Chapter 7

Exercises

Ex. 7-1 100ms = 0.1s , 10mA = 0.01 A

$$v(t) = v(0) + \frac{1}{C}\int_0^t i(t)\,dt \Rightarrow v(.1) = 0 + \frac{1}{10^{-5}}\int_0^{.1}(.01)\,dt$$

$$\underline{v(.1) = 100\text{v}}$$

$$q(.1) = C\,v(.1)$$
$$= 10^{-5}(100) = \underline{10^{-3}\,C = 1\text{mC}}$$

Ex. 7-2 break up into intervals

$$v(t) = \begin{cases} 10t & 0<t<1 \\ 10 & 1<t<2 \\ 10(3-t) & 2<t<3 \\ 0 & t>3 \end{cases}$$

now $i = C\,dv/dt = 10^{-6}\frac{dv}{dt}$

$\therefore$ have

$$\underline{i(t) = \begin{cases} 10^{-6}(10) = 10^{-5}\text{A} & 0<t<1 \\ 0 & 1<t<2 \\ 10^{-6}(-10) = -10^{-5}\text{A} & 2<t<3 \\ 0 & t>3 \end{cases}}$$

Ex. 7-3 break up into intervals

$$i = \begin{cases} 0 & t<1 \\ 1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

now $v(t) = v(0) + 1/C\int_0^t i\,dt$

$$= \frac{1}{10^{-4}}\int_0^t i\,dt$$

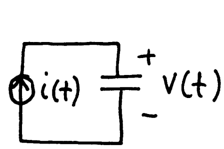
$\therefore$

$0<t<1 \Rightarrow v(t) = 10^4\int_0^t 0\,dt = \underline{0}$

$1<t<2 \Rightarrow v(t) = v(1) + 10^4\int_1^t 1\,dt = 0 + 10^4 t = \underline{10^4 t\ \text{v}}$

$t>2 \Rightarrow v(t) = v(2) + 10^4\int_2^t 0\,dt = 10^4(2) = \underline{2\times 10^4\ \text{v}}$

Ex 7-4



because $i(t) = 0$ when $t<0$

$$v(t) = \frac{1}{C}\int_{-\infty}^t i(t)\,dt = \frac{1}{5}\int_0^t (2+2\cos 5t)\,dt + v(0)$$

$$\underline{= \frac{2}{5}t + \frac{2}{25}\sin 5t + \frac{1}{5}\ \text{v}}$$

Ex. 7-5 $W = 1/2\,C\,V^2 = 1/2\,(2\times 10^{-4})(100)^2 = \underline{1\text{ J}}$

$V_c(0^+) = V_c(0^-) = \underline{100\text{v}}$

Ex. 7-6 $W(t) = W(0) + \int_0^t vi\,dt \Rightarrow W(0) = 0$ since $V(0) = 0$

first find $V(t) = V(0) + 1/C \int_0^t i\,dt$

$$= 10^4 \int_0^t 2\,dt = \underline{2\times10^4\, t}$$

$$\therefore\ w(t) = \int_0^t (2\times10^4 t)(2)\,dt = 2\times10^4\, t^2$$

(a) $W(1s) = 2\times10^4 \text{ J} = \underline{20 \text{ kJ}}$

(b) $W(100s) = 2\times10^4 (100)^2 = 2\times10^8 \text{ J} = \underline{200 \text{ mJ}}$

Ex 7-7 We have $v(0^+) = v(0^-) = 3v$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 5\int_0^t 3e^{5t}\,dt + 3v$$

$$= 3(e^{5t}-1) + 3 = 3e^{5t} \text{ v}$$

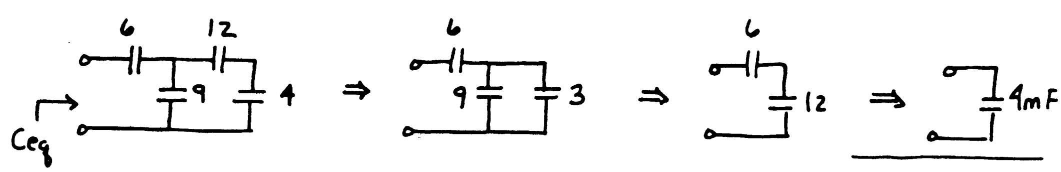
now $v(t) = V_R(t) + V_c(t) = 5i(t) + V_c(t) = 15e^{5t} + 3e^{5t} = \underline{18e^{5t} \text{ V}}$ $0<t<1$

$W_c(t) = \frac{1}{2} C V_c^2(t) = \frac{1}{2}\times.2\,(3e^{5t})^2 = 0.9e^{10t}$ J

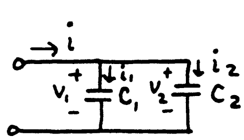
now when $t = 0.2$ s $\Rightarrow W_c(t)\big|_{t=.2s} = \underline{6.65 \text{ J}}$

when $t = .8$s $\Rightarrow W_c(t)\big|_{t=.8s} = \underline{2.68 \text{ kJ}}$

Ex. 7-8



Ex. 7-9



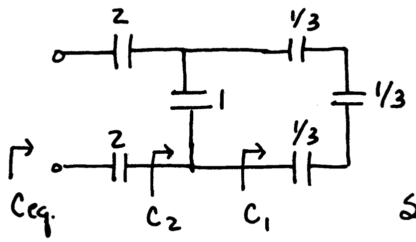
$V_1 = V_2 \Rightarrow \frac{dv_1}{dt} = \frac{dv_2}{dt} \Rightarrow \frac{i_1}{C_1} = \frac{i_2}{C_2}$

$\Rightarrow \underline{i_1 = \frac{C_1}{C_2} i_2}$

from KCL: $i = i_1 + i_2 = \left(\frac{C_1}{C_2} + 1\right) i_2$

$\Rightarrow \underline{i_2 = \frac{C_2}{C_1 + C_2}\, i}$

Ex 7-10



$$\frac{1}{C_1} = \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} + \frac{1}{\frac{1}{3}} \Rightarrow C_1 = 1/9\,\text{mF}$$

$$C_2 = 1 + C_1 = 1 + 1/9 = 10/9\ \text{mF}$$

So $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{10/9}$, $1/C_{eq} = 19/10$

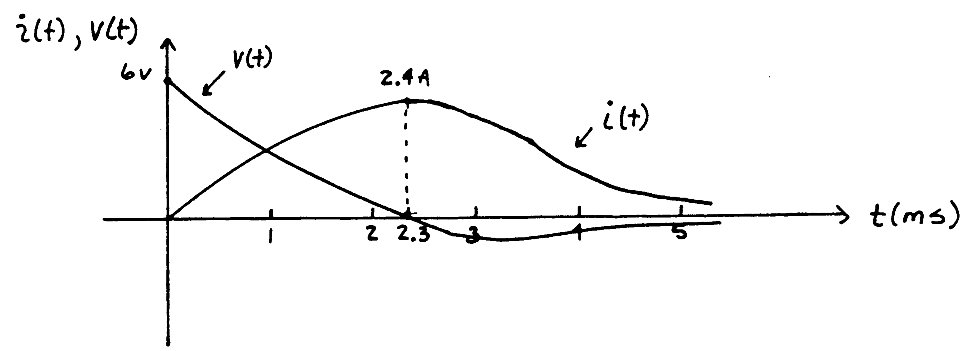
then $C_{eq} = \frac{10}{19}$ mF

Ex. 7-11 $\quad v = L\,di/dt = (.05)\frac{d}{dt}\left[20(1-e^{-100t})\right]$

$= (.05)(20)(100e^{-100t}) = \underline{100e^{-100t}\ \text{V}}$

Ex. 7-12 $\quad v = L\,di/dt = (.002)\left[-1000e^{-200t} + 4000e^{-800t}\right]$

$\underline{= -2e^{-200t} + 8e^{-800t}\ \text{V}}$



Ex. 7-13 $\quad i(t) = i(0) + \frac{1}{L}\int_0^t v(t) = 0 + \frac{1}{1}\int_0^t 9t^2\,dt = \underline{3t^3}$

Ex. 7-14 $\quad v = L\,di/dt = (1/4)\,d/dt(4te^{-t}) = \underline{(1-t)e^{-t}}$

$p = vi = [(1-t)e^{-t}](4te^{-t}) = \underline{4t(1-t)e^{-2t}}$

$W = 1/2\,Li^2 = 1/2(1/4)(4te^{-t})^2 = \underline{2t^2e^{-2t}}$

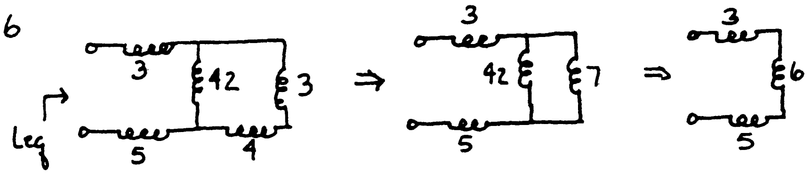
Ex. 7-15 $v(t) = L\,di/dt = \tfrac{1}{2}\,di/dt$

$$i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ -2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases} \qquad \therefore\ v(t) = \begin{cases} 0 & t<0 \\ 1 & 0<t<1 \\ -1 & 1<t<2 \\ 0 & t>2 \end{cases}$$

now $$p(t) = v(t)\,i(t) = \begin{cases} 0 & t<0 \\ 2t & 0<t<1 \\ 2(t-2) & 1<t<2 \\ 0 & t>2 \end{cases}$$

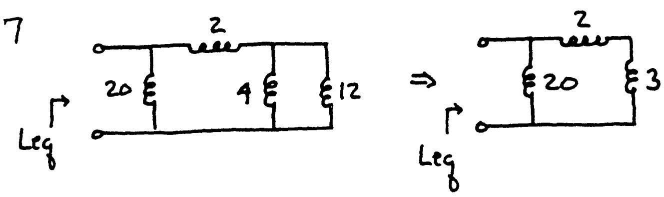
$$w(t) = w(t_0) + \int_{t_0}^{t} p(t)\,dt \quad \Rightarrow \quad \begin{aligned} &t<0:\ w(t) = 0 \text{ since } p(t)=0 \\ &0<t<1:\ w(t) = \int_0^t 4t\,dt = 2t^2 \\ &1<t<2:\ w(t) = w(1) + \int_1^t 4(t-2)\,dt \\ &\qquad = 2t^2 - 8t + 8 \\ &t>2:\ w(t) = w(2) = 0 \end{aligned}$$

Ex. 7-16



$L_{eq} = 3+6+5$
$= 14\text{mH}$

Ex. 7-17



$L_{eq} = \dfrac{(20)(5)}{20+5} = 4\text{mH}$

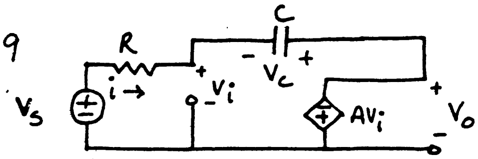
Ex 7-18 $i = i_1 + i_2$

$i_1 = \frac{1}{L_1}\int v\,dt + \cancelto{0}{i_1(0)}$, $\quad i_2 = \frac{1}{L_2}\int v\,dt + \cancelto{0}{i_2(0)}$

so $i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt$, now also $i = \frac{1}{L_p}\int v\,dt + \cancelto{0}{i(0)}$

$$\therefore\ \frac{i_1}{i} = \frac{1/L_1 \int v\,dt}{1/L_p \int v\,dt} = \frac{1/L_1}{1/L_1 + 1/L_2} = \frac{L_2}{L_1+L_2}$$

Ex. 7-19



KVL ↻ left loop : $V_i = V_s - Ri$

also : $i = -C\, dV_c/dt$

$\therefore\ V_i = V_s + RC\, dV_c/dt$

but $V_o = -AV_i = -AV_s - ARC\, dV_c/dt$ (1)

also from KVL ↻ : $V_o = V_i + V_c = -\frac{V_o}{A} + V_c$ (2)

solving for V_c in (2) and plugging into (1) yields

$$\underline{RC(1 + 1/A)\frac{dV_o}{dt} + \frac{V_o}{A} = -V_s}$$

for $A \to \infty$: $(1 + 1/A) = 1$ and $\frac{V_o}{A} = 0$

$\therefore$ left with $RC\frac{dV_o}{dt} = -V_s$ or $\frac{dV_o}{dt} = -\frac{V_s}{RC}$

$\Rightarrow V_o(t) - V_o(0) = -1/RC \int_0^t V_s\, dt$

also for $A \to \infty$, $V_i = 0 \Rightarrow V_o = V_c$

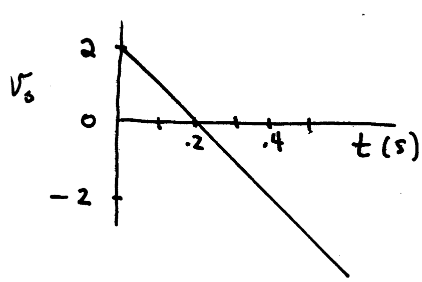
$$\underline{\therefore\ v_o(t) = -\frac{1}{RC}\int_0^t V_s\, dt + V_c(0)}$$

Ex 7-20

$$v_o = -\frac{1}{RC}\int_0^t v_s\, d\tau + v_c(0)$$

$$= -2\int_0^t 5\, d\tau + 2$$

$$= -10t + 2 \quad V \qquad t \geq 0$$

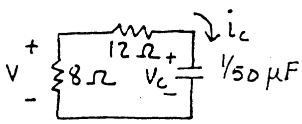


Problems

P. 7-1

$$v(t) = v(0) + 1/C\int_0^t i\,dt = 25 + 2.5\times10^4\int_0^t (6\times10^{-3})e^{-6t}dt$$
$$= 25 + 150\int_0^t e^{-6t}dt$$
$$= 25 + 150\left[-1/6\,e^{-6t}\right|_0^t$$
$$v(t) = \underline{100 - 25e^{-6t}\text{ V}}$$

P. 7-2 at $t = 0^+$ $\quad v_C(0^+) = v_C(0^-) = 40\text{V}$

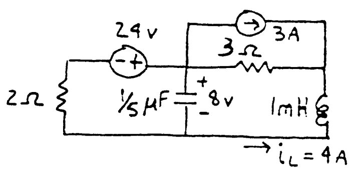


from voltage divider $v(0^+) = v_C(0^+)\left[\frac{8}{8+12}\right]$

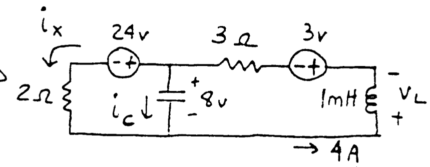
$\underline{v(0^+) = 16\text{ V}}$

now $\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = 50\left(-\frac{16\text{V}}{8\Omega}\right) = \underline{-100\,\mu\text{V/s}}$

P. 7-3 at $t = 0^+ \Rightarrow v_C(0^+) = v_C(0^-) = 8\text{V}$ and $i_L(0^+) = i_L(0^-) = 4\text{A}$



$\Rightarrow$

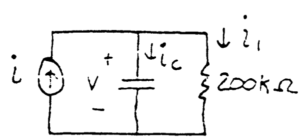


KVL: $v_L + 3 + 3(4) + 8 = 0 \qquad v_L(0^+) = -23\text{V} \Rightarrow \underline{\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = -23\frac{\text{mA}}{\mu\text{s}}}$

KVL: $2i_x = 8 - 24 \Rightarrow i_x = -8\text{A}$

$\therefore i_C = 4 - i_x = 12\text{A} \Rightarrow \underline{\frac{dv_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = 60\text{ V}/\mu\text{s}}$

P. 7-4



$i_1 = v/200 = 1/40(1 - 2e^{-2t})\text{mA} = 25(1 - 2e^{-2t})\ \mu\text{A}$

$i_C = C\,dv/dt = (10)(-2)(-10e^{-2t}) = 200e^{-2t}\ \mu\text{A}$

$\therefore i = i_C + i_1 = 200e^{-2t} + 25 - 50e^{-2t} = \underline{25 + 150e^{-2t}\ \mu\text{A}}$

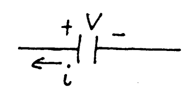
P. 7-5 $v = v(0) + 1/C\int i\,dt$ and $q = Cv$

$\therefore Cv = Cv(0) + \int i\,dt = Cv(0) + it \quad \leftarrow$ since i = constant

$\therefore t = \frac{q - Cv(0)}{i} = \frac{150\mu C - (15\mu F)(5V)}{25\text{mA}}$

$\underline{t = 3\text{ms}}$

P. 7-6 (a) when discharging $\Rightarrow$



$\therefore\ i = -C\,dv/dt$

$\therefore\ v(t) = v(t_0) - \frac{1}{C}\int_{t_0}^{t} i\,dt$

t = 6ms $\quad v(6) = 10 - \frac{1}{(.1)}\int_0^6 (.12)\,dt$

$= \underline{2.8\text{ v}}$

t = 10ms $\quad v(10) = v(6) - \frac{1}{(.1)}\int_6^{10} (.3 - .03t)\,dt$

$= 2.8 - 10[(.3)(10-6) - .015(100-36)]$

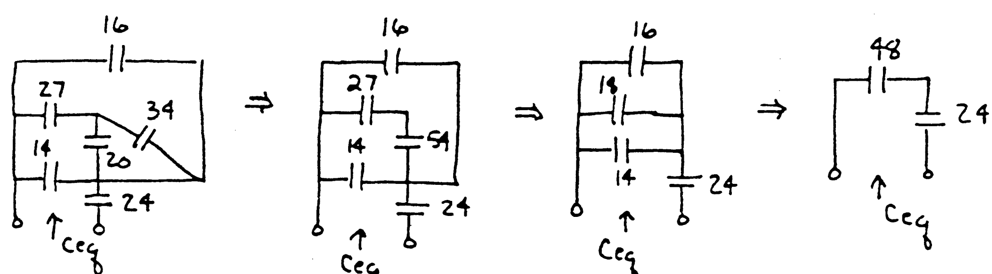
$= \underline{.4\text{ v}}$

(b) $\Delta W = W(0) - W(10\text{ms})$

$= \frac{1}{2} C\,v(0)^2 - \frac{1}{2} C\,v(10\text{s})^2$

$= \frac{1}{2}(.1)[(10)^2 - (.4)^2] = \underline{4.992\text{ mJ}}$

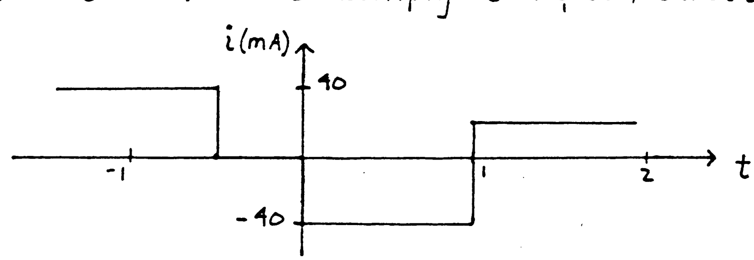
P. 7-7

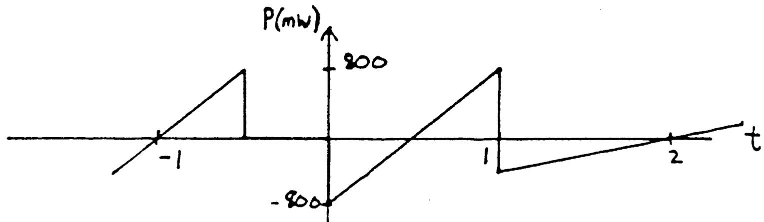


$\frac{1}{C_{eq}} = \frac{1}{48} + \frac{1}{24} \Rightarrow \underline{C_{eq} = 16\text{ mF}}$

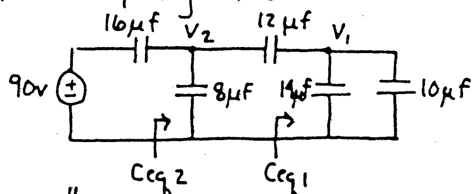
P. 7-8 $i(t) = C\,dv_c/dt$ so read off slope of $v_c(t)$ to get $i(t)$

$p(t) = v_c(t)\,i(t) \Rightarrow$ so multiply $v_c(t)$ & $i(t)$ curves to get $p(t)$



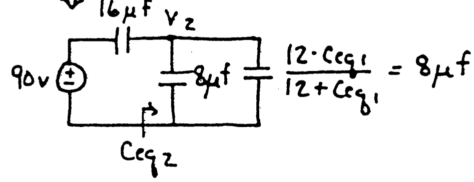


P. 7-9 simplify circuit



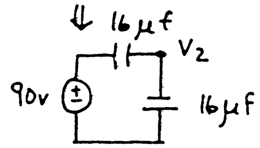
$$C_{eq_1} = 14 + 10 = 24\mu f$$

⇓



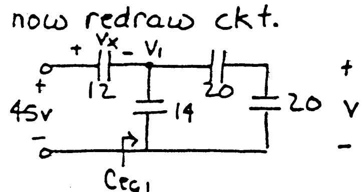
$$C_{eq_2} = 8 + 8 = 16\,\mu f$$

⇓



by inspection $\underline{V_2 = 45\,v}$

now redraw ckt.



now $q = CV$ and since charge remains the same on each plate, $C_1 V_1 = C_2 V_2$

or $\Rightarrow$ $\underline{12 V_x = C_{eq_1} V_1 = 24 V_1}$ (1)

from KVL ↻ : $\underline{45 = V_x + V_1}$ (2)

solving (1) & (2) yields $\underline{V_1 = 15\,v}$

by inspection $\underline{V = \frac{1}{2} V_1 = 7.5\,v}$

P. 7-10

$$V_c(t) = V_c(0) + \frac{1}{C}\int_0^t i\,dt$$

$$= V_c(0) + \frac{1}{2}\int_0^t 50\cos(10t + \pi/6)\,dt$$

$$= [V_c(0) - \frac{5}{2}\sin\pi/6] + \frac{5}{2}\sin(10t + \pi/6)$$

now since $V_c(t)_{ave.} = 0 \Rightarrow V_c(0) = \frac{5}{2}\sin\pi/6$

$\therefore\; V_c(t) = \frac{5}{2}\sin(10t + \pi/6)\ v$

$\therefore\; W_{max} = \frac{1}{2} C V_{c\,max}^2 = \frac{1}{2}(2)(2.5)^2 = \underline{6.25\mu J}$

first non-negative t for max energy occurs:

$$10t + \pi/6 = \pi/2$$

$$\Rightarrow \underline{t = \pi/30 = 0.1047\ s}$$

P. 7-11 $W = \frac{1}{2} C V^2 \Rightarrow V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(4e^{-10t})}{.001}} = 40\sqrt{5}\, e^{-5t}$

$\therefore \underline{V(.1) = 40\sqrt{5}\, e^{-5(.1)} = 54.25\,v}$

$i(t) = C\, dv_c/dt = (10^{-3})(-5)(40\sqrt{5}\, e^{-5t}) = 1/\sqrt{5}\, e^{-5t}$

$\therefore \underline{i(.1) = 1/\sqrt{5}\, e^{-5(.1)} = 0.2712\ A}$

P. 7-12 find max. voltage across coil

$v(t) = L\, di/dt = 200[100(400)\cos 400t]\ v$

$\therefore V_{max} = 8\times10^6\, v$

thus have a field of $8\times10^6\, v/2m = 4\times10^6\ V/m$ which exceeds dielectric strength in air of $3\times10^6\ V/m$ $\therefore$ will get a discharge as the air is ionized.

P. 7-13 $i(t) = 2\times10^{-3}\sin^2 t\ A$

$v(t) = L\, di/dt = (.01)(4\times10^{-3})\sin t \cos t = 4\times10^{-5}\sin t\cos t\ v$

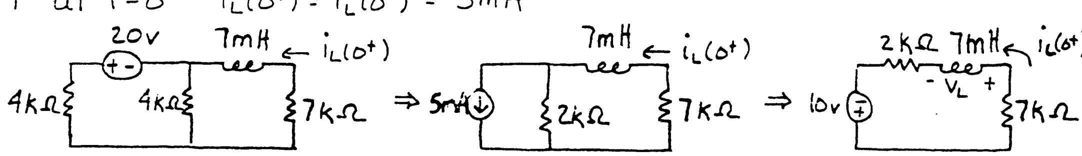
$\therefore p(t) = vi = (4\times10^{-5}\sin t\cos t)(2\times10^{-3}\sin^2 t)$

$= \underline{8\times10^{-8}\sin^3 t\cos t\ W}$

$W(t) = \frac{1}{2} L i^2(t) \Rightarrow W(3\pi/4) = \frac{1}{2}(.01)(4\times10^{-6}\sin^4(3\pi/4))$

$= \underline{5\times10^{-9}\ J = 5\,nJ}$

P. 7-14 at $t=0^+$ $i_L(0^+) = i_L(0^-) = 5\,mA$



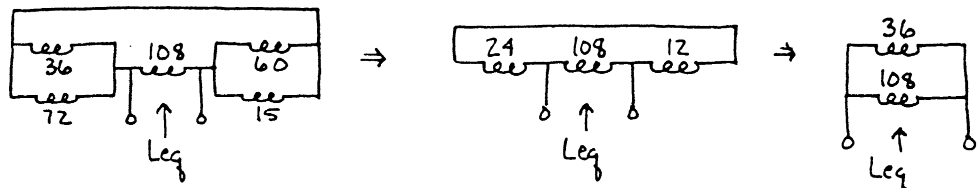
KVL: $7 i_L(0^+) + V_L(0^+) + 2 i_L(0^+) - 10 = 0$

$\Rightarrow V_L(0^+) = 10 - 9(5) = -35$ volts

now $V_L(t) = L\, di/dt$

$\therefore \underline{\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{7} = -\frac{35}{7} = -5\ A/ms}$

P. 7-15



$1/L_{eq} = 1/36 + 1/108 \Rightarrow \underline{L_{eq} = 27 \mu H}$

P. 7-16 $V_L(t) = L\,di/dt$

$= (.01)[(-5t+5)e^{-t}] = (.05)(1-t)e^{-t}$

$\therefore p(t) = V_L i = .25t(1-t)e^{-2t}$

for max $p(t)$ take $dP/dt = 0 \Rightarrow 2t^2 - 4t + 1 = 0$

$\Rightarrow t = .293, 1.707$

$P(.293) = .0288\,W$, $P(1.707) = -.0099\,W$

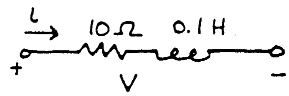
$\therefore \underline{t_{max} = .293 s \quad \text{since } P > 0}$

$w(t) = \int_0^t p(t)dt = \int_0^t .25t(1-t)e^{-2t}dt = 1/8\,t^2 e^{-2t}$

for $w(t)$ max, take $dw/dt = 0 \Rightarrow -2t^2 + 2t = 0$

$\Rightarrow \underline{t_{max} = 1s}$

P. 7-17



$V = L\,di/dt + 10i$

$= (.1)(4e^{-t} - 4te^{-t}) + 10(4te^{-t})$

$\underline{= 0.4e^{-t} + 39.6te^{-t} \text{ volts}}$

P. 7-18 $V_L = L\,di/dt$

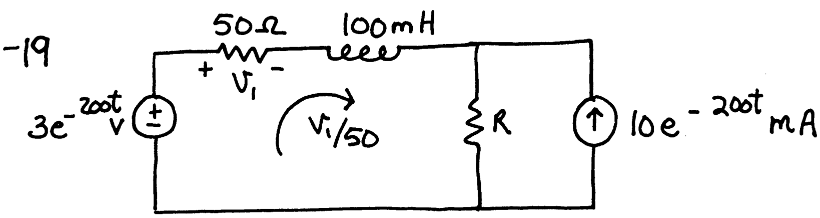
<u>at t = 1ms</u> : $di/dt = \frac{\Delta i}{\Delta t} = \frac{4-2}{2-0} = 1\,A/s$

$\therefore V_L(1ms) = (.02)(1) = \underline{0.02 \text{ volts}}$

<u>t = 6ms</u> : $di/dt = \Delta i/\Delta t = \frac{0-(-4)}{6-4} = 2\,A/s$

$\therefore V_L(6ms) = (.02)(2) = \underline{0.04 \text{ volts}}$

P. 7-19



$V_1(t) = e^{-200t}$ V

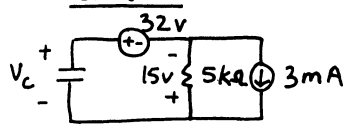
$$\text{KVL} \curvearrowright: \quad -3e^{-200t} + V_1 + .1\frac{d}{dt}\left(\frac{V_1}{50}\right) + R\left[\frac{V_1}{50} + .01e^{-200t}\right] = 0$$

$$-3e^{-200t} + e^{-200t} - .4e^{-200t} + R\left[\tfrac{1}{50} + .01\right]e^{-200t} = 0$$

$$-2.4 + R(.03) = 0$$

$$\Rightarrow \underline{R = \frac{2.4}{.03} = 80\,\Omega}$$

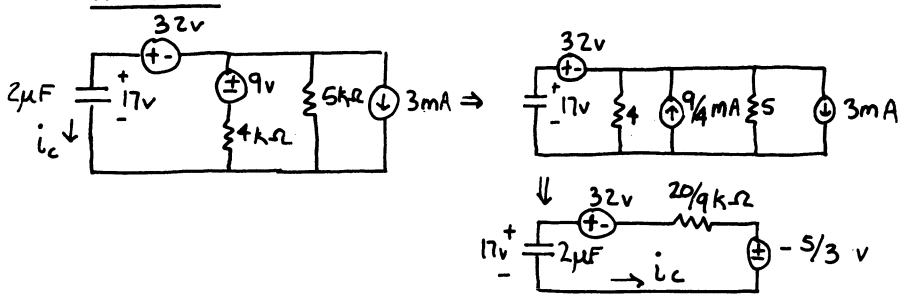
P. 7-20 at $t=0^-$



$\text{KVL} \curvearrowright: \; -V_c(0^-) + 32 - 15 = 0$

$\Rightarrow \underline{V_c(0^-) = V_c(0^+) = 17\text{v}}$

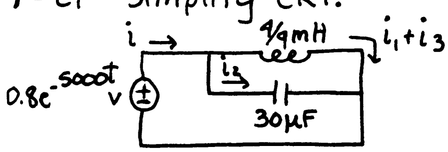
at $t=0^+$



$$\text{KVL}: \quad 5/3 + \tfrac{20}{9}\, i_c - 32 + 17 = 0 \quad \Rightarrow \underline{i_c(0^+) = 6\text{mA}}$$

$$\text{now} \quad i_c(0^+) = C\frac{dV_c(0^+)}{dt} \Rightarrow \frac{dV_c(0^+)}{dt} = \frac{6\text{mA}}{2\mu\text{F}} = \underline{3\text{ V/ms}}$$

P. 7-21 simplify ckt.



$$\text{let } i_4 = i_1 + i_3 = i_1(0) + i_3(0) + \tfrac{9}{4}\times 10^3 \int_0^t .8e^{-5000t}\,dt$$

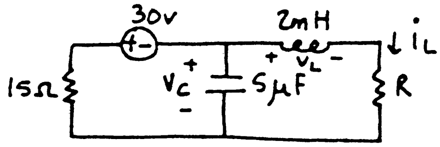
$$= .7 + 1.8\times 10^{-3}\int_0^t e^{-5000t}\,dt$$

$$= 1.06 - 0.36e^{-5000t} \text{ A}$$

$$\text{also } i_2 = C\,dv/dt = (30\times 10^{-6})(4000)e^{-5000t} = -.12e^{-5000t} \text{ A}$$

$$\therefore \underline{i(t) = i_2(t) + i_4(t) = 1.06 - .48e^{-5000t} \text{ A}}$$

P. 7-22

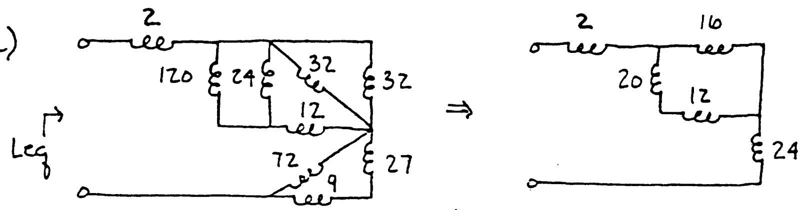


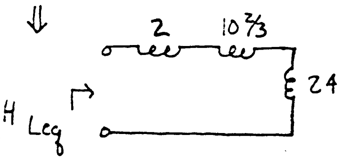
for energies to be the same $\Rightarrow \frac{1}{2}Li_L^2 = \frac{1}{2}CV_c^2$

now in steady state $V_L = 0 \Rightarrow V_c = i_L R$

$\therefore \quad Li_L^2 = C(i_L R)^2 \Rightarrow R = \sqrt{L/C} = \sqrt{\dfrac{2\times10^{-3}}{5\times10^{-6}}} = 20\,\Omega$

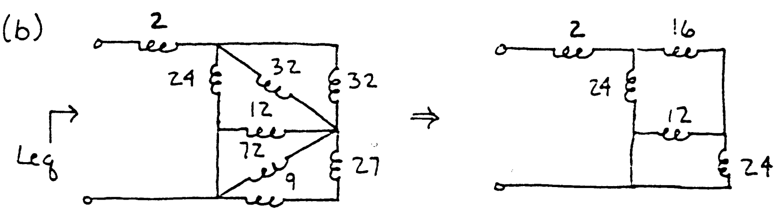
P. 7-23 (a)

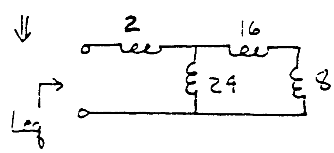




$L_{eq} = 2 + 10\tfrac{2}{3} + 24 = 36\tfrac{2}{3}\,\text{mH}$

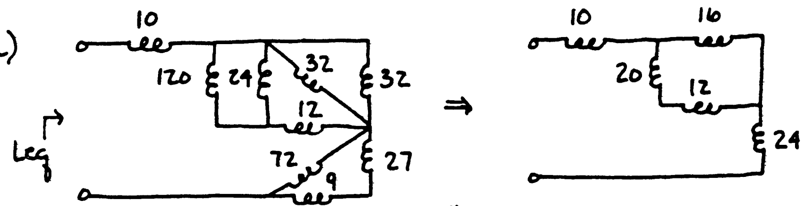
(b)

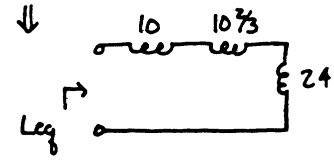




$L_{eq} = 2 + \dfrac{(24)(16+8)}{24+16+8} = 14\,\text{mH}$

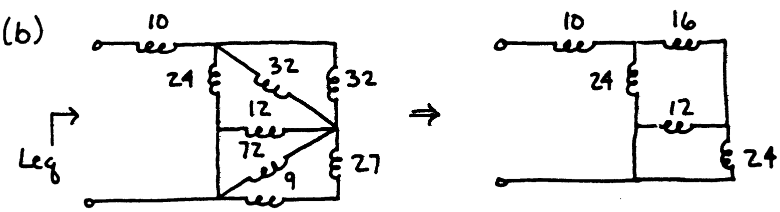
P. 7-24 (a)

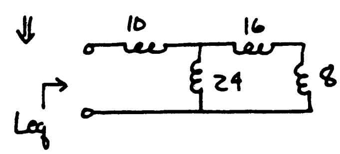




$\underline{L_{eq} = 10 + 10\tfrac{2}{3} + 24 = 44\tfrac{2}{3}\text{ H}}$

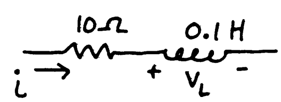
(b)





$$\underline{L_{eq} = 10 + \frac{(24)(16+8)}{24+16+8} = 22\text{ H}}$$

P. 7-25

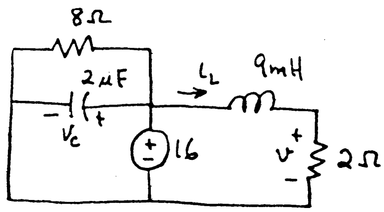


$$P_{10\Omega} = i^2R = (5\times10^{-3}e^{-4t})^2(10) = 2.5\times10^{-4}e^{-8t}\text{ W}$$

$$P_{0.1H} = V_L i = L\,i\,di/dt$$
$$= (.1)(5\times10^{-3}e^{-4t})(-2\times10^{-3}e^{-4t}) = -10^{-5}e^{-8t}\text{ W}$$

$$\therefore\ \underline{P_{Total} = P_{10\Omega} + P_{0.1H} = 2.4\times10^{-4}e^{-8t}\text{ W} = .24e^{-8t}\text{ mW}}$$

P 7-26 at $t = 0^-$

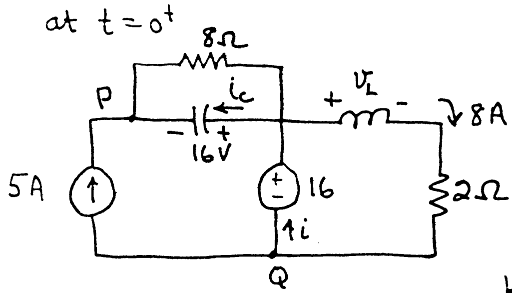


at steady-state

$$v_c(0^-) = v_c(0^+) = 16 \text{ V}$$

and $$i_L(0^-) = i_L(0^+) = \frac{16}{2} = 8 \text{ A}$$

at $t = 0^+$



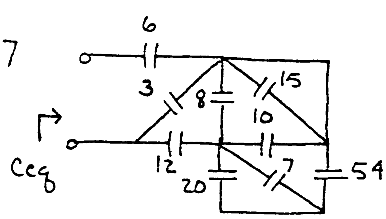
KVL: $-16 + v_L + 8(2) = 0 \Rightarrow v_L(0^+) = 0$

KCL at P: $-5 - i_c - \frac{16}{8} = 0 \Rightarrow i_c(0^+) = -7\text{A}$

KCL at Q: $5 + i - 8 = 0 \Rightarrow i(0^+) = 3 \text{ A}$

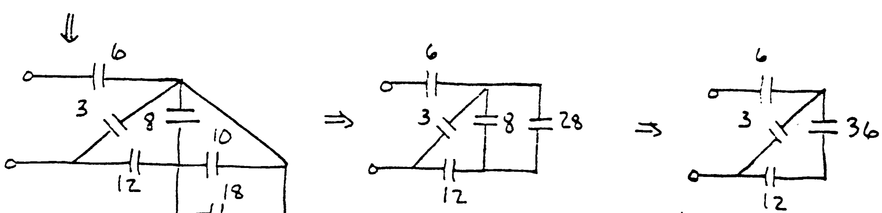
Then $\dfrac{di_L(0^+)}{dt} = \dfrac{v_L(0^+)}{L} = 0$ and $\dfrac{dv_c(0^+)}{dt} = \dfrac{i_c(0^+)}{C} = \dfrac{-7}{2\mu F} = -3.5 \dfrac{A}{\mu s}$

P. 7-27

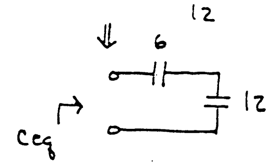


note that a short ckt is across the 15F capacitor. since $V_{15F} = 0 \Rightarrow q_{15} = 0$

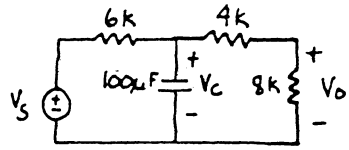
∴ eliminate the 15μF capacitor



$$C_{eq} = \frac{6(12)}{6+12} = 4\,\mu F$$



P. 7-28



from voltage divider $V_o = 2/3\, V_c$

$\underline{t<0}$: $V_c = 0 \quad \therefore \underline{V_o(t) = 0}$

$\underline{0<t<.3}$: $V_c(t) = V_c(\infty) + A e^{-t/RC}$

$\underline{\text{find } R}$: kill source $\Rightarrow R = 6 // (4+8) = 4k\Omega$

$\underline{\text{find } V_c(\infty)}$: at $t=\infty$ capacitor = open ckt.

from voltage divider

$V_c(\infty) = V_s\left(\frac{12}{12+6}\right) = 9\left(\frac{12}{18}\right) = 6\,v$

$\therefore V_c(t) = 6 + A e^{-2.5t}$

$V_c(0) = 0 = 6 + A \Rightarrow A = -6 \Rightarrow V_c(t) = 6 - 6e^{-2.5t}$

$\therefore \underline{V_o(t) = 4 - 4e^{-2.5t}\ v}$

$\underline{t>.3}$ $V_c(.3) = 6 - 6e^{-2.5(.3)} = 3.166\,v$

$\therefore V_c(t) = V_c(.3)\, e^{-2.5(t-.3)}$

$= 3.166\, e^{-2.5(t-.3)}$

$\therefore \underline{V_o(t) = 2.11 e^{-2.5(t-.3)}\ v}$

P. 7-29 Total energy extracted $= P\Delta t = (3\times10^4 W)(3600s) = 1.08\times10^8 J$

Initially we have $W = 1/2\, CV^2$

$\Rightarrow C = \frac{2W}{V^2} = \frac{2(1.08\times10^8)}{100^2} = 21600\,F$

size of capacitor $= 21600\,F\left(\frac{10^6\mu F}{F}\right)\left(\frac{1cm^3}{10\mu F}\right) = 2.16\times10^9 cm^3$

$= \underline{2160\ m^3}$

much too big!

