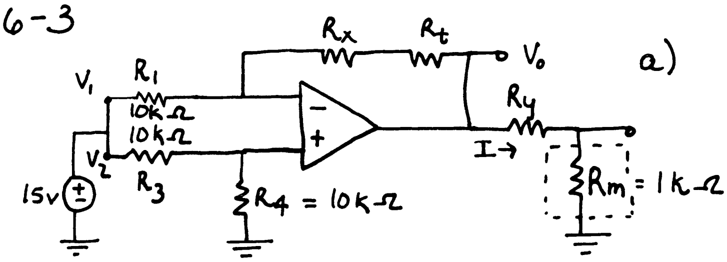
$\Rightarrow$



V_o = constant

$\underline{i_L = \frac{V_o}{R}}$ if $R_L \ll R$

AP 6-3



a) $V_1 = V_2 = 15v$ from result of problem 6-1

$$V_o = \frac{1+(R_x+R_t)/R_1}{1+R_3/R_4}(V_2) - \frac{R_x+R_t}{R_1}V_1$$

when $t = -55°c$ $V_o = 0 \rightarrow R_t = 1000e^{55/25} = 9.025 k\Omega$

$\therefore$ have $0 = \dfrac{(10+R_x+9.025)10(15)}{(10+10)(10)} - \dfrac{R_x+9.025}{10}(15) \Rightarrow \underline{R_x = 988k\Omega}$

b) when $t = 125°c$

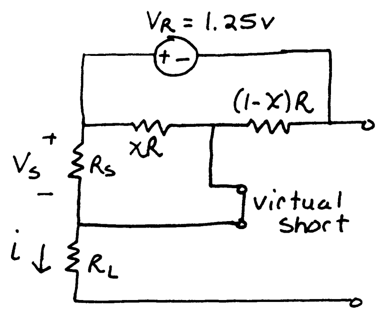
$R_t = 1000e^{-125/25} = 6.74\Omega$

so $\underline{V_o = 6.75V}$

now because want $I = 1mA$ through the meter

$R_y + R_m = \dfrac{V_o}{I} = \dfrac{6.75v}{1mA} = 6.75k\Omega \Rightarrow \underline{R_y = 5.75 k\Omega}$

AP 6-4



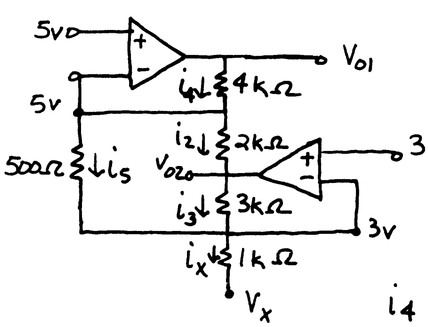
using voltage divider

$$V_s = \frac{XR}{XR+(1-X)R}V_R = XV_R$$

also $\underline{i = \dfrac{XV_R}{R_s}}$

for $X = 1$, $i_{max} = \dfrac{(1)(1.25)}{1} = 1.25A$ so $\underline{0 < i \le 1.25A}$

AP 6-5



$i_5 = \dfrac{5-3}{500} = 4mA$, $i_x = \dfrac{3-V_x}{1}$ (mA)

$\therefore i_3 = i_x - i_5 = -(1+V_x)$ (mA)

$V_{02} = (3k\Omega)i_3 + 3 = -3V_x$ (volts)

$i_2 = \dfrac{5-V_{02}}{2k} = \left(\dfrac{5+3V_x}{2}\right)$ (mA)

$i_4 = i_2 + i_5 = \dfrac{5+3V_x}{2} + 4 = (6.5+1.5V_x)$ (mA)

$V_{01} = (4k\Omega)i_4 + 5 = 26 + 6V_x + 5 = (31+6V_x)$ (volts)

so $V_{01} - V_{02} = 31 + 6V_x - (-3V_x) = 31 + 9V_x = 13v \Rightarrow \underline{V_x = -2v}$

AP 6-6 The ladder structure uses the shunt resistors to generate n binary weighted currents. From the left we have $I_1 = V_R/2R$ $I_2 = I_1/2$, ... $I_n = I_1/2^{n-1}$

Depending on the switch position, each current is diverted to the ground bus I^+ or to the virtual ground bus I^-. Use bit b_k to identify the status of the switch k. Then

$$V_o = -\frac{R_f}{R} V_R \left(b_1 2^{-1} + b_2 2^{-2} + \cdots b_{n-1} 2^{n-1} + b_n 2^{-n}\right)$$