P. 7-30 max. charge on capacitor $= CV$

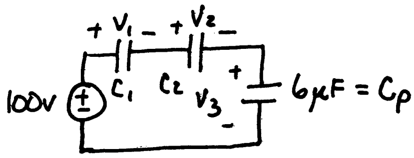
$= (10^{-5}F)(6v) = 6\times10^{-5}\,C$

$\therefore \Delta t = \frac{\Delta q}{i} = \frac{6\times10^{-5}C}{10^{-5}A} = \underline{6\ sec}$

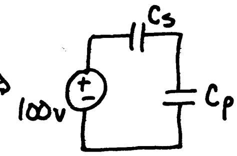
stored energy $= W = 1/2\, CV^2$

$= 1/2(10\mu F)(6v)^2 = \underline{180\mu J}$

P 7-31



$\Rightarrow$



$C_1 = 6\mu F$, $C_2 = 12\mu F$

$$C_s = \frac{C_1C_2}{C_1+C_2} = \frac{6(12)}{6+12} = 4\mu F$$

$$C_{eq} = \frac{C_sC_p}{C_s+C_p} = \frac{4(6)}{4+6} = 2.4\,\mu F$$

$$\therefore\ q_T = C_{eq}(100v) = 2.4\times10^{-6}(100) = 240\mu C$$

so have $240\mu C$ on $6\mu F$ and $12\mu F$ capacitors

$$\Rightarrow V_1 = q_T/C_1 = 240\mu C/6\mu F = \underline{40v}$$

$$V_2 = 240\mu C/12\mu F = \underline{20v}$$

$$V_3 = 240\mu C/6\mu F = \underline{40v}$$

P 7-33

$$v = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \cdots \frac{1}{C_N}\int i\,dt$$

$$\text{so } \frac{v_N}{v} = \frac{1/C_N}{\sum_{n=1}^{N} 1/C_N}$$

$$\text{for } N = 3 \Rightarrow \frac{v_N}{v} = \frac{1/C}{1/C+1/C+1/C} = \frac{1/C}{3/C} = \underline{\frac{1}{3}}$$

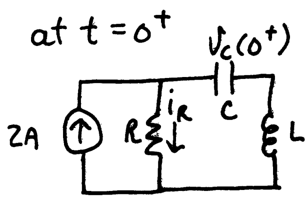
P 7-34

$$\text{for } t>0 \Rightarrow i_c = C\frac{dv_c}{dt} = i \qquad \text{so } \left.\frac{dv_c}{dt}\right|_{t=0^+} = i(0^+)$$

$$\text{KVL: } V_c + L\frac{di}{dt} - Ri_R = 0$$

$$\Rightarrow V_c(0^+) + L\left.\frac{di}{dt}\right|_{t=0^+} - Ri_R(0^+) = 0$$

at $t=0^+$

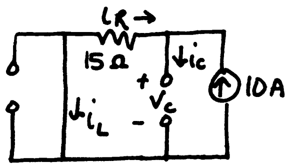


$$\left.\frac{di}{dt}\right|_{0^+} = \frac{R}{L}i_R(0^+) - \frac{V_c(0^+)}{L}$$

$\Rightarrow i_R(0^+) = 2A$ since i cannot change from 0 at $t=0$

$$\text{then } \left.\frac{di}{dt}\right|_{0^+} = \frac{2R}{L} - \frac{(-1)}{L} = \underline{\frac{2R+1}{L}}$$

P 7-35 $\underline{t=0^-}$

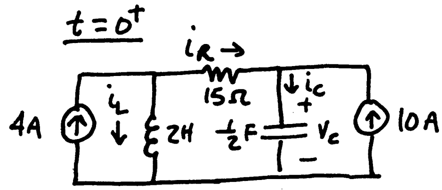


$V_c(0^-) = (15)10 = \underline{150v}$ $\qquad i_c(0^-) = \underline{0}$

$V_L(0^-) = \underline{0v}$ $\qquad i_L(0^-) = \underline{10A}$

$V_R(0^-) = 15(-10) = -\underline{150v}$ $\qquad i_R(0^-) = \underline{-10A}$

$\underline{t=0^+}$

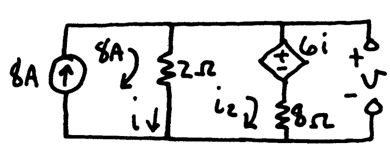


$V_c(0^+) = V_c(0^-) = \underline{150v}$ $\qquad i_L(0^+) = i_L(0^-) = \underline{10A}$

$V_R(0^+) = -6(15) = -\underline{90v}$ $\qquad i_R(0^+) = 4A - 10A = \underline{-6A}$

$V_L(0^+) = V_R(0^+) + V_c(0^+) = \underline{60v}$ $\qquad i_c(0^+) = i_R(0^+) + 10A$
$= \underline{4A}$

P 7-36 $\underline{t=0^-}$



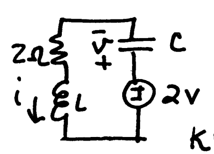
KVL: $-2i + 6i + 8i_2 = 0$ (1)

also: $i_2 = 8 - i$ (2)

Solving (1) & (2) $\Rightarrow$ $i(0^-) = i = 16A$
$i_2(0^-) = i_2 = -8A$

now $v(0^-) = v = 8i_2 + 6i$
$= 8(-8) + 6(16) = \underline{8v}$

$\underline{t=0^+}$ 8A source can be removed from rest of circuit



$i(0^+) = i(0^-) = 16A$

$v(0^+) = v(0^-) = 80v$

KVL: $-2 + v + 2i + L\frac{di}{dt} = 0$

$\therefore \frac{di(0^+)}{dt} = \frac{1}{1/2}\left[2 - v(0^+) - 2i(0^+)\right] = 2(2 - 80 - 2(16)) = \underline{-220\text{ A/s}}$

$i(0^+) = C\frac{dv(0^+)}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{i(0^+)}{C} = \frac{16}{1/5} = \underline{80\text{ V/s}}$

P 7-37 $v(t) = v(0) + \frac{1}{C}\int_0^t i(t)dt = 0 + \frac{1}{.5}\int_0^t i(t)dt = 2\int_0^t 10t\,dt$

$= \underline{10t^2 \qquad 0<t\le 1}$

$v(1) = 10$

$v(t) = 10 + 2\int_1^t 10\,dt = \underline{20t - 10 \qquad 1<t<2}$

$v(2) = 30$

$v(t) = 30 + 2\int_2^t (42 - 6t)\,dt = \underline{84t - 6t^2 - 74 \qquad 2<t<3}$

$v(3) = 34$; $v(t) = 34 + 2\int_3^t (-6)\,dt = \underline{70 - 12t \qquad 3<t<4}$

$v(4) = 22$; $v(t) = 22 + 2\int_4^t (-30 + 6t)\,dt = \underline{6t^2 - 60t + 166 \qquad 4<t<5}$

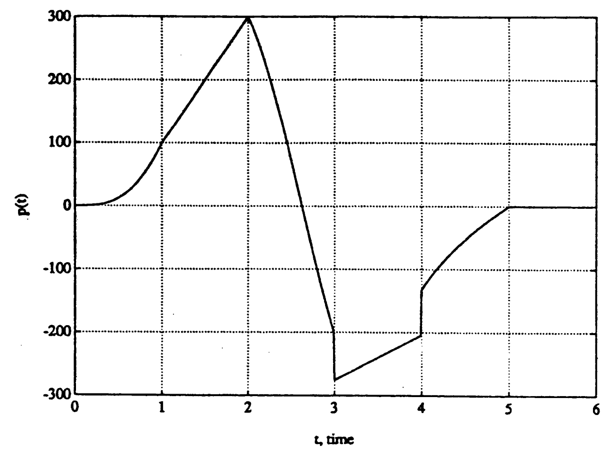
$v(5) = 16$; $v(t) = 16 + 2\int_5^t (0)\,dt = \underline{16 \qquad 5<t<6}$

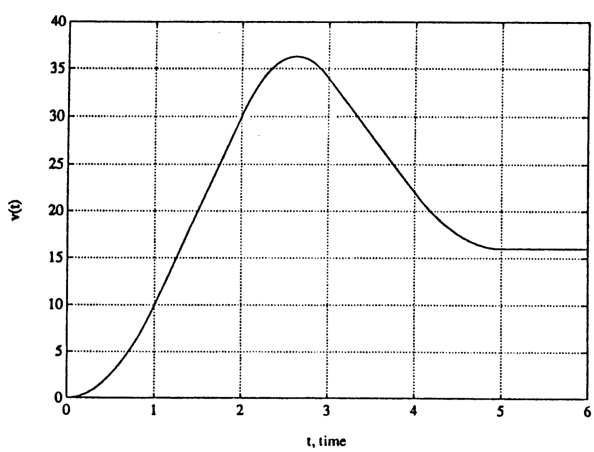
$$p(t) = i(t)v(t) = \begin{cases} 100t^3 & 0<t<1 \\ 200t - 100 & 1<t<2 \\ 256t^3 - 2016t^2 + 4712t - 3108 & 2<t<3 \\ 72t - 420 & 3<t<4 \\ 36t^3 - 540t^2 + 2796t - 4980 & 4<t<5 \\ 0 & 5<t<6 \end{cases}$$

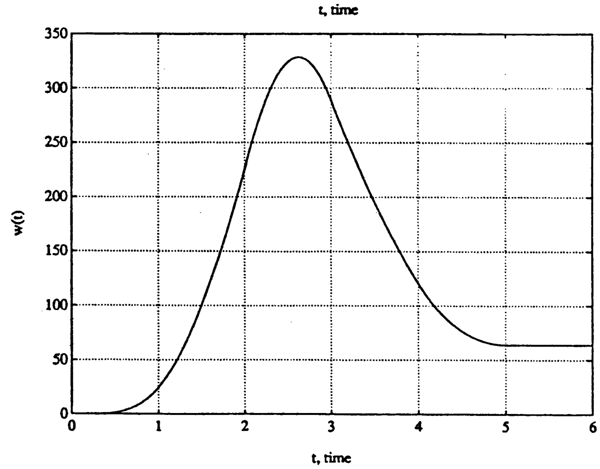
P7-37 (Continued)

$$W_c = \tfrac{1}{2} C v^2 = \tfrac{1}{2}(.5) v^2 = \tfrac{1}{4} v^2$$

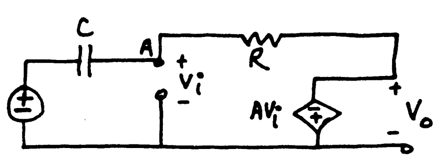
$$= \begin{cases} 25t^4 & 0<t<1 \\ 100t^2 - 100t + 25 & 1<t<2 \\ 64t^4 - 672t^3 + 2356t^2 - 3108t + 1369 & 2<t<3 \\ 36t^2 - 420t + 1225 & 3<t<4 \\ 9t^4 - 180t^3 + 1398t^2 - 4980t + 6889 & 4<t<5 \\ 64 & 5<t<6 \end{cases}$$







P. 7-38



(a) KCL at A: $C\frac{d}{dt}(v_s - v_i) = \frac{v_i - v_o}{R}$ also $v_i = -\frac{v_o}{A}$

$$\Rightarrow C\frac{dv_s}{dt} + \frac{C}{A}\frac{dv_i}{dt} = -\frac{v_o}{AR} - \frac{v_o}{R}$$

assume $A \gg 1$ and $v_s \gg \frac{v_i}{A}$

$$\Rightarrow \underline{v_o = -RC\frac{dv_s}{dt}}$$

(b) $RC = 1/10$

let $R = 10\,k\Omega \Rightarrow \underline{C = 1/10(10^4)} \underline{= 10^{-5}F = 10\,\mu F}$

Advanced Problems

AP 7-1

$$i_s(t) = \begin{cases} t & 0 \le t \le .5 \\ -t+1 & .5 \le t \le 1.5 \\ t-2 & 1.5 \le t \le 2.5 \end{cases} \qquad v = \frac{1}{C}\int_0^t i\,dt$$

$0 \le t \le .5$

$$v = 1/C\int_0^t t\,dt = t^2/2C = \underline{500t^2} \qquad @\,t=.5 \quad v(.5) = 125\text{v}$$

$.5 \le t \le 1.5$

$$v = 1/C\int_{.5}^{t}(-t+1)dt + v(.5) = \frac{1000}{2}\left[-t^2 + 2t - \frac{3}{4}\right] + 125$$

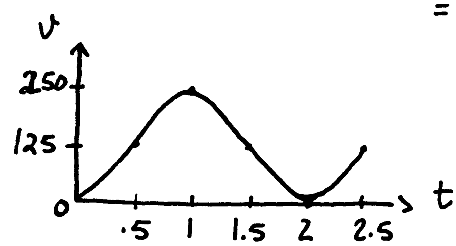
$$= \underline{-500t^2 + 1000t - 250}$$

at $t = 1.5$ $\quad v(1.5) = 125\text{v}$

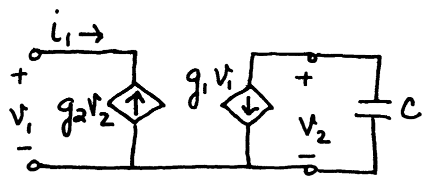
$1.5 \le t \le 2.5$

$$v = 1/C\int_{1.5}^{t}(t-2)dt + v(1.5) = 1000\left[\frac{t^2}{2} - 2t\right]_{1.5}^{t} + 125$$

$$= \underline{500t^2 - 2000t + 2000}$$



AP 7-2



$$v_2 = -\frac{1}{C}\int g_1 v_1\,dt$$

$$i_1 = -g_2 v_2 = \frac{g_1 g_2}{C}\int_0^t v_1\,dt$$

$$\Rightarrow v_1 = \frac{C}{g_1 g_2}\frac{di}{dt}$$

$$\therefore\ \underline{L = \frac{C}{g_1 g_2}}$$

AP 7-3 $\quad 1\,cm^2/100\,\mu F$ for the capacitor or $10^{-6}\,m^3/100\times10^{-6}\,F = \frac{1}{100}\,m^3/F$

with $v_0 = 500v \Rightarrow$ energy stored $= \frac{1}{2} C v_0^2$

we need $W = 1500$ watts $\times$ 3600 sec/hr $= 5.4\times10^6$ J

so $\quad C = \dfrac{2W}{v_0^2} = 43.2\ F$

size: $\dfrac{43.2\ F}{100\ F/m^3} = 0.432\,m^3 = 76cm \times 76cm \times 76cm$

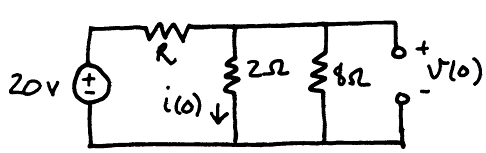
$\therefore$ not very practical!

Design Problems

DP 7-1 $\quad$ we have $\quad i(0) = \dfrac{30}{2+R} = 5 \quad \&\quad v(0) = \dfrac{R}{2+R}\,30 = 20$

both relations above are satisfied for $\underline{R = 4\,\Omega}$

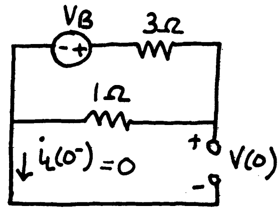
DP 7-2 $\quad$ at $t=0^-$



$i(0) = \dfrac{20}{R + 2\|8} = 3.7 \Rightarrow \underline{R = 3.8\,\Omega}$

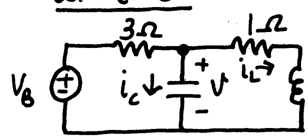
check: $v(0) = 2i(0) = 2(3.7) = \underline{7.4\,v}$ ok

DP 7-3 $\underline{\text{at } t=0^-}$



$v(0) = \frac{1}{4} V_B$

$\underline{\text{at } t=0^+}$



now $\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$ and $i_L(0^-) = i_L(0^+) = 0$

at top node : $\frac{v - V_B}{3} + i_L + i_c = 0$

$$i_c(0^+) = \frac{V_B - v(0^+)}{3} = \frac{V_B - 3}{3}$$

$$\frac{dv_c}{dt} = 24 = \frac{1}{(1/8)} \frac{(V_B - 3)}{3} \Rightarrow \underline{V_B = 12\text{V}}$$

DP 7-4 $\frac{1}{2} L i_L^2 = \frac{1}{2} C v_c^2$ (1) $\Leftarrow$ in steady state

now in dc $i_L = v_c/R$

so (1) becomes $L\left(\frac{v_c}{R}\right)^2 = C v_c^2 \Rightarrow C = \frac{L}{R^2}$

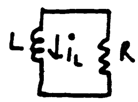
then $R = \sqrt{L/C} = \sqrt{\frac{10^{-2}}{10^{-6}}} = \sqrt{10^4} = 10^2$

so $\underline{R = 100\,\Omega}$

Chapter 8

Exercises

Ex. 8-1



KVL: $L\,di_L/dt + Ri_L = 0$

$di_L/dt + R/L\, i_L = 0$

integrating both sides leads to:

$\ln(i_L(t) + i_L(0)) = -R/L\,t$

$\Rightarrow i_L(t) = i_L(0)\,e^{-R/L\,t}$

from figure 7-9 in text $\Rightarrow$ $i_L(0) = V_0/R$

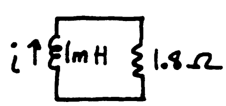
$\therefore$ $\underline{i_L(t) = V_0/R\, e^{-R/L\,t}}$

Ex 8-2 $\tau = RC = (10^6\Omega)(10^{-3}F) = 1000\,s$

$v(t) = V_0\,e^{-t/\tau} = V_0 e^{-t/1000}$

at $t = 100s$ $\quad \dfrac{v}{V_0} = e^{-.1} = 0.905$ or 90% of initial value

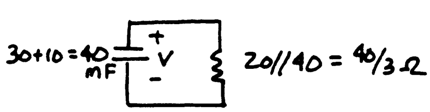
Ex. 8-3 at $t>0$



$i(t) = i(0^+)e^{-R/L\,t} = i(0^+)e^{-1.8/10^{-3}\,t} = i(0^+)e^{-1800t}$

now $i(0^-) = i(0^+) = 8A$ $\quad\therefore$ $\underline{i(t) = 8e^{-1800t}\ A}$

Ex. 8-4 at $t>0$

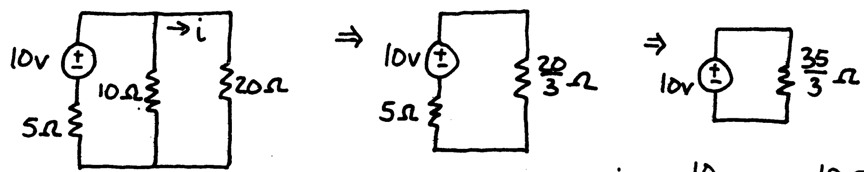


$V(t) = V(0^+)\,e^{-t/RC} = V(0^+)\,e^{-t/(40/3)(.04)}$

$= V(0^+)\,e^{-1.875t}$

now $V(0^-) = 16v = V(0^+)$ $\quad\therefore$ $\underline{V(t) = 16e^{-1.875t}\ v}$

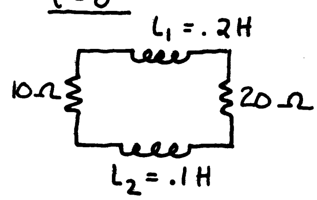
Ex 8-5 $t=0^-$



from current divider $i = \frac{10v}{35/3\,\Omega} \cdot \frac{10\Omega}{10\Omega + 20\Omega} = \frac{2}{7}A$

$\therefore\ i(0^-) = i(0^+) = 2/7\,A$

$t=0^+$



$R/L = \frac{10+20}{.1+.2} = \frac{30}{.3} = 100s^{-1}$

$i(t) = i(0^+)e^{-R/L\,t} = \underline{2/7\,e^{-100t}\ A}$

Ex. 8-6 from Ex. 7-2 $i(t) = 8e^{-1800t}\,A$

$\therefore\ W(0) = 1/2\,L\,i(0)^2 = 1/2(10^{-3})(8)^2 = .032\,J$

want time when

$$1/2\,W(0) = \int_0^t i^2 R\,dt$$

$$.016 = \int_0^t (8e^{-1800t})(1.8)\,dt$$

$$= \frac{115.2}{3600}(1-e^{-3600t})$$

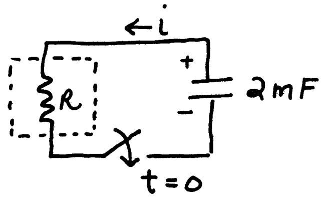
solving for t $\Rightarrow$ $\underline{t = 1.925 \times 10^{-4}\,s}$

Ex. 8-7 from Ex. 7-3 $V(t) = 16e^{-1.875t}\,V$

$$W(t) = \int_0^t V^2/R\,dt = \int_0^t \frac{(16e^{-1.875t})^2}{40/3}\,dt$$

$$= 19.2\int_0^t e^{-3.75t}\,dt = \underline{5.12(1-e^{-3.75t})\,J}$$

Ex 8-8



R = resistance of the switch and line

when $t=0$, the stored energy is $\frac{1}{2} C V_0^2 = 5\,J$

when $t = 6ms$, the transferred energy is

$$W_T = \tfrac{1}{2} C V_0^2 (1 - e^{-2t/RC})$$
$$= 5\left(1 - e^{-2\times 6\times 10^{-4}/150\times10^{-3}\times 2\times 10^{-3}}\right)$$
$$= 5(1 - e^{-4}) = 4.91\ J$$

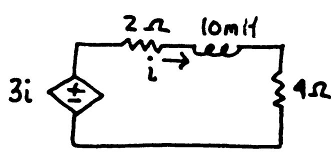
so remaining energy in the capacitor is $W_c = 5 - 4.91 = 0.09\ J$

now $\dfrac{W_c}{\frac{1}{2} C V_0^2} = \dfrac{0.09}{5} \Rightarrow 1.8\%$ energy left remaining

now $i = -C\dfrac{dv}{dt} = \dfrac{CV_0}{RC} e^{-t/RC} = \dfrac{1}{R}\sqrt{\dfrac{2W_T}{C}}\, e^{-t/RC}$

$$\underline{= 32.75\ A}$$

Ex. 8-9



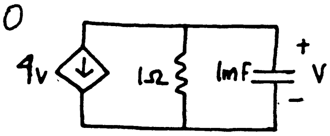
KVL: $-3i + 2i + (.01)\dfrac{di}{dt} + 4i = 0$

$\dfrac{di}{dt} + 300 i = 0$

$\Rightarrow s + 300 = 0 \Rightarrow s = -300$

$\therefore \underline{i(t) = i(0)\, e^{-300t} = 2e^{-300t}\ A}$

Ex. 8-10



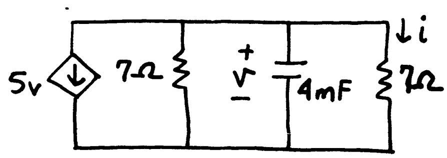
KCL at top node : $4v + v/1 + (.001)\, dv/dt = 0$

$\dfrac{dv}{dt} + 5000\, v = 0$

$s + 5000 = 0 \Rightarrow s = -5000$

$\therefore \underline{v(t) = v(0)\, e^{-5000t} = 10 e^{-5000t}\ v}$

Ex 8-11



KCL: $5v + 2\dfrac{v}{7} + .004\dfrac{dv}{dt} = 0$

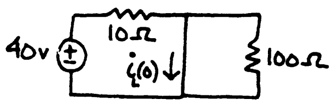
$\dfrac{dv}{dt} + 1321\, v = 0$

$\therefore v(t) = v(0)\, e^{-1321t}$

$= 3\, e^{-1321t}$

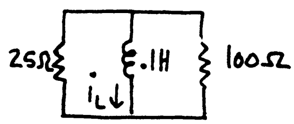
So $\underline{i(t) = \frac{3}{7} e^{-1321t}\ A}$

Ex. 8-12 <u>at $t=0^-$</u> (steady-state)



$i_L(0^-) = 40v/10\Omega = 4A = i_L(0^+)$

<u>$0 < t < 10ms$</u>

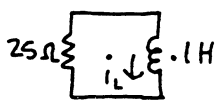


$i(t) = i(0)e^{-R/L\,t}$ where $R = 25 // 100 = 20\Omega$

$= 4e^{-\frac{20}{.1}t}$

$\underline{i(t) = 4e^{-200t}\ A}$

<u>$t > 10ms$</u>



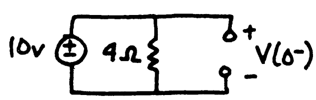
$i(t) = i(10ms)e^{-R/L(t-.01)}$

$i(10ms) = 4e^{-200(.01)} = .54A$

$\therefore\ i(t) = .54\,e^{-25/.1\,(t-.01)}$

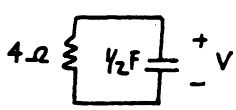
$\underline{i(t) = .54e^{-250(t-.01)}\ A}$

Ex. 8-13 <u>at $t=0^-$</u> (steady-state)



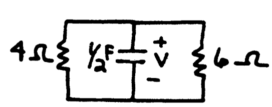
$V(0^-) = V(0^+) = 10v$

<u>$0 < t < 2s$</u>



$\underline{V(t) = V(0)\,e^{-t/(4)(1/2)} = 10e^{-t/2}\ v}$

<u>$t > 2s$</u>



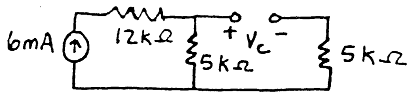
$V(t) = V(2)e^{-(t-2)/RC}$ where $R = 4//6 = 2.4\Omega$

$V(2) = 10e^{-(2)/2} = 3.68v$

$\therefore\ \underline{V(t) = 3.68e^{-(t-2)/(2.4)(1/2)} = 3.68\,e^{-(t-2)/1.2}\ v}$

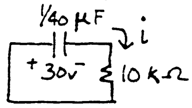
Problems

P. 8-1 at $t=0^-$ (steady-state)



$v_c(0^-) = v_c(0^+) = (6mA)(5k\Omega) = 30V$

at $t=0^+$

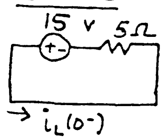


$i(0^+) = -30V/10k\Omega = -3\,mA$

$t>0$

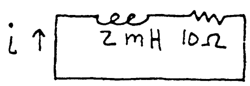
$$i(t) = i(0)\,e^{-t/RC} = -3\,e^{-t/10^4(\frac{1}{40})\times 10^{-6}} = -3\,e^{-4000t}\ mA$$

P. 8-2 at $t=0^-$



$i_L(0^-) = i_L(0^+) = 15V/5\Omega = 3A$

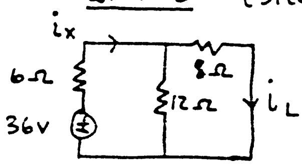
for $t>0$



$i(t) = i(0)\,e^{-R/L\,t} = i(0)\,e^{-5000t}$

now $i(0) = i_L(0) = 3$ $\quad\therefore\quad$ $i(t) = 3e^{-5000t}\ A$

P. 8-3 at $t=0^-$ (steady-state)



KVL $\circlearrowright i_x$: $-36 + 6i_x + 12(i_x - i_L) = 0$

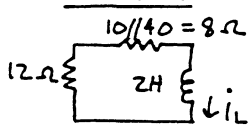
$\quad\hookrightarrow -36 + 18i_x - 12i_L = 0 \quad (1)$

KVL $\circlearrowright i_L$: $8i_L + 12(i_L - i_x) = 0$

$\quad\hookrightarrow 20i_L - 12i_x = 0 \quad (2)$

Solving (1) & (2) yields $i_L(0^-) = i_L(0^+) = 2A$

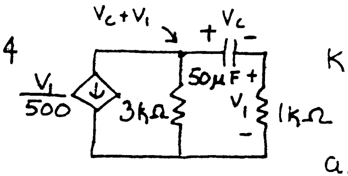
for $t>0$



$i_L(t) = i_L(0)\,e^{-R/L\,t} = 2e^{-10t}$

from current divider $i(t) = i_L(t)\left[\frac{10}{10+40}\right] = 2/5\,e^{-10t}\ A$

P. 8-4



KCL at top node : $\frac{V_1}{500} + \frac{(V_c+V_1)}{3000} + 50\times10^{-6}\, dV_c/dt = 0$

$\underline{140V_1 + 20V_c + 3\, dV_c/dt = 0}$ (1)

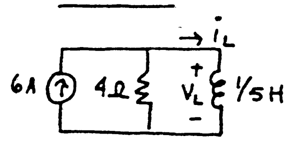
also $\underline{\frac{V_1}{1000} = 50\times10^{-6} \frac{dV_c}{dt}}$ (2)

plugging (2) into (1) yields $\frac{dV_c}{dt} + 2V_c = 0$

$s + 2 = 0 \Rightarrow s = -2$

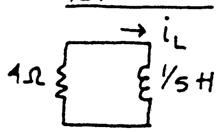
$\therefore \underline{V_c(t) = V_c(0)e^{-2t} = 6e^{-2t}\ V}$

P. 8-5 <u>at t=0⁻</u> (steady - state)



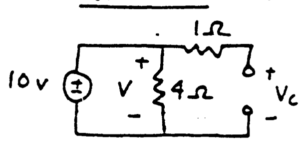
$V_L(0^-) = 0 \quad \therefore i_L(0^-) = 6A = i_L(0^+)$

<u>for t > 0</u>



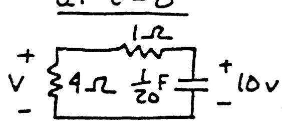
$\underline{i_L(t) = i_L(0)e^{-R/L\, t} = 6e^{-20t}\ A}$

P. 8-6 <u>at t = 0⁻</u> (steady state)



$V_c(0^-) = 10v = V_c(0^+)$

<u>at t = 0⁺</u>



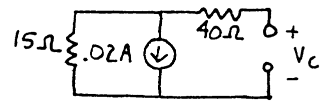
from voltage divider

$V(0^+) = 10v\left[\frac{4}{4+1}\right] = 8v$

<u>for t > 0</u>

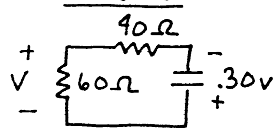
$V(t) = V(0)e^{-t/RC} = 8e^{-t/(5)(1/20)} = \underline{8e^{-4t}\ v}$

P. 8-7 at $t=0^-$ (steady state)



$V_c(0^-) = -(.02A)(15\Omega) = -.30v = V_c(0^+)$

at $t=0^+$



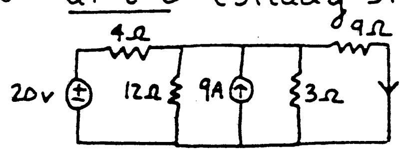
voltage divider

$V(0^+) = -.30v\left[\frac{60}{60+40}\right] = -.18v$

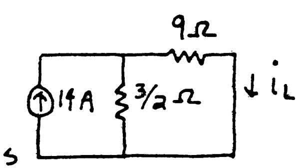
for $t>0$

$V(t) = V(0)e^{-t/RC} = -.18e^{-t/(100)(1\times10^{-4})} = \underline{-.18e^{-100t}\ v}$

P. 8-8 at $t=0^-$ (steady state)



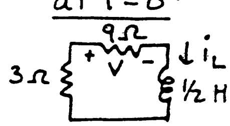
⇒ source transforms



current division

$i_L(0^-) = 14\left(\frac{3/2}{3/2+9}\right)$
$= 2A$

at $t=0^+$

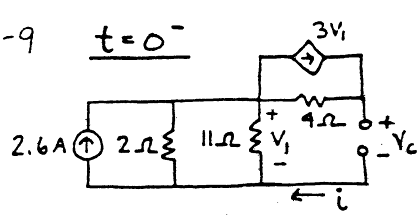


$i_L(0^+) = i_L(0^-) = 2A \quad \therefore\ V(0^+) = 9(2) = 18v$

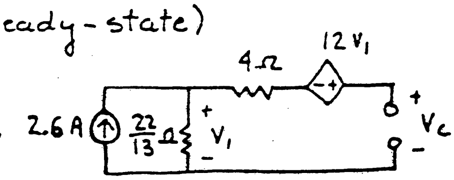
for $t>0$

$V(t) = V(0)e^{-R/L\,t} = 18e^{-12/\frac{1}{2}t} = \underline{18e^{-24t}\ v}$

P. 8-9 $t=0^-$ (steady-state)



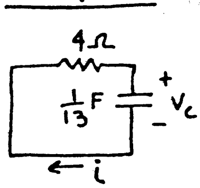
⇒



$V_1(0^-) = (2.6A)\left(\frac{22}{13}\right) = 4.4v$

$\therefore\ V_c(0^-) = V_c(0^+) = V_1 + 12V_1 = 13(4.4) = 57.2v$

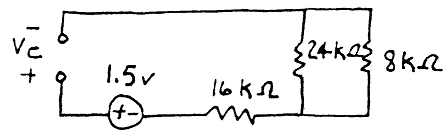
for $t>0$ short ckt across 11Ω $\therefore\ V_1 = 0$



$V_c(t) = V_c(0)e^{-t/RC} = 57.2e^{-3.25t}\ v$

$\underline{i(t) = -\frac{V_c(t)}{4} = -14.3e^{-3.25t}\ A}$

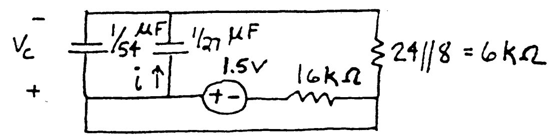
P. 8-10 $\underline{t=0^-}$ (steady state)



since no current flowing

$V_c(0^-) = V_c(0^+) = 1.5\text{v}$

$\underline{t>0}$



$V_c(t) = V_c(0)\, e^{-t/RC}$

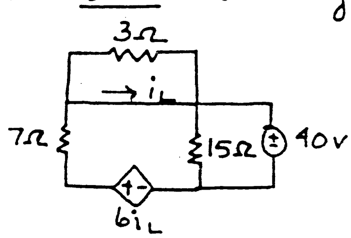
where $R = 6\text{k}\Omega$

$C = 1/54 + 1/27 = 1/18\ \mu\text{F}$

$\therefore\ V_c(t) = 1.5\, e^{-t/(6\text{k}\Omega)(\frac{1}{18}\mu\text{F})} = 1.5\, e^{-3000t}\ \text{V}$

$\therefore\ i = \frac{1}{27}\times 10^{-6}\, dV_c/dt = -\frac{1}{27}\times 10^{-6}(1.5)(-3000)\, e^{-3000t} = \underline{5/3\, e^{-3000t}\ \text{mA}}$

P. 8-11 $\underline{t=0^-}$ (steady state)



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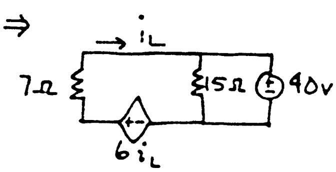
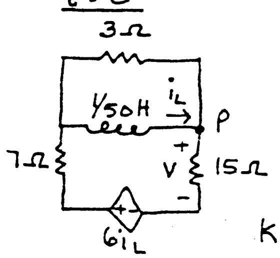


KVL ↻: $-6i_L + 7i_L + 40 = 0$

$\Rightarrow i_L(0^-) = i_L(0^+) = -40\text{A}$

$\underline{t>0}$



KCL at P: $\frac{1/50\, di_L/dt}{3} + i_L - V/15 = 0$

↳ $\underline{1/10\, \frac{di_L}{dt} + 15 i_L - V = 0}$ (1)

KVL ↻: $V - 6i_L + 7(V/15) + 1/50\, di_L/dt = 0$

↳ $\underline{22V - 90 i_L + 3/10\, di_L/dt = 0}$ (2)

Solving for V in (1) and plugging into (2) yields

$\frac{di_L}{dt} + 96\, i_L = 0$

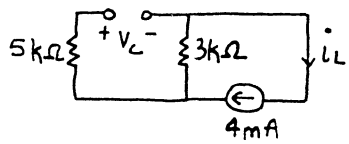
$\Rightarrow\ s + 96 = 0\ \Rightarrow\ s = -96$

$i_L(t) = i_L(0)\, e^{-96t} = -40\, e^{-96t}\ \text{A}$

but $v(t) = 15 i_L(t) + 1/10\, di_L/dt$

$v(t) = -600 e^{-96t} + 384 e^{-96t} = \underline{-216 e^{-96t}\ \text{V}}$

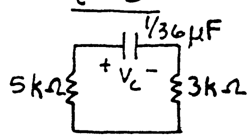
P. 8-12 $\underline{t=0^-}$ (steady state)



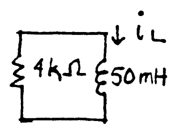
$i_L(0^-) = i_L(0^+) = 4\text{mA}$

$V_C(0^-) = V_C(0^+) = 3k\Omega\,(4\text{mA}) = 12\text{ V}$

$\underline{t>0}$



and



$\frac{R}{L} = \frac{4\times10^3}{50\times10^{-3}} = 8\times10^4$

$V_C(t) = V_C(0)\,e^{-t/RC}$

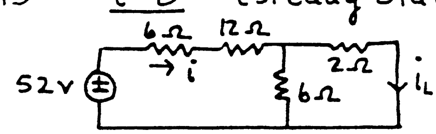
$= 12\,e^{-t/(8\times10^3)(\frac{1}{36}\times10^{-6})}$

$\underline{V_C(t) = 12\,e^{-4500t}}\text{ V}$

$i_L(t) = i_L(0)\,e^{-R/L\,t}$

$\underline{i_L(t) = 4\,e^{-80{,}000t}}\text{ mA}$

P. 8-13 $\underline{t=0^-}$ (steady state)

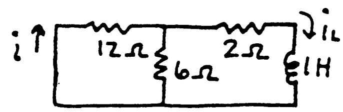


KVL ↻: $-52 + 18i + {}^{12}/_{8}\,i = 0$

$\Rightarrow i(0^-) = 104/39\text{ A}$

$\therefore i_L = i\left(\frac{6}{6+2}\right) = \underline{2\text{A} = i_L(0^+)}$

$\underline{0<t<51\text{ms}}$



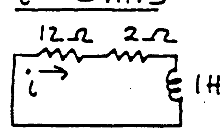
$i_L(t) = i_L(0)\,e^{-R/L\,t}$

$i_L(t) = 2\,e^{-6t}\text{ A}$

$R = 6\|12 + 2 = 6\Omega$

$\therefore \underline{i(t) = i_L(t)\left(\frac{6}{6+12}\right) = {}^2/_3\,e^{-6t}\text{ A}}$

$\underline{t>51\text{ms}}$



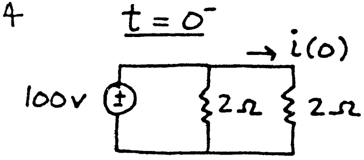
$i(t) = i(51\text{ms})\,e^{-R/L(t-.051)}$

$R = 14\Omega$

$i(51\text{ms}) = {}^2/_3\,e^{-6(.051)} = 1.47$

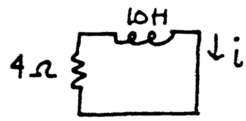
$\therefore \underline{i(t) = 1.47\,e^{-14(t-.051)}\text{ A}}$

P. 8-14 $\quad \underline{t=0^-}$ (steady state)



$$i(0^-) = i(0^+) = \frac{100\text{v}}{2\Omega} = 50\text{A}$$

$\underline{t>0}$

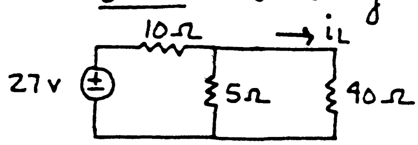


$$i(t) = i(0)e^{-4/10\,t} = 50e^{-.4t}$$

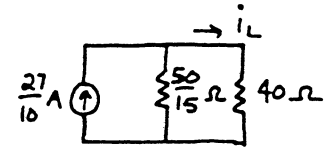
$$\therefore\ W = \int_0^{60} i^2 R\,dt = (50)^2(4)\int_0^{60} e^{-.8t}dt$$

$$= 10^4\left(\frac{e^{-.8t}}{-.8}\right)\Bigg|_0^{60} = \underline{1.25\times10^4\ \text{J}}$$

P. 8-15 $\quad \underline{t=0^-}$ (steady state)

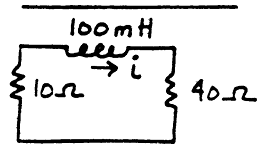


⇒



current divider $\quad i_L(0^-) = i_L(0^+) = \frac{27}{10}\left(\frac{50/15}{50/15+40}\right) = .2077\text{ A}$

$\underline{0<t<4\text{ms}}$

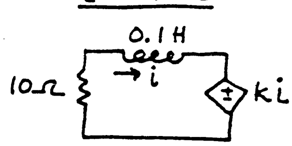


$i(t) = i(0)e^{-R/L\,t}$ $\quad R = 50\Omega,\ L = .1\text{H}$

$i(t) = .2077e^{-500t}$ A

and $i(4\text{ms}) = .2077e^{-500(.004)} = .0281$ A

$\underline{t>4\text{ms}}$



KVL ↻: $10i + (.1)\,di/dt + ki = 0$

$$\frac{di}{dt} + 10(10+k)i = 0$$

$$\therefore\ i(t) = i(4\text{ms})\,e^{-10(10+k)(t-.004)}\ \text{A}$$

if $k=10$: $i(t) = .0281e^{-200(t-.004)}$ A

if $k=-11$: $i(t) = .0281e^{+10(t-.004)}$ A

P.8-15 (continued)

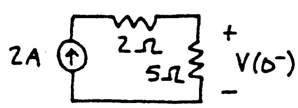
$\therefore$ for $k=10$

$$i(t) = \begin{cases} 0.2077\,e^{-500t} & 0<t<4\text{ms} \\ 0.0281\,e^{-200(t-.004)} & t>4\text{ms} \end{cases}$$

for $k=-11$

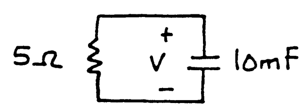
$$i(t) = \begin{cases} 0.2077\,e^{-500t} & 0<t<4\text{ms} \\ 0.0281\,e^{10(t-.004)} & t>4\text{ms} \end{cases}$$

P. 8-16 $t=0^-$ (steady state)



$V(0^-) = V(0^+) = (2A)(5\Omega) = 10\text{v}$

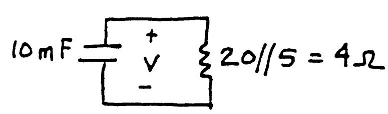
$0<t<100\text{ms}$



$V(t) = v(0)\,e^{-t/RC}$

$$\underline{V(t) = 10\,e^{-t/(5)(.01)} = 10\,e^{-20t}\text{ Volts}}$$

$t>100\text{ms}$

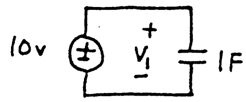


$$V(t) = V(100\text{ms})\,e^{-\frac{(t-.1)}{(4)(.01)}}$$

$$V(100\text{ms}) = 10\,e^{-20(.1)} = 1.35\text{ v}$$

$$\therefore \underline{V(t) = 1.35\,e^{-25(t-.1)}\text{ Volts}}$$

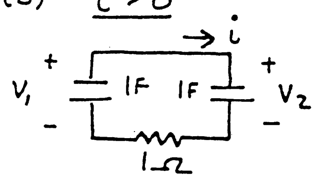
P. 8-17 (a) $t=0^-$ (steady state)



$V_1(0^-) = 10\,\text{v}$

$W_1(0^-) = \frac{1}{2} C_1 V_1^2(0^-) = \frac{1}{2}(1)(10)^2 = \underline{50\,\text{J}}$

(b) $\underline{t>0}$



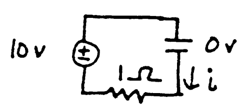
KVL: $-V_1 + V_2 + 1\,dV_2/dt = 0$

$\frac{d}{dt}$: $\underline{-\frac{dV_1}{dt} + \frac{dV_2}{dt} + \frac{d^2V_2}{dt^2} = 0}$ (1)

now $i = i_{C_2} = -i_{C_1} \Rightarrow \underline{dV_1/dt = -dV_2/dt = -i}$ (2)

from (1) & (2) $\Rightarrow \frac{di}{dt} + 2i = 0 \Rightarrow i(t) = i(0)e^{-2t}$

$\underline{t=0^+}$



$i(0^+) = 10\text{v}/1\Omega = 10\,\text{A}$ $\therefore\ i(t) = 10e^{-2t}$

with $i = dV_2/dt = -dV_1/dt$

$V_2(t) = V_2(0) + \frac{1}{1}\int_0^t i\,dt = 10\int_0^t e^{-2t}dt = 5(1-e^{-2t})\,\text{v}$

$V_1(t) = V_1(0) + \frac{1}{1}\int_0^t (-i)\,dt = 10 - 5(1-e^{-2t}) = 5(1+e^{-2t})\,\text{v}$

$\therefore\ W_{C_1}(t) = \frac{1}{2} C_1 V_1^2 = (\frac{1}{2})(1)[5(1+e^{-2t})]^2 = \underline{12.5(1+e^{-2t})^2\,\text{J}}$

$W_{C_2}(t) = \frac{1}{2} C_2 V_2^2 = \underline{12.5(1-e^{-2t})^2\,\text{J}}$

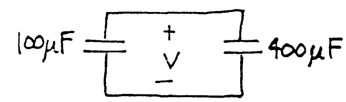
$W_R(t) = \int_0^t i^2 R\,dt = \int_0^t (10e^{-2t})^2(1)\,dt = \underline{25(1-e^{-4t})\,\text{J}}$

P. 8-18 $\underline{t=0^-}$

assume V_1 = voltage across 10mF capacitor = 3 volts

$\underline{0 < t < 10ms}$

with R negligibly small, we may assume a static steady-state situation is obtained in the circuit nearly instantaneously ($t=0^+$). Thus with both capacitors in parallel, the common voltage is obtained by considering charge conservation.



$$\text{at } t=0^-, \quad q_{100\mu F} = CV = (100\mu F)(3v) = 300\mu C$$

$$q_{400\mu F} = CV = (400\mu F)(0) = 0$$

$$q_{TOT} = q_{100} + q_{400} = 300\mu C$$

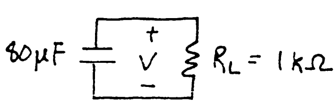
$$\text{at } t=0^+, \quad q_{100} + q_{400} = 300\mu C$$

now using $q = CV \Rightarrow (100\mu F)(v) + (400\mu F)(v) = 300\mu C$

$$\Rightarrow \underline{V = 0.6 \text{ volts}}$$

$\underline{10ms < t < 1s}$

combine 100μF & 400μF in parallel to obtain



with $V(10ms) = 0.6v$

$$v(t) = V(10ms)\, e^{-1/RC\,(t-.01)}$$

$$= 0.6\, e^{-(t-.01)/(10^3)(8\times10^{-5})}$$

$$\underline{V(t) = 0.6\, e^{-12.5(t-.01)} \text{ volts}}$$

P. 8-19 After the TV set is switched off,

$$V_c(t) = 20e^{-t/RC} \text{ kV}$$
$$= 20e^{-t/(10^8)(10^{-6})} = 20e^{-.01t} \text{ kV}$$

After 30 sec,

$$V_c(30) = 20e^{-.3} = 14.82 \text{ kV} = \text{shock voltage}$$

If in contact with circuit for only 1/2 sec. ⇒ can assume voltage is approximately constant.

$$\therefore\ W = P\Delta t = \frac{V^2}{R}\Delta t$$
$$= \frac{(14.82\text{ kV})^2}{(10\text{k}\Omega)}(1/2\text{ s})$$
$$W \cong 11 \text{ kJ}$$

P 8-20

$$\left.\begin{array}{l} v_0 = 300\text{v} \\ v(\infty) = 0\text{v} \end{array}\right\} \quad v(t) = 300e^{-t/\tau}$$

at $t = .25\text{s}$ ⇒ $v(.25) = 300e^{-1/4\tau} = 189\text{v}$

$$e^{-1/4\tau} = 189/300 = .63$$
$$\Rightarrow 1/4\tau = .462 \quad \Rightarrow \tau = 0.541\text{s}$$

now $\tau = .541 = RC = 270\times10^3\, C$

$$\Rightarrow C = \frac{.541}{270\times10^3} = 2\times10^{-6}\text{F} = 2\mu\text{F}$$

P 8-21 $i = I_0 e^{-t/\tau}$ $\quad \tau = L/R = \frac{10^{-4}}{10^{-10}} = 10^6 \text{ sec}$

1 day $= 60\times60\times24 = 86{,}400$ sec

so $i = I_0 e^{-8.64\times10^4/10^6} = .917\, I_0$ or drops 9.83 %

P 8-22 $\tau = L/R = 1/10$ s

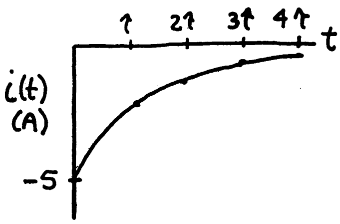
$i(t) = i(0)e^{-t/\tau}$

now $i(0) = -5A$

$\Rightarrow i(t) = -5e^{-10t}$

$di/dt = 50e^{-10t}$

at $t=0$ $\frac{di(0)}{dt} = \underline{50 \text{ A/s}}$



P 8-23 $\tau = RC$, $i(0) = 6mA$; $i(t) = i(0)e^{-t/\tau}$

for $t_1 = \tau \Rightarrow i(t_1) = i(0)e^{-t_1/\tau} = i(0)e^{-1} = (6\times10^{-3})(.368)$
$= 2.2mA$

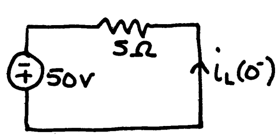
so $\tau = t_1 = 2ms$ from curve

$\therefore i(t) = 6e^{-500t}$ mA

and $RC = 2ms$

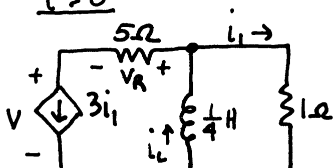
so $C = \frac{2\times10^{-3}}{10^3} = \underline{2\mu F}$

P 8-24 <u>t<0</u>



$i_L(0^-) = \frac{50}{5} = 10A = i_L(0^+)$

<u>t>0</u>



using KVL: $V + \frac{1}{4}\frac{di_L}{dt} + 5(3i_1) = 0$ (1)

& $\frac{1}{4}\frac{di_L}{dt} + i_1 = 0$ (2)

KCL at top node: $i_L = i_1 + 3i_1 = 4i_1$ (3)

from (3) & (2) get $\Rightarrow \frac{di_L}{dt} + i_L = 0$

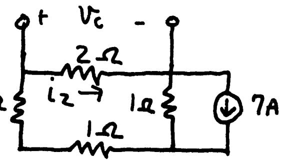
so have $i_L(t) = i_L(0)e^{-t/\tau}$ here $\tau = 1s$

$i_L(t) = \underline{10e^{-t}}$ A

$\therefore i_1(t) = \frac{1}{4}i_L(t) = \underline{\frac{5}{2}e^{-t} A}$

so from (1) get $\underline{v(t) = -5/4 e^{-t} \text{ volts}}$

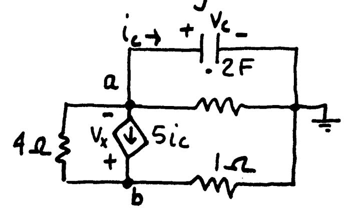
P 8-25 $\underline{t=0^-}$ (steady state)



$$i_2 = \frac{1}{1+4+2+1}\,7 = \frac{7}{8}\,A$$

$$V_c(0^-) = V_c(0^+) = 2i_2 = 7/4\ V$$

$\underline{0<t<.35}$ closing the right most switch shorts out 1Ω in parallel with 7A source and isolates 7A source leaving,



KCL at a: $-\frac{V_x}{4} + 5i_c + i_c + \frac{V_c}{2} = 0$ (1)

KCL at b: $\frac{V_c+V_x}{4} - 5i_c + \frac{V_c+V_x}{1} = 0$ (2)

also: $i_c = .2\,dV_c/dt$ (3)

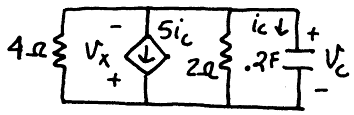
plugging (3) into (1) & (2) & then eliminating V_x yields ⇒ $\frac{dV_c}{dt} + \frac{3}{4}V_c = 0$

so have $V_c(t) = V_c(0)e^{-3/4t} = \underline{7/4\,e^{-3/4t}}$, $i_c = .2\frac{dV_c}{dt} = -\frac{21}{40}e^{-3/4t}$

so from (1) have $V_x(t) = 24i_c + 2V_c = \underline{-9.1\,e^{-3/4t}\ v}$

$\underline{t>.35}$

$$V_c(.35) = 7/4\,e^{-3/4(.35)} = 1.35\ v$$



now $\underline{V_x = -V_c}$

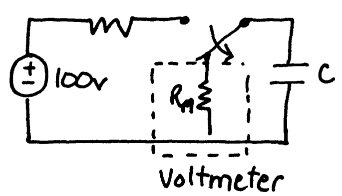
KCL: $V_c/4 + 5i_c + V_c/2 + i_c = 0$ (1)

also: $i_c = .2\,dV_c/dt$ (2)

from (1) & (2) ⇒ $dV_c/dt + 5/8\,V_c = 0$ ⇒ $V_c(t) = V_c(.35)\,e^{-(t-.35)5/8}$

$= \underline{1.35\,e^{-5/8(t-.35)}\ v}$

P 8-26



$$V_c(t) = 100e^{-t/R_mC}$$

$$36.7 = 100e^{-13.5/R_mC}$$

or $R_mC = 13.5$

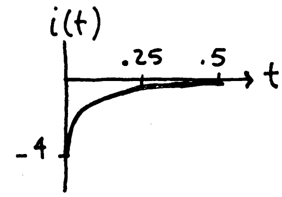
⇒ $C = 13.5/3\times10^4 = \underline{450\mu F}$

Advanced Problems

AP 8-1 $i_L(0^-) = \frac{36}{2+6\|12}\left(\frac{12}{18}\right) = 4 = i_L(0^+)$ so $i(0) = -4\,A$

$i_\infty = 0$, $R_{TH} = 18\,\Omega$, $\tau = L/R_{TH} = 2/18 = 1/9\ s$

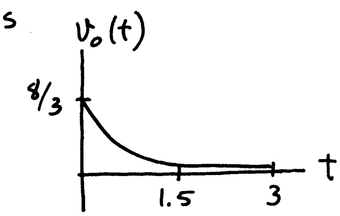
$i(t) = -4e^{-9t}\ A$



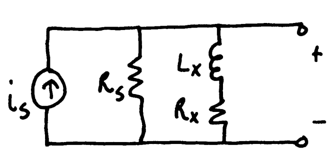
AP 8-2 $v_c(0^-) = v_c(0^+) = \frac{6}{6+3}(12) = 8\,v$ $\therefore\ v_o(0^+) = \frac{2}{2+4}(8) = \frac{8}{3}\,v$

$R_{TH} = 6000\,\Omega$, $\tau = R_{TH}C = 6000(10^{-4}) = .6s$

$\therefore\ \underline{v_o(t) = 8/3\, e^{-1.667t}\ v}$



AP 8-3

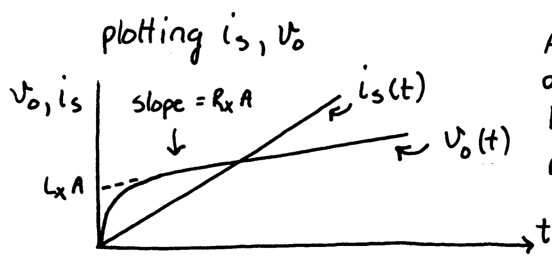


ignoring the large internal resistance R_s of the current source, we have

$$v_o = R_x i_s + L_x\, di_s/dt$$

with $i_s = At$ $\Rightarrow$ $v_o = R_x At + L_x A$

plotting i_s, v_o



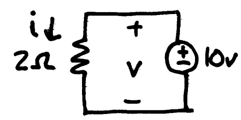
After a short rise time (due to R_s), v_o approaches linear relation above $\therefore R_x = A/\text{slope}$ $L_x = A/v_o(t=0)$, but because of the finite rise time above expression may not be too accurate. For a more accurate method see Hewlett Packard Journal, October 1990, pg 75.

AP 8-4 using equation 8-18 $\Rightarrow w = \frac{1}{2} C V_o^2 (1 - e^{-2t/RC})$

or $t = -\frac{RC}{2} \ln\left[1 - \frac{2w}{CV_o^2}\right]$

so $t = -\frac{(100)(10^{-4})}{2} \ln\left[1 - \frac{2(.025)}{(10^{-4})(24)^2}\right] = \underline{.0101\,s = 10.1\,ms}$

AP 8-5 $\underline{t = 0^-}$



$v(0) = 10v$

$i(0) = 10v/2\Omega = 5A$

$\underline{0 < t < .15s}$

Left ckt. have

$R_t = 2 \| 4 = 4/3\,\Omega$

$\tau = L/R_t = .2/\tfrac{4}{3} = 3/20$

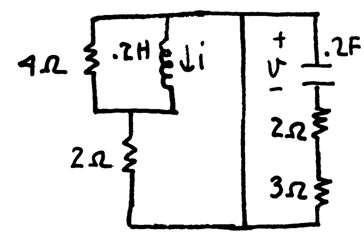
$\underline{i(t) = 5e^{-20/3\,t}\ A}$

right ckt. have

$R_t = 5\Omega$

$\tau = R_t C = (5)(.2) = 1$

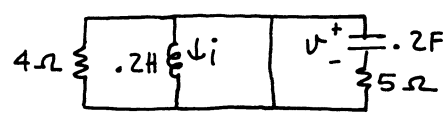
$\underline{v(t) = 10e^{-t}\,v}$



$\underline{.15 < t < 1s}$

$i(.15) = 1.84\,A$

$v(.15) = 8.61\,v$



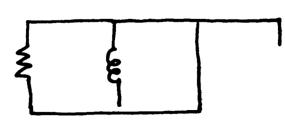
right ckt is unchanged $\Rightarrow v(t) = v(.15)e^{-(t-.15)} = \underline{8.61e^{-(t-.15)}\,v}$

left ckt have $\tau = .2/4 = .05 \Rightarrow i(t) = i(.15)e^{-20(t-.15)}$

$= \underline{1.84e^{-20(t-.15)}\ A}$

$\underline{t > 1s}$

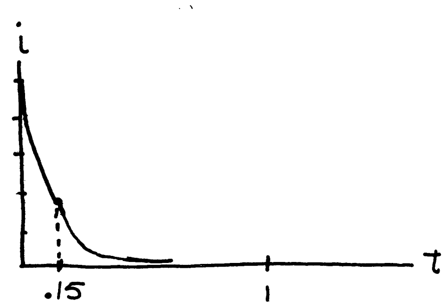
$i(1) \approx 0 \quad \therefore \underline{i(t) \approx 0}$

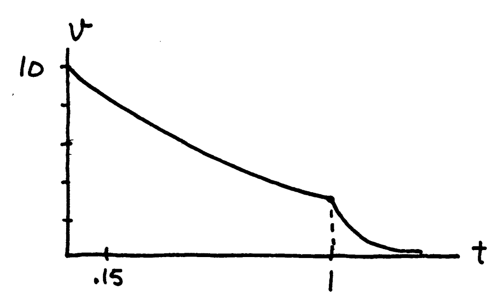


$v(1) = 3.68v$

$\tau = RC = (2)(.2) = .4$

$\therefore v(t) = 3.68e^{-(t-1)/.4} = \underline{3.68e^{-2.5(t-1)}\,v}$





AP 8-6 $\quad v(0) = 20\,kV \qquad C = 10^{-6}\,F$

$\tau = RC = (10^8)(10^{-6}) = 100\,s$

assume he get inside the TV in 3 sec., so most of the voltage remains on the capacitor.

He touches across the capacitor so ignoring $100\,M\Omega$ in parallel with $1\,M\Omega$ have:

$$i(0) = \frac{20\times10^3}{10^6} = 0.02\,A = 20\,mA$$

stored energy $= W = \frac{1}{2} C\, v(0)^2$

$$= \frac{1}{2}(10^{-6})(20\times10^3)^2 = 200\,J$$

body: $\tau = 10^{-6}(10^6) = 1\,sec$

so energy drain goes like $e^{-t/2\tau}$

so if he holds on for $8\tau = 8$ sec, he absorbs virtually all the energy.

AP 8-7 $\quad \tau = RC = 10^6 \times 10^{-6} = 1\,s$

$v_c(0) = 5\,v$ $\quad \Big\} \Rightarrow V_c(t) = 5e^{-t/\tau}$

now $5/2 = v_c(t_1) = 5\,e^{-t_1/\tau}$

$e^{-t_1/\tau} = .5 \quad \Rightarrow \underline{t_1 = .693\,s}$

$$i(t_1) = \frac{v(t_1)}{100\,k\Omega} = \frac{5/2}{10^5} = \underline{25\,\mu A}$$

Design Problems

DP 8-1 $\quad$ desire $\tau = 120\,ms$

now R_T of circuit $\Rightarrow R_T = 8 + 20\|(9 + 70\|30) = 20\,k\Omega$

(all in $k\Omega$)

so $\tau = C \times 20\,k\Omega = 120 \times 10^{-3} \quad \therefore \underline{C = 6\,\mu F}$

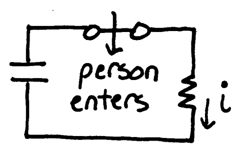
DP 8-2 $\quad i(0) = \dfrac{36}{(2\|4+1)}\left(\dfrac{1/4}{1/4+1/2}\right) = 5.13\text{ A}$

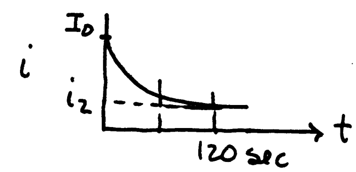
at $t = .1 \Rightarrow i(t) = 5.13e^{-t/\tau} = 5.13e^{-.1/\tau} = 1.14$

so have $1.5 = .1/\tau$ or $1/\tau = 15$

$\therefore\ 1/\tau = R/L = 5/L = 15 \Rightarrow \underline{L = 1/3\text{H}}$

DP 8-3 Design : spring activated switch will reopen if $i < .135\ I_0$ unless override switch is pressed





at 2τ, $i_2 = .135\ I_0$

let $\tau = 60\text{sec}$, need $RC = 60$

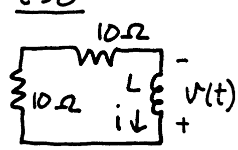
so let $R = 1\text{M}\Omega = 10^6\,\Omega$

$C = 60\mu\text{F} = 60\times10^{-6}\text{F}$

switch opens when $i < .135\,I_0$

DP 8-4 $\quad$ inductor : $i(0) = \dfrac{10}{10+10}\left(\dfrac{60}{15+10\|10}\right) = 3/2\text{ A}$

$\underline{t>0}$



$i(t) = 3/2\,e^{-t/\tau}$

$v(t) = -L\,di/dt = \dfrac{3}{2\tau}e^{-t/\tau}$

need $\quad 3/2\,e^{-.025/\tau} > .5$

$\dfrac{3}{2\tau}e^{-.025/\tau} > 25$

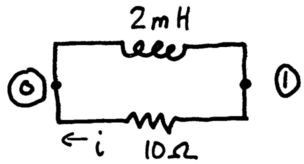
try $\tau = .025$, then have $\quad 3/2\,e^{-1} = .552 > .5$

$\dfrac{3}{2(.025)}e^{-1} = 22.1 > 20$

$\therefore\ L = R\tau = (20\Omega)(.025\text{s}) = \underline{.5\text{H}}$

Spice Problems

SP 8-1



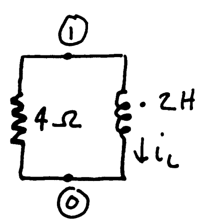
input file:

```
L1   0 1  2E-3  IC=3
R1   1 0  10
.TRAN  .0001  1.5ms  uic
.PRINT  TRAN  I(L1)
.probe
.end
```

output file:

time	I(L1)
0.0 E0	3.000 E 0
1.0 E-4	1.819 E0
2.0 E-4	1.104 E0
3.0 E-4	6.687 E-1
4.0 E-4	4.043 E-1
5.0 E-4	2.454 E-1
6.0 E-4	1.486 E-1

SP 8-2



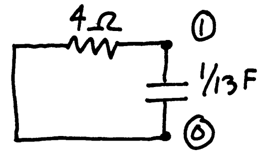
input file:

```
R1   1 0 4
L1   1 0 0.2H  IC=6A
.TRAN  .1 5 0 .01  UIC
.PRINT  TRAN I(L1)
.end
```

output file:

Time	I(L1)
0.0	6.000 E0
1.0 E-1	8.088 E-1
2.0E-1	1.085 E-1
3.0E-1	1.454 E-2
4.0E-1	1.950 E-3
5.0E-1	2.615 E-4
6.0E-1	3.506 E-5

SP 8-3

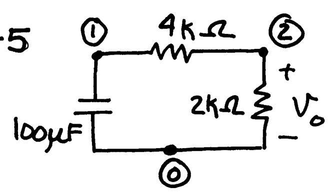


input file:
```
r3   0 1 4
c1   1 0 .07692 ic=57.2
.tran .5 10 0 .01 uic
.print tran i(c1)
.end
```

output file:

time	I(C1)
0.0	- 1.430 E 1
5.0E-1	- 2.815 E 0
1.0E 0	- 5.542 E -1
1.5 E0	- 1.091 E-1
2.0E0	- 2.148 E-2
2.5E0	- 4.228 E-3
3.0E0	- 8.324 E-4

SP 8-5



input file:
```
C1   1  0  100u  IC=2.667
R1   1  2  4K
R2   2  0  2K
.TRAN  .1 3s 0 .01 UIC
.PRINT  TRAN  V(2)
.END
```

time	V(2)
0	.889
.3	.539
.6	.327
.9	.198
1.2	.120
2.0	.032
3.0	.006

SP 8-6 input file :

```
R1   1 0 1K
C1   1 0 20u   IC=12
.TRAN   2MS   25MS   UIC
.PRINT  TRAN  V(1)
.END
```

output file :

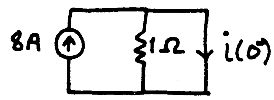
time	V(1)
1.60 E-2	5.392 E0
1.80 E-2	4.879 E0
2.00 E-2	4.415 E0
2.20 E-2	3.994 E0
2.40 E-2	3.614 E0
2.50 E-2	3.438 E0

$\therefore \underline{t_1 \simeq .022\,s = 22ms}$

Chapter 9

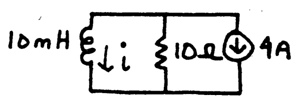
Exercises

Ex. 9-1 $t=0^-$ (steady state)



$i(0^-) = i(0^+) = 8\,A$

$t>0$



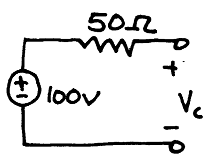
$$i(t) = i(\infty) + Ae^{-10t/.01}$$
$$= i(\infty) + Ae^{-1000t}$$

now at $t=\infty$, inductor becomes a short $\therefore\ i(\infty) = -4A$

$i(t) = -4 + Ae^{-1000t} \Rightarrow i(0) = 8 = -4 + A \Rightarrow A = 12$

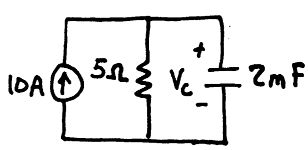
$$\therefore\ \underline{i(t) = -4 + 12e^{-1000t}\ A}$$

Ex 9-2 $t<0$ (steady state)

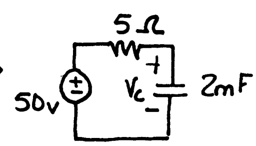


$V_c(0^-) = 100v = V_c(0^+)$

$t>0$



$\Rightarrow$



KVL yields

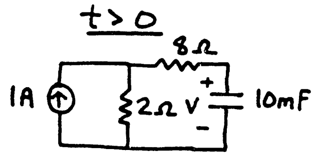
$$V_c - 50 + 5\left(C\frac{dV_c}{dt}\right) = 0$$
$$\Rightarrow \frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{50}{RC}$$

now $RC = 5(2\times10^{-3}) = 10ms$

$\therefore\ V_c(t) = 50 + Ae^{-t/RC}$ since $V_c(0) = 100 = 50 + A \Rightarrow A = 50$

$$V_c(t) = 50 + 50e^{-100t} = \underline{50\left(1+e^{-100t}\right)v}$$

Ex. 9-3 $t>0$

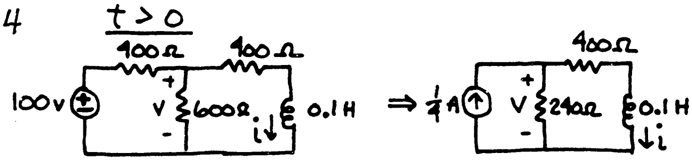


$$V(t) = V(\infty) + Ae^{-t/RC}$$ where $R = 2+8 = 10\Omega$
$$= V(\infty) + Ae^{-10t}$$

at $t=\infty$, capacitor becomes an open $\therefore\ V(\infty) = (1A)(2\Omega) = 2v$

$\Rightarrow v(0) = 10 = 2 + A \Rightarrow A = 8$ $\quad \therefore\ \underline{v(t) = 2 + 8e^{-10t}\ v}$

Ex. 9-4 $\underline{t > 0}$



KCL at top node:

$V/240 = 1/4 - i$

$\Rightarrow \underline{V = 60 - 240i}$ (1)

$i(t) = i(\infty) + Ae^{-R/L\, t}$ where $R = 240 + 400 = 640\Omega$

$i(\infty) = 1/4\left(\frac{240}{240+400}\right) = 0.09375A$

$\therefore\ i(t) = .09375 + Ae^{-6400t} \Rightarrow i(0) = 1/2 = .09375 + A$

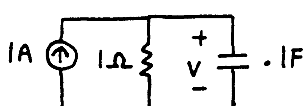
$\Rightarrow A = .40625$

$\therefore\ i(t) = .09375 + .40625e^{-6400t}$ A

from (1) : $V(t) = 60 - 22.5 - 97.5e^{-6400t}$

$\underline{V(t) = 37.5 - 97.5e^{-6400t} \text{ Volts}}$

Ex. 9-5 $\underline{0 < t < t_1}$



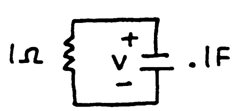
$V(t) = V(\infty) + Ae^{-t/RC}$

where $V(\infty) = (1A)(1\Omega) = 1v$

$V(t) = 1 + Ae^{-t/(1)(.1)} = 1 + Ae^{-10t}$

now $V(0^-) = V(0^+) = 0 = 1 + A \Rightarrow A = -1$ $\quad \underline{\therefore V(t) = 1 - e^{-10t} \text{ v}}$

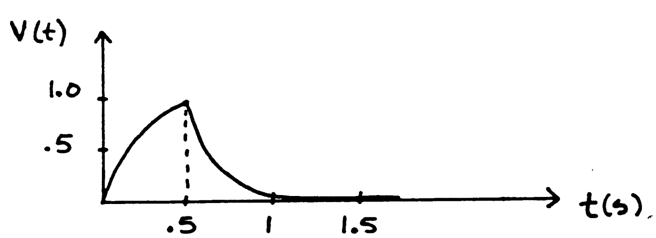
$\underline{t > t_1}$



$t_1 = .5s$ $\quad V(t) = V(t_1)e^{-\frac{t-.5}{(1)(.1)}} = V(.5)e^{-10(t-.5)}$

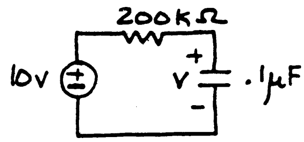
now $V(.5) = 1 - e^{-10(.5)} = .993$ v

$\therefore\ \underline{V(t) = .993e^{-10(t-.5)} \text{ v}}$



Ex. 9-6 $\quad \underline{t<0}$ no sources $\therefore V(0^-) = V(0^+) = 0$

$\underline{0<t<t_1}$



$V(t) = V(\infty) + Ae^{-t/RC} = V(\infty) + Ae^{-t/2\times10^5(10^{-7})}$

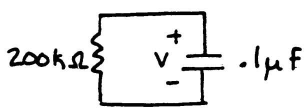
where for $t=\infty$ (steady-state)

$\therefore$ capacitor becomes an open $\Rightarrow V(\infty) = 10v$

$V(t) = 10 + Ae^{-50t}$

now $V(0) = 0 = 10 + A \Rightarrow A = -10 \quad \therefore \underline{V(t) = 10(1-e^{-50t})\,v}$

$\underline{t>t_1}$, $t_1 = .1s$



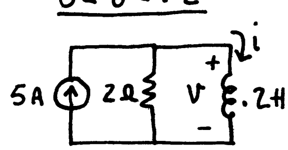
$V(t) = V(.1)e^{-50(t-.1)}$

where $V(.1) = 10(1-e^{-50(.1)}) = 9.93\,v$

$\therefore \underline{v(t) = 9.93\,e^{-50(t-.1)}\,v}$

Ex 9-7 $\quad \underline{\text{for } t<0} \quad i = 0$

$\underline{0<t<.2}$



$$\left.\begin{array}{l} \text{KCL: } -5 + v/2 + i = 0 \\ \text{also: } v = .2\dfrac{di}{dt} \end{array}\right\} \frac{di}{dt} + 10i = 50$$

$\therefore i(t) = 5 + Ae^{-10t}$

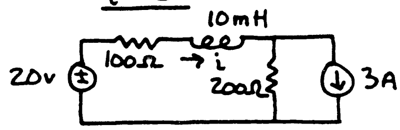
$i(0) = 0 = 5 + A \Rightarrow A = -5$

So have $\underline{i(t) = 5(1-e^{-10t})\,A}$

$\underline{t>.2}$

$i(.2) = 4.32\,A \qquad \therefore \underline{i(t) = 4.32\,e^{-10(t-.2)}\,A}$

Ex. 9-8 $\underline{t>0}$

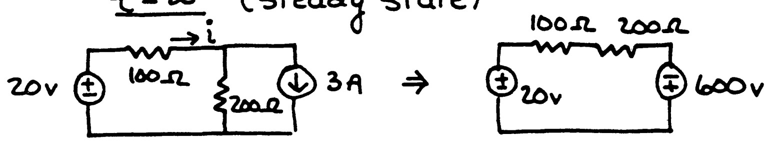


$i(t) = i(\infty) + Ae^{-R/L\,t}$

$\Rightarrow R = 100 + 200 = 300\,\Omega$

$i(t) = i(\infty) + Ae^{-30{,}000t}$

$\underline{t=\infty}$ (steady state)

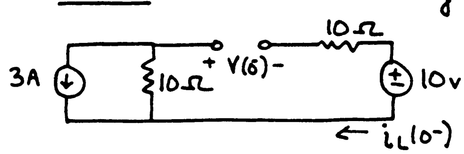


$$i(\infty) = \frac{600v + 20v}{100\Omega + 200\Omega} = 2.067\ A$$

now $i(0^-) = i(0^+) = 0 = 2.067 + A \quad \Rightarrow A = -2.067$

$$\therefore\ \underline{i(t) = 2.067(1 - e^{-30{,}000t})\ A}$$

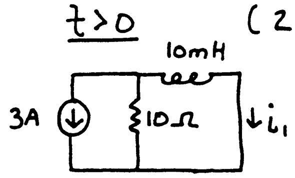
Ex. 9-9 $\underline{t=0^-}$ (assume steady state)

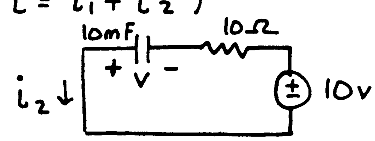


by inspection $i_L(0^-) = 0 = i_L(0^+)$

KVL: $3(10) + V + 10 = 0 \quad \Rightarrow V(0^-) = V(0^+) = -40v$

$\underline{t>0}$ (2 circuits where $i = i_1 + i_2$)





$i_1 = i_1(\infty) + Ae^{-10t/.01}$

$i_1 = -3 + Ae^{-1000t}$

now $i_1(0^+) = i_L(0^+) = 0 = -3 + A$

$\therefore\ i_1(t) = -3 + 3e^{-1000t}\ A$

$i_2 = i_2(\infty) + Be^{-t/RC}$

$\quad = 0 + Be^{-t/10(.01)}$

$i_2 = Be^{-10t}$

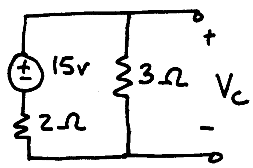
now $i_2(0^+) = \dfrac{V(0^+) + 10}{10} = -3A$

$\therefore\ -3 = B$

$\therefore\ i_2(t) = -3e^{-10t}\ A$

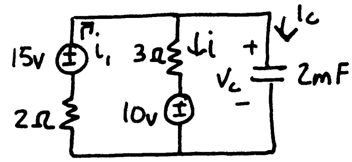
$$\therefore\ \underline{i(t) = -3 + 3e^{-1000t} - 3e^{-10t}\ A}$$

Ex 9-10 $t<0$



$$V_c(0^-) = \frac{3}{2+3}\,15 = 9\text{v} = V_c(0^+)$$

$t>0$



KCL yields: $i_1 = i + 2\times10^{-3}\dfrac{dV_c}{dt}$ (1)

KVL : $15 = V_c + 2i_1$ (2)

KVL : $10 + 3i = V_c$ (3)

plugging (2) & (3) into (1) yields $\dfrac{dV_c}{dt} + \dfrac{5000}{12}V_c = \dfrac{65000}{12}$

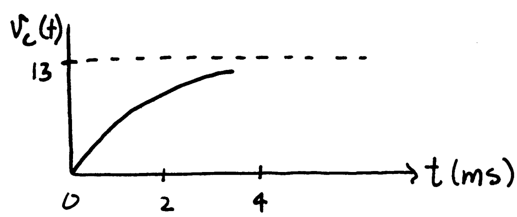
$\tau = 2.4\,\text{ms}$

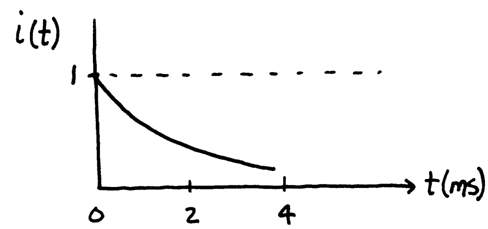
$\therefore V_c(t) = 13 + Ae^{-5000/12\,t}$

now $V_c(0) = 9 = 13 + A \Rightarrow A = -4$

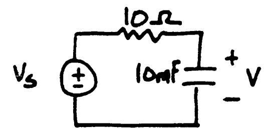
$V_c(t) = 13 - 4e^{-\frac{5000}{12}t}$ V

& $i(t) = \frac{1}{3}V_c - \frac{10}{3} = 1 - \frac{4}{3}e^{-\frac{5000}{12}t}$ A





Ex. 9-11



$V_s = 10 \sin 20t$ v

KVL: $-10\sin 20t + 10\left(.01\frac{dv}{dt}\right) + V = 0$

$\Rightarrow \frac{dv}{dt} + 10V = 100\sin 20t$

<u>natural response</u> : $s + 10 = 0 \Rightarrow s = -10 \quad \therefore V_n(t) = Ae^{-10t}$

<u>forced response</u> : try $V_f(t) = B_1 \cos 20t + B_2 \sin 20t$

plugging $V_f(t)$ into the diff. eqn. and equating like terms yields : $B_1 = -40$ & $B_2 = 20$

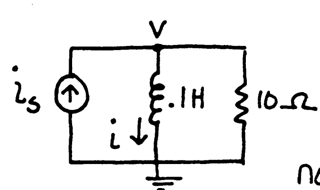
<u>complete response</u> : $V(t) = V_n(t) + V_f(t)$

$V(t) = Ae^{-10t} - 40\cos 20t + 20\sin 20t$

now $V(0^-) = V(0^+) = 0 = A - 40 \quad \therefore A = 40$

$\therefore V(t) = 40e^{-10t} - 40\cos 20t + 20\sin 20t$ v

Ex. 9-12



$i_s = 10e^{-5t}$

KCL at top node: $-10e^{-5t} + i + V/10 = 0$

now $V = .1\frac{di}{dt} \Rightarrow \frac{di}{dt} + 100i = 1000e^{-5t}$

natural response : $s + 100 = 0 \Rightarrow s = -100 \quad \therefore i_n(t) = Ae^{-100t}$

forced response : try $i_f(t) = Be^{-5t}$ & plug into D.E.

$\Rightarrow -5Be^{-5t} + 100Be^{-5t} = 1000e^{-5t}$

$\Rightarrow B = 10.53$

complete response : $i(t) = Ae^{-100t} + 10.53e^{-5t}$

now $i(0^-) = i(0^+) = 0 = A + 10.53 \Rightarrow A = -10.53$

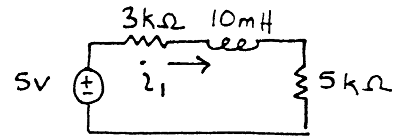
$\therefore i(t) = 10.53(e^{-5t} - e^{-100t})$ A

Ex 9-13 A current $i_L = V_s/1$ flows in the inductor with the switch closed. When the switch opens i_L cannot change instantaneously. Thus, the energy stored in the inductor dissipates in a spark. Add a resistor (say 1 kΩ) accross the switch terminals.

Problems

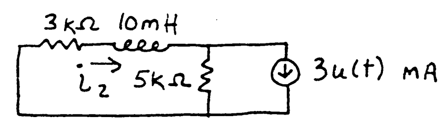
P. 9-1

Consider 5v source only



no changes occur between $-\infty < t < \infty$, so have steady state condition $\Rightarrow i_1 = \frac{5v}{8k\Omega} = 5/8$ mA for all time t

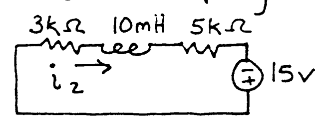
Consider $3u(t)$ source only



for $t<0$, have no source

$\therefore i_2 = 0$

for $t>0$, use source transformation to simplify ckt.



$$i_2(t) = i_2(\infty) + Ae^{-8000t/.01}$$

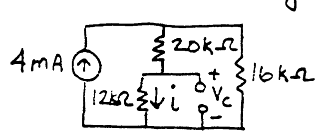
$$= \frac{15v}{8k\Omega} + Ae^{-8\times10^5 t}$$

now $i_2(0) = 0 = \frac{15}{8} + A \Rightarrow A = -15/8 \quad \therefore i_2(t) = 15/8\left(1 - e^{-8\times10^5 t}\right)$ mA

$$\therefore i(t) = i_1(t) + i_2(t) = \begin{cases} 5/8 \text{ mA} & t<0 \\ \frac{5}{2} - \frac{15}{8}e^{-8\times10^5 t} \text{ mA} & t>0 \end{cases}$$

P. 9-2

$t<0$ (steady state)

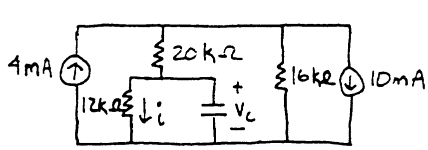


current divider $i(0^-) = 4\left(\frac{16}{16+32}\right) = 4/3$ mA

$\therefore V_c(0^-) = 12\,i(0^-) = 16v = V_c(0^+) = 12\,i(0^+)$

$\therefore i(0^+) = 4/3$ mA

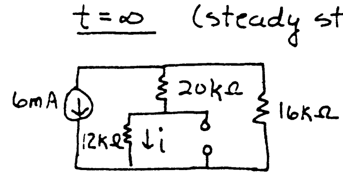
$t>0$



$$i(t) = i(\infty) + Ae^{-t/RC} \qquad R = 12//32 = 9k\Omega$$

$$= i(\infty) + Ae^{-2000t} \qquad C = 1/18\ \mu F$$

$t=\infty$ (steady state)



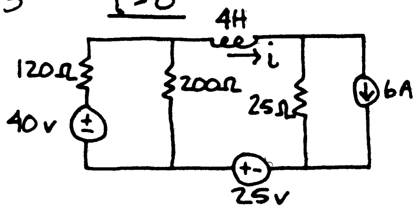
$$i(\infty) = -6\left[\frac{16}{16+32}\right] = -2\text{mA}$$

$$\therefore i(t) = -2 + Ae^{-2000t}$$

now $i(0) = 4/3 = -2 + A \Rightarrow A = 10/3$

$$\therefore i(t) = 10/3\, e^{-2000t} - 2 \text{ mA} \qquad t>0$$

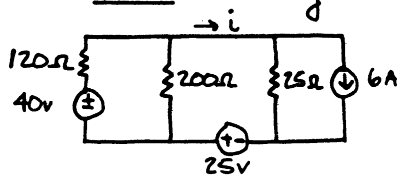
P. 9-3 $\underline{t>0}$



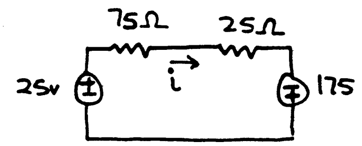
$i(t) = i(\infty) + Ae^{-R/L\,t}$, $R = 120 \| 200 + 25$

$= i(\infty) + Ae^{-25t}$ $R = 100\Omega$

$\underline{t=\infty}$ (steady state)

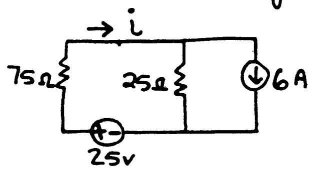


source transforms $\longrightarrow$

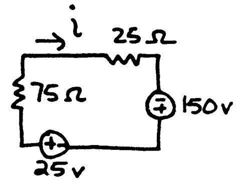


$i(\infty) = \dfrac{200v}{100\Omega} = 2A$

$\underline{t<0}$ (steady state)



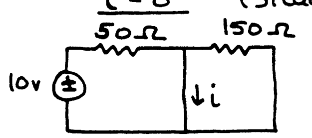
$\Rightarrow$



$i(0) = \dfrac{175v}{100\Omega} = 7/4 \text{ A}$

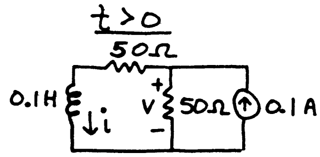
now, $i(0) = 7/4 = 2 + A \Rightarrow A = -1/4$ $\quad \therefore \underline{i(t) = 2 - 1/4\, e^{-25t} u(t) \text{ A}}$

P. 9-4 $\underline{t=0^-}$ (steady state)



$i(0^-) = \dfrac{10v}{100\Omega} = 0.1\text{ A} = i(0^+)$

$\underline{t>0}$



$i(t) = i(\infty) + Ae^{-100/.1\, t} = i(\infty) + Ae^{-1000t}$

by inspection at $t=\infty$ $\quad i(\infty) = .1A/2 = .05A$

now $i(0) = 0.1 = .05 + A \Rightarrow A = .05$

$\therefore \underline{i(t) = .05(1 + e^{-1000t}) \text{ A}}$

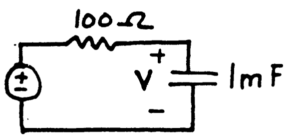
KCL at top node : $i + V/50 - 0.1 = 0$

$\Rightarrow V = 5 - 50i = 5 - 2.5 - 2.5e^{-1000t}$ v

$\underline{v(t) = 2.5(1 - e^{-1000t}) \text{ volts}}$

P. 9-5 $\underline{t<0}$, capacitor is uncharged $\Rightarrow V=0$

$\underline{0<t<.2s}$



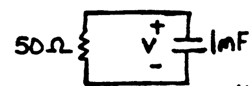
$v(t) = v(\infty) + Ae^{-t/RC}$

$= 200 + Ae^{-t/100(10^{-3})}$

$v(t) = 200 + Ae^{-10t}$; $v(0) = 0 = 200 + A \Rightarrow A = -200$

$\therefore \underline{v(t) = 200(1-e^{-10t})\ v}$

$\underline{t>.2s}$

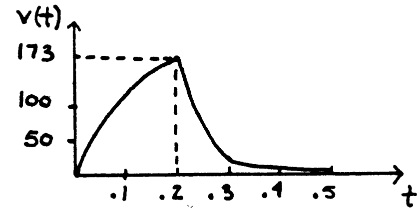


$v(t) = v(.2)e^{-\frac{(t-.2)}{(50)(10^{-3})}}$

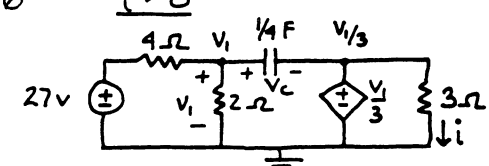
$= v(.2)e^{-20(t-.2)}$

$v(.2) = 200(1-e^{-10(.2)}) = 173$

$\therefore \underline{v(t) = 173e^{-20(t-.2)}}$



P. 9-6 $\underline{t>0}$



KCL at v_1: $(v_1-27)/4 + v_1/2 + 1/4\frac{dv_c}{dt} = 0$

$\underline{3v_1 + \frac{dv_c}{dt} - 27 = 0}$ (1)

also: $\underline{v_c = v_1 - 3i}$ (2) and $3i = v_1/3 \Rightarrow \underline{v_1 = 9i}$ (3)

(2) into (1) $\Rightarrow$ $\underline{3v_1 + dv_1/dt - 3\,di/dt - 27 = 0}$ (4)

(3) into (4) $\Rightarrow$ $\underline{di/dt + 9/2\, i = 9/2}$

$\therefore i(t) = A_1e^{-9/2t} + A_2 \Rightarrow$ with $i_f(t) = A_2$ & plugging back into the diff. eqn. $\Rightarrow A_2 = 1$

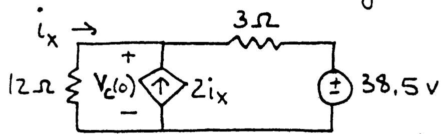
$\therefore i(t) = A_1e^{-9/2t} + 1$

now $v_c(0) = 12v$ $\therefore$ from (2) $\Rightarrow v_1 - 3i = 12$
" (3) $\Rightarrow v_1 = 9i$ $\Big\}$ $i(0) = 2A$

now $i(0) = 2 = A_1 + 1 \Rightarrow A_1 = 1$

$\therefore \underline{i(t) = 1 + e^{-9/2t}\ A}$

P. 9-7 $\underline{t=0^-}$ (steady-state)

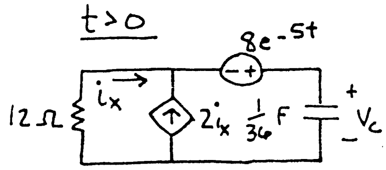


KVL: $12i_x + 3(3i_x) + 38.5 = 0$

$\Rightarrow i_x = -1.8\bar{3}$ A

also $\underline{V_c(0^-) = -12i_x = 22\text{v} = V_c(0^+)}$

$\underline{t>0}$



KVL: $\underline{12i_x - 8e^{-5t} + V_c = 0}$ (1)

KCL: $-i_x - 2i_x + \frac{1}{36}\frac{dV_c}{dt} = 0$

$\underline{i_x = \frac{1}{108}\frac{dV_c}{dt}}$ (2)

(2) into (1) yields

$\frac{dV_c}{dt} + 9V_c = 72e^{-5t} \Rightarrow V_{cn}(t) = Ae^{-9t}$

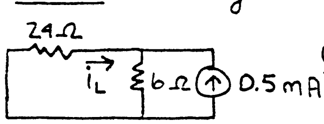
try $V_{cf}(t) = Be^{-5t}$ & plug into D.E.

$\Rightarrow B = 18$

$\therefore V_c(t) = Ae^{-9t} + 18e^{-5t}$

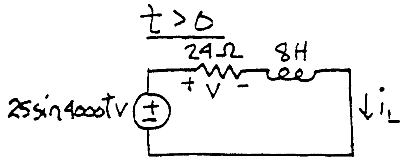
$V_c(0) = 22 = A + 18 \Rightarrow A = 4$ $\quad$ $\underline{\therefore V_c(t) = 4e^{-9t} + 18e^{-5t}\text{ v}}$

P. 9-8 $\underline{t=0^-}$ (steady-state)



current divider $\Rightarrow i_L(0^-) = -.5\left(\frac{6}{6+24}\right) = -0.1$ mA

$\underline{t>0}$



KVL: $-25\sin 4000t + 24i_L + .008\frac{di_L}{dt} = 0$

$\underline{\frac{di_L}{dt} + 3000i_L = \frac{25}{.008}\sin 4000t}$

$i_{L_n}(t) = Ae^{-3000t}$, try $i_{L_f}(t) = B\cos 4000t + C\sin 4000t$ & plug into D.E.

and equate like terms $\Rightarrow B = -1/2$, $C = 3/8$

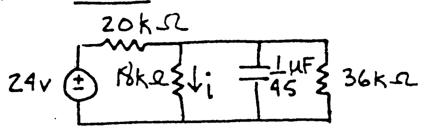
$\therefore i_L(t) = Ae^{-3000t} - 1/2\cos 4000t + \frac{3}{8}\sin 4000t$

$i_L(0^+) = i_L(0^-) = -.1 = A - 1/2 \Rightarrow A = 2/5$ mA

$\therefore i_L(t) = 2/5\, e^{-3000t} - 1/2\cos 4000t + 3/8\sin 4000t$ mA

but $\underline{V(t) = 24i_L(t) = 9.6\,e^{-3000t} - 12\cos 4000t + 9\sin 4000t\text{ v}}$

P. 9-9 $\underline{t>0}$



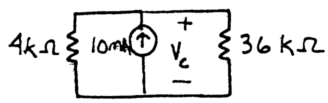
$$i(t) = i(\infty) + Ae^{-t/RC} \qquad \text{where } R = 20\|18\|36$$
$$= i(\infty) + Ae^{-6000t} \qquad = 7.5\,k\Omega$$

at $t=\infty$ (steady-state) $\therefore$ from voltage division $18i = 24\left(\frac{18\|36}{20+18\|36}\right)$

$\Rightarrow i(\infty) = 1/2\ mA$

$\therefore i(t) = 1/2 + Ae^{-6000t}$

$\underline{t=0^-}$ (steady-state)



current division $\Rightarrow \frac{V_c(0^-)}{36} = 10\left(\frac{4}{4+36}\right) = 1$

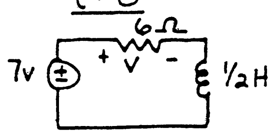
$\therefore V_c(0^-) = 36v = V_c(0^+)$

$\underline{t=0^+}$

$V_c(0^+) = 18\,i(0^+) \quad \Rightarrow i(0^+) = \frac{36}{18} = 2mA = 1/2 + A \quad \Rightarrow A = 3/2$

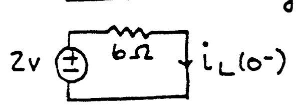
$\therefore \underline{i(t) = 3/2\, e^{-6000t} + 1/2 \ \ mA}$

P. 9-10 $\underline{t>0}$



$$V(t) = V(\infty) + Ae^{-R/L\,t} = 7 + Ae^{-12t}$$

$\underline{t=0^-}$ (steady-state)

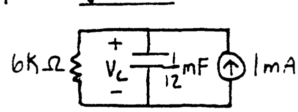


$i_L(0^-) = i_L(0^+) = \frac{2v}{6\Omega} = 1/3\,A$

$\therefore V(0^+) = 6\,i_L(0^+) = 2v$

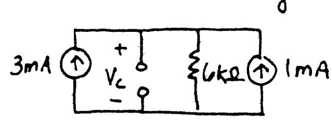
$\therefore V(0) = 2 = 7 + A \Rightarrow A = -5 \qquad \therefore \underline{V(t) = 7 - 5e^{-12t}\ v}$

P. 9-11 $\underline{t>0}$



$$V_c(t) = V(\infty) + Ae^{-t/RC} = (6k\Omega)(1mA) + Ae^{-2t}$$

$\underline{t=0^-}$ (steady-state)



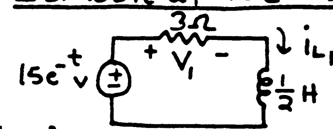
$V_c(0^-) = (4mA)(6k\Omega) = 24\,v = V_c(0^+)$

now $V_c(0) = 24 = 6 + A \Rightarrow A = 18$

$\therefore \underline{V_c(t) = 6 + 18e^{-2t}\ v}$

P. 9-12 use superposition, <u>1st look at $15e^{-t}u(t)$ source</u>

$t>0$

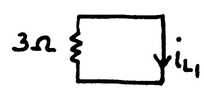


KVL: $-15e^{-t} + 3i_{L_1} + \frac{1}{2}\frac{di_{L_1}}{dt} = 0 \Rightarrow \underline{\frac{di_{L_1}}{dt} + 6i_{L_1} = 30e^{-t}}$

$\therefore\ i_{L_1}(t) = Ae^{-6t} + i_{Lp}(t) \Rightarrow$ try $i_{Lp}(t) = Be^{-t}$ & plug into D.E.