Design Problems

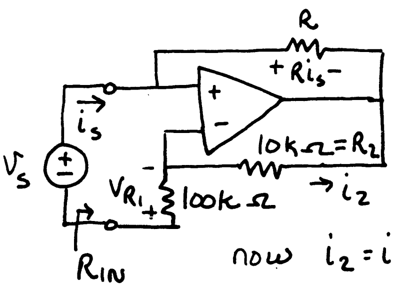
OP 6-1 $v_o = -\left[\frac{12}{4} v_1 + \frac{12}{6}(-10)\right]$

range is: $\pm 12 = -[3v_1 - 20]$

$v_1 = \left(\frac{20 \pm 12}{3}\right)$, so $v_1^+ = \frac{32}{3}$ & $v_1^- = \frac{8}{3}$

so have $\underline{\frac{8}{3} < v_1 < \frac{32}{3}}$ linear range

OP 6-2



$R_{IN} = \frac{v_s}{i_s}$

$V_{R_2} = Ri_s \quad \therefore \ i_2 = i_s R/R_2$

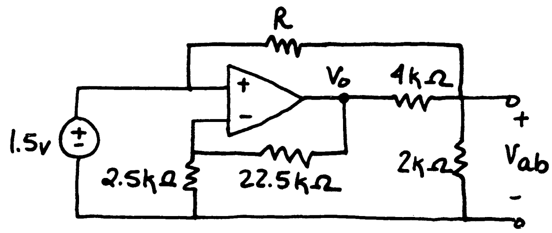
now $i_2 = i_1$ so $V_{R_1} = i_1 R_1 = \frac{i_s R}{R_2} R_1$

KVL around left mesh: $V_s + V_{R_1} = 0$ or $V_s + i_s \frac{RR_1}{R_2} = 0$

so $R_{in} = \frac{v_s}{i_s} = -\frac{RR_1}{R_2} = -\frac{R(100)}{10} = -10R$

need $\left|\frac{v_s}{i_s}\right| = 1M\Omega$ or $\underline{R = 100k\Omega}$ * note it is a negative resistance

DP 6-3



$$V_o = \left(1 + \frac{22.5}{2.5}\right)1.5 = 15\text{ v}$$

KCL at terminals a-b $\quad \dfrac{V_{ab}}{2} + \dfrac{V_{ab}-1.5}{R} + \dfrac{V_{ab}-15}{4} = 0 \qquad (1)$

$$\Rightarrow V_{ab} = \frac{6+15R}{3R+4}$$

now find i_{sc} at a-b

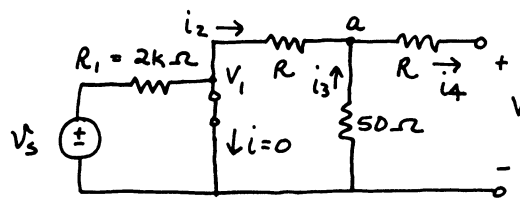
$$i_{sc} = \frac{1.5}{R} + \frac{15}{4} \qquad (2)$$

now $R_{out} = \dfrac{V_{ab}}{i_{sc}} = \dfrac{\frac{6+15R}{(3R+4)}}{\frac{6+15R}{4R}} = \dfrac{4R}{3R+4}$

$= .57\text{ k}\Omega$

$\therefore\ \underline{R = 1\text{k}\Omega}$

DP 6-4



$V_1 = 0$

$i_1 = \dfrac{V_s}{R_1}$; $i_2 = i_1$

at a : $\quad V_a = V_1 - i_2 R = -i_2 R = -\dfrac{V_s R}{R_1}$

$$i_3 = -\frac{V_a}{50} = \frac{V_s R}{50 R_1}$$

at a : $\quad i_4 = i_2 + i_3 = \dfrac{V_s}{R_1} + \dfrac{R}{50 R_1} V_s = \dfrac{50+R}{50 R_1} V_s$

$$V_o = V_a - R i_4 = -\frac{R}{R_1} V_s = -\frac{R(50+R)}{50 R_1} V_s$$

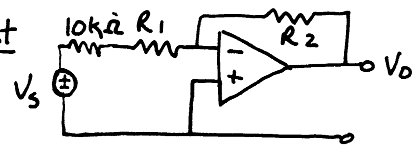
so $\dfrac{V_o}{V_s} = -\dfrac{R}{R_1}\left(1 + \dfrac{50+R}{50}\right)$

given $R_1 = 2\text{k}\Omega$ find R if $V_o/V_s = -1000$

$\Rightarrow -1000 = \dfrac{R}{2000}\left(1 + \dfrac{50+R}{50}\right)$ yields $\underline{R = 9950\ \Omega}$