$i_{L_1}(t) = Ae^{-6t} + 6e^{-t}$ $\Rightarrow B = 6$

<u>$t=0^-$</u> (steady state)



$i_{L_1}(0^-) = 0 = i_{L_1}(0^+)$

$\therefore\ i_{L_1}(t) = -6e^{-6t} + 6e^{-t}$

now $\underline{V_1(t) = 3i_{L_1}(t) = -18e^{-6t} + 18e^{-t}}$ v $u(t)$

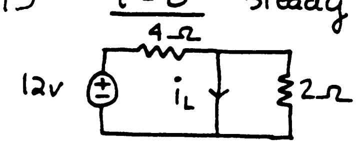
<u>now consider $-15e^{-t}\,u(t-1)$ source</u>

just take previous result and change $t \rightarrow t-1$ and flip signs

$\therefore\ \underline{V_2(t) = 18e^{-6(t-1)} - 18e^{-(t-1)}}\ u(t-1)$

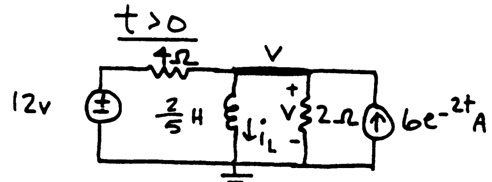
$\therefore\ \underline{V(t) = [-18e^{-6t} + 18e^{-t}]u(t) + [18e^{-6(t-1)} - 18e^{-(t-1)}]\,u(t-1)}$

P. 9-13 <u>$t=0^-$</u> steady state



$i_L(0^-) = \frac{12v}{4\Omega} = 3A = i_L(0^+)$

<u>$t>0$</u>



KCL at V: $\frac{(V-12)}{4} + i_L + \frac{V}{2} = 6e^{-2t}$ (1)

also: $V = \frac{2}{5}\,di_L/dt$ (2)

plugging (2) into (1) $\Rightarrow$ $di_L/dt + \frac{10}{3}i_L = 10 + 20e^{-2t}$

$\therefore\ i_{Ln}(t) = Ae^{-10/3t}$, try $i_{Lf}(t) = B + Ce^{-2t}$ & plugging into D.E.

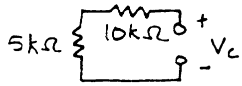
& equating like terms $\Rightarrow B = 3,\ C = 15$

$\therefore\ i_L(t) = Ae^{-10/3t} + 3 + 15e^{-2t}$, $i_L(0) = 3 = A + 3 + 15 \Rightarrow A = -15$

$\therefore\ i_L(t) = -15e^{-10/3t} + 3 + 15e^{-2t}$

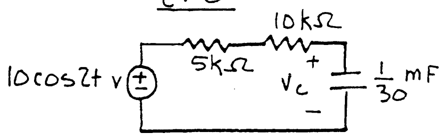
now $\underline{V(t) = \frac{2}{5}\frac{di_L}{dt} = 20e^{-10/3t} - 12e^{-2t}}$ v

P. 9-14 $\underline{t=0^-}$ steady state



$v_c(0^-) = 0 = v_c(0^+)$

$\underline{t>0}$



KVL: $-10\cos 2t + 15\left(\frac{1}{30}\frac{dv_c}{dt}\right) + v_c = 0$

$\hookrightarrow \frac{dv_c}{dt} + 2v_c = 20\cos 2t$

$v_{cn}(t) = Ae^{-2t}$, try $v_{cf}(t) = B\cos 2t + C\sin 2t$ & plug into D.E.

$\Rightarrow B = C = 5$

$\therefore v_c(t) = Ae^{-2t} + 5\cos 2t + 5\sin 2t$

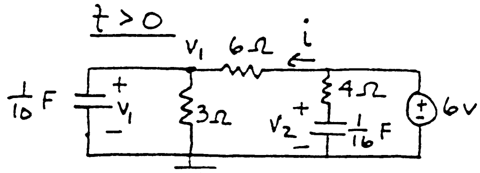
now $v_c(0) = 0 = A + 5 \Rightarrow A = -5$

$\therefore \underline{v_c(t) = -5e^{-2t} + 5\cos 2t + 5\sin 2t \text{ V}}$

P. 9-15 $\underline{t=0^-}$ steady state

no sources present $\therefore v_1(0^-) = v_2(0^-) = i(0^-) = 0$

$\underline{t>0}$



KVL around right loop

$v_2 - 6 + 4\left(\frac{1}{16}\frac{dv_2}{dt}\right) = 0$

$\hookrightarrow \frac{dv_2}{dt} + 4v_2 = 24$

$\therefore v_{2n}(t) = Ae^{-4t}$, try $v_{2f}(t) = B$ & plug into D.E. $\Rightarrow B = 6$

$\Rightarrow v_2(t) = Ae^{-4t} + 6 \Rightarrow v_2(0) = 0 = A + 6$ $\therefore \underline{v_2(t) = 6(1 - e^{-4t}) \text{ V}}$

KCL at v_1: $\frac{1}{10}\frac{dv_1}{dt} + \frac{v_1}{3} + \frac{(v_1 - 6)}{6} = 0 \Rightarrow \frac{dv_1}{dt} + 5v_1 = 10$

$\therefore v_{1n}(t) = Ae^{-5t}$, try $v_{1f}(t) = B$ & plug into D.E. $\Rightarrow B = 2$

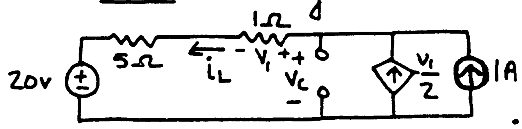
$\therefore v_1(t) = Ae^{-5t} + 2 \Rightarrow v_1(0) = 0 = A + 2$ $\therefore \underline{v_1(t) = 2(1 - e^{-5t}) \text{ V}}$

now $i(t) = \frac{6 - v_1(t)}{6} = 1 - \frac{1}{6}v_1(t)$

$= 1 - \frac{1}{3}(1 - e^{-5t})$

$\underline{i(t) = \frac{1}{3}(2 + e^{-5t}) \text{ A}}$

P. 9-16 $\underline{t=0^-}$ steady state



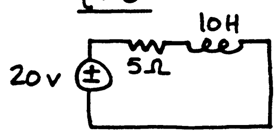
KCL: $i_L(0^-) = V_1/2 + 1 = V_1/1$

$\Rightarrow V_1 = 2v$

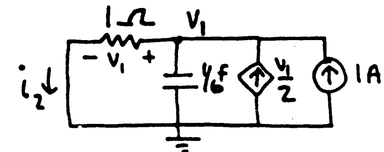
$\therefore i_L(0^-) = i_L(0^+) = 2A$

KVL: $-20 + 5(-2) - 2 + V_c = 0 \Rightarrow V_c(0^-) = V_c(0^+) = 32v$

$\underline{t>0}$



and



$i_1(t) = i_1(\infty) + Ae^{-1/2t}$

$= 4 + Ae^{-1/2t}$

$i_1(0) = -2 = 4 + A \Rightarrow A = -6$

$\therefore \underline{i_1(t) = 4 - 6e^{-1/2t} \text{ A}}$

KCL at V_1: $i_2 + 1/6 \frac{dv_1}{dt} - \frac{V_1}{2} - 1 = 0$ (1)

also: $V_1 = (1)(i_2)$ (2)

(2) into (1) yields

$\frac{di_2}{dt} + 3i_2 = 6$

$\therefore i_{2n}(t) = Ae^{-3t}$, try $i_{2f}(t) = B$ & plug into the D.E. $\Rightarrow B = 2$

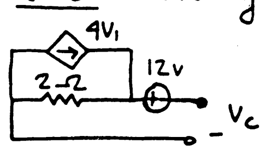
$\therefore i_2(t) = Ae^{-3t} + 2$, $i_2(0) = 32 = A + 2$

$\Rightarrow A = 30$

$\Rightarrow \underline{i_2(t) = 30e^{-3t} + 2 \text{ A}}$

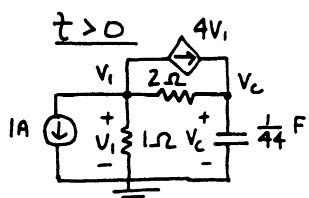
$\therefore \underline{i(t) = i_1(t) + i_2(t) = 6 - 6e^{-t/2} + 30e^{-3t} \text{ A}}$

P. 9-17 $\underline{t=0^-}$ (steady state)



$V_1 = 0 \quad \therefore V_c(0^-) = -12v = V_c(0^+)$

$\underline{t>0}$



KCL at V_1 : $1 + V_1 + (V_1 - V_c)/2 + 4V_1 = 0$ (1)

KCL at V_c : $-4V_1 + (V_c - V_1)/2 + 1/44 \frac{dV_c}{dt} = 0$

$\Rightarrow V_1 = 1/9\left(V_c + 1/22 \frac{dV_c}{dt}\right)$ (2)

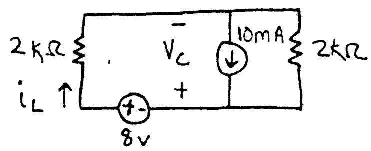
plugging (2) into (1) yields $\frac{dV_c}{dt} + 4V_c = -36$

$V_{cn}(t) = Ae^{-4t}$ and try $V_{cf}(t) = B \Rightarrow 4B = -36 \Rightarrow B = -9$

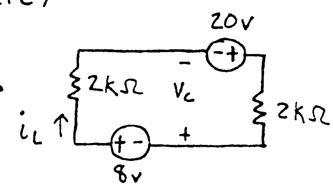
$\therefore V_c(t) = Ae^{-4t} - 9$, now $V_c(0) = -12 = A - 9 \Rightarrow A = -3$

$\underline{V_c(t) = -3e^{-4t} - 9 \text{ v}}$

P. 9-18 $t = 0^-$ (steady state)



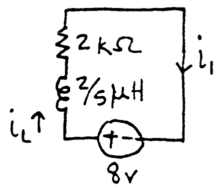
$\Rightarrow$



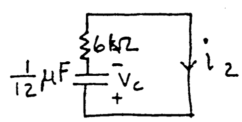
$i_L(0^-) = i_L(0^+)$
$= \frac{28v}{4\Omega} = 7mA$

$V_c(0^-) = V_c(0^+) = 2i_L(0) - 8 = 6v$

$t > 0$ get 3 independent circuits



$i_1(t) = i_1(\infty) + A_1 e^{-5000t}$ where $i_1(\infty) = 8v/2k\Omega = 4mA$
$= 4 + A_1 e^{-5000t}$
now $i_1(0) = i_L(0) = 7 = 4 + A_1 \Rightarrow$ $\underline{i_1(t) = 4 + 3e^{-5000t}\ mA}$



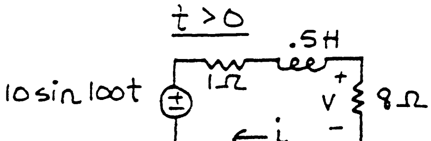
$i_2(t) = A_2 e^{-2000t}$
now $i_2(0) = {-V_c(0)}/{6k\Omega} = -6/6 = -1mA$
$\therefore$ $\underline{i_2(t) = -e^{-2000t}\ mA}$

$\underline{i_3(t) = -10mA}$

$\therefore$ $\underline{i(t) = i_1(t) + i_2(t) + i_3(t) = 3e^{-5000t} - e^{-2000t} - 6\ mA}$

P. 9-19 $t < 0$ no sources present $\therefore i(0) = 0$

$t > 0$



KVL: $-10\sin 100t + i + .5\,di/dt + V = 0$ (1)
also : $i = V/8$ (2)
(2) into (1) yields $\underline{\frac{dv}{dt} + 18V = 160\sin 100t}$

$\therefore V_n(t) = Ae^{-18t}$, try $V_f(t) = B\cos 100t + C\sin 100t$ & plug into the D.E. & equate like terms
$\Rightarrow B = -1.55$ & $C = 0.279$

$\therefore V(t) = Ae^{-18t} - 1.55\cos 100t + 0.279\sin 100t$
now from (2) : $V(0) = 8i(0) = 0$
$\therefore V(0) = 0 = A - 1.55 \Rightarrow A = 1.55$
so $\underline{V(t) = 1.55e^{-18t} - 1.55\cos 100t + 0.279\sin 100t\ \ V}$

P. 9-20 (a) KVL ↻: $-V_o + L\,di/dt + \alpha i = 0 \Rightarrow \underline{di/dt + \frac{\alpha}{L} i = \frac{V_o}{L}}$

$\therefore\ i(t) = i(\infty) + Ae^{-\alpha/L\,t}$

choose $i(\infty) = i_f(t) = B$ & plug into D.E. $\Rightarrow B = V_o/\alpha$

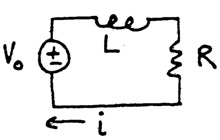
$\therefore\ i(t) = V_o/\alpha + Ae^{-\alpha/L\,t}$ but $i(0) = I_o = V_o/\alpha + A$

$\Rightarrow A = I_o - V_o/\alpha$

so $\underline{i(t) = V_o/\alpha + (I_o - V_o/\alpha)e^{-\alpha/L\,t}}$

(b) for a series RL circuit we have

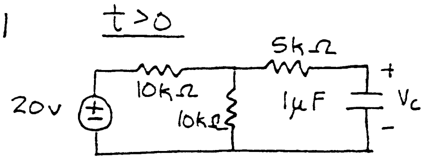
$i(t) = i_f(t) + Ae^{-R/L\,t}$



Comparing this expression with the one in part a), see that the two become equivalent when $\alpha = R$ $\therefore$ under this specification, the above circuit is equivalent to the given circuit.

(c) If α is less than zero, then the circuit is no longer a decay circuit but rather an exponentially growing circuit. Therefore it would be impossible to realize an equivalent circuit with only passive circuit elements since a negative resistance would be required.

P. 9-21 $\underline{t>0}$



$V_c(t) = V_c(\infty) + Ae^{-t/RC}$ where $R = 5k\Omega + 10k\Omega \| 10k\Omega = 10k\Omega$

from voltage divider

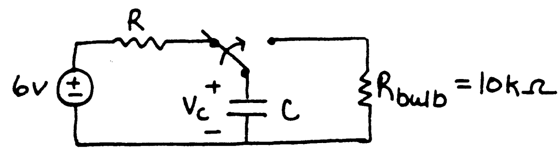
$V(\infty) = 20\left(\frac{10}{10+10}\right) = 10\text{V}$

$\therefore\ V_c(t) = 10 + Ae^{-100t}$

now $V_c(0) = 0 = 10 + A \Rightarrow A = -10$

$\underline{V_c(t) = 10(1 - e^{-100t})\ \text{V}}$

P. 9-22



Assume capacitor is charged at $t = 5\tau$, ie. $\frac{V_c}{V_{cmax}} = 1 - e^{-5} = .993$

Similarly, assume the capacitor is discharged when $\frac{V_c}{V_{cmax}} = e^{-5} = 6.74 \times 10^{-3}$.

Now determine C from discharging condition

$V_c(t) = V_c(t_0)e^{-(t-t_0)/CR_{bulb}} \Rightarrow 6.74 \times 10^{-3} = e^{-0.5/CR_{bulb}}$

$\Rightarrow \underline{C = 10^{-5}\,f = 10\mu f}$

Now determine a condition for R from charging circuit at the instant $V_c = 0 \Rightarrow \frac{6v}{R} < 100 \times 10^{-6}$ A

$\Rightarrow R > 60k\Omega$

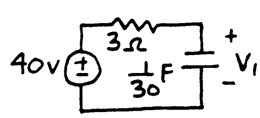
then for the charging ckt.

$.993 = 1 - e^{-5/RC}$

$-4.96 = -5/RC \quad \Rightarrow \underline{R = \frac{5}{(4.96)(10^{-5})} = 100.8\,k\Omega}$

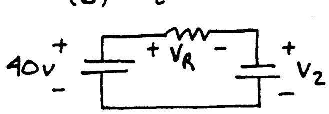
and see that $R \simeq 100k\Omega > 60k\Omega$.

P. 9-23 (a) $\underline{t = 0^-}$ (steady state)



no current flows $\therefore \underline{V_1(0^-) = 40\text{ v}}$

(b) $t = 0^+$



$V_1(0^-) = V_1(0^+) = 40v$

$V_2(0^-) = V_2(0^+) = 0$

$\therefore V_R(0^+) = V_1(0^+) = 40v$

(c) $V_R(t) = V_R(0)e^{-t/RC}$ where $R = 5\Omega$ & $C = (30 + 60)^{-1} = 1/90\,F$

$\underline{V_R(t) = 40e^{-18t}\text{ v}}$

(d) $\underline{i(t) = V_R(t)/5 = 8e^{-18t}\text{ A}}$

P. 9-23 cont.

(e) $V_1(t) = V_1(0) + \frac{1}{1/30}\int_0^t -i(t)dt = 40 - 30\int_0^t 8e^{-18t}dt$

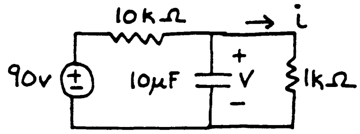
$= 40 + 40/3\,(e^{-18t}-1)$

$\underline{V_1(t) = 80/3 + 40/3\,e^{-18t} \text{ V}}$

$V_2(t) = V_2(0) + \frac{1}{1/60}\int_0^t i(t)\,dt = 0 + 60\int_0^t 8e^{-18t}dt$

$\underline{V_2(t) = 80/3 - 80/3\,e^{-18t} \text{ V}}$

P. 9-24 Let $t=0$ be the instant the bulb turns on. $\therefore V(0) = 65\text{v}$ in the circuit with the bulb as a $1k\Omega$ resistor.



$v(t) = v(\infty) + Ae^{-t/RC}$

where $R = 1k\Omega \| 10k\Omega = 0.909\,k\Omega$

$v(\infty) = 90\left(\frac{1}{10+1}\right) = 8.18\text{v}$

$\therefore v(t) = 8.18 + Ae^{-110t}$, $v(0) = 65 = 8.18 + A \Rightarrow A = 56.82$

so $\underline{v(t) = 8.18 + 56.82e^{-110t} \text{ v}}$, now $i(t) = v(t)/1k\Omega$

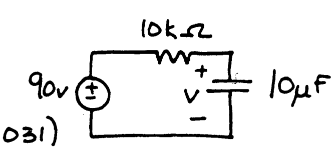
$\therefore \underline{i(t) = 8.18 + 56.82e^{-110t} \text{ mA}}$

for flashing rate, find t when $i = I_s = 10\text{mA}$

$10 = 8.18 + 56.82e^{-110t} \Rightarrow t = 0.031\text{ s}$

at this instant, $v = (10\text{mA})(1k\Omega) = 10$ volts

now treat $v = 10$ volts as initial condition for circuit when the bulb is open



$\underline{t > 0.031\text{s}}$

$v(.031) = 10$ volts

$v(\infty) = 90\text{v}$, $R = 10k\Omega$ $\Rightarrow v(t) = 90 + Be^{-10(t-.031)}$

$v(.031) = 10 = 90 + B \Rightarrow B = -80$

$\therefore \underline{v(t) = 90 - 80e^{-10(t-.031)} \text{ volts}}$ & $\underline{i(t) = 0}$

now determine when $v(t)$ reaches 65v

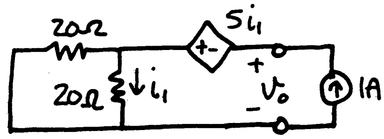
$\Rightarrow 65 = 90 - 80e^{-10(t-.031)} \Rightarrow t = 0.147\text{s}$

$\therefore$ flashing rate $= \frac{1}{\text{period}} = \frac{1}{.147\text{s}} = 6.8\text{ s}^{-1}$

P 9-25 $v_{oc} = v_T = \left(40 \times \frac{20}{20+20}\right) - 5i_1$

$= 20 - 5\left(\frac{40}{40}\right) = \underline{15\text{ v}}$

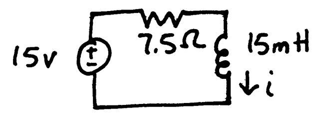
for R_T, kill source with $v = 0$



note $i_1 = \frac{1}{2}A$

$R_T = \frac{v_o}{1} = 1(10\Omega) - 5\left(\frac{1}{2}A\right) = \underline{7.5\Omega}$

forced response



$i = 2A$

$\tau = L/R = \frac{15 \times 10^{-3}}{7.5} = 2\text{ms}$

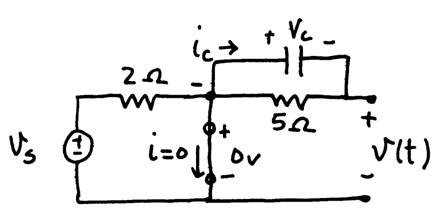
natural : $i = Be^{-t/\tau} = Be^{-500t}$

total : $i = Be^{-500t} + 2$ now $i(0) = 0 \Rightarrow B = -2$

$\underline{i(t) = 2(1 - e^{-500t})\text{ A}}$

for $e^{-500t} = .01$ or $500t = 4.605 \Rightarrow \underline{t = 9.2\text{ms}}$

P 9-26



$v_s = 4(1 - u(t))$

$\underline{t<0}$: KCL at input terminal '-' : $4/2 = \frac{v_c}{5} + i_c$

now $i_c(t) = 0$ $\therefore$ $\underline{v_c(t) = 10\text{v}}$ & $\underline{v(t) = -v_c = -10\text{v}}$

$\underline{t \geq 0}$: $v_s = 0$

so have $C\frac{dv_c}{dt} + \frac{v_c}{5} = 0$ with $C = 1/20$

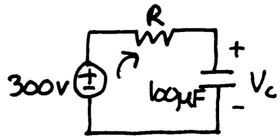
$v_c(t) = v_c(0)e^{-4t} = 10e^{-4t}\text{v}$

and $\underline{v(t) = -v_c(t) = -10e^{-4t}\text{v}}$

So for all t have

$\underline{v(t) = -10 + 10(1 - e^{-4t})u(t) \text{ V}}$

P 9-27 $\quad \underline{t=0}$



$V_c(0^-) = V_c(0^+) = 30v$ = voltage when conduction of tube terminates

KVL : $\frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{300}{RC} \quad \Rightarrow V_c(t) = 300 + Ae^{-t/RC}$

$V_c(0) = 30 = 300 + A \quad \Rightarrow A = -270$

so $V_c(t) = 300 - 270e^{-t/RC}$

Because the resistance of photoflash is equal to zero, the discharge of the capacitor takes no time. So when $t = 10s$, $V_c(t)$ should equal 240v $\Rightarrow \; 240 = 300 - 270e^{-10/RC}$

$\therefore \; RC = -10/\ln \frac{2}{9} = 6.65$

$\Rightarrow R = 6.65/10^{-4} = \underline{66.5 \text{ k}\Omega}$

Energy transferred to the tube $= \frac{1}{2}C(V_i^2 - V_f^2)$

$= \frac{1}{2}10^{-4}\left((240)^2 - (30)^2\right) = 2.84 \text{ J}$

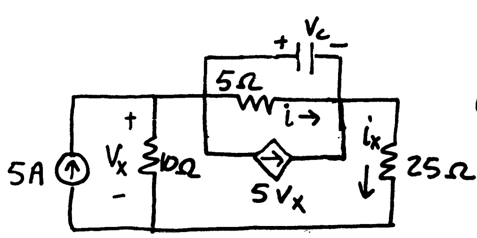
If wait 20s instead of 10s

$V_c(t) = 300 - 270e^{-t/RC}\Big|_{t=20s} = 300 - 270e^{-20/6.65} = 286.7v$

Then energy transfer $= \frac{1}{2}10^{-4}\left[(286.7)^2 - (30)^2\right] = 4.06 \text{ J}$

% increase in energy delivered $= \frac{4.06 - 2.84}{2.84} \times 100\% = \underline{42.9\%}$

P 9-28



using KVL, KCL :

$5 = \frac{V_x}{10} + i + 2\times10^{-3}\frac{dV_c}{dt} + 5V_x \qquad (1)$

$V_x = V_c + 25i_x \qquad (2)$

$i_x = 5 - V_x/10 \qquad (3)$

$V_c = 5i \qquad (4)$

solving (1)-(4) yields

$\frac{dV_c}{dt} + 828.5V_c = -8.855\times10^4 \quad \Rightarrow V_c(t) = -106.9 + Ae^{-828.5t}$

now $V_c(0) = 2.14 = -106.9 + A \quad \Rightarrow A = 109.0$

$\therefore \; V_c(t) = -106.9 + 109.04e^{-828.5t} \; V$

$\underline{i(t) = \frac{V}{5} = -21.38 + 21.808e^{-828.5t} \; A}$

P 9-29 $i_s = 40[u(t) - u(t-t_o)]$ A $\qquad t_o = 1$ms

for $t<0$ $\quad i_s = 0$

so for $t>0$ $\quad v = v_n + v_f \qquad$ where $v_n = Ae^{-t/\tau}$

$$\tau = L/R = \frac{50\times10^{-3}}{50\Omega} = 1\times10^{-3}\text{s} = 1\text{ms}$$

$$v_f = 20\left(\frac{30}{30+20}\right)40 = 480\text{v}$$

$\therefore v(t) = Ae^{-1000t} + 480$

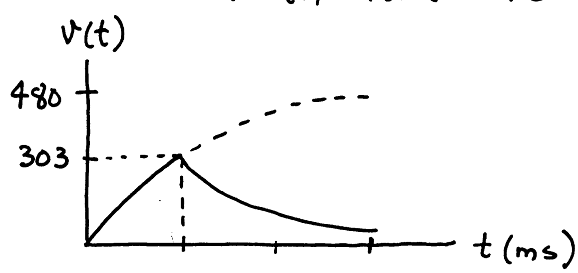
now $i_L(0) = 0$ so $v(0) = 0 \quad \Rightarrow A = -480$

So have $v = \begin{cases} 0 & t<0 \\ 480(1-e^{-1000t}) & 0<t<1\text{ms} \end{cases}$

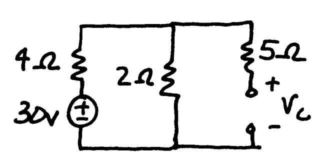
for $t > 1$ms $\quad v = Be^{-1000(t-t_o)}$

$v(1\text{ms}) = B = 480(1-e^{-1})$

$\Rightarrow v(t) = 480(1-e^{-1})e^{-1000(t-t_o)}$



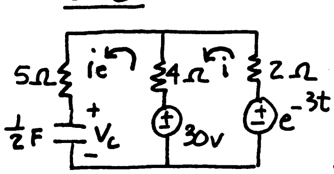
P 9-30 $t=0^-$ (steady state)



$V_c(0^-) = \frac{2}{4+2}30 = 10\text{v}$

$\therefore V_c(0^+) = 10\text{v}$

$t>0$



KVL: $\frac{5}{2}\frac{dV_c}{dt} + V_c + 4\left(\frac{1}{2}\frac{dV_c}{dt} - i\right) = 30 \qquad (1)$

$2i + 4\left(i - \frac{1}{2}\frac{dV_c}{dt}\right) + 30 = e^{-3t} \qquad (2)$

Solving for i in (2) and plugging into (1) yields

$dV_c/dt + 6/19\,V_c = 6/19\,(10 + 2/3\,e^{-3t})$

So $V_{cn} = Ae^{-6/19t}$; try $V_{cf} = B + Ce^{-3t}$ & plug into D.E.

$\Rightarrow -3Ce^{-3t} + \frac{6}{19}(B + Ce^{-3t}) = \frac{60}{19} + \frac{4}{19}e^{-3t}$

equating coeffs and like terms yields

$B = 10, \; C = -4/51 \quad \Rightarrow V_c = 10 - \frac{4}{51}e^{-3t} + Ae^{-6/19t}$

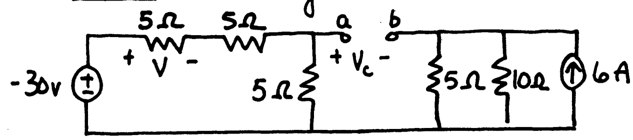
when $t=0$, $V_c(0^+) = 10$v

$\therefore 10 = 10 - 4/51 + A$

$\Rightarrow A = 4/51$

$\therefore V_c(t) = 10 + \frac{4}{51}\left(e^{-6/19t} - e^{-3t}\right)$ V

P 9-31 $\underline{t<0}$ (steady state)

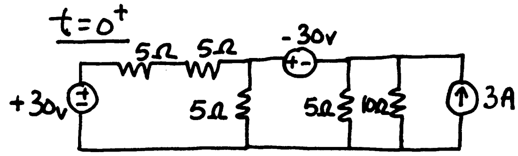


$v_a = \frac{5}{15}(-30) = -10\text{v}$

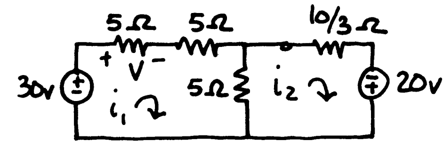
$v_b = (5\|10)6 = 20\text{v}$

$v_c(0^-) = v_a - v_b = -30\text{v} = v_c(0^+)$

$\underline{t=0^+}$



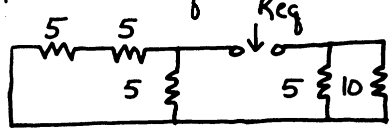
$\Rightarrow$



KVL: $\left.\begin{aligned}(5+5)i_1 + 5(i_1 - i_2) &= 30 \\ 5(i_2 - i_1) + \tfrac{10}{3} i_2 &= 20\end{aligned}\right\}$ yields $i_1(0^+) = 7/2\ \text{A}$

so $v(0^+) = 5i_1 = 5(7/2) = \underline{\frac{35}{2}\ \text{v}}$

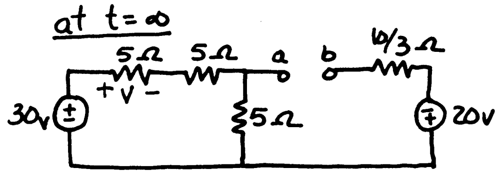
now find R_{eq}



$R_{eq} = 10\|5 + (5+5)\|5 = \frac{20}{3}\,\Omega$

$\therefore\ \tau = R_{eq}C = \frac{20}{3}\left(\frac{1}{2}\right) = \frac{10}{3}\,\text{s}$

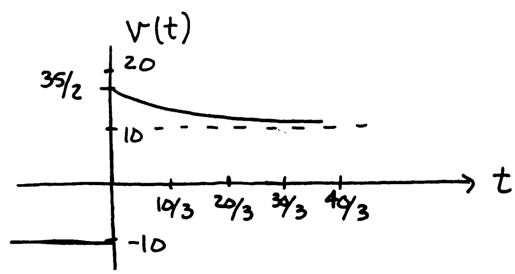
$\underline{\text{at } t=\infty}$



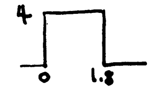
$v(\infty) = \frac{5}{15} \times 30 = 10\text{v}$

$\underline{\text{when } t \geq 0}$: $v(t) = 10 + \left(\frac{35}{2} - 10\right)e^{-3/10\,t} = 10 + \frac{15}{2}e^{-3/10\,t}\ \text{v}$

$\underline{\text{when } t<0}$: $v(t) = -10\text{v}$

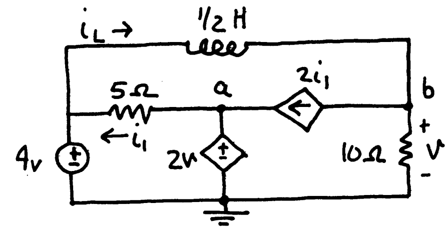


P 9-32 $t=0^-$ (steady state) $\Rightarrow i_L(0)=0$ (undriven ckt)



use superposition; consider $v_s = 4[u(t)-u(t-1.8)]$ v as two separate sources.

4u(t) source at t>0



KVL bottom loop

$$-4-5i_1+2v=0 \qquad (1)$$

KVL around supermesh formed by open-circuiting $2i_1$ source

$$\frac{1}{2}\frac{di_L}{dt}+v-2v+5i_1=0 \qquad (2)$$

KCL at b : $-i_L+2i_1+v/10=0 \qquad (3)$

solving (1) - (3) for i_L yields : $\frac{di_L}{dt}+\frac{20}{9}i_L=\frac{40}{9}$

$\therefore i_L(t)=2+Ae^{-20/9\,t}$; $i(0)=0=2+A \Rightarrow A=-2$

$i_L(t)=2(1-e^{-20/9\,t})$

eliminating i_1 from (1) & (3) yields $v=\frac{16}{9}+\frac{10}{9}i_L$

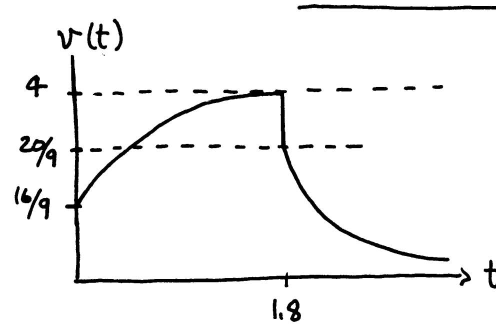
$$=\frac{16}{9}+\frac{10}{9}\left[2(1-e^{-20/9\,t})\right]$$

$$v(t)=4-\frac{20}{9}e^{-20/9\,t}\text{ v}$$

now using superposition, the solution from $-4u(t-1.8)$ source is

$$v(t)=-\left[4-\frac{20}{9}e^{(t-1.8)}\right]u(t-1.8)$$

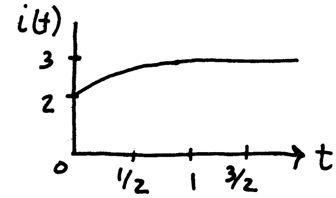
$\therefore$ complete solution is $v(t)=\left[4-\frac{20}{9}e^{-20/9\,t}\right]u(t)-\left[4-\frac{20}{9}e^{-\frac{20}{9}(t-1.8)}\right]u(t-1.8)$



P 9-33 $\quad i(0) = \frac{12}{2+4} = 2A \;;\; \tau = L/R = 2/4 = 1/2\ s$

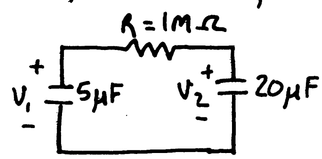
$i_n = Ae^{-2t},\; i_f = 12/4 = 3A \qquad \therefore\ i(t) = 3 + Ae^{-2t}$

now $i(0) = 2 = 3 + A \Rightarrow A = -1$



so $\underline{i(t) = 3 - 1e^{-2t}} \quad t > 0$

P 9-34 $\quad v_1(0) = 100v,\; v_2(0) = 0$



KVL ⟳ : $-v_1 + Ri + v_2 = 0 \qquad (1)$

now $v_1 = 100 - 1/C_1 \int_0^t i\,dt$

$v_2 = 1/C_2 \int_0^t i\,dt$

so (1) becomes: $-100 + \frac{1}{C_1}\int_0^t i\,dt + Ri + \frac{1}{C_2}\int_0^t i\,dt = 0$

or $-100 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{i}{s} + Ri = 0$

where $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ and $RC_{eq} = (10^6)\left(\frac{20 \times 5}{20+5} \times 10^{-6}\right) = 4$

$\therefore$ have $(1 + C_{eq}Rs)i = s(100C_{eq}) \Rightarrow (1 + C_{eq}Rs)i = 0$

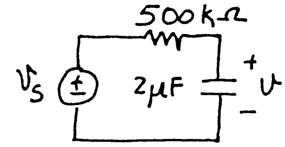
$\therefore\ i_f = 0$ and $i_n = Ae^{-t/RC_{eq}} = Ae^{-t/4} = i$

now $i(0) = \frac{100v}{10^6\Omega} = 10^{-4}A$, so have $\underline{i(t) = 10^{-4}e^{-t/4}A}$

$\therefore\ v_1(t) = 100 - \frac{1}{5 \times 10^{-6}}\int_0^t i\,dt = \underline{20 + 80e^{-t/4}\ V}$

$v_2(t) = \frac{1}{20 \times 10^{-6}}\int_0^t i\,dt = \underline{20(1 - e^{-t/4})\ V}$

P 9-35 $\quad v_s = \begin{cases} 0 & t<1 \\ 4 & 1<t<2 \\ 0 & t>2 \end{cases}$



$RC = (5\times10^5)(2\times10^{-6}) = 1$

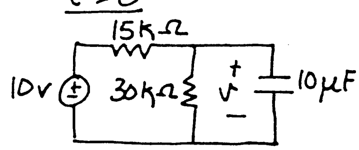
now $v(2) = 4 - 4e^{-1}$

so $v(t) = (4-4e^{-1})e^{-(t-2)} \quad t>2$

$\therefore$ have $v(t) = \begin{cases} 0 & t<1 \\ 4-4e^{-(t-1)} & 1<t<2 \\ (4-4e^{-1})e^{-(t-2)} & t>2 \end{cases}$

P 9-36 $\quad v_c(0) = \frac{30}{35}(12) = 10.29\text{v}$

$\underline{t>0}$



$v(\infty) = \frac{30}{45}(10) = 6.667\text{v}$

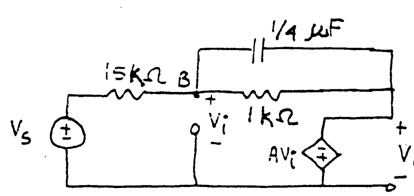
$\tau = RC = \frac{15(30)\times10^3\times10^{-5}}{(15+30)} = 0.1\,\text{s}$

$v = v_f + v_n = 6.667 + A\,e^{-10t}$

$v(0) = 10.29 = 6.667 + A \quad \Rightarrow \quad A = 3.62$

so $\underline{v(t) = 6.667 + 3.62\,e^{-10t}}$

P.9-37



KCL at B:

$(V_s - V_i)/15000 = \frac{V_i - V_o}{1000} + 2.5\times10^{-7}\frac{d}{dt}(V_i - V_o) \qquad (1)$

also: $V_i = -V_o/A \qquad (2)$

(2) into (1) yields $\quad V_s/15000 = \frac{16}{15000}\frac{V_o}{A} - \frac{V_o}{1000} + 2.5\times10^{-7}\left(-\frac{\dot V_o}{A} - \dot V_o\right)$

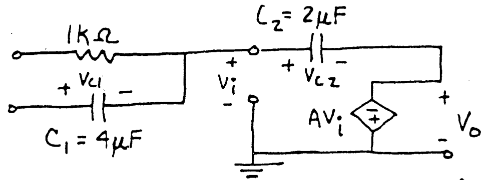
assuming $A \gg 1 \Rightarrow V_s/15000 = -\frac{V_o}{1000} - 2.5\times10^{-7}\frac{dV_o}{dt}$

$\Rightarrow \underline{dV_o/dt + 4000V_o = -4000e^{-1000t}}$

$\therefore V_{on}(t) = Ae^{-4000t}$, try $V_{of}(t) = Be^{-1000t}$ & plug into D.E. $\Rightarrow B = -4/3$

$\Rightarrow V_o(t) = V_{on}(t) + V_{of}(t) = Ae^{-4000t} - 4/3e^{-1000t}$

now $V_o(0) = 4 = A - 4/3 \quad \Rightarrow A = 16/3$ so $\underline{V_o(t) = \frac{4}{3}(4e^{-4000t} - e^{-1000t})}$

P. 9-38 

(a) from KVL $V_{c1} = V_1 - V_i$

$V_o = V_i - V_{c2}$

KCL at '−' terminal to input of amplifier : $4\times10^{-6}\frac{dV_{c1}}{dt} + \frac{(V_2 - V_i)}{1000} = 2\times10^{-6}\frac{dV_{c2}}{dt}$

$$\Rightarrow \quad 4\times10^{-6}\frac{d(V_1 - V_i)}{dt} + \frac{V_2 - V_i}{1000} = 2\times10^{-6}\frac{d(V_i - V_o)}{dt}$$

assume $V_i \simeq 0 \quad \Rightarrow \quad \frac{dV_o}{dt} = -2\frac{dV_1}{dt} -$

integrating both sides yields :

$$V_o(t) = V_o(0) - 2[V_1(t) - V_1(0)] - 500\int_0^t V_2(t)\,dt$$

now for $V_i \simeq 0 \Rightarrow V_o(0) = -V_{c2}(0)$ & $V_1(0) = V_{c1}(0)$

$$\therefore \; \underline{V_o(t) = -2V_1(t) - 500\int_0^t V_2(t)\,dt}$$

(b) $V_1 = 10e^{-2000t}$, $V_2 = 5e^{-1000t}$

$$\Rightarrow V_o(t) = -20e^{-2000t} - 500\int_0^t 5e^{-1000t}\,dt$$

$$\underline{V_o(t) = -20e^{-2000t} + 2.5(-1 + e^{-1000t})} \text{ V}$$

Advanced Problems

AP 9-1 a) at $t=0$, switch 1 is closed & switch 2 is closed

$i(0^-) = 0$, $i_\infty(t) = \frac{6}{.3} = 20\text{ A}$ $\quad 0<t<3$

approaches this final value

$\tau = \frac{L}{R} = \frac{1.2}{.3} = 4\text{s} \Rightarrow i(t) = i_\infty + [i(0) - i_\infty]e^{-t/4} = \underline{20-20e^{-t/4}\text{ A}}$

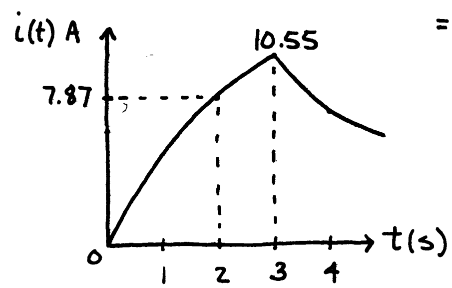
so have $i(3) = 20-20e^{-3/4} = 10.55\text{ A}$

b) switch 1 is closed & switch 2 is open

$i_\infty = \frac{6}{1.5} = 4\text{A}$, $\tau = \frac{1.2}{1.5} = 0.8\text{s}$

$\Rightarrow i(t) = 4 + (10.55-4)e^{-(t-3)/.8}$

$= \underline{4 + 6.55e^{-1.25(t-3)}\text{ A}}$



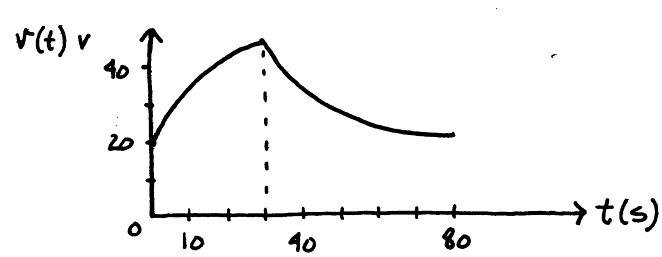
AP 9-2 $v(0) = 20\text{v}$, $v(\infty) = 70\text{ v}$, $\tau = 2\times10^{-6}(20\times10^{+6}) = 40\text{s}$

$\Rightarrow v(t) = 70 + (20-70)e^{-t/40} = \underline{70-50e^{-t/40}\text{v}}$ $\quad 0<t<30$

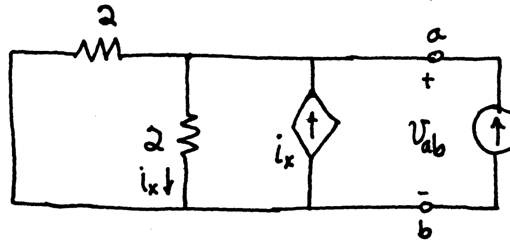
at $t = 30\text{s}$ return switch

$v(30) = 46.4\text{v}$, $V(\infty) = 20\text{v}$, $\tau = 2\times10^{-6}(5\times10^{6}) = 10\text{s}$

$\Rightarrow v(t) = 20 + (46.4-20)e^{-(t-30)/10} = \underline{20 + 26.4e^{-(t-30)/10}\text{v}}$ $\quad t>30$



AP 9-3 Find Norton equivalent, remove C & short v_s



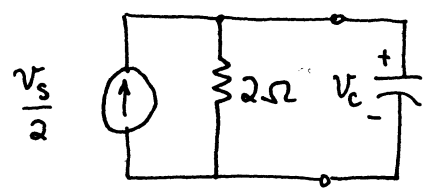
with v_s connected

$i_n = i_{sc} = v_s/2$

$$R_T = \frac{v_{ab}}{1} \qquad \text{note } i_x = 0$$

so $R_T = 2$

Norton:



$v_c(0) = 0$

$$\frac{dv}{dt} + \frac{v}{2} = \frac{v_s}{2}$$

$$v_s = \begin{cases} 5t & 0 \le t \le 2 \\ 10 & t > 2 \end{cases}$$

$$v = 5t + 10\left(e^{-t/2} - 1\right) \qquad 0 < t < 2$$

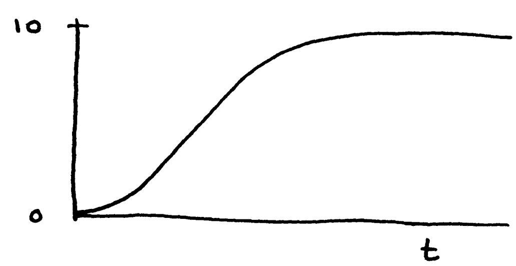
at $t = 2$ $\quad v = 10e^{-1} = 3.68$

$t > 2$

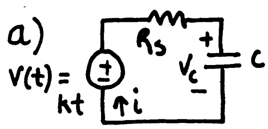
$$v = Ae^{-t/2} + 10$$

at $t = 2$ $\quad 3.68 = Ae^{-1} + 10 \quad$ or $\quad A = 10 - 10e$

$$v = 10e^{-t/2} - 10e^{-\frac{(t-2)}{2}} + 10 \qquad t \ge 2$$



AP 9-4 a)



KVL: $-kt + R_s i + v_s = 0$ and $i = C\,dv_c/dt$

$\Rightarrow \frac{dv_c}{dt} + \frac{1}{R_s C} v_c = \frac{k}{R_s C} t$

$v_c = v_{cn} + v_{cf}$, now $v_{cn} = Ae^{-t/R_sC}$, try $v_{cf} = B_0 + B_1 t$ & plug into D.E.

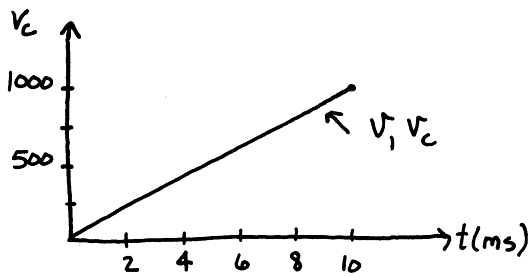
$\Rightarrow B_1 + \frac{1}{R_s C}[B_0 + B_1 t] = \frac{k}{R_s C} t$

thus $B_0 = -kR_sC$, $B_1 = k$

so have $v_c(t) = Ae^{-t/R_sC} + k(t - R_sC)$, $v_c(0) = 0 = A - kR_sC \Rightarrow A = kR_sC$

$\therefore v_c(t) = k[t - R_sC(1 - e^{-t/R_sC})]$ plugging in $k = 1000$, $R_s = 625\,k\Omega$ & $C = 2000\,pF$ get

$v_c(t) = 1000[t - 1.25\times10^{-5}(1 - e^{-80{,}000t})]$



v & v_c track well on millisecond time scale

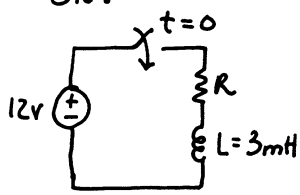
$v(.1ms) - v_c(.1ms) = .012\,V$

b) $F = qE$, $E = v_c/s$, $q = Cv_c$ $\Rightarrow \therefore F = \frac{C}{s} v_c^2$

now $F = m\ddot{y}$ where y = transverse deflection of beam and $y(t)$ is described by the differential eqn. $\frac{d^2y}{dt^2} = \frac{C}{ms} v_c^2(t)$

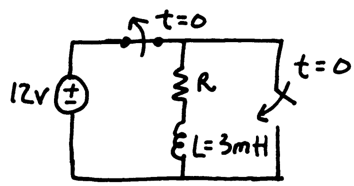
where $v_c(t)$ is given in part a)

AP 9-5 ON:



$$i_{ON} = \frac{12}{R}\left(1-e^{-Rt/0.003}\right)$$

OFF:



$$i_{OFF} = \frac{12}{R} e^{-Rt/0.003}$$

first solve for R using i_{ON} : $0.3 = \frac{12}{R}\left(1-e^{-R(0.0001)/0.003}\right)$

$$\Rightarrow R = 40\left(1-e^{-R/30}\right)$$

solving by iteration yields $R \simeq 18\Omega$

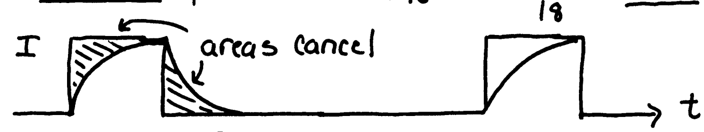
test for i_{ON} : $i(0.0001) = \frac{12}{18}\left(1-e^{-18(.0001)/.003}\right) = 0.3\ A$ ✓

test for i_{OFF} : $i(0.0002) = \frac{12}{18} e^{-18(.0002)/.003} = 0.200\ A$

This exceeds the requirement so $\underline{R = 18\Omega}$ is good. (A larger R would satisfy i_{OFF} timing but not i_{ON} timing).

So $I_{max} = \frac{12}{18} = \underline{667\ mA}$, $P_{ave} = .25\ V^2/R = .25\ \frac{(12)^2}{18} = \underline{2W}$

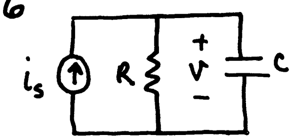
Assumption:



so we use $P_{ave.} \simeq .25\ \frac{V_{max}^2}{R}$

An open circuit turnoff will cause an infinite voltage across the switch causing arcing across a mechanical switch, or destruction of a solid state switch.

AP 9-6



assume $v(0) = 0$

$$\therefore\ v(t) = \frac{1}{C}\int_0^t i(t)\,dt = \frac{1}{C}\int_0^t \left(i_s - \frac{v}{R}\right)dt$$

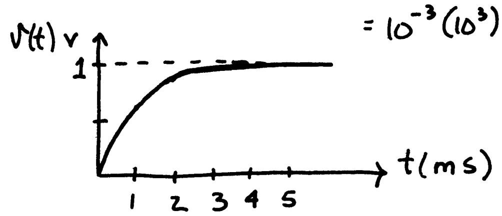
differentiating $\Rightarrow$ $\dfrac{dv}{dt} = \dfrac{1}{C}\left(i_s - \dfrac{v}{R}\right)$ or $\dfrac{dv}{dt} + \dfrac{v}{RC} = \dfrac{i_s}{C}$

so have $v(t) = i_s R + Ae^{-t/RC}$

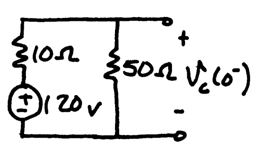
$= I_m R + Ae^{-t/RC}$; now $v(0) = 0 \Rightarrow A = -I_m R$

$$\therefore\ v(t) = I_m R\left(1 - e^{-t/RC}\right)$$

$$= 10^{-3}(10^3)\left(1 - e^{-t/10^3(10^{-6})}\right) = \underline{1 - e^{-1000t}} \qquad 0<t<5\text{ms}$$

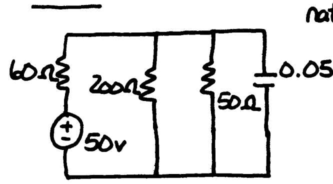


AP 9-7 $\underline{t = 0^-}$

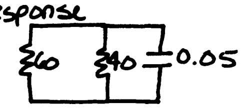


$$v_c(0^-) = \frac{50}{60}(120) = 100\text{v} = v_c(0^+)$$

$\underline{t > 0}$



natural response $\Rightarrow$



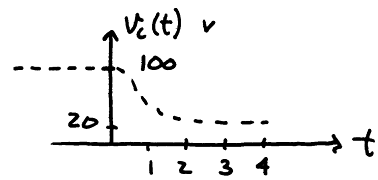
$\tau = RC = 1.2$

$v_n(t) = Ae^{-t/1.2}$

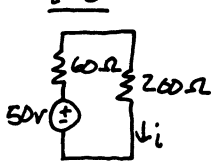
now $v_f(t) = \dfrac{40(50)}{60+40} = 20\text{v}$

$\therefore\ v_c(t) = Ae^{-t/1.2} + 20$; now $v_c(0) = 100 = A + 20 \Rightarrow A = 80$

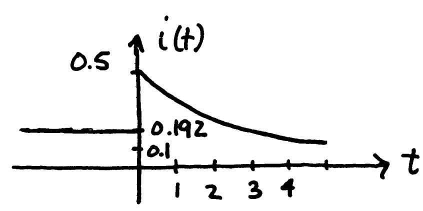
$v_c(t) = 20 + 80e^{-t/1.2}$



$\underline{t < 0}$



$i(t) = 50/260 = 0.192\text{ A}$



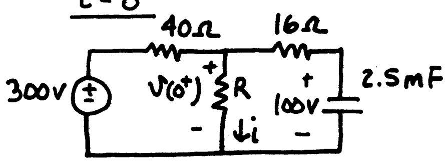
$\underline{t > 0}$

$$i(t) = \frac{v(t)}{20} = .1 + .4e^{-t/1.2}$$

Design Problems

DP 9-1 $v(0) = 100\text{v}$

$t = 0^+$



KCL: $\dfrac{v(0^+) - 300}{40} + \dfrac{v(0^+)}{R} + \dfrac{v(0^+) - 100}{16} = 0$

$\Rightarrow v(0^+) = \dfrac{1100R}{7R+80}$

$\therefore i(0^+) = \dfrac{v(0^+)}{R} = \dfrac{1100}{7R+80}$

now $i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$ where $i(\infty) = \dfrac{300}{40+R}$, $\tau = R_T C$

Kill 300v source $\Rightarrow R_T = 16 + \dfrac{40R}{40+R} = \dfrac{56R+640}{40+R}$ so $\tau = \dfrac{.14R+1.6}{40+R}$

$\therefore i(t) = \dfrac{300}{40+R} + \left[\dfrac{1100}{7R+80} - \dfrac{300}{40+R}\right] e^{-t(40+R)/(.14R+1.6)}$

require $i = 2.5\text{A}$ at $t = 47\text{ms}$

so have $2.5 = \dfrac{300}{40+R} + \left[\dfrac{1100}{7R+80} - \dfrac{300}{40+R}\right] e^{-\frac{.047(40+R)}{(.14R+1.6)}}$

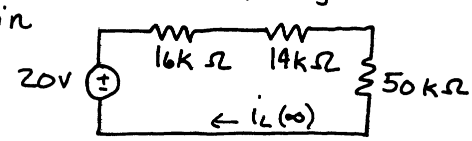
solving iteratively yields $\underline{R = 60\Omega}$

DP 9-2 find R_T at inductor terminals

$R_T = (50+30)\|20 + 14 + 75\|150 = 16 + 14 + 50 = 80\text{k}\Omega$

So $5\tau = 5L/R = 3.1\,\mu\text{s} \Rightarrow L = \dfrac{80\times10^3(3.1\times10^{-6})}{5} = \underline{49.6\text{mH}}$

in steady state with inductor shorted & using multiple source transformations, obtain



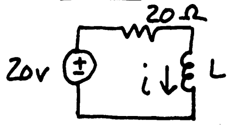
$i_L(\infty) = \dfrac{20}{16+14+50} = 0.25\text{mA}$

$\therefore W = \frac{1}{2} L\, i_L^2(\infty) = \frac{1}{2}(4.96\times10^{-2})(2.5\times10^{-4})^2$

$= \underline{1.55\text{nJ}}$

DP 9-3 $i(0^-) = \frac{50V}{10\Omega} = 5A$

$\underline{t>0}$



$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$; $i(\infty) = \frac{20v}{20\Omega} = 1A$

so have $i(t) = 1 + 4e^{-t/\tau}$

require $3 = 1 + 4e^{-.14/\tau}$ $\Rightarrow$ $\tau = .202$

$\therefore$ $L/R = L/20 = .202$ $\Rightarrow$ $\underline{L = 4.04H}$

at $t = 1s$ $i(1) = 1 + 4e^{-1/.202} = \underline{1.03A}$

DP 9-4 $i(0) = 0$

$i(t) = Ae^{-t/\tau} + B$ where $B = \frac{12}{R_s + R}$

$\tau = \frac{L}{R + R_s}$, need $5\tau \leq 100ms$ or $\tau \leq 20ms$

$f_{max} = i_{max}$ $\Rightarrow$ $.5 = i_{max}$ or $i_{max} = 500mA$

in steady state $i_{ss} = \frac{12}{R_s + R} = .50\,A$ or $R_s + R = 12(2) = 24$

$R_s = 1\Omega \Rightarrow R = 23\Omega$

then $\frac{L}{R_s + R} = .02$ or $\underline{L = .02(24) = 0.48H}$

DP 9-5 $v(0) = \frac{30}{37.5}\,80 = 64\text{v}$; $v(\infty) = \frac{30}{37.5}\,20 = 16\text{v}$

$R_{eq} = 2\text{k}\Omega + 7.5\text{k}\Omega \| 30\text{k}\Omega = 8\text{k}\Omega$

so $\tau = RC = 8\times10^3\,C$

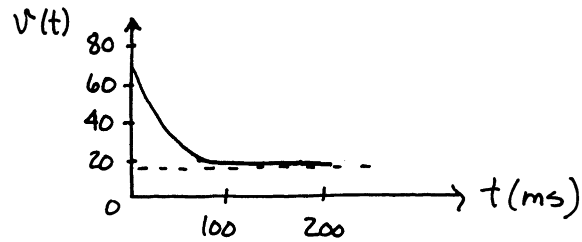
now $v(t) = v(\infty) + [v(0) - v(\infty)]\,e^{-t/\tau} = 16 + 48e^{-t/\tau}$

require $\frac{v(.2) - v(\infty)}{v(\infty)} = .01 \Rightarrow v(.2) = 1.01\,v(\infty)$

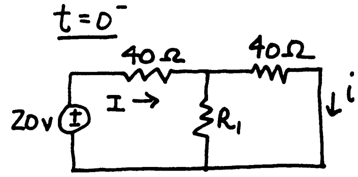
so $16 + 48e^{-.2/\tau} = 1.01\,(16)$

$e^{-.2/\tau} = .00333 \Rightarrow \tau = .0351\text{ s}$

$\therefore\; C = \frac{.0351}{8\times10^3} = 4.39\times10^{-6}\text{F} = 4.39\,\mu\text{F}$

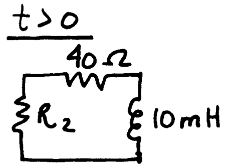


DP 9-6 $\underline{t=0^-}$



current division $i(0^-) = \frac{20\text{v}}{40 + \frac{40R_1}{40+R_1}} \cdot \frac{R_1}{R_1 + 40}$

$\underline{t>0}$



$i(t) = i(0)\,e^{-t/\tau}$ where $\tau = \frac{L}{R} = \frac{10^{-2}}{40 + R_2}$

at $t < 200\mu\text{s}$ need $i > 60\text{mA}$ and $i < 180\text{mA}$

find R_1 to set $i(0) < 180\text{mA}$

try a solution: if $\underline{R_1 = 40\Omega}$, then $i(0) = \frac{1}{6}\text{A} = 166.6\text{mA}$ ok

then $i(t) = 166.6\text{mA}\;e^{-t/\tau}$

at $t_1 = 200\mu\text{s} = .2\text{ms} \rightarrow i(t_1) > 60\text{mA}$ required

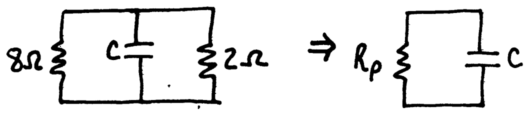
try $\underline{R_2 = 10\Omega}$, then $\tau = \frac{10^{-2}}{50} = .2\text{mS} = \frac{1}{5000}\text{s}$

$i(t_1) = 166.6\times10^{-3}e^{-5000t_1} = 166.6\times10^{-3}e^{-1}$

$= 61.2\text{mA}$ ok

DP 9-7 $v(t) = v_n(t) + v_f(t)$

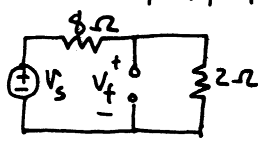
to find $v_n(t)$, need τ so short circuit the voltage source



$\tau = R_p C$

$$v_n(t) = A e^{-t/\tau} = A e^{-t/1.6C}$$

to find $v_f(t)$, open circuit the capacitor



$$v_f = \frac{2}{2+8}(20) = 4\text{V}$$

$$\therefore v(t) = 4 + A e^{-t/1.6C}$$

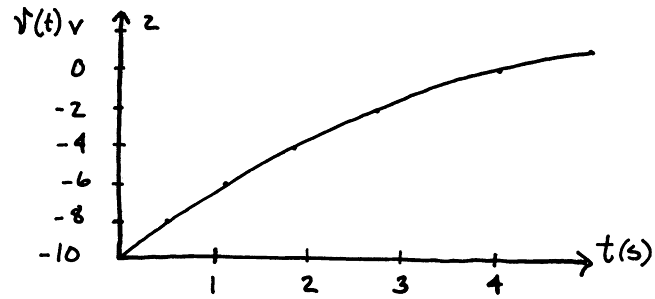
now $v(0) = -10 = 4 + A \Rightarrow A = -14\text{V}$

so $v(t) = 4 - 14e^{-t/1.6C}$

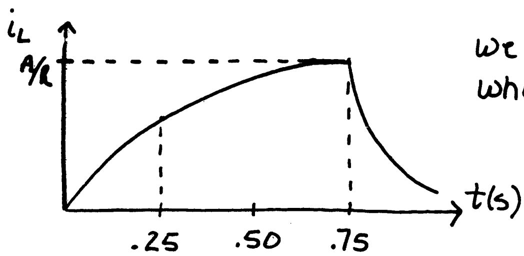
at $t = 4\text{s}$ want $v(4) = 0$

$\Rightarrow 0 = 4 - 14e^{-4/1.6C}$

above yields $\underline{C = 2.0\text{F}}$



DP 9-8 The current waveform will look like:



We need only consider the rise time where $i_L(t) = \frac{V_s}{R}(1 - e^{-t/\tau})$

$$= \frac{A}{R}(1 - e^{-t/\tau})$$

where $\tau = \frac{L}{R_T} = \frac{.2}{3} = \frac{1}{15}$

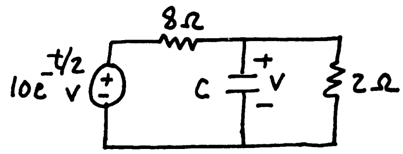
$$\therefore i_L(t) = \frac{A}{3}(1 - e^{-15t})$$

now find A so that $i_L^2 R_{fuse} \geq 10\text{W}$ during $.25 \leq t \leq .75$

$\therefore$ want $[i_L^2(.25)]R_{fuse} = 10\text{W}$

$$\frac{A^2}{9}(1 - e^{-15(.25)})^2(1) = 10 \Rightarrow \underline{A = 9.715 \text{ volts}}$$

DP 9-9 $\underline{t>0}$



KCL: $\dfrac{v - 10e^{-t/2}}{8} + C\dfrac{dv}{dt} + \dfrac{v}{2} = 0$

$\Rightarrow \dfrac{dv}{dt} + \dfrac{5}{8C}v = \dfrac{10e^{-t/2}}{8C}$

$v(t) = v_n + v_f$ now $v_n = Ae^{-\frac{5}{8C}t}$

try $v_f = Be^{-t/2}$ & plug into D.E.

$\Rightarrow -\dfrac{B}{2} + \dfrac{5}{8C}B = \dfrac{10}{8C}$ yields $B = \dfrac{10}{5-4C}$

$\therefore v(t) = Ae^{-\frac{5}{8C}t} + \dfrac{10}{5-4C}e^{-t/2}$

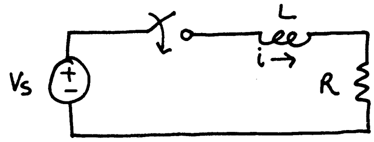
now $v(0) = -10 \Rightarrow A = -10 - \dfrac{10}{5-4C}$

so $v(t) = \dfrac{10}{5-4C}e^{-t/2} - \left(\dfrac{10}{5-4C} + 10\right)e^{-5t/8C}$

now $v(1) = 0 = \dfrac{10}{5-4C}e^{-1/2} - \left(\dfrac{10}{5-4C} + 10\right)e^{-5/8C}$

solving iteratively yields $\underline{C = .303\text{ F}}$

DP 9-10



$i(0) = 0$

desire $i(t)$ peak magnitude $\geq .6A$

period where $|i| \geq \frac{i_{max}}{4}$ is > 1 sec.

choose $v_s = V_0 e^{-bt}$ select V_0, b to meet specs

determine energy delivered to R for $t > 10s \Rightarrow \infty$

natural response: $i_n(t) = Be^{-R/Lt} = Be^{-2t}$ where $R/L = 4/2 = 2$

forced response: $L\frac{di}{dt} + Ri = V_0 e^{-bt}$

substitute $i_f = Ae^{-bt}$ into above $\Rightarrow A = \frac{V_0}{(4-2b)}$

$\therefore i(t) = i_n + i_f = Be^{-2t} + \frac{V_0}{(4-2b)} e^{-bt}$

need $b > 2$ so that 2nd term has faster decay but $(4-2b) \neq 0$

if $b = 4$ then $i(t) = Be^{-2t} + \frac{V_0}{-4} e^{-4t}$

select $V_0 = 12$ (note $i(0) = 0$) so have $i(t) = Be^{-2t} - 3e^{-4t}$

$i(0) = 0 \Rightarrow B = 3$

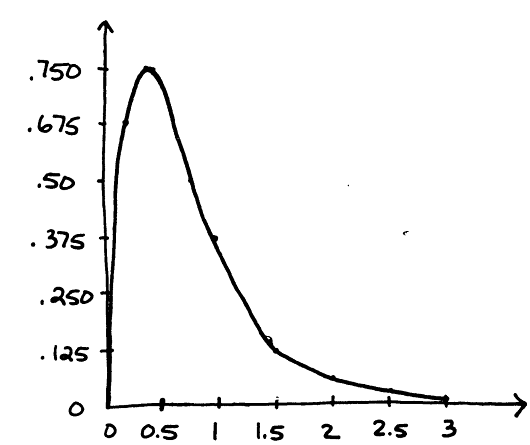
thus have $i(t) = 3[e^{-2t} - e^{-4t}]$ A

maximum is at: $-2e^{-2t} + 4e^{-4t} = 0$ or $\underline{t = .35}$

then $i(t)_{max} = 3[.4966 - .2466] = .75$ A

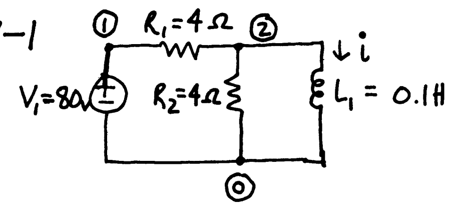
$p = i^2R = 9[e^{-4t} - 2e^{-6t} + e^{-8t}](4)$

$W = 36\int_0^\infty [e^{-4t} - 2e^{-6t} + e^{-8t}]dt = 36[\frac{1}{4} - \frac{2}{6} + \frac{1}{8}] = \underline{1.50 \text{ J}}$



Spice Problems

SP 9-1

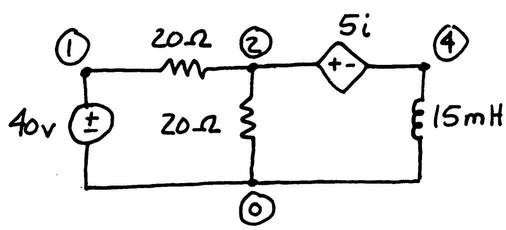


input file

```
* Inductor Transient
R1  1 2 4
R2  2 0 4
L1  2 0 0.1  IC=3
V1  1 0  Pulse (0 80)
.TRAN  2.5m  25m  UIC
.PLOT  TRAN  I(L1)
.PROBE
.END
```

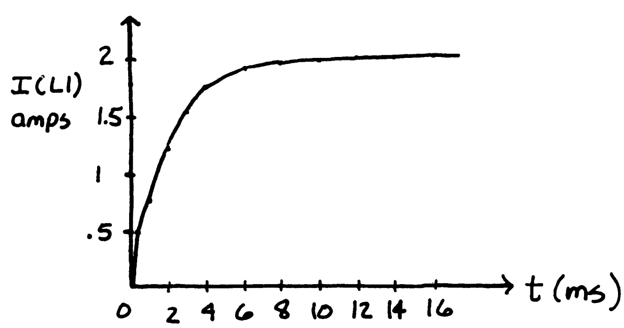
Ans. $i(0) = 3A$

SP 9-2

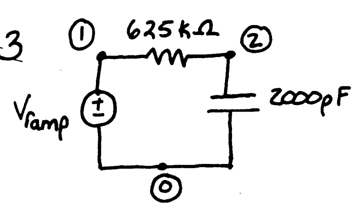


input file

```
V1      1 0  40v
R1      1 2  20
R2      2 3  20
VDUMMY  3 0 0
H1   2 4  VDUMMY  5
L1   4 0  15E-3H  IC=0
.TRAN 1ms 15ms 0.0001 UIC
.PRINT  TRAN  I(L1)
.PROBE
.END
```

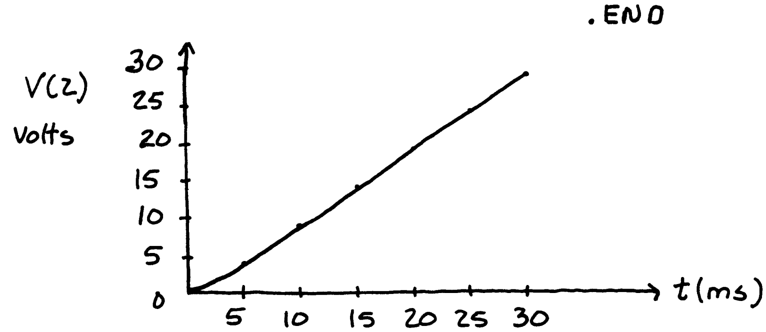


SP 9-3

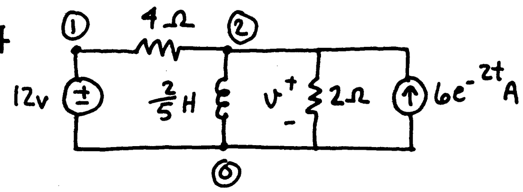


input file

```
VRAMP  1 0  PWL(0 0 1 1000)
RS   1 2  625K
C1   2 0  2000PF
.TRAN 1ms 30ms  0  .0001
.PROBE
.END
```

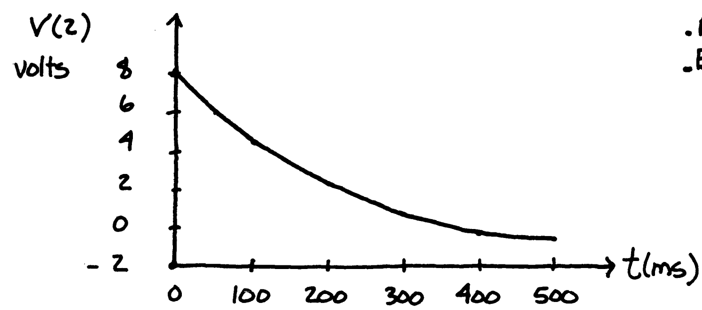


SP 9-4

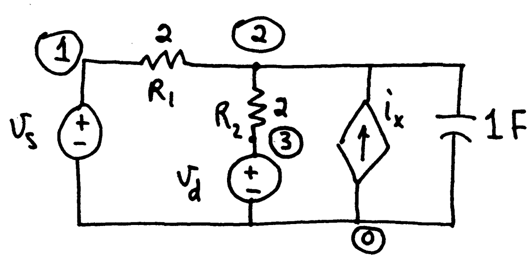


input file

```
VI     1  0  12
RI     1  2  4
LI     2  0  .4  IC=3
R2     2  0  2
IFORCE 0  2  EXP(6 0 0 .5)
.TRAN  .5S  1S  0  .3mS  UIC
.PROBE
.END
```



SP 9-5

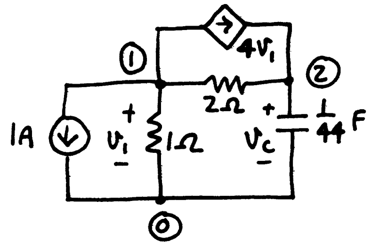



```
VS  1  0   PWL (0  0  2  10  20  10)
R1  1  2   2
R2  2  3   2
VD  3  0   0
F1  0  2   VD  1
C1  2  0   1   IC=0
.TRAN   0.2  20   UIC
.PRINT   TRAN   V(2)
.PLOT    TRAN   V(2)
.END
```

Answer:

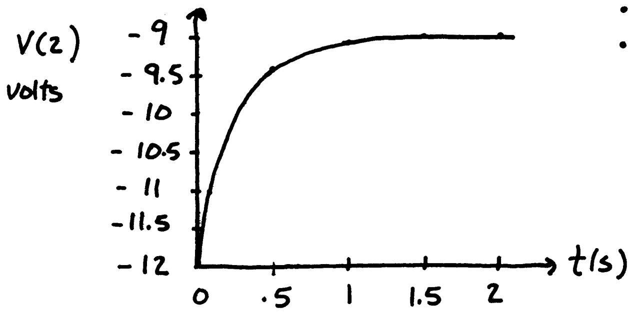
t	v_c (V)
1	1.08
2	3.67
3	6.16
4	7.67
⋮	
10	9.89
50	10

SP 9-6

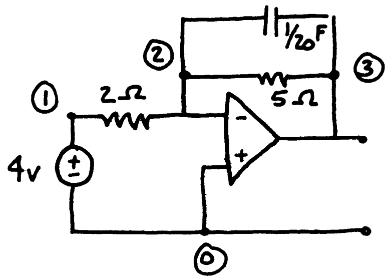


input file

```
IFORCE 1 0 1
R1 1 0 1
R2 1 2 2
G1 1 2 1 0 4
C1 2 0 .023 IC=-12
.TRAN .1 2 0 10ms UIC
.PROBE
.END
```

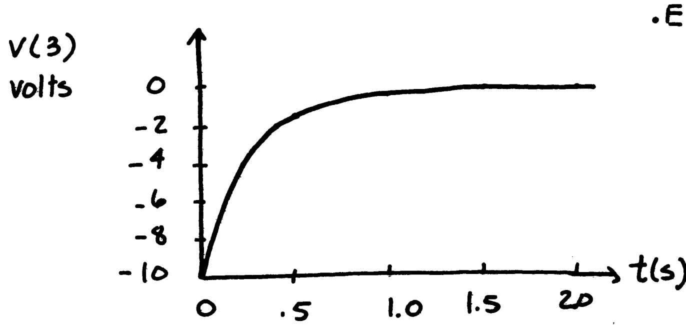


SP 9-8



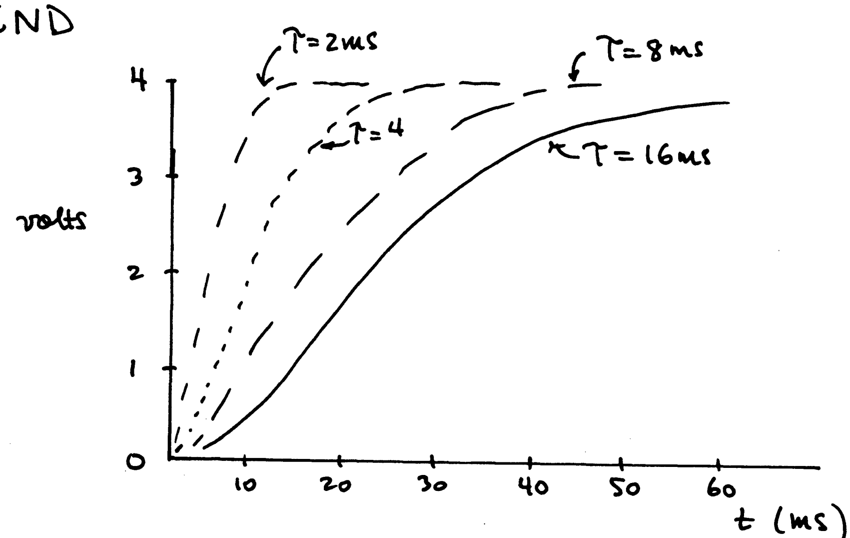
input file

```
;VS 1 0 4
R1 2 0 2
R2 2 3 5
C1 2 3 .05 IC=10
XAMP 2 0 3 IDEALOPAMP
.TRAN .1 2 0 10E-3S UIC
;.AC DEC 100 100 1000
.PROBE
.END
```



SP 9-9

```
R1   1   2   2K
C1   2   0   .001M      IC=0
R2   1   3   4K
C2   3   0   .001M      IC=0
R3   1   4   8K
C3   4   0   .001M      IC=0
R4   1   5   16K
C4   5   0   E-6        IC=0
V1   1   0   PULSE (0 4 0 0 0 100M 100M)
.TRAN   2M   60M   UIC
.PROBE  V(2)  V(3)  V(4)  V(5)
.Options nopage
.END
```



Chapter 11

Exercises

Ex. 11-1 (a) $T = 2\pi/\omega = \underline{2\pi/4}$

(b) v leads i by $30-(-70) = \underline{100^\circ}$

Ex. 11-2 $v = 3\cos 4t + 4\sin 4t$

$v = \sqrt{(3)^2+(4)^2}\cos(4t - \tan^{-1} 4/3)$

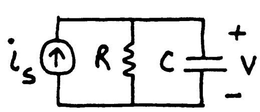
$v = \underline{5\cos(4t-53^\circ)}$

Ex. 11-3 $i = -5\cos 5t + 12\sin 5t$

$i = \sqrt{(-5)^2+(12)^2}\cos(5t-\tan^{-1} 12/-5)$

$\underline{i = 13\cos(5t-112.6^\circ)}$

Ex. 11-4



KCL : $i_s = v/R + C\,dv/dt$

$\frac{dv}{dt} + \frac{v}{RC} = \frac{I_m}{C}\cos\omega t$

try $v_f(t) = A\cos\omega t + B\sin\omega t$ & plug into above D.E.

$\Rightarrow -\omega A\sin\omega t + \omega B\cos\omega t + \frac{1}{RC}(A\cos\omega t + B\sin\omega t) = \frac{I_m}{C}\cos\omega t$

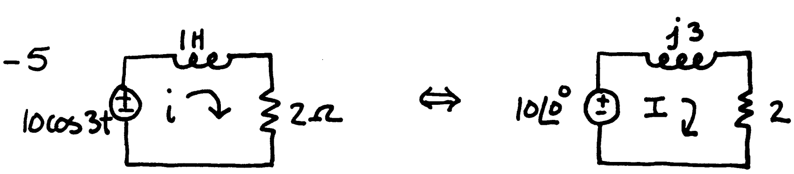
equating $\sin\omega t$ & $\cos\omega t$ terms yields $A = \frac{R I_m}{1+\omega^2R^2C^2}$

$B = \frac{\omega R^2 C I_m}{1+\omega^2R^2C^2}$

$\therefore v_f(t) = \frac{R I_m}{1+\omega^2R^2C^2}\cos\omega t + \frac{\omega R^2 C I_m}{1+\omega^2R^2C^2}\sin\omega t$

$\underline{v_f(t) = \frac{R I_m}{\sqrt{1+\omega^2R^2C^2}}\cos[\omega t - \tan^{-1}(\omega RC)]}$

Ex 11-5



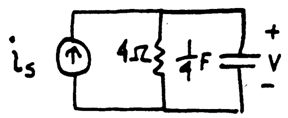
$j\omega L = j\,3\cdot 1 = j3$

KVL : $-10 + j3I + 2I = 0$

$\Rightarrow I = \frac{10}{2+j3} = \frac{10\angle 0^\circ}{\sqrt{13}\angle 56.3^\circ} = \frac{10}{\sqrt{13}}\angle -56.3^\circ$

$\therefore \underline{i(t) = \frac{10}{\sqrt{13}}\cos(3t-56.3^\circ)}$

Ex. 11-6



$i_s = 10\cos 4t = 10\,\mathrm{Re}[e^{j4t}]$

KCL : $\frac{1}{4}\frac{dv}{dt} + \frac{1}{4}v = 10e^{j4t}$

assume $v = Ae^{j4t}$ & plug into above D.E.

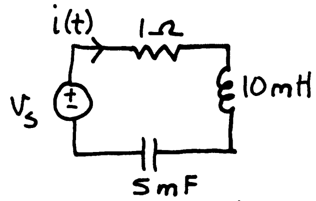
$\Rightarrow A = \frac{10}{\frac{1}{4}+j} = \frac{40}{\sqrt{17}}e^{-j76^\circ}$

$\therefore V = \frac{40}{\sqrt{17}}e^{-j76^\circ}e^{j4t} = \frac{40}{\sqrt{17}}e^{j(4t-76^\circ)}$

now actual response $\Rightarrow$ $v(t) = \mathrm{Re}[V]$

$$\underline{v(t) = \frac{40}{\sqrt{17}}\cos(4t-76^\circ)}$$

Ex 11-7



$V_s = 40\cos 100t = \mathrm{Re}\{4e^{j100t}\}$

KVL : $i(t) + 10\times10^{-3}\frac{di(t)}{dt} + \frac{1}{5\times10^{-3}}\int_{-\infty}^{t} i(t)\,dt = v_s$

assume $i(t) = Ae^{j100t}$ where is complex number to be determined

plugging into D.E. yields

$Ae^{j100t} + jAe^{j100t} + (-j2A)e^{j100t} = 4e^{j100t}$ $\Rightarrow A = \frac{4}{1-j} = 2\sqrt{2}e^{j45^\circ}$

so $\beta = \tan^{-1}\frac{1}{1} = 45^\circ$

$$\underline{i(t) = \mathrm{Re}\,2\sqrt{2}\,e^{j100t}e^{j45^\circ} = \mathrm{Re}\{2\sqrt{2}e^{j(100t+45^\circ)}\} = 2\sqrt{2}\cos(100t+45^\circ)}$$

Ex. 11-8

(a) $i = 4\cos(\omega t - 80^\circ) = \mathrm{Re}\{4e^{j\omega t}e^{-j80^\circ}\}$

$\therefore$ $\underline{I = 4e^{-j80^\circ} = 4\angle -80^\circ}$

(b) $i = 10\cos(\omega t + 20^\circ) = \mathrm{Re}\{10e^{j\omega t}e^{j20^\circ}\}$

$\therefore$ $\underline{I = 10e^{j20^\circ} = 10\angle 20^\circ}$

(c) $i = 8\sin(\omega t - 20^\circ) = 8\cos(\omega t - 110^\circ) = 8\,\mathrm{Re}\{e^{j\omega t}e^{-j110^\circ}\}$

$\therefore$ $\underline{I = 8e^{-j110^\circ} = 8\angle -110^\circ}$

Ex. 11-9

(a) $V = 10\angle -140^\circ = 10e^{-j140^\circ}$

$\therefore v(t) = \mathrm{Re}\{10e^{-j140^\circ}e^{j\omega t}\} = \underline{10\cos(\omega t - 140^\circ)}$

(b) $V = 80 + j75 = 109.7\angle 43.2^\circ = 109.7e^{j43.2^\circ}$

$\therefore v(t) = \mathrm{Re}\{109.7e^{j43.2^\circ}e^{j\omega t}\} = \underline{109.7\cos(\omega t + 43.2^\circ)}$

Ex. 11-10 (a) $V = Ri = 10(5\cos 100t) = \underline{50\cos 100t}$

(b) $V = L\frac{di}{dt} = 0.01[5(-100)\sin 100t]$

$= -5\sin 100t = \underline{5\cos(100t + 90°)}$

(c) $V = \frac{1}{C}\int i\,dt = 10^3\int 5\cos 100t\,dt$

$= 50\sin 100t = \underline{50\cos(100t - 90°)}$

Ex. 11-11 $i = C\frac{dv}{dt} = 10\times10^{-6}[100(-500)\sin(500t + 30°)]$

$= -0.5\sin(500t + 30°)$

$= 0.5\sin(500t + 210°) = \underline{0.5\cos(500t + 120°)}$

Ex 11-12 from figure Ex 11-12 we get $i(t) = I_m \sin\omega t$ (A) ; $I = I_m\angle{-90°}$ A

$v(t) = V_m\cos\omega t$ (V) ; $V = V_m\angle{0°}$ V

$i(t) = I_m\sin\omega t = I_m\cos(\omega t - 90°)$

the voltage leads the current by 90°, ∴ it is an inductor

$\Rightarrow Z_{eq} = \frac{V}{I} = \frac{V_m\angle 0°}{I_m\angle{-90°}} = \frac{V_m}{I_m}\angle 90°\ \Omega$

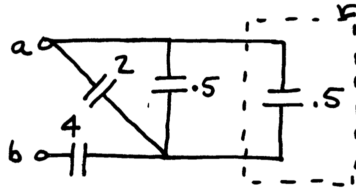
also $Z_{eq} = j\omega L = \omega L\angle 90°$ $\Rightarrow \omega L = \frac{V_m}{I_m}$

or $\underline{L = \frac{V_m}{\omega I_m}}$ (H)

Ex. 11-13 $Z = j\omega L = j(1000)(.01) = \underline{j10\,\Omega}$

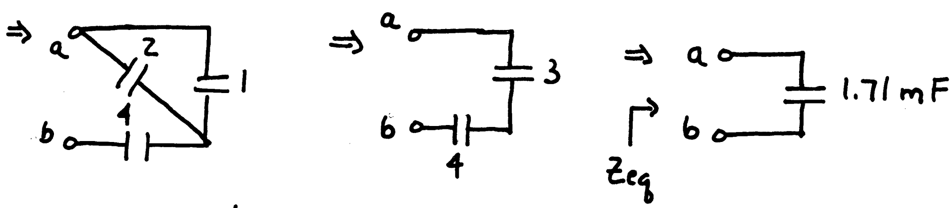
$Y = 1/Z = -j/10 = -j0.1 = \underline{0.1\angle{-90°}}$ ℧

Ex 11-14



$C_{eq_1} = .5mF = \left(\frac{1}{3mF} + \frac{1}{1.5mF} + \frac{1}{1mF}\right)^{-1}$

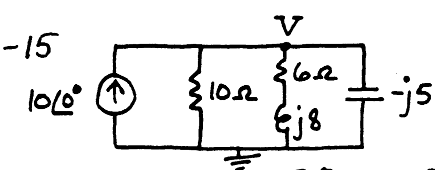
(all capacitors in mF)



$$Z_{eq} = -j/\omega C = -j/1000 \times 1.71 \times 10^{-3} = -0.585\,j\ \Omega$$
$$= 0.585\angle{-90^\circ}\ \Omega$$

$$Y_{eq} = \frac{1}{Z_{eq}} = 1.709\angle{90^\circ}\ S$$

Ex. 11-15



$Z_L = j(10^5)(80 \times 10^{-6}) = j8$

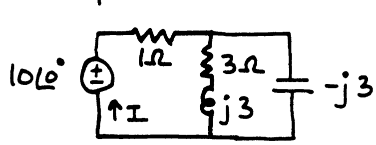
$Z_C = -j/(10^5)(2 \times 10^{-6}) = -j5$

KCL at top : $\frac{V}{6+j8} + \frac{V}{10} + \frac{V}{-j5} = 0$

$$\Rightarrow V = \frac{1000}{16+j12} = \frac{1000}{20\angle{36.9^\circ}} = 50\angle{-36.9^\circ}$$

$\therefore$ $\underline{v(t) = 50 \cos(10^5 t - 36.9^\circ)}$

Ex. 11-16 phasor ckt



$$I = \frac{10\angle{0^\circ}}{1 + \frac{(3+j3)(-j3)}{3+j3-j3}} = \frac{10\angle{0^\circ}}{4-j3} = \frac{10\angle{0^\circ}}{5\angle{-36.9^\circ}}$$
$$= 2\angle{36.9^\circ}$$

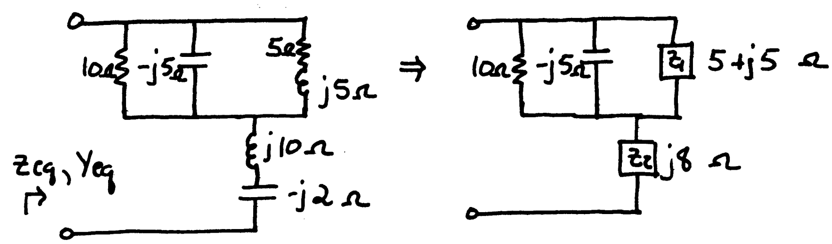
$\therefore$ $\underline{i(t) = 2\cos(3t + 36.9^\circ)}$

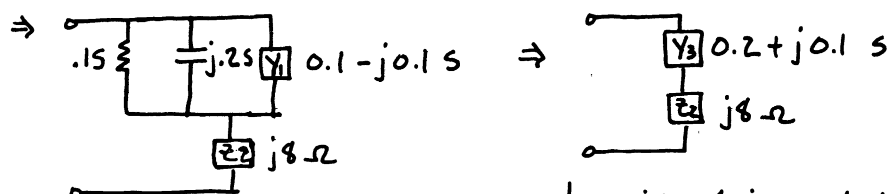
Ex. 11-17 $I_1 = I\left[\frac{3+j3}{3+j3-j3}\right] = (2\angle{36.9^\circ})(1+j) = (2\angle{36.9^\circ})(\sqrt{2}\angle{45^\circ})$

$$= 2\sqrt{2}\angle{81.9^\circ}$$

$\therefore$ $\underline{i_1(t) = 2\sqrt{2}\cos(3t + 81.9^\circ)}$

Ex 11-18 express circuit using impedance

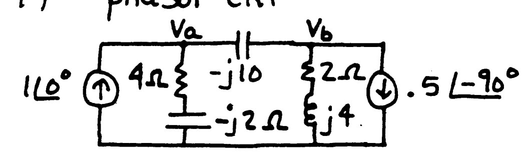




$$Z_{eq} = Z_2 + \frac{1}{Y_3} = j8 + 4 - j2 = \underline{4 + j6 = 7.2\angle 56.3^\circ\ \Omega}$$

$$Y_{eq} = \frac{1}{Z_{eq}} = \underline{0.14\angle{-56.3^\circ}\ S}$$

Ex. 11-19 phasor ckt



KCL at Va: $\dfrac{V_a}{4-j2} + \dfrac{V_a - V_b}{-j10} = 1$

$$\Rightarrow \underline{(4-j12)V_a + (-4+j2)V_b = -20-j40}$$

KCL at Vb: $\dfrac{V_b - V_a}{-j10} + \dfrac{V_b}{2+j4} + .5\angle{-90^\circ} = 0$

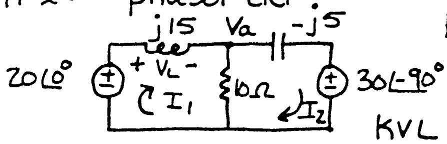
$$\Rightarrow \underline{(-2-j4)V_a + (2-j6)V_b = 10+j20}$$

using Cramer's rule $V_a = \dfrac{\begin{vmatrix} (-20-j40) & (-4+j2) \\ (10+j20) & (2-j6) \end{vmatrix}}{\begin{vmatrix} (4-j12) & (-4+j2) \\ (-2-j4) & (2-j6) \end{vmatrix}} = \dfrac{-200+j100}{-80-j60}$

$$= \sqrt{5}\angle 296.5^\circ$$

$\therefore$ $\underline{V_a(t) = \sqrt{5}\cos(100t + 296.5^\circ) = \sqrt{5}\cos(100t - 63.5^\circ)}$

Ex. 11-20 phasor ckt.