DP 6-5 need gain $V_o/V_s = \dfrac{4}{20\times10^{-3}} = 200$

inverting ckt

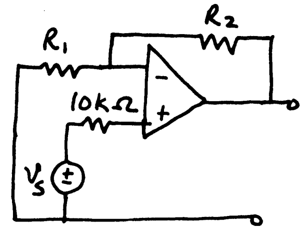


$$\frac{V_o}{V_s} = -\frac{R_2}{R_1+R_m}$$

use $R_2 = 2\,M\Omega$, $R_1 = 0$

$R_m = 10k\Omega$

non inverting ckt



$$\frac{V_o}{V_s} = \left(1+\frac{R_2}{R_1}\right)V_s$$

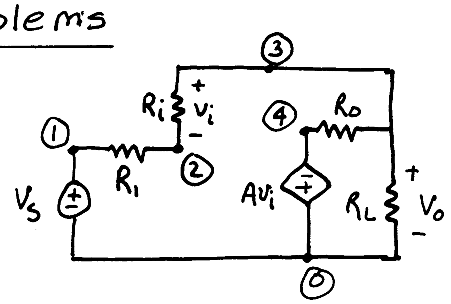
R_{IN} = higher for noninverting compared to inverting

choose $(1+R_2/R_1) = 200$ use $R_1 = 1k\Omega$, $R_2 = 199\,k\Omega$

so use noninverting for higher R_{IN} for microphone

Spice Problems

SP 6-1



a) input file

```
VS  1 0  DC  10.0000768
R1  1 2  1K
Ri  2 3  10meg
RO  4 3  1e-99
RL  3 0  5K
E1  0 4  3 2  1meg
.TF V(3) VS
.END
```

$\Rightarrow$ V(3)/VS = 1.0

b) input file

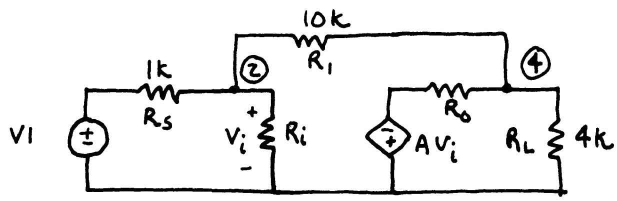
```
VS  1 0  DC  10.0000768
R1  1 2  1K
Ri  2 3  80K
RO  4 3  100
RL  3 0  5K
E1  0 4  3 2  1E4
.TF V(3) VS
.END
```

$\Longrightarrow$ V(3)/VS = .9999

$R_{in} = 7.8\times10^{8}\,\Omega$

$R_{out} = 10\,m\Omega$

SP 6-2



a) <u>ideal</u>

$A = 10^8$, $R_i = 10M\Omega$, $R_o = 0$

```
VI  1 0  DC 1
RS  1 2  1000
RI  2 0  10meg
R1  2 4  10k
E1  0 3  2 0  10meg
RO  3 4  0
RL  4 0  4K
.DC  VI 1 1 1
.PRINT DC V(4)
.END
```

ANS. $V(4) = -9.9999$ v

b) <u>actual</u>

$A = 10^4$, $R_i = 200k\Omega$, $R_o = 5k\Omega$

```
VI  1 0  DC 1
RS  1 2  1000
RI  2 0  200K
R1  2 4  10K
E1  0 3  2 0  1E04
RO  3 4  5K
RL  4 0  4K
.DC  VI 1 1 1
.PRINT DC V(4)
.END
```

ANS. $V(4) = -9.9697$ v

SP 6-3 See figure in SP 6-1

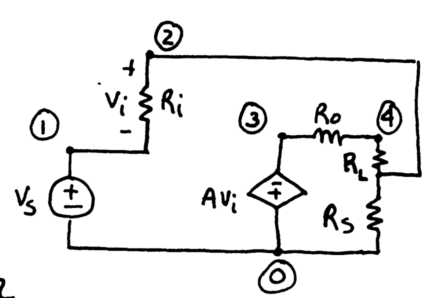
input file:

```
VS  1 0  DC 10.0000768
R1  1 2  1k
Ri  2 3  100k
Ro  4 3  1000
RL  3 0  10k
E1  0 4  3 2  1e4
.TF  V(3)  VS
.END
```

ANS. $V(3) = 9.9990$, Rin $= 9.092E+08\ \Omega = 909$ Megaohm

$V(3)/VS = .9999$

SP 6-4



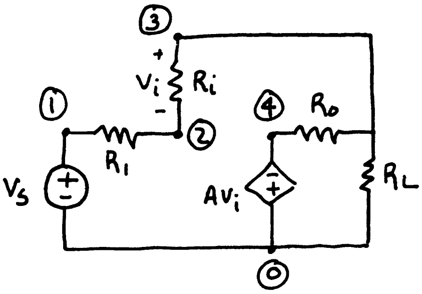
$R_L = 10k\Omega$

input file:

```
RL  4 2  10k
VS  1 0  DC 1
RI  1 2  100K
E1  0 3  2 1  1E5
RO  3 4  1K
RS  2 0  10K
.TF  V(4,2)  VS
.END
```