KVL $\curvearrowright I_1$: $j15 I_1 + 10(I_1 - I_2) = 20$

$\Rightarrow (10+j15)I_1 - 10 I_2 = 20$ (1)

KVL $\curvearrowright I_2$: $-j5 I_2 + 10(I_2 - I_1) = -30\angle{-90^\circ}$

$\Rightarrow -10 I_1 + (10-j5) I_2 = j30$ (2)

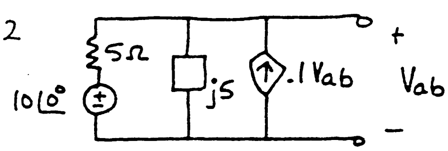
from Cramer's rule

$$I_1 = \frac{\begin{vmatrix} 20 & -10 \\ j30 & 10-j5 \end{vmatrix}}{\begin{vmatrix} 10+j15 & -10 \\ -10 & 10-j5 \end{vmatrix}} = \frac{200+j200}{-75+j100} = \frac{8\sqrt{2}}{5}\angle{-82^\circ}$$

now $V_L = (j15) I_1 = (15\angle 90^\circ)(8\sqrt{2}/5 \angle{-82^\circ}) = 24\sqrt{2}\angle 8^\circ$

$\therefore$ $\underline{V_L(t) = 24\sqrt{2}\cos(\omega t + 8^\circ)\text{ v}}$

Ex. 11-22

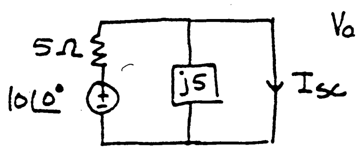


find Voc (= Vab here)

KCL at top : $\frac{V_{oc} - 10}{5} + \frac{V_{oc}}{j5} = .1 V_{oc}$

$\Rightarrow V_{oc} = \frac{20}{5\angle{-63.4^\circ}} = \underline{8.9\angle 63.4^\circ}$

now find Isc

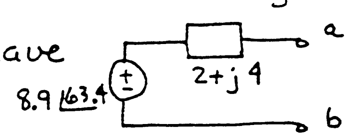


$V_{ab} = 0$ $\therefore$ no current source

$I_{sc} = \frac{10\angle 0^\circ}{5\Omega} = \underline{2\angle 0^\circ}$

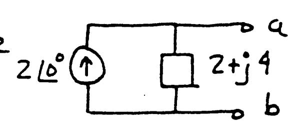
$\therefore Z_T = \frac{V_{oc}}{I_{sc}} = \frac{8.9\angle 63.4^\circ}{2} = 4.45\angle 63.4^\circ = 2+j4$

$\therefore$ have

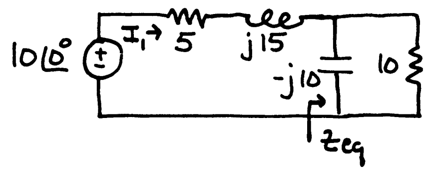


Ex. 11-23 from Ex. 10-16 $I_n = I_{sc} = 2\angle 0^\circ$ & $Z_T = 2+j4$

$\therefore$ have



Ex 11-24 a) turn off current source, use phasors with $\omega = 10$ rad/s



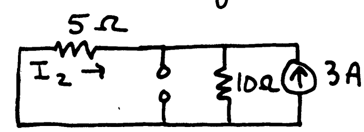
$$Z_{eq} = \frac{-j10 \cdot 10}{10-j10} = 5(1-j)$$

KVL: $-10 + 5I_1 + j15I_1 + 5(1-j)I_1 = 0$

$$\Rightarrow I_1 = \frac{10}{10+j10} = 0.707\angle{-45^\circ}$$

$\therefore$ $\underline{i_1(t) = 0.707\cos(10t - 45^\circ)}$ A

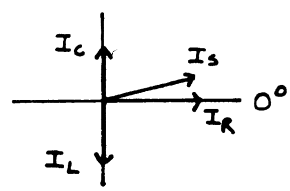
b) turn off voltage source, $\omega = 0$ rad/s



current divider $I_2 = -\frac{10}{15} 3 = -2$A

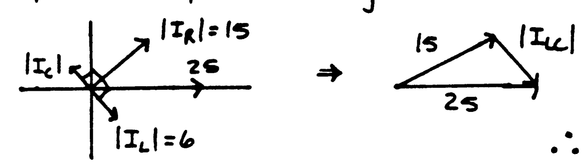
so by superposition $\underline{i(t) = 0.707\cos(10t - 45^\circ) - 2 \quad \text{A}}$

Ex. 11-25 Diagram drawn with relative magnitudes arbitrarily chosen



Ex. 11-26 2 possible phasor diagrams for currents

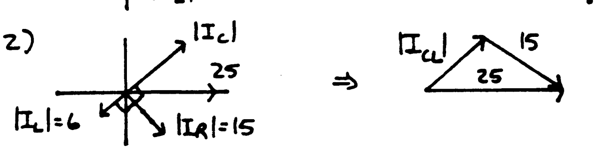
(1)



$\therefore |I_{LC}| = \sqrt{(25^2) - (15^2)} = 20$

$= |I_{CL}|$

(2)



now if $|I_{Lc}| = |I_L| - |I_c| \Rightarrow |I_c| = 6 - 20 = -14$ (impossible)

$\therefore$ from case (2) $|I_{CL}| = |I_c| - |I_L| \Rightarrow \underline{|I_c| = 20 + 6 = 26}$

Ex. 11-28 $Z_1 = \dfrac{R_1 X_1 (X_1 - jR_1)}{R_1^2 + X_1^2}$ and $R_1 = 1\text{k}\Omega$

$$X_1 = \frac{1}{\omega C_1} = \frac{1}{(1000)(10^{-6})} = 1\text{k}\Omega$$

$$\therefore Z_1 = \frac{(1)(1)(1-j1)}{1+1} = \frac{1}{2} - j\frac{1}{2} \ \text{k}\Omega$$

$$Z_2 = R_2 = 1\text{k}\Omega$$

$$\therefore \underline{\frac{V_o}{V_s} = -\frac{Z_2}{Z_1} = \frac{-1}{1/2 - j1/2} = -1-j}$$

Problems

P. 11-1

(a) $V = 15\cos(400t + 30°)$

$i = 3\sin(400t + 30°) = 3\cos(400t - 60°)$

$\therefore$ V leads i by 90° $\Rightarrow$ element is an <u>inductor</u>

now $|Z_L| = \dfrac{V_{peak}}{i_{peak}} = \dfrac{15}{3} = 5 = \omega L = 400L$

$\Rightarrow \underline{L = .0125\,H = 12.5\,mH}$

(b) i leads v by 90° $\therefore$ <u>capacitor</u>

$|Z_c| = \dfrac{V_{peak}}{i_{peak}} = \dfrac{8}{2} = 4 = \dfrac{1}{\omega C} = \dfrac{1}{900C} \Rightarrow \underline{C = 277.77\,\mu F}$

(c) $V = 20\cos(250t + 60°)$

$i = 5\sin(250t + 150°) = 5\cos(250t + 60°)$

since v & i are in phase $\Rightarrow$ element is a <u>resistor</u>

$\therefore R = \dfrac{V_{peak}}{i_{peak}} = \dfrac{20}{5} = \underline{4\Omega}$

P. 11-2

(a) $i(t) = 2\cos(6t + 120°) + 4\sin(6t - 60°)$

$= 2(\cos 6t + \cos 120° - \sin 6t \sin 120°) + 4(\sin 6t \cos 60° - \cos 6t \sin 60°)$

$= -(1 + 2\sqrt{3})\cos 6t + (2 - \sqrt{3})\sin 6t$

$= \sqrt{(1+2\sqrt{3})^2 + (2-\sqrt{3})^2}\ \cos\left[6t - \tan^{-1}\dfrac{2-\sqrt{3}}{-(1+2\sqrt{3})}\right]$

$i(t) = \underline{2\sqrt{5}\,\sin(6t - 86.6°)}$

(b) $v(t) = 5\sqrt{2}\cos 8t + 10\sin(8t + 45°)$

$= 5\sqrt{2}\cos 8t + 10[\sin 8t \cos 45° + \cos 8t \sin 45°]$

$= 10\sqrt{2}\cos 8t + 5\sqrt{2}\sin 8t$

$v(t) = \sqrt{250}\,\cos(8t - 26.56°) = \underline{5\sqrt{10}\,\sin(8t + 63.4°)}$

P. 11-3

(a) rotate 45° $\Rightarrow$ $I = 6 + j8 = 10\angle 53.1°$

subtract 45°

$\underline{I' = 10\angle 8.1° = 7\sqrt{2} + j\sqrt{2}}$

(b) rotate 90° $\Rightarrow$ $I = 10\angle 53.1°$

add 90°

$\underline{I' = 10\angle 143.1° = -8 + j6}$

P. 11-4 (a) $\mathbf{V}_1 = 3\angle 60^\circ = 1.5 + j\,2.598$

$\mathbf{V}_2 = 8\angle -22.5^\circ = 7.391 - j\,3.061$

$\therefore\ \mathbf{V}_1 + \mathbf{V}_2 = 8.891 - j\,0.463 = 8.90\angle -2.98^\circ$

$\Rightarrow\ \underline{v_1 + v_2 = 8.90\cos(2t - 2.98^\circ)}$

(b) $v_1 = 2\sqrt{2}\cos(4t - 90^\circ)\ \Rightarrow\ \mathbf{V}_1 = 2\sqrt{2}\angle -90^\circ = -j\,2\sqrt{2}$

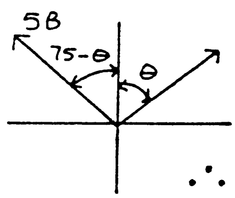
$\mathbf{V}_2 = 10\angle 30^\circ = 5\sqrt{3} + j5$

$\therefore\ \mathbf{V}_1 + \mathbf{V}_2 = 5\sqrt{3} + j(5 - 2\sqrt{2}) = 8.93\angle 14.1^\circ$

$\Rightarrow\ \underline{v_1 + v_2 = 8.93\cos(4t + 14.1^\circ)}$

P. 11-5 $$\frac{(5\angle 36.9^\circ)(10\angle -53.1^\circ)}{(4+j3)(6-j8)} = \frac{50\angle -16.2^\circ}{10 - j5} = \frac{10\angle -16.2^\circ}{\sqrt{5}\angle -26.56^\circ} = \underline{2\sqrt{5}\angle 10.36^\circ}$$

P. 11-6



2A + 5B is pure imaginary and on the '+' imaginary axis

$\therefore\ 2|A|\sin\theta = 5B\sin(75^\circ - \theta)$

$2(5\sqrt{2})\sin\theta = 5(4)\sin(75^\circ - \theta)$

$= 20[\sin 75^\circ\cos\theta - \cos 75^\circ\sin\theta]$

$\Rightarrow\ \tan\theta = \dfrac{\sin 75^\circ}{1/\sqrt{2} + \cos 75^\circ} = 1 \quad \therefore\ \theta = 45^\circ$

so $\underline{A = 5\sqrt{2}\angle 45^\circ}$ and $\underline{B = 4\angle 90^\circ + (75^\circ - 45^\circ) = 4\angle 120^\circ}$

P. 11-7 $$5\angle +81.87^\circ\left[4 - j3 + \frac{3\sqrt{2}\angle -45^\circ}{5\sqrt{2}\angle -8.13^\circ}\right] = 5\angle +81.87^\circ\left[4 - j3 + \tfrac{3}{5}\angle -36.87^\circ\right]$$

$= 5\angle +81.87^\circ\,(4.48 - j\,3.36)$

$= 5\angle +81.87^\circ\,(5.6\angle -36.87^\circ)$

$= 28\angle +45^\circ$

$= \underline{14\sqrt{2} + j\,14\sqrt{2}}$

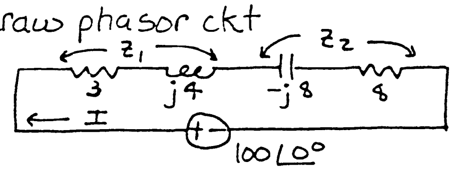
P. 11-8 $\quad I = 72\sqrt{3} + 36\sqrt{3}\,\angle 140^\circ - 90^\circ + 144\,\angle 210^\circ + 25\,\angle\phi$

$$= 40.08 - j\,24.23 + 25\,\angle\phi$$

$$= 46.83\,\angle -31.15^\circ + 25\,\angle\phi$$

clearly for $|I|$ to be max. the above 2 terms must add in same direction (in phase) $\Rightarrow$ $\underline{\phi = -31.15^\circ}$

P. 11-9 $\quad$ draw phasor ckt



(a) $\underline{Z_1 = 3 + j4 = 5\,\angle 53.1^\circ}$ $\quad$ $\underline{Z_2 = 8 - j8 = 8\sqrt{2}\,\angle -45^\circ}$

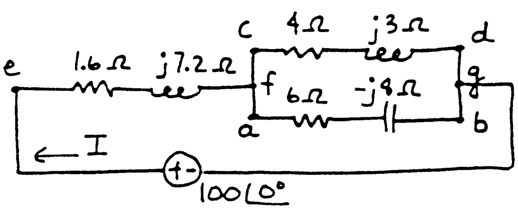
(b) total impedance $= Z_1 + Z_2$

$$= 3 + j4 + 8 - j8 = 11 - j4 = \underline{11.7\,\angle -20.0^\circ}$$

(c) $I = \dfrac{100\angle 0^\circ}{Z_1 + Z_2} = \dfrac{100}{11.7\,\angle -20^\circ} = \underline{100/11.7\,\angle 20.0^\circ}$

$\therefore\ \underline{i(t) = 8.55\cos(1250t + 20.0^\circ)}$

P. 11-10



(a) $Z_{eq} = Z_{ef} + Z_{fg} = 1.6 + j7.2 + \dfrac{(4+j3)(6-j8)}{(4+j3)+(6-j8)}$

$$= 1.6 + j7.2 + 4.4 + j0.8$$

$$\underline{Z_{eq} = 6 + j8 = 10\,\angle 53.1^\circ}$$

(b) $I = \dfrac{100\angle 0^\circ}{Z_{eq}} = \dfrac{100\angle 0^\circ}{10\,\angle 53.1^\circ} = \underline{10\,\angle -53.1^\circ}$

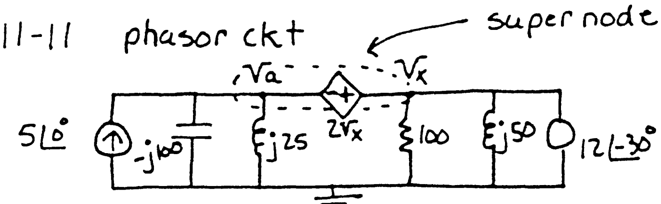
(c) $V_{ef} = I\,Z_{ef} = (10\,\angle -53.1^\circ)(1.6 + j7.2) = (10\,\angle -53.1^\circ)(4\sqrt{17}/5\,\angle 77.47^\circ)$

$$= \underline{40\sqrt{17}/5\,\angle 24.3^\circ}$$

(d) $V_{fg} = I\,Z_{fg} = (10\,\angle -53.1^\circ)(2\sqrt{5}\,\angle 10.3^\circ) = \underline{20\sqrt{5}\,\angle -42.8^\circ}$

(e) $I_{cd} = \dfrac{V_{fg}}{Z_{cd}} = \dfrac{20\sqrt{5}\,\angle -42.8^\circ}{5\,\angle 36.9^\circ} = \underline{4\sqrt{5}\,\angle -79.7^\circ}$

P. 11-11 phasor ckt



KCL at supernode: $-5 - \frac{V_a}{-j100} + \frac{V_a}{j25} + \frac{V_x}{100} + \frac{V_x}{j50} - (6\sqrt{3} - j6) = 0$

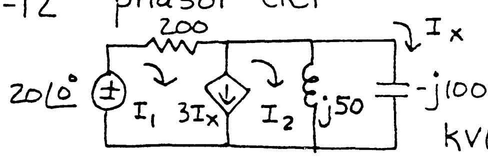
$\Rightarrow 3V_a + (2+j)V_x = 600 + j(500 + 600\sqrt{3}) \quad (1)$

also: $V_x - V_a = 2V_x \Rightarrow V_a = -V_x \quad (2)$

(2) into (1) yields: $V_x = \frac{600 + j(500 + 600\sqrt{3})}{-1 + j} = 1168\angle{-66.3^\circ}$

$\therefore \; V_x(t) = 1168 \cos(500t - 66.3^\circ)$

P. 11-12 phasor ckt



KVL I_x: $-j100 I_x + j50(I_x - I_2) = 0$

$\Rightarrow I_x + I_2 = 0 \quad (1)$

KVL outer loop: $-20 + 200 I_1 + j50(I_2 - I_x) = 0$

$\Rightarrow 200 I_1 + j50 I_2 - j50 I_x = 20 \quad (2)$

also $I_1 - I_2 = 3 I_x \quad (3)$

solving for I_2 in (1) & plugging into (2) and (3) yields

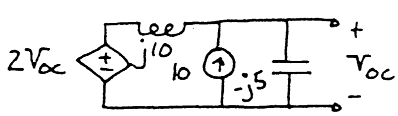
$-20 + 200 I_1 - j100 I_x = 0 \quad (4)$

$I_1 = 2 I_x \quad (5)$

(5) into (4) yields $I_x = \frac{20}{400 - j100} = .0485\angle{14.0^\circ}$

$\therefore \; i_x(t) = .0485 \cos(1000t + 14.0^\circ)$ A

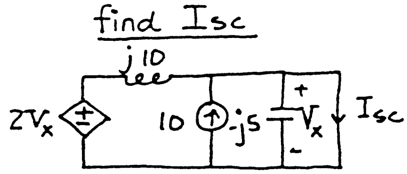
P. 11-13 find Voc



KCL at top: $\frac{V_{oc} - 2V_{oc}}{j10} - 10 + \frac{V_{oc}}{-j5} = 0$

$\Rightarrow V_{oc} = -j\,100/3 = 100/3\angle{-90^\circ}$ V

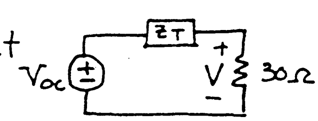
find Isc



$V_x = 0 \Rightarrow I_{sc} = 10\angle{0^\circ}$ A

$\therefore \; Z_T = \frac{V_{oc}}{I_{sc}} = \frac{100/3\angle{-90^\circ}}{10\angle{0^\circ}} = 10/3\angle{-90^\circ}\ \Omega$

so have the equivalent ckt

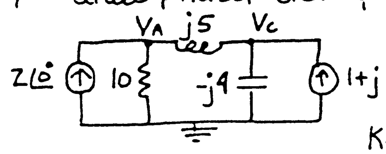


using voltage divider

$V = V_{oc}\left(\frac{30}{30 + Z_T}\right) = \frac{-j300}{9 - j} = 33.13\angle{-83.66^\circ}$ v

so $V(t) = 33.13 \cos(20t - 83.66^\circ)$ v

P. 11-14 draw phasor ckt & use nodal analysis



KCL at V_A : $-2 + V_A/10 + (V_A - V_c)/j5 = 0$

$\Rightarrow (2+j)V_A - 2V_c = j20$ (1)

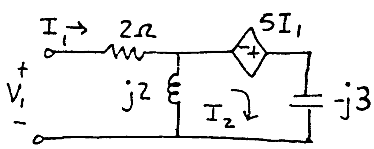
KCL at V_c : $(V_c - V_A)/j5 + V_c/{-j4} - (1+j) = 0$

$\Rightarrow 4V_A + V_c = 20 - j20$ (2)

using Cramer's rule

$$V_c = \frac{\begin{vmatrix} (2+j) & j20 \\ 4 & 20-j20 \end{vmatrix}}{\begin{vmatrix} (2+j) & -2 \\ 4 & 1 \end{vmatrix}} = \frac{60-j100}{10+j} = \frac{116.6\angle -59^\circ}{\sqrt{101}\angle 5.7^\circ} = \underline{11.6\angle 64.7^\circ \text{ V}}$$

P. 11-15



apply test voltage, $V_1 \Rightarrow Z_{IN} = V_1/I_1$

KVL $\circlearrowright I_1$: $-V_1 + 2I_1 + j2(I_1 - I_2) = 0 \Rightarrow -V_1 + I_1(2+j2) - j2I_2 = 0$ (1)

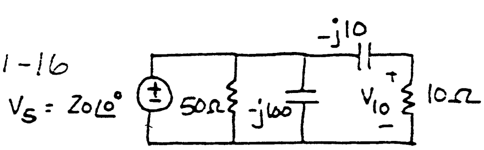
KVL $\circlearrowright I_2$: $j2(I_2 - I_1) - 5I_1 - j3I_2 = 0 \Rightarrow I_2 = -\frac{(5+j2)I_1}{j}$ (2)

plugging (2) into (1) yields

$$V_1 = (12+j6)\,I_1$$

$$\therefore\ \underline{Z_{IN} = \frac{V_1}{I_1} = 12 + j6 = 6\sqrt{3}\angle 26.6^\circ\ \Omega}$$

P. 11-16

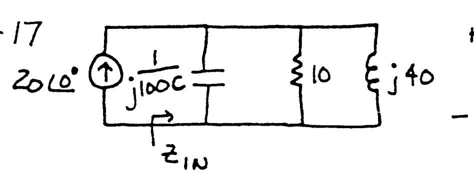


using voltage divider

$$V_{10} = V_S\left(\frac{10}{10-j10}\right) = 20\angle 0^\circ\left(\frac{10}{10\sqrt{2}\angle -45^\circ}\right) = \underline{10\sqrt{2}\angle 45^\circ}$$

$$\underline{\therefore\ V_{10}(t) = 10\sqrt{2}\cos(100t + 45^\circ)\ \text{V}}$$

P. 11-17



$V = 100 - j100 = 100\sqrt{2}\angle -45^\circ$

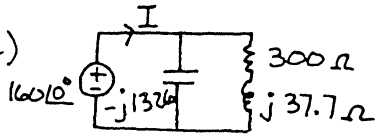
$$Y_{IN} = 1/Z_{IN} = I/V = \frac{20\angle 0^\circ}{100\sqrt{2}\angle -45^\circ} = (.1)\sqrt{2}\angle 45^\circ = .1 + .1j$$

also $Y_{IN} = j100c + \frac{1}{j40} + \frac{1}{10} = j(100c - 1/40) + 1/10$

equating the imaginary terms

$$100c - 1/40 = .1 \qquad \Rightarrow \underline{c = 1/800\ \text{F}}$$

P. 11-18 (a)

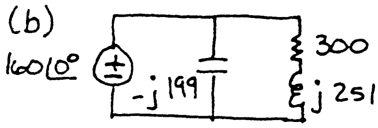


$$I = \frac{160\angle 0°}{\dfrac{(-j1326)(300+j37.7)}{-j1326+300+j37.7}}$$

$$= \frac{160\angle 0°}{303\angle -5.9°} = 0.53\,\underline{\angle 5.9°}$$

$$\therefore\ i(t) = 0.53\cos(120\pi t + 5.9°)\ \text{A}$$

(b)

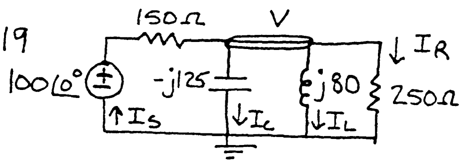


$$I = \frac{160\angle 0°}{\dfrac{(-j199)(300+j251)}{-j199+300+j251}}$$

$$= \frac{160\angle 0°}{256\angle -59.9°} = 0.625\,\underline{\angle 59.9°}$$

$$\therefore\ i(t) = 0.625\cos(800\pi t + 59.9°)\ \text{A}$$

P. 11-19



KCL at V: $\dfrac{(V-100)}{150} + \dfrac{V}{-j125} + \dfrac{V}{j80} + \dfrac{V}{250} = 0$

$\Rightarrow V = 57.6\,\underline{\angle 22.9°}$

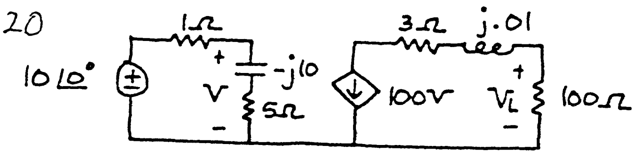
$\therefore\ I_s = \dfrac{100-V}{150} = .667 - .384\angle 22.9° = \underline{.347\angle -25.5°}$

$I_c = \dfrac{V}{125\angle -90°} = \underline{0.461\angle 112.9°}$

$I_L = \dfrac{V}{80\angle 90°} = \underline{0.720\angle -67.1°}$

$I_R = \dfrac{V}{250} = \underline{0.230\angle 22.9°}$

P. 11-20



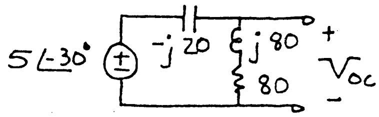
using Voltage divider $V = 10\angle 0° \left[\dfrac{5-j10}{1+5-j10}\right]$

$= 9.59\,\underline{\angle -4.4°}$

now $V_L = -100(100\,V) = -9.59\times 10^4\,\underline{\angle -4.4} = 9.59\times 10^4\,\underline{\angle 175.6°}$

$$\therefore\ V_L(t) = 9.59\times 10^4\cos(10^8 t + 175.6°)$$

P. 11-21 find V_{oc}

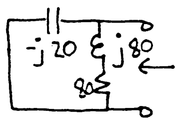


using voltage divider

$$V_{oc} = 5\angle{-30°}\left(\frac{80+j80}{80+j80-j20}\right) = 5\angle{-30°}\left(\frac{80\sqrt{2}\angle{-21.9°}}{100\angle{36.9°}}\right)$$

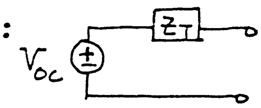
$$= \underline{4\sqrt{2}\angle{-21.9°}\ \text{V}}$$

find Z_T (kill voltage source)

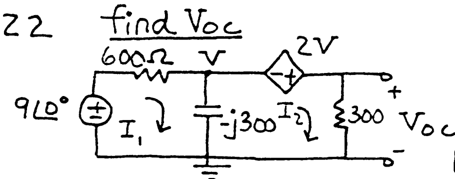


$$Z_T = \frac{(-j20)(80+j80)}{-j20+80+j80} = \underline{23\angle{-81.9°}\ \Omega}$$

∴ have the equivalent ckt:



P. 11-22 find V_{oc}



KVL I_1: $600I_1 - j300(I_1 - I_2) = 9$

$\Rightarrow (600-j300)I_1 + j300I_2 = 9$ (1)

KVL I_2: $-2V + 300I_2 - j300(I_2 - I_1) = 0$

also $V = -j300(I_1 - I_2)$

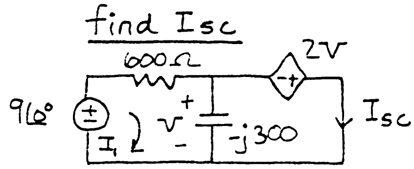
$\Rightarrow j3I_1 + (1-j3)I_2 = 0$ (2)

using Cramer's rule for eqns. (1) and (2)

$\Rightarrow I_2 = .0124\angle{-16°}$

$$\therefore V_{oc} = 300I_2 = \underline{3.71\angle{-16°}\ \text{V}}$$

find I_{sc}

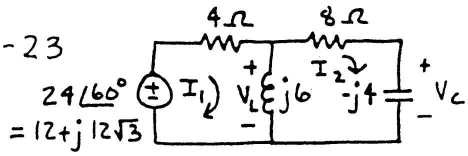


KVL I_{sc}: $-2V - V = 0 \Rightarrow V = 0$

$$\therefore I_{sc} = \frac{9\angle{0°}}{600} = \underline{.015\angle{0°}}$$

so $$Z_T = \frac{V_{oc}}{I_{sc}} = \frac{3.71\angle{-16°}}{.015} = \underline{247\angle{-16°}\ \Omega}$$

P. 11-23



$\mathrm{KVL}\ \circlearrowright I_1: \quad (4+j6)I_1 - j6 I_2 = 12+j12\sqrt{3} \qquad (1)$

$\mathrm{KVL}\ \circlearrowright I_2: \quad -j6 I_1 + (8+j2)I_2 = 0 \qquad (2)$

using Cramer's rule to solve I_1

$$I_1 = \frac{(12+j12\sqrt{3})(8+j2)}{(4+j6)(8+j2)-(-j6)(-j6)} = \underline{2.5\angle 29^\circ = 2.2+j1.2 \text{ A}}$$

from (2) $$I_2 = \frac{j6}{8+j2}(2.5\angle 29^\circ) = \frac{6\angle 90^\circ}{\sqrt{68}\angle 14^\circ}(2.5\angle 29^\circ) = \underline{1.82\angle 105^\circ}$$

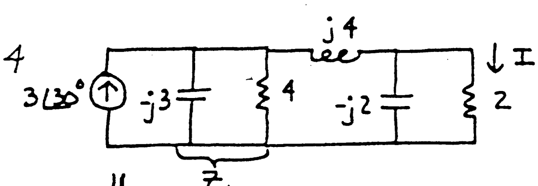
now $V_L = j6(I_1 - I_2) = (6\angle 90^\circ)(2.5\angle 29^\circ - 1.82\angle 105^\circ)$

$= (6\angle 90^\circ)(2.71\angle -11.3^\circ)$

$\underline{V_L = 16.3\angle 78.7^\circ \text{ V}}$

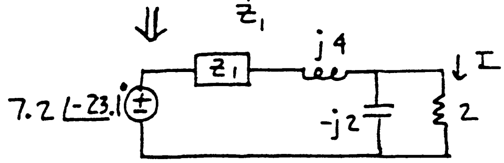
and $V_c = -j4 I_2 = (4\angle -90^\circ)(1.82\angle 105^\circ) = \underline{7.28\angle 15^\circ \text{ V}}$

P. 11-24

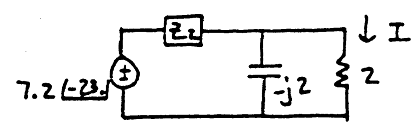


$$Z_1 = \frac{(-j3)(4)}{-j3+4} = 2.4\angle -53.1^\circ = 1.44 - j1.92$$

$\Downarrow$

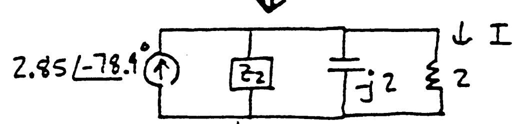


$\Rightarrow$



$Z_2 = Z_1 + j4 = 1.44 + j2.08$
$= 2.53\angle 55.3^\circ$

$\Downarrow$



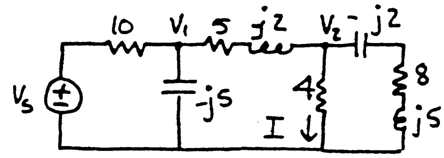
$Z_3 = 3.51\angle -37.9^\circ = 2.77 - j2.16$

this is wrong.

$$\therefore I = 2.85\angle -78.4^\circ \left(\frac{3.51\angle -37.9^\circ}{2.77 - j2.16 + 2}\right)$$

$$I = 2.85\angle -78.4^\circ \frac{(3.51\angle -37.9^\circ)}{(5.24\angle -24.4^\circ)} = \underline{1.9\angle -92^\circ \text{ A}}$$

P. 11-25



KCL at V_1: $\frac{V_1-V_s}{10}+\frac{V_1}{-j5}+\frac{V_1-V_2}{5+j2}=0$

$\Rightarrow (11+j2)V_1-(5+j2)V_s=10V_2$ (1)

KCL at V_2: $\frac{V_2-V_1}{5+j2}+I+\frac{V_2}{8+j3}=0$

$\Rightarrow (8+j3)V_1=(13+j5)V_2+(34+j31)I$ (2)

also $V_2=4I=4(3\angle 45^\circ)=12\angle 45^\circ=6\sqrt{2}+j6\sqrt{2}$ (3)

plugging I and (3) into (2) yields

$(8+j3)V_1=74.24+j290.62$

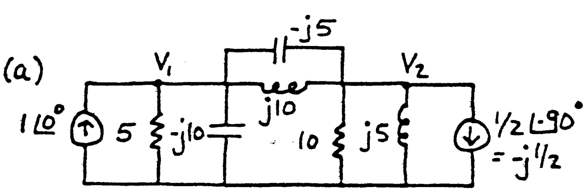
$\therefore V_1=\frac{300\angle 75.7^\circ}{8.54\angle 20.6^\circ}=35.1\angle 55.1^\circ=20.1+j28.8$

now plugging V_1 and V_2 into (1) yields

$(5+j2)V_s=-209.4+j473.1$

$\therefore V_s=\frac{517.4\angle 113.9^\circ}{5.38\angle 21.8^\circ}=96.1\angle 92.1^\circ$ V

P. 11-26 (a)



KCL at V_1: $(1/5+1/{-j10}+1/{j10}+1/{-j5})V_1-(1/{j10}+1/{-j5})V_2=1\angle 0^\circ$

$\Rightarrow (2+j2)V_1-jV_2=10$ (1)

KCL at V_2: $(1/{-j5}+1/{j10}+1/10+1/{j5})V_2-(1/{-j5}+1/{j10})V_1=j1/2$

$\Rightarrow V_1+(1+j)V_2=-5$ (2)

using Cramer's rule

$V_1=\frac{10(1+j)-(-j)(-5)}{(2+j2)(1+j)-(-j)(1)}=\frac{10+j5}{j5}=2.24\angle -63.4^\circ$

from (1) $V_2=\frac{(2+j2)V_1-10}{j}=(2-j2)(2.24\angle -63.4^\circ)+j10$

$=-2+j4=4.47\angle 116.6^\circ$

$\therefore v_1(t)=2.24\cos(100t-63.4^\circ)$ V

$v_2(t)=4.47\cos(100t+116.6^\circ)$ V

P. 11-26 (Continued)

(b) Use superposition since have a DC and AC source

<u>1st consider AC source</u> (open DC current source)

use eqn's. (1) & (2) from part (a) but set RHS of (1) to zero

$(2+j2)V_1' - jV_2' = 0 \quad (1)$

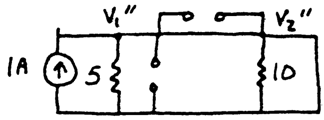
$V_1' + (1+j)V_2' = -5 \quad (2)$

Solving for V_2' in (1) and plugging into (2) $\Rightarrow V_1' = -1 = 1\angle 180^\circ$

$\therefore$ from (1) $V_2' = (2-j2)(1\angle 180^\circ) = 2\sqrt{2}\angle 135^\circ$

so $v_1'(t) = \cos(100t + 180^\circ)$ and $v_2'(t) = 2\sqrt{2}\cos(100t + 135^\circ)$

<u>now consider DC source</u> (open AC current source)



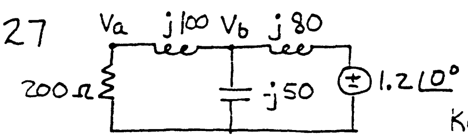
$\omega = 0$ here

$\therefore V_1'' = V_2'' = 0$ because of short circuit

$\therefore v_1(t) = v_1'(t) + v_1''(t) = \cos(100t + 180^\circ)$ V

$v_2(t) = v_2'(t) + v_2''(t) = 2\sqrt{2}\cos(100t + 135^\circ)$ V

P. 11-27



KCL at Va : $\dfrac{V_a}{200} + \dfrac{V_a - V_b}{j100} = 0 \quad (1)$

KCL at Vb : $\dfrac{V_b - V_a}{j100} + \dfrac{V_b}{-j50} + \dfrac{V_b - 1.2}{j80} = 0$

$\Rightarrow V_a = \frac{1}{4}V_b - \frac{3}{2} \quad (2)$

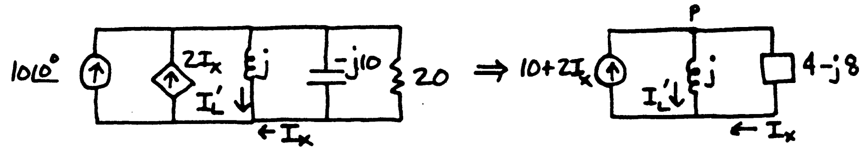
(2) into (1) yields $V_b = 2.21\angle -144^\circ$

and from (2) $V_a = 0.55\angle -144^\circ - 1.5 = 1.97\angle -171^\circ$

$\therefore v_a(t) = 1.97\cos(4000t - 171^\circ)$ V

$v_b(t) = 2.21\cos(4000t - 144^\circ)$ V

P. 11-28 Use superposition, consider the current source, i_1, first



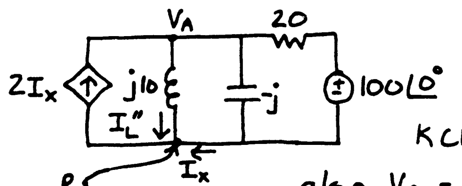
(1) from KCL at P: $I_L' = 10 + 2I_x - I_x = 10 + I_x$

(2) from current divider: $I_L' = (10 + 2I_x)\left(\frac{4-j8}{j+4-j8}\right) = 11.07 - j.61 + I_x(2.21 - j.12)$

solving for I_x in (1) & plugging into (2) yields $I_L' = 9.09\angle 2.59°$

$\therefore\ i_L'(t) = 9.09\cos(100t + 2.59°)$

now consider the voltage source, V_1



KCL at V_A: $-2I_x + \frac{V_A}{j10} + \frac{V_A}{-j} + \frac{V_A - 100}{20} = 0$ (1)

KCL at P: $I_x = I_L''$ (2)

also $V_A = j10\, I_L''$ (3)

plugging (2) & (3) into (1) yields

$-2I_L'' + I_L'' - 10I_L'' + j\tfrac{1}{2}I_L'' - 5 = 0$

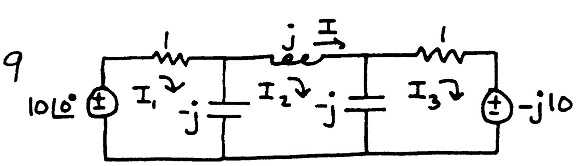
$\Rightarrow I_L'' = .45\angle -177.4°$

$\therefore\ i_L''(t) = .45\cos(1000t - 177.4°)$

so $i_L(t) = i_L'(t) + i_L''(t)$

$i_L(t) = 9.09\cos(100t + 2.59°) + .45\cos(1000t - 177.4°)$

P. 11-29



$I_2 = I$

KVL I_1: $(1-j)I_1 + jI_2 = 10$

KVL I_2: $jI_1 - jI_2 + jI_3 = 0$

KVL I_3: $jI_2 + (1-j)I_3 = j10$

Cramer's rule yields $I_2 = \frac{10 - j10}{(1-j)} = 10\text{A}$

$\therefore\ i(t) = 10\cos 1000t$ A

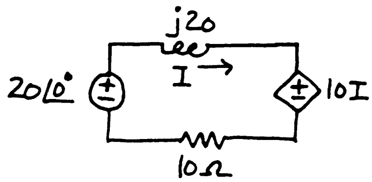
P. 11-30 $\quad Z = jX_L + \dfrac{R(-jX_C)}{R - jX_C} = \dfrac{RX_C^2}{R^2+X_C^2} + j\left[X_L - \dfrac{R^2 X_C}{R^2+X_C^2}\right]$

want $\operatorname{Im}\{Z\} = 0 \Rightarrow X_L = \dfrac{R^2 X_C}{R^2+X_C^2}$

want $\operatorname{Re}\{Z\} = aR \Rightarrow \underline{X_C = R\sqrt{\dfrac{a}{1-a}}}$

$\therefore\ X_L = \dfrac{R^2\left(R\sqrt{\frac{a}{1-a}}\right)}{R^2 + R^2\left(\frac{a}{1-a}\right)} = \underline{R\sqrt{a(1-a)}}$

P. 11-31

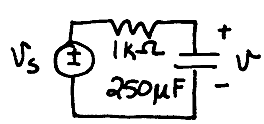


KVL: $-20 + j20I + 10I + 10I = 0$

$\Rightarrow I = \dfrac{20}{20+j20} = \dfrac{1}{\sqrt{2}\angle 45^\circ} = 1/\sqrt{2}\ \angle -45^\circ$

so $\underline{i(t) = 1/\sqrt{2}\cos(100t - 45^\circ)}$

P 11-32 $\quad \underline{t>0}$



KVL: $-v_s + Ri + v = 0$

KCL: $i = C\dfrac{dv}{dt}$

$\Big\}\ \dfrac{dv}{dt} + 4v = 4v_s$

$V(\infty) = 20v$

char. eqn $\Rightarrow (s+4) = 0$

so $v(t) = Ae^{-4t} + 20$

$\underline{t<0}$ (use phasors) $\omega = 3$ rad/s

$Z_{Tot} = 1000 + \dfrac{1}{j(.75\times10^{-3})} = 1000 - j\dfrac{4000}{3}$

$Z_C = \dfrac{1}{j(.75\times10^{-3})} = -j\dfrac{4000}{3}$

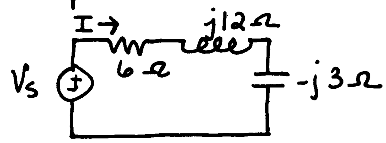
so $V = \dfrac{Z_C V_s}{Z_T} = \dfrac{(-j4000)\,50}{3000 - j4000} = 40\angle -36.9^\circ$

or $v(t) = 40\cos(3t - 36.9^\circ)$ so $v(t=0) = 32v = v(0^+)$

now $v(0) = 32 = A + 20 \quad \therefore A = 12$

thus $\underline{v(t) = 12e^{-4t} + 20\ v \qquad t>0}$

P 11-33 phasor circuit

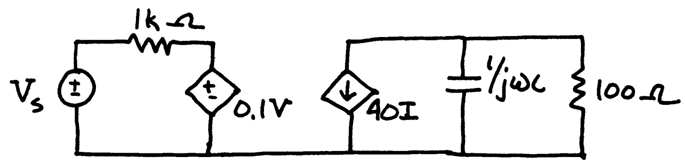


$$V_s = 2\angle 30^\circ$$

$$I = \frac{2\angle 30^\circ}{6 + j12 + 3/j} = .185\angle -26.3^\circ$$

$$\therefore\ \underline{i(t) = .185 \cos(4t - 26.3^\circ)\ A}$$

P 11-34



$$\omega = 200\ \text{rad/s}$$

$$V_s = 5\angle 0^\circ$$

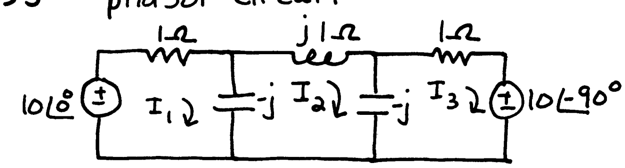
KCL at output : $\frac{V}{100} + 40I + V(j\omega C) = 0$ (1)

also $I = \frac{V_s - .1V}{1000}$ (2)

from (1) and (2) get $V = \frac{-200}{6 + j2} = 31.62\angle 161.6^\circ$

$$\therefore\ \underline{v(t) = 31.62 \cos(200t + 161.6^\circ)\ V}$$

P 11-35 phasor circuit



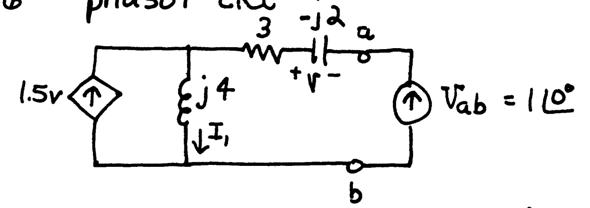
three mesh eqns :

$$(1-j)I_1 + (j)I_2 + 0I_3 = 10$$
$$jI_1 \quad -jI_2 + jI_3 = 0$$
$$0I_1 + jI_2 + (1-j)I_3 = j10$$

$$\text{so } I_2 = \frac{\begin{vmatrix} (1-j) & 10 & 0 \\ j & 0 & j \\ 0 & j10 & (1-j) \end{vmatrix}}{\begin{vmatrix} (1-j) & j & 0 \\ j & -j & j \\ 0 & j & (1-j) \end{vmatrix}} = \frac{10 - j10}{1-j} = 10$$

$$\therefore\ \underline{i(t) = 10 \cos 10^3 t\ A}$$

P 11-36 phasor ckt



$Z_T = \dfrac{V_{ab}}{1}$

$V = -(-j2)(1) = j2 \text{ V}$

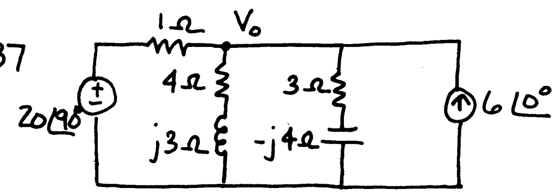
$I_1 = 1 + 1.5V = 1 + j3$

then $V_{ab} = (3 - j2)1 + j4(1 + j3) = -9 + j2$

$\therefore\ Z_T = -9 + j2 = \underline{-9.22\angle -12.5°\ \Omega}$

Thev. equiv. is $V_{oc} = 0$ & $Z_T = -9.22\angle -12.5°\ \Omega$

P 11-37

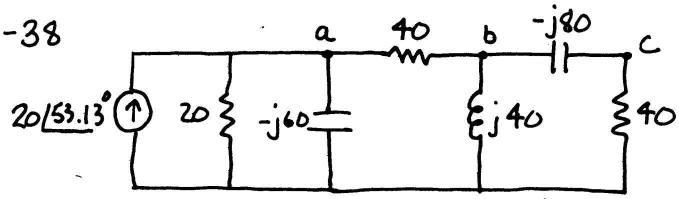


KCL at V_o:

$$\frac{V_o}{4+j3} + \frac{V_o}{3-j4} + \frac{V_o - j20}{1} = 6\angle 0°$$

$\Rightarrow\ \underline{V_o = 16.31\angle 71.5°\ \text{V}}$

P 11-38



$\omega = 10^4 \text{ rad/s}$

KCL at a: $\left(\frac{1}{20} + \frac{1}{40} + \frac{j}{60}\right)V_a + \left(-\frac{1}{40}\right)V_b = 20\angle 53.13°$ (1)

KCL at b: $\left(-\frac{1}{40}\right)V_a + \left(\frac{1}{40} - j/40 + \frac{j}{80}\right)V_b - j/80\,V_c = 0$ (2)

KCL at c: $-\frac{j}{80}V_b + \left(\frac{1}{40} + j/80\right)V_c = 0$ (3)

Solving (1) – (3) simultaneously for V_a

$V_a = \sqrt{2}\cdot 240\angle 45°$ thus $\underline{v_a(t) = 240\sqrt{2}\cos(\omega t + 45°)}$

P 11-39 for algebraic addition, the rectangular form is convenient,

$V_1 = 150\cos(-30°) + j150\sin(-30°) = 130 - j75$ V

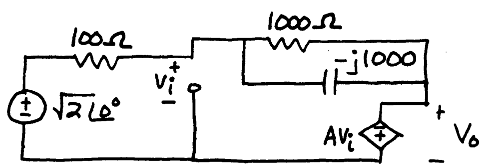
$V_2 = 200\cos 60° + j200\sin 60° = 100 + j173$ V

By the rules for equality and addition

$$V = V_1 + V_2 = 230 + j98 = 250\angle 23.1°\ \text{V}$$

thus $\underline{v(t) = v_1(t) + v_2(t) = 250\cos(377t + 23.1°)}$ V

P. 11-40



KCL at '−' input to op amp : $\dfrac{\sqrt{2}\angle 0° - V_i}{100} = (V_i - V_o)\left(\dfrac{1}{1000} + \dfrac{1}{-j1000}\right)$

also $V_i = -V_o/A$ plugging V_i into above yields

$$V_o\left[1 + \frac{11}{A} + j\left(1 + \frac{1}{A}\right)\right] = -\sqrt{2}\,10\angle 0°$$

assume $A \gg 1$ $\Rightarrow$ $V_o(1+j) = -10\sqrt{2}\angle 0°$

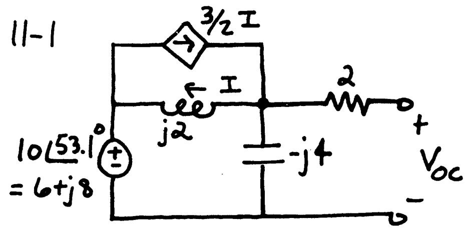
$\Rightarrow$ $V_o = 10\angle 135°$

so $\underline{V_o(t) = 10\cos(1000t + 135°) = 10\cos(1000t - 225°)}$

3 problems missing!

Advanced Problems

AP 11-1

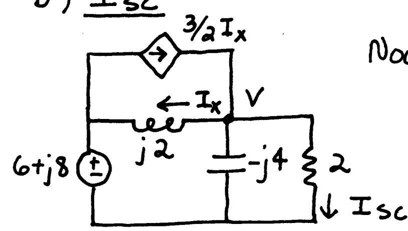


a) $\underline{V_{oc}}$ Node equation:

$$\frac{V_{oc}}{-j4}+\frac{V_{oc}-(6+j8)}{j2}-\frac{3}{2}\left(\frac{V_{oc}-(6+j8)}{j2}\right)=0$$

yields $\underline{V_{oc}=3+j4=5\angle 53.1^\circ}$

b) $\underline{I_{sc}}$



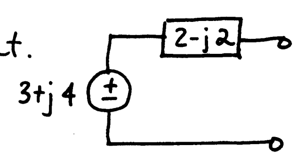
Node eqn:

$$\frac{V}{2}+\frac{V}{-j4}+\frac{V-(6+j8)}{j2}-\frac{3}{2}\left[\frac{V-(6+j8)}{j2}\right]=0$$

yields $V=\dfrac{3+j4}{1-j}$

thus $I_{sc}=\dfrac{V}{2}=\dfrac{3+j4}{2-j2}$ $\therefore$ $Z_T=\dfrac{V_{oc}}{I_{sc}}=3+j4\left(\dfrac{2-j2}{3+j4}\right)=\underline{2-j2}$

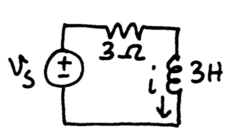
so Thev. equiv. ckt.



AP 11-2

1) from $t<0$ circuit $\Rightarrow i(0)=2$

2) switch 2 closes @ $t=0$



$\tau=L/R=3/3=1\text{s}$ $\therefore$ $i_n(t)=Ae^{-t}$

3) forced response $\Rightarrow$ use phasors, $V_s=3\angle 0^\circ$

now $Z=3+j\omega L=3+j6$ $\therefore$ $I_f=\dfrac{3\angle 0^\circ}{3+j6}=.447\angle -63.44^\circ$

so $i_f(t)=.447\sin(2t-63.4^\circ)$

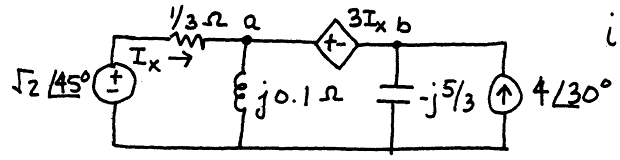
$\therefore$ $i(t)=Ae^{-t}+.447\sin(2t-63.4^\circ)$

$i(0)=2=A+.447\sin(-63.4^\circ)$ $\Rightarrow A=2.4$

so $\underline{i(t)=2.4e^{-t}+.447\sin(2t-63.4^\circ)}$ A

AP 11-3 $\quad v_s = \sqrt{2}\sin(\omega t + 135°) = \sqrt{2}\cos(\omega t + 45°)$; $\omega = 100$ rad/s

$i_s = 4\cos(\omega t + 30°)$



form supernode with a–b & do KCL

$$\frac{V_a - \sqrt{2}\angle 45°}{1/3} + \frac{V_a}{j0.1} + \frac{V_b}{-j5/3} - 4\angle 30° = 0 \qquad (1)$$

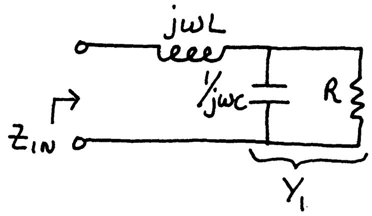
also: $V_a - V_b = 3I_x = 3\left[\frac{\sqrt{2}\angle 45° - V_a}{1/3}\right] = 9(1+j) - 9V_a \qquad (2)$

solving for V_b in (2) & subbing into (1) yields

$$V_a = 2.09\angle 137.3°$$

thus $\underline{v_a(t) = 2.09\cos(\omega t + 137.3°)\,\text{V}}$ $\quad \omega = 100$ rad/s

AP 11-4



$Y_1 = \frac{1}{R} + j\omega C$

$Z_1 = \frac{1}{Y_1} = \frac{1}{1/R + j\omega C} = \frac{R}{1 + j\omega RC}$

now $Z_{IN} = j\omega L + \frac{R}{1 + j\omega RC} = \frac{R(1-\omega^2 LC) + j\omega L}{1 + j\omega RC}$

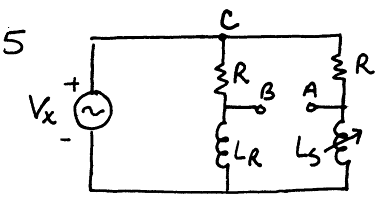
with $L = 97.5\,\text{mH}$, $C = 39\,\text{pF}$, $\omega = 10^8$ rad/s

$$Z_{IN} = \frac{R(1 - .038) + j9.75}{1 + j0.0039R} = \frac{.962R + j9.75}{1 + j0.0039R}$$

$\underline{\text{for } R = 25\,\Omega}$ $\quad Z_{IN} = \underline{25.8\angle 16.5° = 24.7 + j7.33\ \Omega}$

for $R = 50\,\Omega$ $\quad Z_{IN} = \underline{48.2\angle 0.43° = 48.2 + j0.36\ \Omega}$

AP 11-5



$V_x = \sin(2\pi \cdot 400t)$, $\omega = 2\pi \cdot 400$

$R = 100\,\Omega$

$L_R = 40\text{mH}$

$L_s = 60\text{mH}$, door opened

$= 60\text{mH}$, door closed

with the door open $\rightarrow |V_A - V_B| = 0$ since bridge ckt is balanced

with the door closed $\rightarrow Z_{L_R} = j(800\pi)(0.04) = j\,100.5\,\Omega$

$Z_{L_S} = j(800\pi)(0.06) = j\,150.8\,\Omega$

using nodal analysis

node B: $\dfrac{V_B - V_C}{R} + \dfrac{V_B}{Z_{LR}} = 0 \quad \Rightarrow V_B = \dfrac{j100.5}{j100.5 + 100} V_C$

for $V_C = |V_x| = 1 \quad V_B = 0.709 \angle 44.86^\circ$

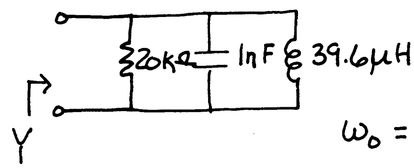
node A: $\dfrac{V_A - V_C}{R} + \dfrac{V_A}{Z_{LS}} = 0 \quad \Rightarrow V_A = 0.833 \angle 33.55^\circ$ for $V_C = |V_x| = 1$

$\therefore\ V_A - V_B = .833 \angle 33.55^\circ - .709 \angle 44.86^\circ$

$= (.694 + j\,.460) - (.503 + j0.500) = .191 - j0.040$

$V_A - V_B = \underline{0.195 \angle -11.83^\circ}$

AP 11-6



$Y = G + Y_L + Y_c$

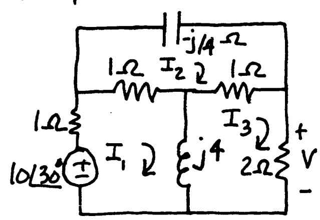
$Y = G$ when $Y_L + Y_c = 0$ or $\frac{1}{j\omega L} + j\omega C = 0$

$\omega_o = \frac{1}{\sqrt{LC}}$, $f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{39.6\times10^{-15}}}$

$= .07998 \times 10^7$ Hz $= 800$ kHz

(80 on the dial of the radio)

AP 11-7 phasor ckt



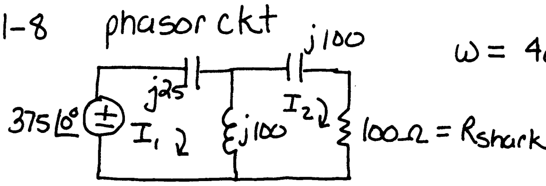
$$\begin{bmatrix} (2+j4) & -1 & -j4 \\ -1 & (2+1/j4) & -1 \\ -j4 & -1 & (3+j4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 30° \\ 0 \\ 0 \end{bmatrix}$$

using Cramer's rule $I_3 = \frac{2+j8}{12+j22.5} 10\angle 30° = 3.225\angle 44°$

$V = 2I_3$

$\therefore$ $v(t) = 6.45\cos(10^5 t + 44°)$ V

AP 11-8 phasor ckt



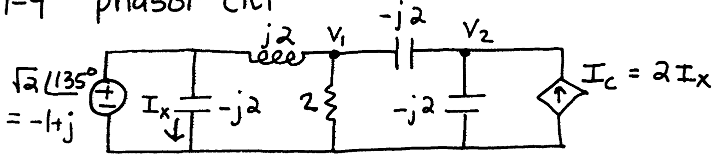
$\omega = 400$ rad/s

KVL: $j75I_1 - j100I_2 = 375$ (1)

KVL: $-j100I_1 + (100+j100) = 0$ (2)

solving for I_2 yields $I_2 = 4.5 + j1.5$ $\Rightarrow i_2(t) = 4.74\angle 18.4°$ A

AP 11-9 phasor ckt



node V_1 : $\dfrac{V_1 - (-1+j)}{j2} + \dfrac{V_1}{2} + \dfrac{V_1 - V_2}{-j2} = 0$ (1)

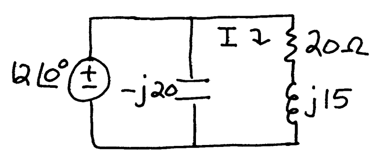
node V_2 : $\dfrac{V_2 - V_1}{-j2} + \dfrac{V_2}{-j2} - I_c = 0$ (2)

also : $I_c = 2I_x = 2\left[\dfrac{-1+j}{-2j}\right] = -1-j$ (3)

Solving (1) thru (3) yields $V_2 = \dfrac{-3-j}{1+j2} = \sqrt{2}\angle -45°$

$\therefore$ $\underline{v(t) = v_2(t) = \sqrt{2}\cos(40t - 45°)\ V}$

AP 11-10 $\omega = 2\pi(796) = 5000$



$Z_C = \dfrac{-j}{5000 \times 10^{-5}} = -j20$

$Z_L = 15\angle 90° = -j15$

$Z_{RL} = Z_R + Z_L = 20 + j15 = 25\angle 37°\ \Omega$

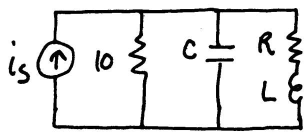
$Z_{eq} = \dfrac{Z_{RL} \cdot Z_C}{Z_{RL} + Z_C} = \dfrac{25\angle 37° \cdot 20\angle -90°}{20 + j15 - j20} = \dfrac{500\angle -53°}{20.6\angle -14°} = 24.3\angle -39°\ \Omega$

so $I = \dfrac{V}{Z_{eq}} = \dfrac{12\angle 0°}{24.3\angle -39°} = 0.49\angle 39°\ A$

$\underline{i(t) = 0.49\cos(5000t + 39°)\ A}$

Design Problems

DP 11-1



$Z_1 = 10 \qquad Y_1 = 1/10$

$Z_2 = 1/j\omega C \qquad Y_2 = j\omega C$

$Z_3 = R + j\omega L \qquad Y_3 = 1/R+j\omega L$

use the fact that admittances in parallel add and $V(\Sigma Y) = I$

so $V(Y_1 + Y_2 + Y_3) = I_s$ with $v(t) = 80\cos(1000t - \theta) \Rightarrow V = 8\angle{-\theta}$

$i_s(t) = 10\cos 100t \Rightarrow I_s = 10\angle{0^\circ}$

so have $80\angle{-\theta}\left[\frac{1}{10} + \frac{1}{R+j\omega L} + j\omega C\right] = 10\angle{0^\circ}$

$\Rightarrow R + 10 - 10\omega^2 LC + j(\omega L + 10\omega RC) = 1.25R + j1.25\omega L$

equate real part : $40 - 40\omega^2 LC = R \qquad (1)$

" imaginary part : $40RC = L \qquad (2)$

plugging (2) into (1) yields $R = 40(1 - 4\times10^7 RC^2) \qquad \omega = 1000\ \text{rad/s}$

now try $\underline{R = 20\Omega} \quad \Rightarrow \quad 1 - 2(1 - 4\times10^7(20)C^2)$

which yields $C = 2.5\times10^{-5}\text{F} = \underline{25\mu\text{F}}$

$\therefore L = 40RC = 0.02\text{H} = \underline{20\text{mH}}$

now check θ : $Y_1 = 1/10 = 0.1$

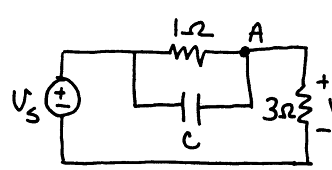
$Y_2 = j0.25$

$Y_3 = 1/20+j20 = .025 - j.025$

$\therefore Y = Y_1 + Y_2 + Y_3 = .125$, so $V = YI_s = (.125\angle{0^\circ})(10\angle{0^\circ}) = 1.25\angle{0^\circ}$

$\therefore \underline{\theta = 0^\circ}$ meets the design spec.

DP 11-2



$\omega = 3\ \text{rad/s}$

$Z_C = 1/j\omega C$

Nodal analysis at A :

$$\frac{V_o - V_s}{1} + \frac{V_o - V_s}{1/j\omega C} + \frac{V_o}{3} = 0$$

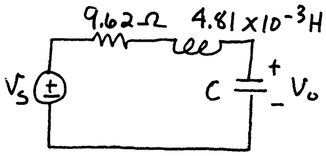
$$\Rightarrow \frac{V_o}{V_s} = \frac{3 + j3\omega C}{4 + j3\omega C}$$

now $\frac{V_o}{V_s} = \frac{9\angle{\phi}}{\frac{15}{\sqrt{2}}\angle{0^\circ}} = \frac{3 + j9C}{4 + j9C} = \frac{\sqrt{9 + (9C)^2}\ \angle{\tan^{-1}(9C/3)}}{\sqrt{16 + (9C)^2}\ \angle{\tan^{-1}(9C/4)}}$

equate magnitudes & angles $\quad \frac{\sqrt{2}\cdot 9}{15} = \frac{\sqrt{9 + (9C)^2}}{\sqrt{16 + (9C)^2}} \Rightarrow \underline{C = 1/3\ \text{F}}$

$$\phi = \frac{\tan^{-1}\left(\frac{9\cdot 1/3}{3}\right)}{\tan^{-1}\left(\frac{9\cdot 1/3}{4}\right)} = \underline{8.13^\circ}$$

DP 11-3



$Z_L = j\omega L = j\,48.1$, $V_s = I\left(9.62 + j48.1 - j10^4/C\right)$

$Z_C = -j10^{-4}/C$

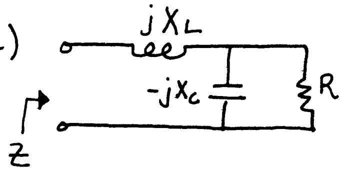
now $V_o = IZ_C = \dfrac{1}{j10^4C}\left[\dfrac{10}{9.62 + j(48.1 - 1/10^4C)}\right] = \dfrac{10}{1 - 48.1\times10^4C + 9.62\times10^4Cj}$

or $|V_o| = \dfrac{10}{\sqrt{(1-48.1\times10^4C)^2 + (9.62\times10^4C)^2}}$

C	Vo
.5μF	13.14 v
1.0	18.95
1.5	31.88
2.0	50.99 ← $|V_o|_{max}$
2.5	31.81
3.0	18.91

$\simeq 51$ V at $C = 2\mu F$

DP 11-4 a)



$Z = jX_L + Z_1$

$Z_1 = \dfrac{R(-jX_c)}{R - jX_c}\cdot\left(\dfrac{R+jX_c}{R+jX_c}\right) = \dfrac{RX_c^2 - jR^2X_c}{R^2 + X_c^2}$

want $Z_1 = aR$, so $aR = \dfrac{RX_c^2}{R^2+X_c^2} \Rightarrow \underline{X_c = R\sqrt{\dfrac{a}{1-a}}}$ (1)

need $Im\{Z_1\} = -X_L$ or $\dfrac{R^2X_c}{R^2+X_c^2} = X_L$ using (1) above get

$\underline{X_L = R\sqrt{a(1-a)}}$ (2)

b) $a = .2$, $R = 100$, $\omega = 1000$

from (1) $1/\omega C = 100\sqrt{.2/.8} = \dfrac{100}{2} \Rightarrow \underline{C = 20\mu F}$

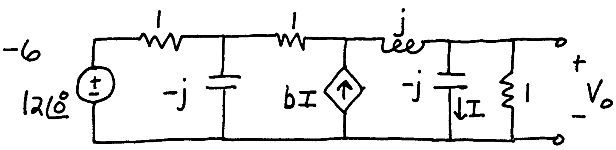
from (2) $\omega L = 100\sqrt{.2(.8)} = 40 \Rightarrow \underline{L = 40\text{mH}}$

DP 11-5 want $X_L = X_C$, $C = .25$ mF

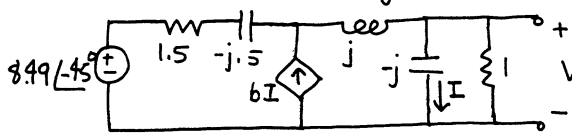
$X_c = 1/\omega C = \dfrac{1}{400\times250\mu F} = 10$

then $X_L = 10 = \omega L = 400L \Rightarrow \underline{L = 10/400 = 25\text{mH}}$

DP 11-6



⇓ use Thev. equiv.



node eqns:

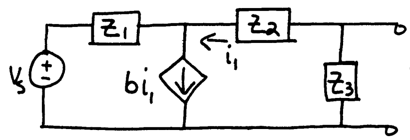
$$\frac{V_1 - 8.49\angle -45^\circ}{1.5 - j.5} + \frac{V_1 - V_o}{j} = bI \qquad (1)$$

$$\frac{V_o - V_1}{j} + \frac{V_o}{-j} + \frac{V_o}{1} = 0 \qquad (2)$$

also have $I = \frac{V_o}{-j}$ (3)

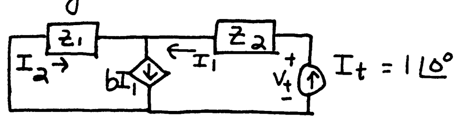
Solving (1), (2) and (3) with $V_o = 6$ yields $\underline{b = 2}$

DP 11-7



$\leftarrow Z_{Th} = 14\angle -45^\circ = 9.899 - j\,9.899$

apply test source $I_t = 1\angle 0^\circ$ A and short V_s



$I_2 = 1\angle 0^\circ$

KCL: $I_2 + I_1 = bI_1 = 2I_1$

so $I_2 = I_1 = 1\angle 0^\circ$

$\therefore V_t = -I_2 Z_1 + I_1 Z_2 = (Z_2 - Z_1)\, 1\angle 0^\circ$

so $Z_t' = \frac{V_t}{I_t} = Z_2 - Z_1 \Rightarrow Z_{Th} = Z_t' // Z_3 = (Z_2 - Z_1) // Z_3$

now have 1 R, 1 L and 1 C, need to choose location of elements for $Z_{Th} = 14\angle -45^\circ$.

let $R = Z_3$, need to choose Z_2 & Z_1 $\Rightarrow Z_2 = (Cs)^{-1}$, $Z_1 = Ls$

$$\therefore Z_{Th} = R // \left[\frac{1}{j\omega C} - j\omega L\right]$$

choose $1/\omega C = \omega L = X$ due to -45° given

$Z_{Th} = R // [-jX - jX] = \frac{j2RX}{R - j2X}$, also $Z_{Th} = \frac{14}{\sqrt{2}} - j\frac{14}{\sqrt{2}}$

now need $R = 2X$, then have

$$\frac{14}{\sqrt{2}} - j\frac{14}{\sqrt{2}} = \frac{2R(R/2)\angle -90^\circ}{R\angle -45^\circ} = \frac{R}{\sqrt{2}}(1 - j)$$

$\therefore \underline{R = 14\,\Omega}$

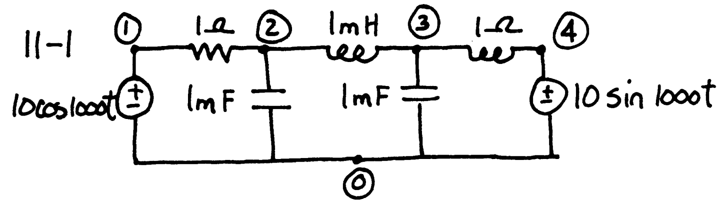
thus $X = R/2 = 7\,\Omega$, with $\omega = 2\pi(60) = 377$ rad/s

$\omega L = 7 \Rightarrow \underline{L = 18.6\text{ mH}}$

$1/\omega C = 7 \Rightarrow \underline{C = 379\,\mu F}$

Spice Problems

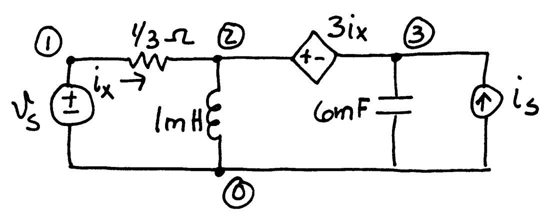
SP 11-1



input file :
```
V1  1 0  ac 10 90 ; cos10wt
R1  1 2  1
C1  2 0  1m
L1  2 3  1m
C2  3 0  1m
R3  3 4  1
V2  4 0  ac 10 0 ; sin10wt
.ac lin 1 159.15  159.15
.print ac  IM(L1)  IP(L1)
.end
```

output : IM (L1) = 10 at f = 159.15 Hz
IP (L1) = 9

SP 11-2

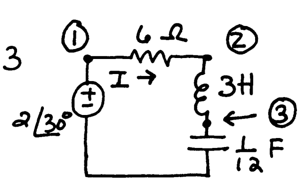


ans.

<u>$V(t) = 2.09\cos(100t + 137.3°)$</u>

input file :
```
Vs  1 0  ac 1.414  135 ; 1.414sin(100t+135)
R1  1 2  .33333
L1  2 0  1e-3
H1  3 2  Vs 3
C1  3 0  6e-3
I1  0 3  ac 4  120 ; 4cos(100t+30)
.ac lin 1 15.915  15.915
.print  ac  vm(L1) vp(L1)
.end
```

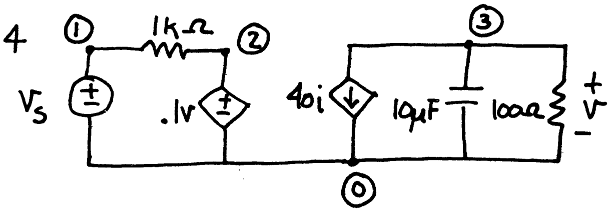
SP 11-3




```
RLC  Series circuit
V1   1 0  ac  2  30
R1   1 2  6
L1   2 3  3
C1   3 0  0.08333
.AC  DEC  1  0.6366   0.6366
.PRINT  AC  IM(R1)  IP(R1)
.END
```

ans: IM = .1849
IP = -26.31
} <u>$i(t) = .1849\cos(\omega t - 26.3°)$</u>

SP 11-4

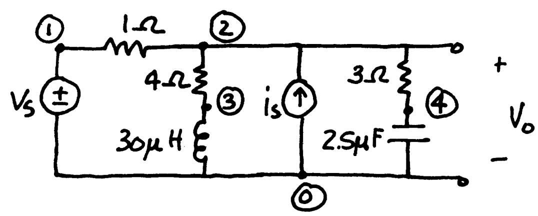



```
VS  1 0 ac 5sin(0 5 31.831 0 0 90)
R1  1 2  1E3
E1  2 0  3 0 0.1 ; this is where you change B
F1  0 3 Vs 40
R2  3 0 100
C1  3 0 10E-6 ic 0
.ac lin 1 31.831 31.831
.print ac vm(3) VP(3)
.end
```

Ans. Vm(3) = 31.62 VP(3) = −18.44

so $v(t) = 31.62 \cos(200t + 161.6°)$

SP 11-5

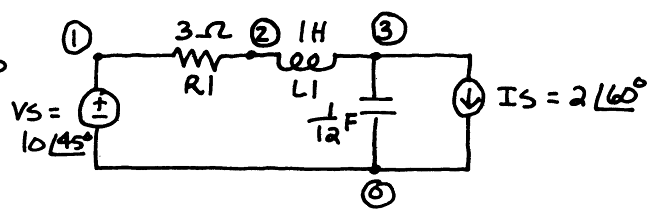



```
VS  1 0 AC 20 90
IS  0 2 AC  6  0
R1  1 2 1
R2  2 3 4
R3  2 4 3
L1  3 0 30E-6
C1  4 5 2.5E-6
.AC LIN 1 15915.5  15915.5
.PRINT AC Vm(2) VP(2)
.END
```

ANS. Vm(2) = 16.31

VP(2) = 71.51°

SP 11-6



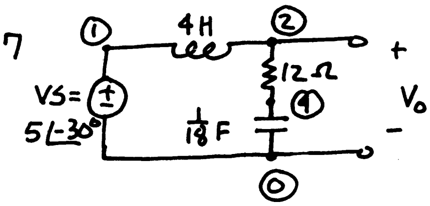
$\omega = 6$ rad/s

$f = .9549$ Hz

```
VS 1 0  AC  10 45
R1 1 2  3
L1 2 3  1
IS  3 0  AC 2 60
C1  3 0  0.0833
.AC DEC 1 0.9549  0.9549
.PRINT AC IM(L1) IP(L1)
.end
```

ANS. IM(L1) = 2.339 IP(L1) = −27.43°

SP 11-7



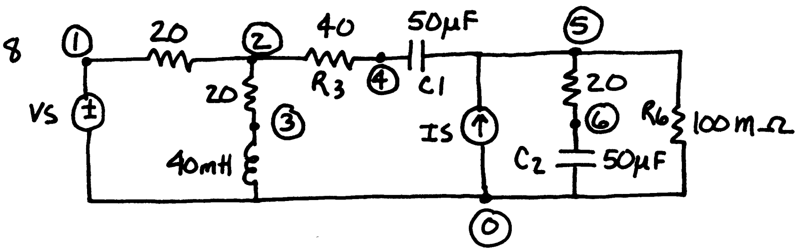
input file

```
VS   1  0   AC  5  -30
L1   1  2   4
R1   2  3   12
C1   3  0   0.0556
.AC  DEC  1  0.4775  0.4775
.PRINT  AC  VM(2)  VP(2)
.END
```

ANS. VM(2) = 4.998

VP(2) = -83.13°

SP 11-8



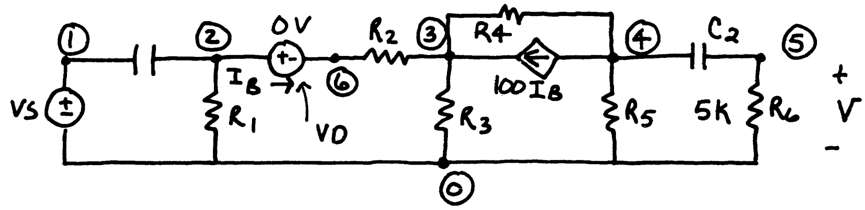
R6 provides dc path to ground with negligible effect for node 5

input file :

```
VS   1  0   AC  200  0
R1   1  2   20
R2   2  3   20
L1   3  0   40m
R3   2  4   40
C1   4  5   50μ
R4   5  6   20
C2   6  0   50u
IS   0  6   AC  8  90
R6   5  0   1E8 ; path to ground
.AC  LIN 1  159.155  159.155
.PRINT  AC  IR(R3)  II(R3)
.END
```

output : IR(R3) = 0.4525   II(R3) = -1.256

SP 11-9



use VO to measure IB

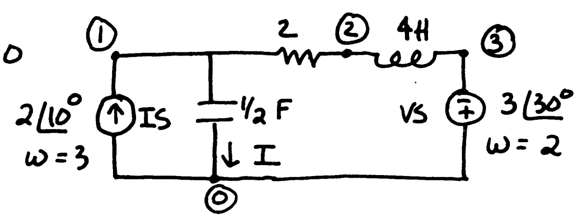
input file:

```
VS  1 0  AC  4  0
C1  1 2  10N
R1  2 0  20K
VO  2 6  AC  0  0
R2  6 3  1K
R3  3 0  100
R4  3 4  100K
F1  4 3  VO  100 ; CCCS
R5  4 0  10K
C2  4 5  20N
R6  5 0  5K
.AC LIN 1  1000  1000
.PRINT  VM(5)  VP(5)
.END
```

ANS. VM(5) = 42.45 VP(5) = - 84.7

$V_o = 42.45 \underline{/-84.7^\circ}$

SP 11-10



2 different frequencies
∴ use superposition

PART A with ω = 2

```
VS 0 3 AC 3 30
C1 1 0 0.5
R1 1 2 2
L1 2 3 4
.AC DEC 1 0.318 0.318
.PRINT AC IM(C1) IP(C1)
.END
```

ANS. $I_1 = 0.412 \underline{/135.9^\circ}$

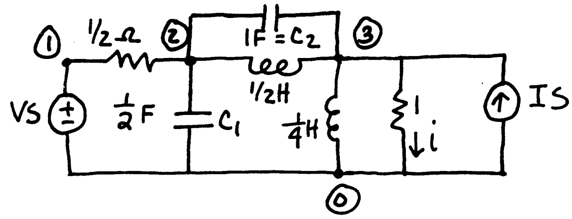
PART B with ω = 3

```
IS 0 1 AC 2 10
C1 1 0 0.5
R1 1 2 2
L1 2 3 4
.AC DEC 1 0.4775 0.4775
.PRINT AC IM(C1) IP(C1)
.END
```

Ans. $I_2 = 2.114 \underline{/10.55^\circ}$

$\therefore \; i(t) = 0.412\cos(2t + 135.9^\circ) + 2.114\cos(3t + 10.55^\circ)$

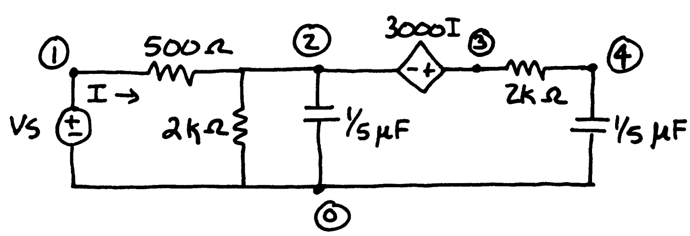
SP 11-13




```
VS  1 0  AC  5 0
R1  1 2  0.5
C1  2 0  0.5
C2  2 3  1
L1  2 3  0.5
L2  3 0  0.25
R2  3 0  1
IS  0 3  AC  5 0
.AC  LIN 1  .3183  .3183
.PRINT  VM(2)  VP(2)  IM(R2)  IP(R2)
```

ANS. VM(2) = 2.236   VP(2) = -26.57
IM(R2) = 4.472   IP(R2) = 63.43

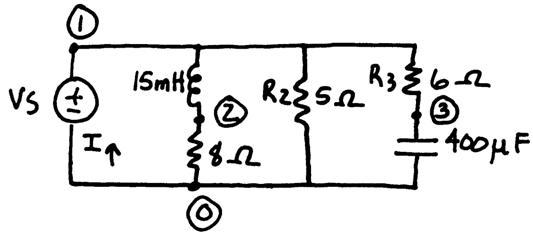
SP 11-14




```
VS  1 0  AC  4 0
R1  1 2  500
R2  2 0  2K
C1  2 0  .2U
H1  3 2  VS  -3000
R3  3 4  2K
C2  4 0  .2U
.AC  LIN 1  795.77  795.77
.PRINT  AC  IM(R1)  IP(R1)
.END
```

ANS. IM(R1) = 0.024   IP(R1) = 53.13

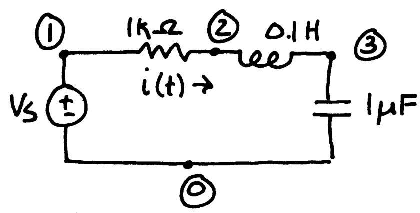
SP 11-15



let VS = cos 2πft   f = 60 Hz

```
VS    1  0   AC   1   0
R1    2  0   8
L     1  2   0.015
R2    1  0   5
R3    1  3   6
C     3  0   400u
.AC   LIN  1  60  60
.PRINT  IM(VS)  IP(VS)
.END
```

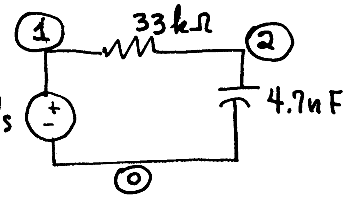
SP 11-16



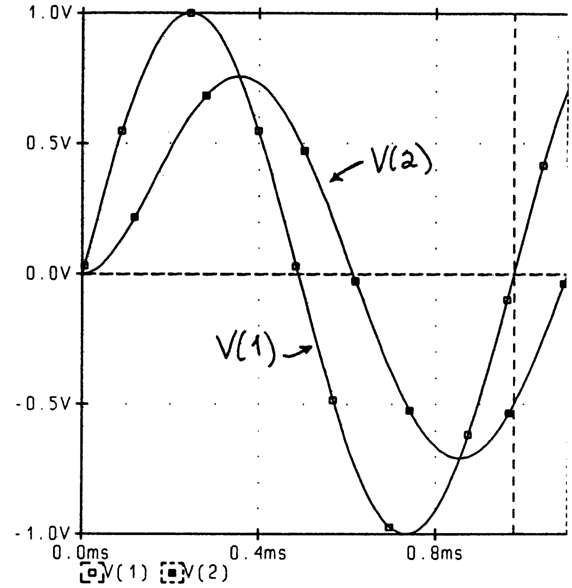

```
VS    1   0   AC   120   30
R1    1   2   1K
L     2   3   0.1
C     3   0   1u
.AC   LIN 1   10K  10K
.PRINT  AC  IM(L)  IP(L)
```

ANS. IM(L) = 1.89 E-02   IP(L) = 129.1

SP 11-17(a)




```
R1  1 2  33k
C1  2 0  4.7E-9
VS  1 0   sin (0 1  1023)
.TRAN   9.8u   1.96m   0   9.8u
.PLOT   V(2)   V(1)
.PROBE
.END
```



Chapter 13

Exercises

Ex. 13-1 (a) $dB = 20\log 2 = \underline{6.02\ dB}$

(b) $dB = 20\log(.5) = \underline{-6.02\ dB}$

Ex. 13-2 $20\log H = 20\log(1/\omega^2) = 20\log(\omega)^{-2} = -40\log\omega$

$\text{slope} = 20\log H(\omega_2) - 20\log H(\omega_1)$

$= -40\log\omega_2 + 40\log\omega_1 = -40\log(\omega_2/\omega_1)$

let $\omega_2 = 10\omega_1$ to consider 1 decade, then

$\underline{\text{slope} = -40\log 10 = -40\ dB/\text{decade}}$

Ex. 13-3 (a) define break freq. as the point where H is $1/\sqrt{2}$ from its value at $\omega = \infty$, since $H \to$ constant as $\omega \to \infty$.

so $H(\infty) = \lim_{\omega\to\infty} H(\omega) = \dfrac{A\omega}{\sqrt{C}\,\omega} = \dfrac{A}{\sqrt{C}}$

now set $H(\omega) = \dfrac{1}{\sqrt{2}} H(\infty) = \dfrac{A}{\sqrt{2C}}$

$\Rightarrow \dfrac{A}{\sqrt{2C}} = \dfrac{A\omega}{(B+C\omega^2)^{1/2}} \quad \Rightarrow \underline{\omega = \sqrt{B/C}}$

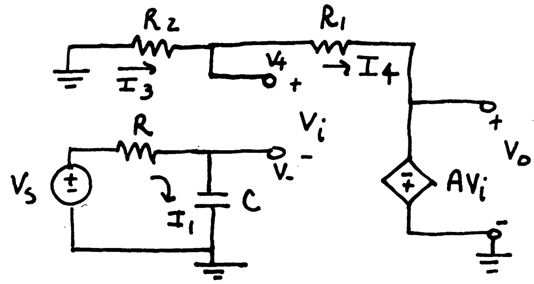
(b) $H(\omega) \to$ constant $\Rightarrow$ $\underline{\text{asymptotic slope} = 0}$

(c) Below the break frequency, treat $(B+C\omega^2)^{1/2}$ as a constant.