answer: $V(4,2)/V(S) = 2$, $R_{out} = 10k\Omega$

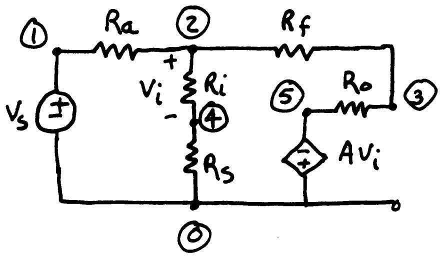
SP 6-5



input file:

```
VS  1 0  DC  10.0000768
R1  1 2  10K
Ri  2 3  100K
RO  4 3  100
RL  3 0  10K
E1  0 4  3 2  1E4
.TF  V(3)  VS
.END
```

SP 6-6

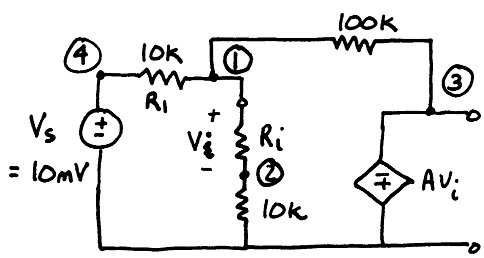


input file:

```
VS  1  0  DC  1
RA  1  2  25K
RF  2  3  50K
Ri  2  4  500K
RO  5  3  100
RS  4  0  10K
E1  0  5  2  4  1E5
.TF  V(3)  VS
.END
```

	Ideal	Actual
v_o/v_s	−2	−2
R_{in}	25 kΩ	25 kΩ
R_{out}	0	3.2 mΩ

SP 6-7



ideal op amp so let $A = 10^6$
$R_i = 10^9\,\Omega$

```
VS   4  0   DC   .01
R1   4  1   10K
R2   1  3   100K
R3   2  0   10K
RI   1  2   1E9
E1   0  3  1  2  1E6
.DC  VS  10M  10M  1
.TF  V(3)  VS
.PRINT  DC  V(3)
.END
```

output:

$\frac{V(3)}{VS} = -1.00E+01$

RIN = 1.00E04

ROUT = 0.00E00

input resistance seen by V_s is 10 kΩ

Chapter 7

Exercises

Ex. 7-1 100 ms = 0.1 s, 10 mA = 0.01 A

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(t)\,dt \Rightarrow v(.1) = 0 + \frac{1}{10^{-5}}\int_0^{.1}(.01)\,dt$$

$$\underline{v(.1) = 100\text{ V}}$$

$$q(.1) = C\,v(.1)$$
$$= 10^{-5}(100) = \underline{10^{-3}\text{ C} = 1\text{ mC}}$$

Ex. 7-2 break up into intervals

$$v(t) = \begin{cases} 10t & 0<t<1 \\ 10 & 1<t<2 \\ 10(3-t) & 2<t<3 \\ 0 & t>3 \end{cases}$$

now $i = C\,dv/dt = 10^{-6}\frac{dv}{dt}$

$\therefore$ have

$$i(t) = \begin{cases} 10^{-6}(10) = 10^{-5}\text{ A} & 0<t<1 \\ 0 & 1<t<2 \\ 10^{-6}(-10) = -10^{-5}\text{ A} & 2<t<3 \\ 0 & t>3 \end{cases}$$

Ex. 7-3 break up into intervals

$$i = \begin{cases} 0 & t<1 \\ 1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

now $v(t) = v(0) + 1/C\int_0^t i\,dt$
$= \frac{1}{10^{-4}}\int_0^t i\,dt$

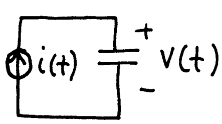
$\therefore$

$0<t<1 \Rightarrow v(t) = 10^4\int_0^t 0\,dt = \underline{0}$

$1<t<2 \Rightarrow v(t) = v(1) + 10^4\int_1^t 1\,dt = 0 + 10^4 t = \underline{10^4 t\text{ V}}$

$t>2 \Rightarrow v(t) = v(2) + 10^4\int_2^t 0\,dt = 10^4(2) = \underline{2\times10^4\text{ V}}$

Ex 7-4



because $i(t) = 0$ when $t<0$

$$v(t) = \frac{1}{C}\int_{-\infty}^t i(t)\,dt = \frac{1}{5}\int_0^t (2+2\cos 5t)\,dt + v(0)$$

$$= \underline{\tfrac{2}{5}t + \tfrac{2}{25}\sin 5t + \tfrac{1}{5}\text{ V}}$$

Ex. 7-5 $W = \frac{1}{2}Cv^2 = \frac{1}{2}(2\times10^{-4})(100)^2 = \underline{1\text{ J}}$

$v_c(0^+) = v_c(0^-) = \underline{100\text{ V}}$