$\therefore H(\omega) \simeq A'\omega$ where $A' =$ constant

so $\underline{\text{slope} = 20\ dB/\text{decade}}$

Ex 13-4 use ideal model



$I_1 = \dfrac{V_s}{R + Z_c}$

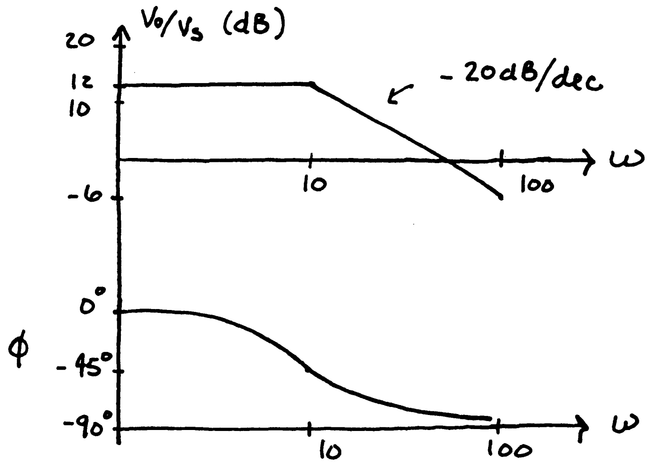
$\therefore V_- = I_1 Z_c = \dfrac{V_s Z_c}{R + Z_c}$

$I_3 = -\dfrac{V_+}{R_2} = I_4$

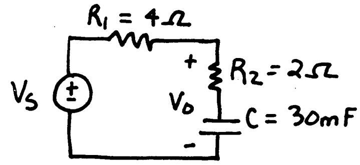
now $V_o = V_+ - I_4 R_1 = V_- - I_3 R_1$

$= \dfrac{V_s Z_c}{R + Z_c} + \dfrac{V_s Z_c}{R + Z_c} \cdot \dfrac{R_1}{R_2} = \left(1 + \dfrac{R_1}{R_2}\right)\left(\dfrac{V_s}{1 + j\omega RC}\right)$

thus $A_v = \dfrac{V_o}{V_s} = \left(1 + \dfrac{R_1}{R_2}\right)\dfrac{1}{j\omega RC + 1}$



Ex 13-5 a)

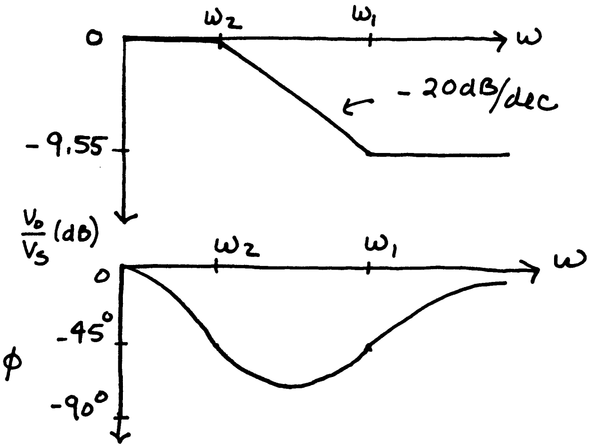


$$Z_o = R_2 + 1/j\omega C$$

$$\frac{V_o}{V_s} = \frac{Z_o}{R_1+Z_o} = \frac{R_2 + 1/j\omega C}{R_1+R_2+1/j\omega C} = \frac{1+j\omega/\omega_1}{1+j\omega/\omega_2}$$

where $\omega_1 = 1/R_2C = 16.7$ rad/s

$\omega_2 = \frac{1}{(R_1+R_2)C} = 5.56$ rad/s



b) $v_s = 10 \cos 20t$ or $V_s = 10\angle 0°$

$$\therefore \frac{V_o}{V_s} = \frac{1+j(20/16.7)}{1+j(20/5.56)} = \frac{1+j1.20}{1+j3.60} = 0.417\angle -24.3°$$

So $V_o = 4.17\angle -24.3°$ $\Rightarrow$ $\underline{v_o(t) = 4.17\cos(20t - 24.3°)\text{ V}}$

Ex. 13-6 (a) $H = \frac{1}{1+jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} = \frac{1}{1+j100\left(\frac{850}{800} - \frac{800}{850}\right)} = \frac{1}{1+j12.1}$

$\Rightarrow$ $\underline{|H| = 0.08}$

(b) $B = \omega_o/Q = \frac{2\pi(800)}{100} = \underline{50k \text{ rad/s} = 8 \text{kHz}}$

Ex. 13-7 $Q\delta = \left[\frac{810-800}{800}\right](100) = 1.25$

from figure 13-17

$\underline{H = 0.37}$ and $\underline{\phi = -68°}$

Ex. 13-8 (a) $Q = \omega_o RC = R\sqrt{C/L} = 8000\sqrt{\dfrac{2.5\times10^{-7}}{40\times10^{-3}}} = \underline{20}$

(b) $B = \dfrac{\omega_o}{Q} = \dfrac{1}{Q\sqrt{LC}} = \dfrac{1}{20\sqrt{(40\times10^{-3})(2.5\times10^{-7})}} = \underline{500\ \text{rad/s}}$

Ex. 13-9 $Q = \omega_o/B = 10^7/2\times10^5 = \underline{50}$

now $\omega_o = \dfrac{1}{\sqrt{LC}} \Rightarrow L = 1/\omega_o^2 C = \dfrac{1}{(10^7)^2(10\times10^{-12})} = \underline{1\text{mH}}$

Ex. 13-10 $\omega_o = 1/\sqrt{LC} = \dfrac{1}{[(10^{-3})(10^{-5})]^{1/2}} = 10^4\ \text{rad/s}$

$Q = \omega_o/B = 10^4/2\pi(15.9) = \underline{100}$

$R = \dfrac{\omega_o L}{Q} = \dfrac{(10^4)(10^{-3})}{100} = \underline{0.1\Omega}$

Ex. 13-11 (a) $\omega_o = 1/\sqrt{LC} \Rightarrow C = 1/(10^6)^2(0.01) = \underline{100\text{pF}}$

$Q = \omega_o/B = 1/\omega_o RC \Rightarrow R = \dfrac{B}{\omega_o^2 C} = \dfrac{10^3}{(10^6)^2(10^{-10})} = \underline{10\Omega}$

(b) $Q = \omega_o/B = 10^6/10^3 = 1000$

$H = \dfrac{1}{1+jQ\left(\dfrac{\omega}{\omega_o}-\dfrac{\omega_o}{\omega}\right)} = \dfrac{1}{1+j1000\left[\dfrac{1.05\times10^6}{10^6}-\dfrac{10^6}{1.05\times10^6}\right]}$

$\underline{H = \dfrac{1}{1+j97.6}}$

Ex 13-12 resonant frequency $\omega_o = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(10\times10^{-6})(5\times10^{-3})}} = 4.47\times10^3\ \text{rad/s}$

Quality factor $Q = \dfrac{\omega_o L}{R} = \dfrac{(4.47\times10^3)(5\times10^{-3})}{5} = 4.47$

So bandwidth is $B = \dfrac{\omega_o}{Q} = 1\times10^3\ \text{rad/s} \Rightarrow \underline{159\ \text{Hz}}$

now $\delta = \dfrac{\omega-\omega_o}{\omega_o} = \dfrac{5\times10^3-4.47\times10^3}{4.47\times10^3} = 0.119$

$Q\delta = (4.47)(0.119) = 0.532$

from figure 13-17 have $|H| = \left|\dfrac{V_o}{V_s}\right| = 0.66$ and $\phi = -50°$

$\therefore\ V_o(t) = 0.66\times10\cos(5\times10^3 t-50°) = \underline{6.6\cos(5\times10^3 t-50°)\text{V}}$

analytical calculation yields

$V_o = 7.07\cos(\omega t-45°)\ \text{V}$

Ex. 13-13 $\omega_b = \frac{1}{(R+R_L)C} \Rightarrow C = \frac{1}{(R+R_L)\omega_b} = \frac{1}{(100+400)(2\pi)(80)}$

$$\underbrace{C \simeq 4\mu F}$$

so $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow L = \frac{1}{\omega_0^2 C}$$

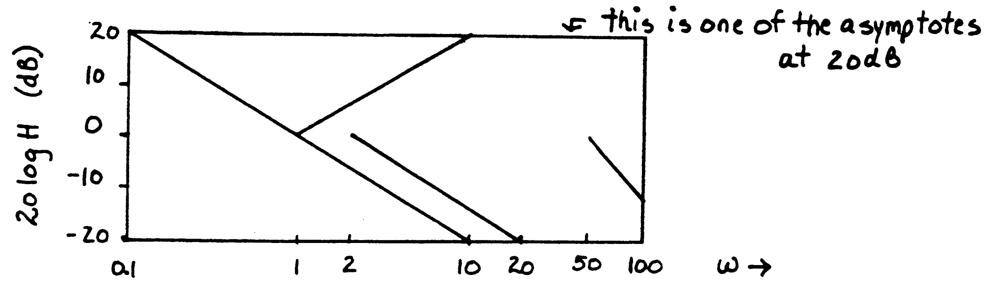
$$= \frac{1}{[2\pi(80)]^2(3.98\times10^{-6})}$$

$$= .0398 H$$

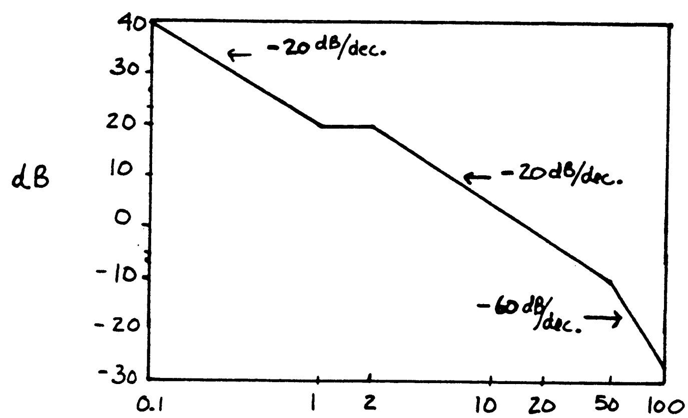
$$\underbrace{L \simeq 40 mH}$$

Ex. 13-14 $H(j\omega) = \dfrac{10(1+j\omega)}{j\omega(1+j0.5\omega)\left[1+j0.6(\omega/50)+(\frac{j\omega}{50})^2\right]}$

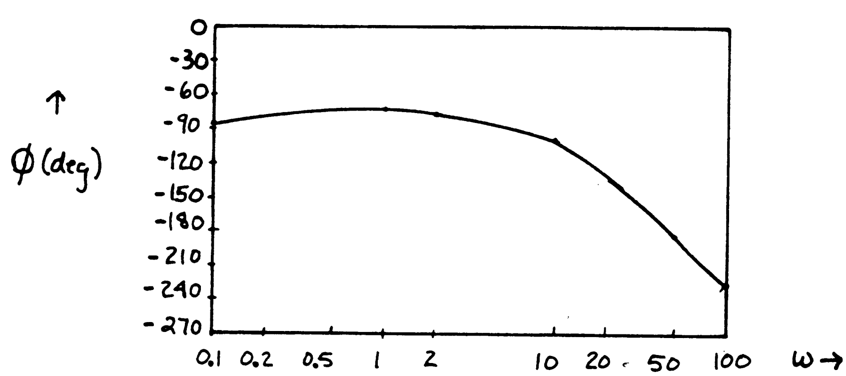
first construct magnitude asymptotes of poles & zero



now add the above for the resultant magnitude response



now $\phi = -90 - \tan^{-1}(0.5\omega) + \tan^{-1}\omega - \tan^{-1}\left[\dfrac{2(0.6)\omega/50}{1-(\omega/50)^2}\right]$



Problems

P. 13-1

$$\omega_o = 1/\sqrt{LC} = \frac{1}{\sqrt{(1/20)(1/30\times 10^{-6})}} = \underline{60{,}000 \text{ rad/s} = 60\text{k rad/s}}$$

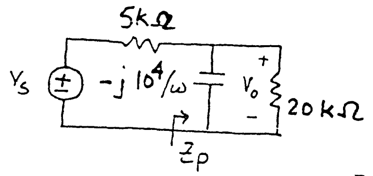
$$Q = \omega_o R C = (60{,}000)(10{,}000)(1/30\times 10^{-6}) = \underline{20}$$

$$\omega_1 = \omega_o\sqrt{1+(1/2Q)^2} - \omega_o/2Q$$

$$\underline{\omega_1 = 60\sqrt{1+(1/40)^2} - 60/40 = 58.519 \text{ k rad/s}}$$

$$B = \omega_o/Q = 60/20 = \underline{3 \text{ k rad/s}}$$

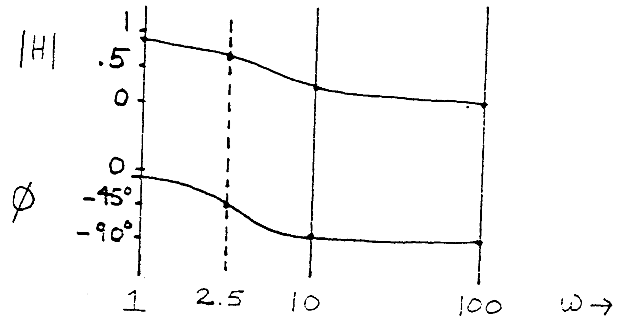
P. 13-2



$$Z_p = \frac{(2\times 10^4)(-j\,10^4/\omega)}{2\times 10^4 - j\,10^4/\omega} = \frac{2\times 10^4}{1+j2\omega}$$

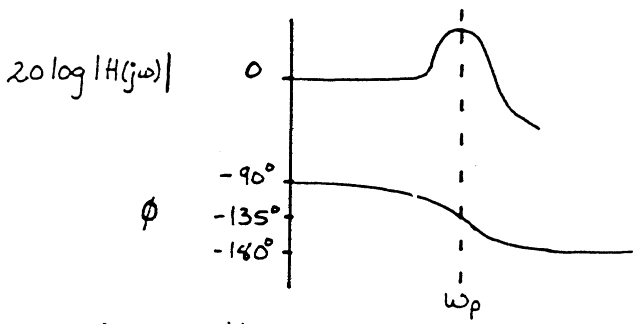
$$\therefore\ \underline{H(j\omega) = V_o/V_s = \frac{Z_p}{Z_p+1} = \frac{4}{[5+j2\omega]}}$$

$$\Rightarrow\ \underline{|H(j\omega)| = \frac{4}{[(2\omega)^2+5^2]^{1/2}}} \qquad \Rightarrow\ \underline{\phi(\omega) = -\tan^{-1}\frac{2\omega}{5}}$$



P.13-3 (a) $H(j\omega) = \frac{V_o}{V_s} = \frac{1/sC}{R+Ls+1/sC} = \underline{\frac{1}{1-(\omega/\omega_o)^2 + j\frac{(\omega/\omega_o)}{Q}}}$ where: $Q = \frac{\omega_o L}{R}$, $s = j\omega$

(b) $|H(j\omega)| = \frac{1}{\sqrt{[1-(\omega/\omega_o)^2]^2 + (\omega/\omega_o)^2/Q^2}}$ $\quad \phi = -\tan^{-1}\frac{(\omega/\omega_o)}{Q[1-(\omega/\omega_o)^2]}$



(c) $\omega_o = 1/\sqrt{LC}$

(d) $\omega_p = \omega_o\sqrt{1-2\zeta^2}$ where $2\zeta = 1/Q \Rightarrow \zeta = 1/10$

$= \omega_o\sqrt{1-2/100}$

$\underline{\omega_p = (1000)(.9899) = 989.9}$

P.13-4 $R = V/|I| = 8\text{v}/0.2\text{mA} = \underline{40\text{ k}\Omega}$

@ 22000rad/s, $|H| = 1/2 = \frac{1}{\sqrt{1+R^2(\omega C - 1/\omega L)^2}} = \frac{1}{\sqrt{1+(40{,}000)^2(22000C - \frac{1}{22000L})^2}}$

$\Rightarrow 22{,}000\, C - \frac{1}{22{,}000 L} = \pm\sqrt{3}/40{,}000$ (1)

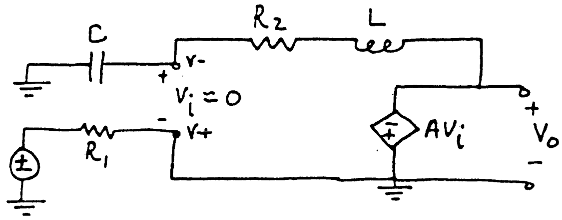
also $1/\sqrt{LC} = 25000 \Rightarrow L = \frac{1}{(25{,}000)^2 C}$ (2)

(2) into (1) and choosing the '−' root of (1) yields

$\underline{C = 6.76\times10^{-9}\text{ F} = 6.76\text{ nF}}$

$\therefore \underline{L = \frac{1}{(25{,}000)^2(6.76\times10^{-9})} = 0.237\text{ H} = 237\text{ mH}}$

P. 13-5 assume ideal op amp



(a) with $V_i = 0$, $V- = V+ = V_s$

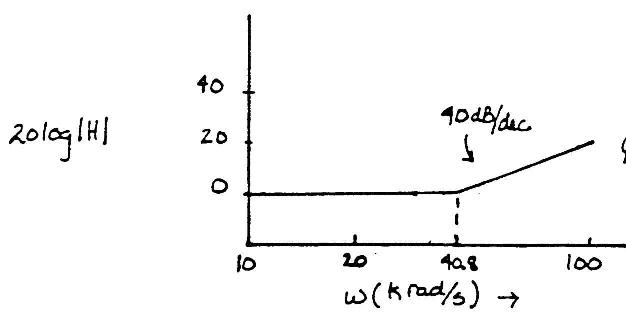
so KCL at V- : $\dfrac{V}{1/sC} + (V_s - V_o)/(R_2 + sL) = 0$

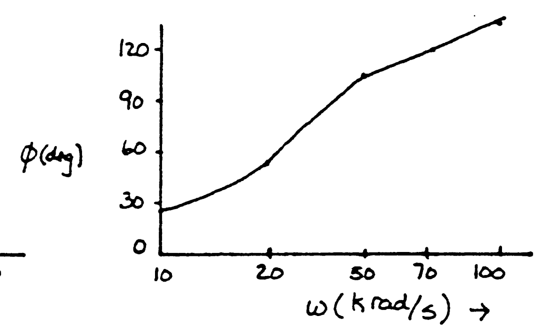
$$\Rightarrow \quad \frac{V_o}{V_s} = 1 + sC[R_2 + sL] \qquad s = j\omega$$

$$\underline{H(j\omega) = \frac{V_o}{V_s} = LC\left[(j\omega)^2 + j\frac{R_2\omega}{L} + \frac{1}{LC}\right]}$$

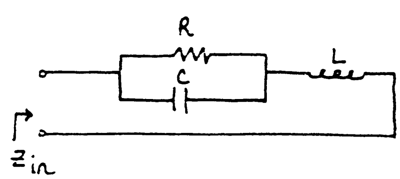
(b) $H(j\omega) = 1 + j\,(1.96)\left(\dfrac{\omega}{4.08\times10^4}\right) + \left(\dfrac{j\omega}{4.08\times10^4}\right)^2$

$$\phi = \tan^{-1}\left[\frac{1.96\left(\frac{\omega}{4.08\times10^4}\right)}{1 - \left(\frac{\omega}{4.08\times10^4}\right)^2}\right]$$





P. 13-6

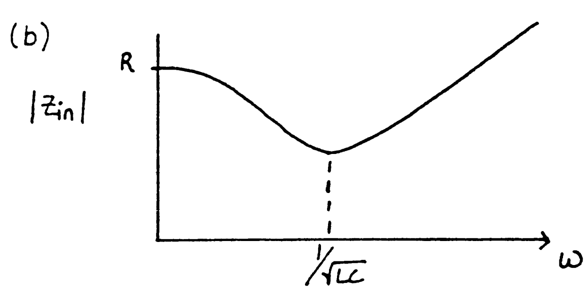


(a) $Z_{in} = j\omega L + \dfrac{R/j\omega C}{R + 1/j\omega C}$

$$Z_{in} = \frac{(R - \omega^2 RLC) + j\omega L}{1 + j\omega RC}$$

$$\therefore \; \underline{|Z_{in}| = \sqrt{\frac{(R - \omega^2 RLC)^2 + (\omega L)^2}{1 + (\omega RC)^2}}}$$

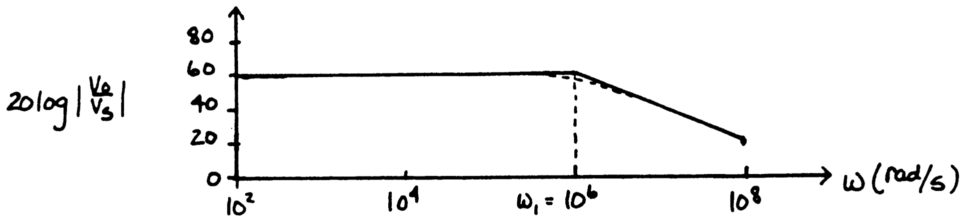
(b)



(c) at $\omega = 1/\sqrt{LC}$, $\underline{|Z_{in}| = \dfrac{1}{\sqrt{C/L\,(1 + R^2 C/L)}}}$

P. 13-7 $\quad V_o = AV_i Z_p \quad$ where $Z_p = \dfrac{R(1/cs)}{R + 1/cs}$ and $V_i = V_s$

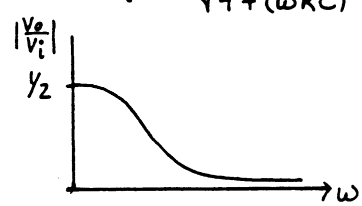
$$\therefore \frac{V_o}{V_s} = \frac{10R}{RSC+1} \Rightarrow \underline{\frac{V_o}{V_s}(j\omega) = \frac{10^3}{j\omega/\omega_1 + 1}} \quad \text{where } \omega_1 = 10^6$$



bandwidth = 10^6 rad/s $\Rightarrow$ $\underline{\text{gain bandwidth} = (1000)(10^6) = 10^9}$

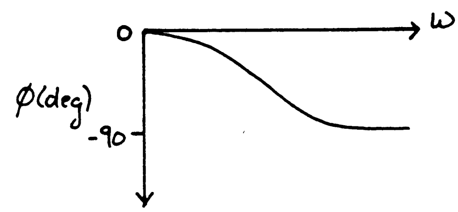
P. 13-8 (a) $\dfrac{V_o}{V_i} = \dfrac{\dfrac{R/sc}{R+1/sc}}{R + \dfrac{R/sc}{R+1/sc}} = \dfrac{1}{2+RSC} = \underline{\dfrac{1}{2+j\omega RC}}$

(b) $\Rightarrow \left|\dfrac{V_o}{V_i}\right| = \dfrac{1}{\sqrt{4+(\omega RC)^2}}$



Above gives correct limiting results. At $\omega = 0$, the cap is open so dc voltage divider yields $V_o/V_i = 1/2$. At $\omega = \infty$ cap is a short $\Rightarrow V_o/V_i = 0$

(c)



At $\omega = 0$, V_i and V_o are in phase. At $\omega = \infty$ V_o lags $I_c = V_i/R$ by 90°

P. 13-9 From the Bode magnitude plot, $|Z| = \dfrac{2}{\sqrt{1+(\omega/10^4)^2}}$

$$\therefore\ Z = \frac{\pm 2}{1 \pm j\,\omega/10^4}$$

choose $+2$ for positive resistance, so $Z = \dfrac{2}{1 \pm j\,\omega/10^4}$ (1)

now the magnitude response is clearly that of a parallel RC combination, $\therefore$ can also write

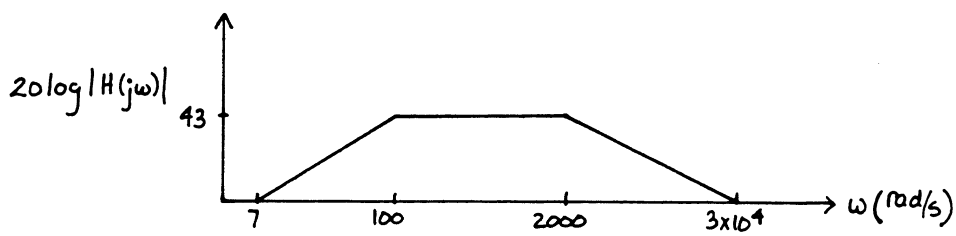
$$Z = \frac{R}{1 + j\omega RC} \qquad (2)$$

comparing (1) & (2) and using the (+) sign in (1)

$\Rightarrow$ $\underline{R = 2\,\Omega}$

$RC = 1/10^4 \ \Rightarrow\ \underline{C = 5\times10^{-5}\,F = 50\,\mu F}$

P. 13-10 A reasonable approximation (asymptotic) for the Bode diagram is:



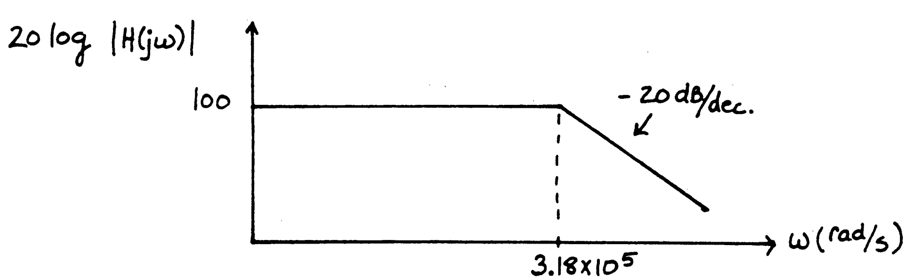
$$\therefore\ H(j\omega) = \frac{A(1+j\omega/7)(1+j\omega/3\times10^4)}{(1+j/100)(1+j\omega/2000)}$$

at the peak, $H(j\omega) \simeq \dfrac{A\,j\omega/7}{j\omega/100} = \dfrac{100}{7}A$

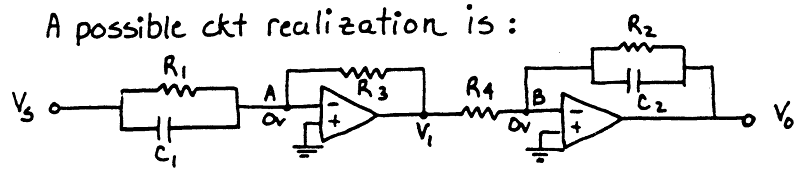
$\therefore\ 43 = 20\log\left(\frac{100}{7}A\right) \ \Rightarrow\ A = 10$

$$\Rightarrow\ \underline{H(j\omega) = \frac{10(1+j\omega/7)(1+j\omega/3\times10^4)}{(1+j/100)(1+j\omega/2000)}}$$

P. 13-11 $H(j\omega) = \dfrac{k}{j\omega\tau+1} = \dfrac{10^5}{1+j\,\omega/3.18\times10^5}$ where $\tau = 1/\omega_0 = 2\pi/f_0$



P. 13-12 A possible ckt realization is:



assuming ideal op amps

KCL at A yields: $\dfrac{V_1}{V_s} = \dfrac{-R_3}{\dfrac{R_1/sC_1}{R_1+1/sC_1}} = -R_3/R_1\,(1+sR_1C_1)$

KCL at B yields: $\dfrac{V_o}{V} = -\dfrac{\dfrac{R_2/sC_2}{R_2+1/sC_2}}{R_4} = -R_2/R_4\left(\dfrac{1}{1+sR_2C_2}\right)$

$\therefore\ H(s) = \dfrac{V_o}{V_s} = \dfrac{V_o}{V_1}\cdot\dfrac{V_1}{V_s} = \dfrac{R_2R_3}{R_1R_4}\cdot\dfrac{1+sR_1C_1}{1+sR_2C_2}$

$\Rightarrow H(j\omega) = \dfrac{R_2R_3}{R_1R_4}\,\dfrac{(1+j\,\omega/\omega_1)}{(1+j\,\omega/\omega_2)}$ where $\omega_1 = 1/R_1C_1$, $\omega_2 = 1/R_2C_2$

now $\dfrac{R_2R_3}{R_1R_4} = 10$ so let $\underline{R_4 = 1\,k\Omega}$ and $\underline{R_3 = 100\,k\Omega}$

and thus $R_2/R_1 = 0.1$ so let $\underline{R_1 = 10\,k\Omega} \Rightarrow \underline{R_2 = 1\,k\Omega}$

now $\omega_1 = 1/R_1C_1 = 100 \Rightarrow \underline{C_1 = 1\,\mu F}$

$\omega_2 = 10^5 = 1/R_2C_2 \Rightarrow \underline{C_2 = 0.01\,\mu F}$

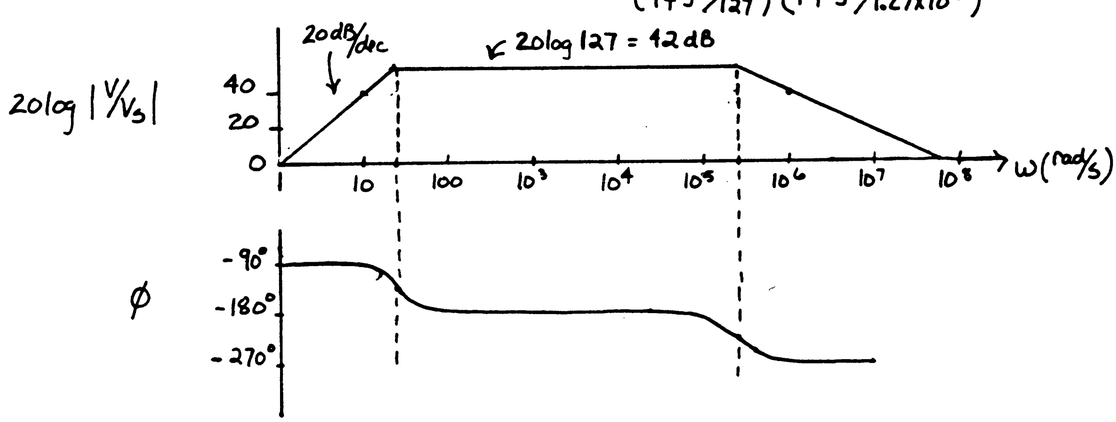
P. 13-13 Use the band stop circuit of figure 13-20. Require $\omega_0 = 2\pi(90\,MHz)$ and $B/2 = 2\pi(90-88\,MHz) = 2.52\times10^7$ rad/s

now $B = \dfrac{1}{(R+R_L)C} = 2.52\times10^7$

let $\underline{R = R_L = 100\,\Omega}$ $\therefore$ $\underline{C = 2\times10^{-10}\,F = 200\,pF}$

and $\underline{L = 1/\omega_0^2C = 1.57\times10^{-8}\,H = 15.7\,nH}$

P. 13-14 let $a = 2\pi(20) = 127 \Rightarrow V/V_s = \dfrac{j\omega}{(1+j\omega/127)(1+j\omega/1.27\times10^5)}$

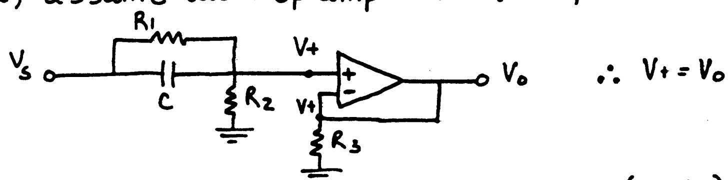


P. 13-15 $\quad B = 10.1 - 9.9 = \underline{0.2 \text{ MHz}}$

at $\omega = \omega_0 \Rightarrow 20\log|H| = 10 \Rightarrow \underline{|H| = 3.16}$

$\underline{Q = \omega_0/B = 10/.2 = 50}$

P. 13-16 (a) assume ideal op amp with $V_i \simeq 0$, $A = \infty$ and i into op amp $\simeq 0$



$\therefore V_+ = V_o$

KCL at V+ yields: $\quad V_o/R_2 + (V_o - V_s)/R_1 + (V_o - V_s)/\tfrac{1}{sC} = 0$

$$\Rightarrow V_o\left[1/R_1 + 1/R_2 + sC\right] = V_s\left[1/R_1 + sC\right]$$

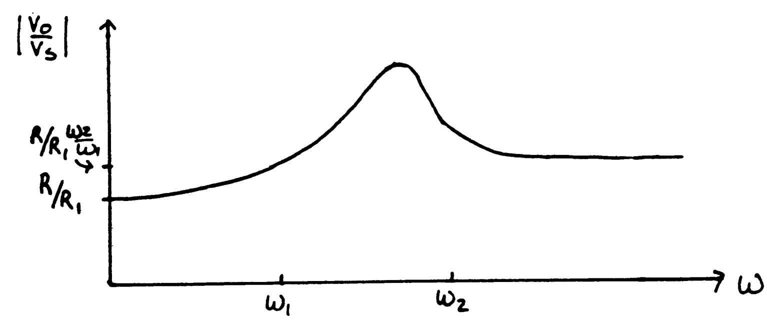
letting $R = \dfrac{1}{1/R_1 + 1/R_2} \quad \Rightarrow V_o\left[1/R + sC\right] = V_s\left[1/R_1 + sC\right]$

with $s = j\omega \quad \Rightarrow \underline{V_o/V_s = \dfrac{1 + j\omega R_1 C}{1 + j\omega RC} \cdot \dfrac{R}{R_1}}$

(b) let $\omega_1 = 1/R_1C$, $\omega_2 = 1/RC$

now since $R = R_1 \| R_2 \Rightarrow R < R_1 \quad \therefore \omega_1 < \omega_2$

now $\dfrac{V_o}{V_s} = \dfrac{1 + j\,\omega/\omega_1}{1 + j\,\omega/\omega_2} \cdot \dfrac{R}{R_1}$



(c) From part (b), $\omega_1 = 1/R_1C$

Let $\underline{C = 0.1\,\mu F}$

then $R_1 = 1/\omega_1 C = 1/(10^4)(10^{-7}) \; \underline{= 10^3\,\Omega = 1\,k\Omega}$

P. 13-17 By inspection,

$$H(j\omega) = \frac{A(1+j\omega/100)}{j\omega(1+j\omega/1000)}$$

from the magnitude plot for $100 < \omega < 1000$

$$|H(j\omega)| \approx \frac{A\,j\,\omega/100}{j\omega} = A/100$$

$$\therefore\ 20\log A/100 = 0 \quad \Rightarrow A = 100$$

so $$\underline{H(j\omega) = \frac{100(1+j\omega/100)}{j\omega(1+j\omega/1000)}}$$

P. 13-18 (a) Assume ideal op amp (inverting), then $H = \frac{V_o}{V_s} = -\frac{Z_2}{Z_1}$
where $Z_1 = R_1 - j/\omega C_1$ and $Z_2 = \dfrac{R_2\, 1/j\omega C_2}{R_2 + 1/j\omega C_2}$

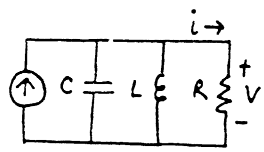
$$\therefore\ \underline{H = -\frac{j\omega R_2 C_1}{(1+j\omega/\omega_2)(1+j\omega/\omega_1)}}$$ where $\omega_1 = 1/R_1C_1$, $\omega_2 = 1/R_2C_2$

(b) $\omega_1 = 1/R_1C_1$ and $\omega_2 = 1/R_2C_2$

(c) at $\omega_1 << \omega << \omega_2$

$$\underline{|H| \approx \frac{\omega R_2 C_1}{\omega/\omega_1} = \omega_1 R_2 C_1 = R_2/R_1}$$

P. 13-19



let $i = I_m \cos\omega_o t$

$\therefore\ V = Ri = R I_m \cos\omega_o t$

now stored energy $= \frac{1}{2}CV^2 = \frac{1}{2}C[R I_m \cos\omega_o t]^2$

$\therefore$ max stored energy $= \frac{1}{2}CR^2 I_m^2 = W_1$

energy dissipated per cycle is

$$W_2 = \int_0^T R i^2 dt = \int_0^{2\pi/\omega_o} R(I_m \cos\omega_o t)^2 dt = \frac{\pi R I_m^2}{\omega_o}$$

$$\therefore\ \underline{Q = 2\pi W_1/W_2 = \omega_o RC}$$

P.13-20 Note that each stage is the same as that in prob. 13-19 with C_1 shorted out. There obtain the transfer function of each stage by setting $j\omega C_1 \to \infty$ (same as shorting C_1) in the transfer function of prob. 13-19.

$$\therefore \quad H_{1,2} = -\frac{R_2/R_1}{1+j\omega R_2 C} \quad \Rightarrow \quad H_{Total} = H_{1,2}^2 = (R_2/R_1)^2 \left(\frac{1}{1+j\omega R_2 C}\right)^2$$

(a) At low frequency,

$H_T = (R_2/R_1)^2 \quad \therefore$ for $H_T = 1$, need $R_1 = R_2$

now also $\omega_1 = 1000 = 1/R_2 C$, so let $\underline{C = 1\mu F}$

$$\therefore \quad \underline{R_1 = R_2 = 1/(1000)(10^{-6}) = 1k\Omega}$$

(b) At $\omega = 10{,}000$

$$|H| = \frac{1}{1+(\omega R_2 C)^2} = \frac{1}{1+[(10^4)(10^3)(10^{-6})]^2} = 10^{-2}$$

$$\Rightarrow \quad 20\log|H| = 20\log 10^{-2} = \underline{-40dB}$$

P. 13-21

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{(10^{-2})(10^{-8})} = \underline{10^5}$$

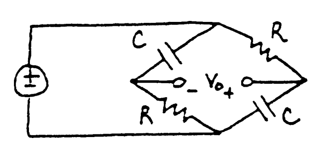
$$Q = \frac{\omega_0 L}{R} = \frac{(10^5)(10^{-2})}{100} = \underline{10}$$

$$B = \omega_0/Q = 10^5/10 = \underline{10^4}$$

$$\delta = \frac{W-\omega_0}{\omega_0} = \frac{1.1\times10^5 - 10^5}{10} = 0.1 \qquad \therefore \quad Q\delta = 1$$

from Fig. 13-17 $\underline{|H| = 0.44}$

P. 13-22



(a) using voltage divider,

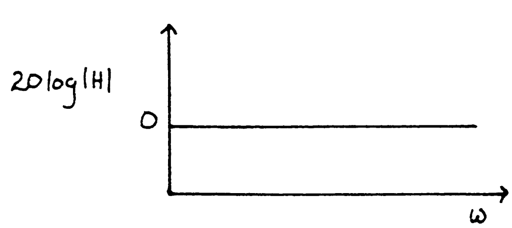
$$V_{o+} = V_s \frac{1/SC}{R+1/SC} = V_s \frac{1}{1+SRC}$$

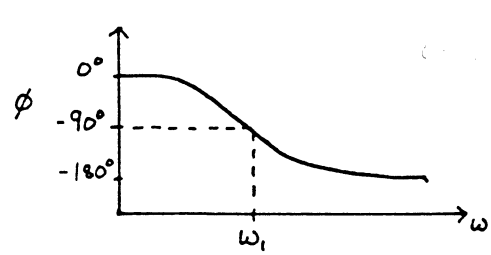
$$V_{o-} = V_s \frac{R}{R+1/SC} = V_s \frac{SRC}{1+SRC}$$

$$\therefore \quad V_o = V_{o+} - V_{o-} = V_s \frac{(1-SRC)}{(1+SRC)}$$

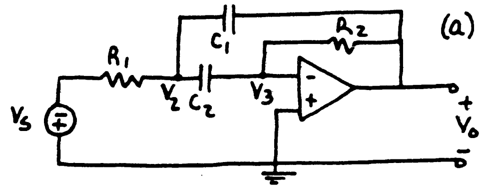
$$\Rightarrow \quad \underline{H(j\omega) = V_o/V_s = \frac{1-j\omega/\omega_1}{1+j\omega/\omega_1}} \qquad \text{with } \omega_1 = 1/RC$$

(b) now $|H(j\omega)| = 1$ and $\phi = -\tan^{-1}(\omega/\omega_1)$





P. 13-23



(a) let $G_1 = 1/R_1$, $G_2 = 1/R_2$, $s = j\omega$

KCL at V_2: $(-V_s - V_2)G_2 + (V_o - V_2)sC_1 - V_2 sC_2 = 0$

KCL at V_3: $V_2 sC_2 + V_o G_2 = 0$

where $V_3 = 0$

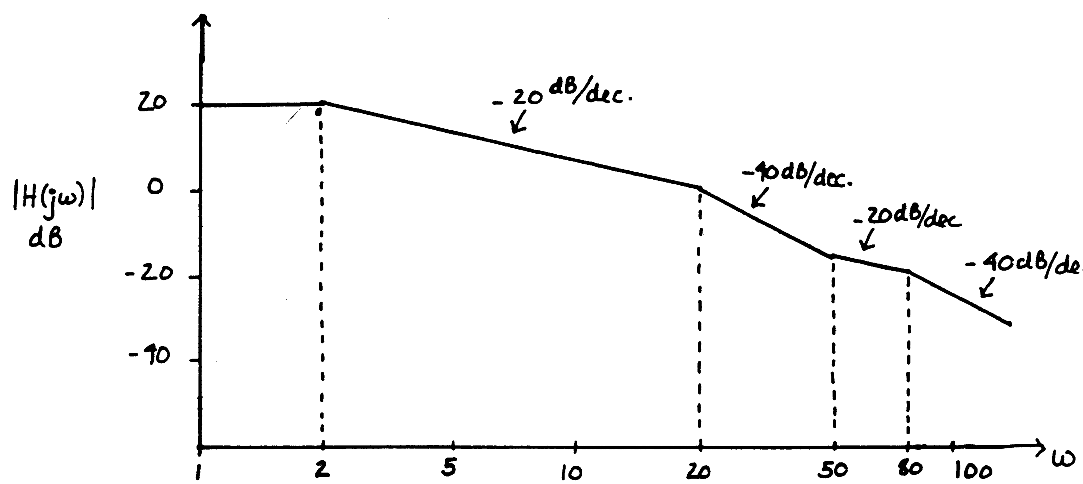
eliminating V_2 and solving for V_o/V_s yields

$$H(j\omega) = V_o/V_s = \frac{j\omega/R_1C_1}{(j\omega)^2 + (j\omega)\dfrac{(C_1+C_2)}{C_1C_2R_2} + \dfrac{1}{R_1R_2C_1C_2}}$$

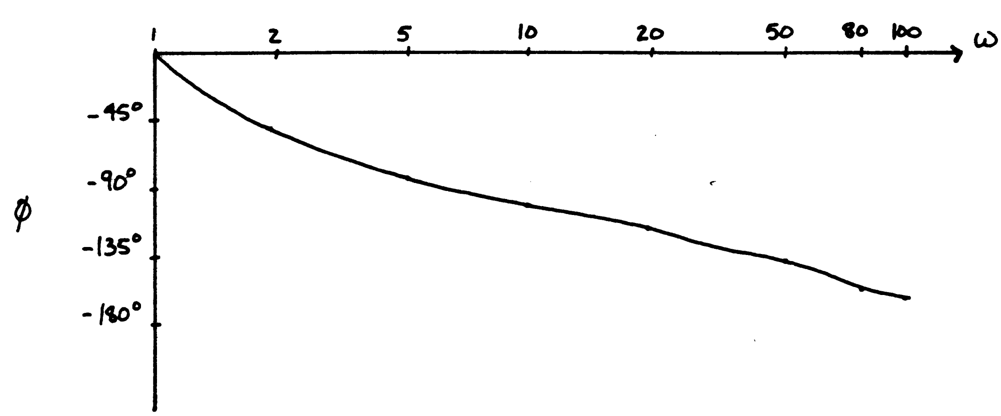
(b) from $H(j\omega)$ in part (a) $\omega_0^2 = 1/R_1R_2C_1C_2$

and $B = \dfrac{(C_1+C_2)}{R_2C_1C_2}$

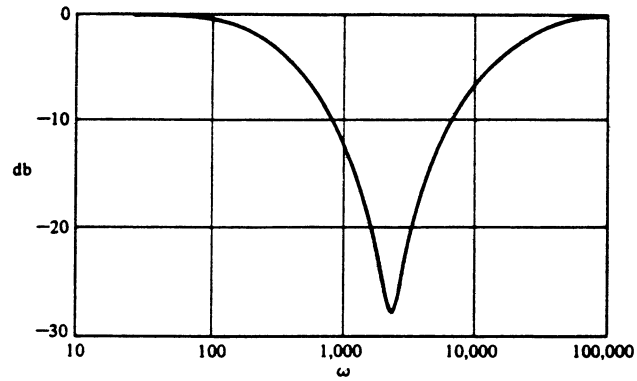
P. 13-24 $H(j\omega) = \dfrac{10(j\omega/50+1)}{(j\omega/2+1)(j\omega/50+1)(j\omega/80+1)}$

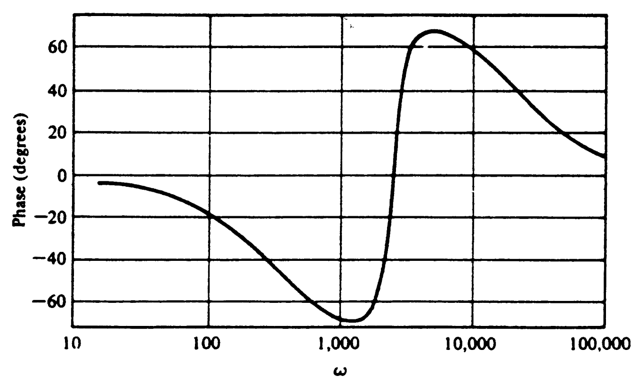


$\phi = \tan^{-1}\omega/50 - \tan^{-1}\omega/2 - \tan^{-1}\omega/20 - \tan^{-1}\omega/80$



P. 13-25 (a)

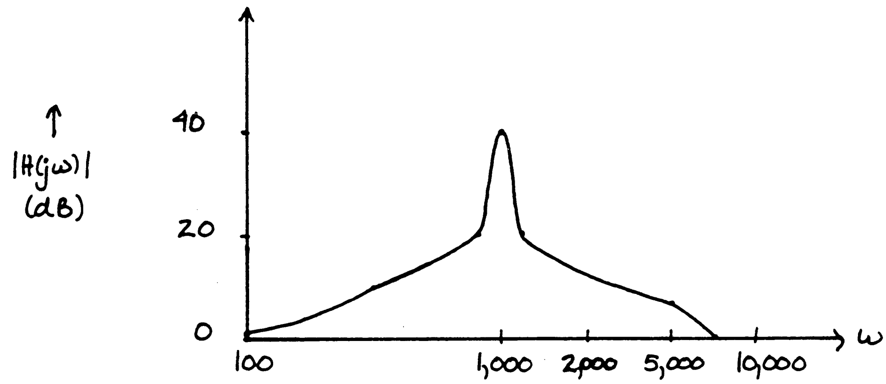


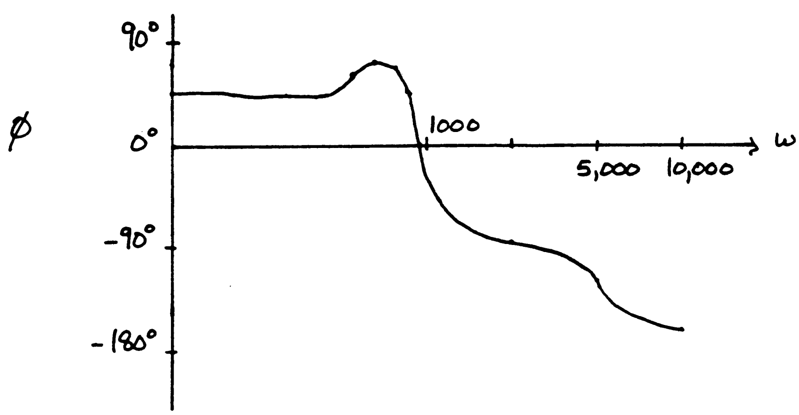


(b) a bandstop filter

(c) for − 20 dB or greater $\underline{\omega_1 = 2000}$ and $\underline{\omega_2 = 5000}$

P. 13-26 (a)





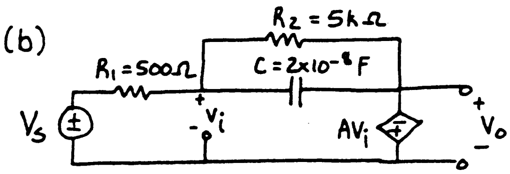
(b) $B = \omega_0/Q = 1000/10 = \underline{100}$

(c) From the Bode diagram, it is clear that the overall Q of the circuit is dictated by the Q of the 2nd order factor. Thus the overall $\underline{Q = 10}$

(d) From the plot, the gain at ω_0 $\underline{= 40\,dB}$

P. 13-27 (a) @ $\omega = 0$, $V_o/V_s = -R_2/500 = -5000/500 = \underline{-10}$

(b)



$s = j\omega$

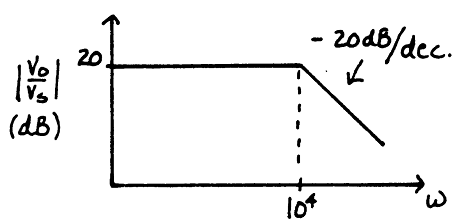
KCL at neg. input : $(V_i - V_s)/R_1 + (V_i - V_o)/R_2 + (V_i - V_o)sC = 0$ (1)

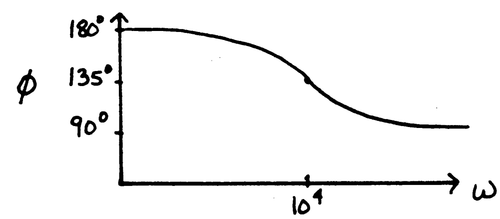
also $V_i = -V_o/A$ (2)

(2) into (1) yields $-\frac{V_o}{A}\left(\frac{1}{R_1} + \frac{1}{R_2} + sC\right) = V_o\left(\frac{1}{R_2} + sC\right) + \frac{V_s}{R_1}$

assume $\frac{1}{R_2} + sC \gg \frac{1}{A}\left(\frac{1}{R_1} + \frac{1}{R_2} + sC\right)$

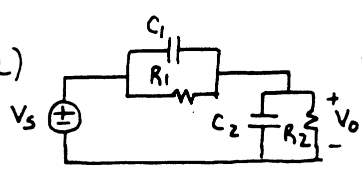
$$\therefore \frac{V_o}{V_s} = -\frac{1}{R_1\left(\frac{1}{R_2} + sC\right)} = -\frac{R_2/R_1}{1 + j\omega R_2 C} = \frac{-10}{1 + j\,\omega/10^4}$$





(c) $\left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{2}}\left|\frac{V_o}{V_s}\right|_{max}$ at $\underline{\omega = 10^4 \text{ rad/s}}$

P. 13-28 (a)



from voltage divider

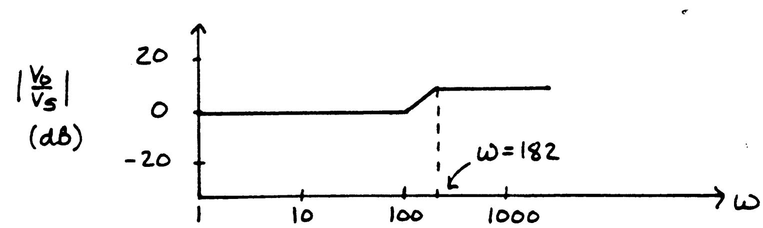
$$V_o = \frac{\frac{R_2/sC_2}{R_2 + 1/sC_2}}{\frac{R_2/sC_2}{R_2 + 1/sC_2} + \frac{R_1/sC_1}{R_1 + 1/sC_1}} V_s$$

$$\Rightarrow \frac{V_o}{V_s} = \left(\frac{R_2}{R_1 + R_2}\right) \frac{1 + R_1C_1 s}{1 + \left(\frac{R_1R_2}{R_1 + R_2}\right)(C_1 + C_2)s}$$

$$\therefore \frac{V_o}{V_s} = \frac{R_2}{R_1 + R_2} = k \quad \text{when } R_1C_1 = \frac{R_1R_2}{R_1 + R_2}(C_1 + C_2)$$

$$\Rightarrow \underline{R_1C_1 = R_2C_2}$$

(b) $\underline{C_1 = 1\mu F}$ $\quad V_o/V_s = 1/2 \; \frac{1 + j\omega(10^{-2})}{1 + j\omega(5.5\times10^{-3})}$

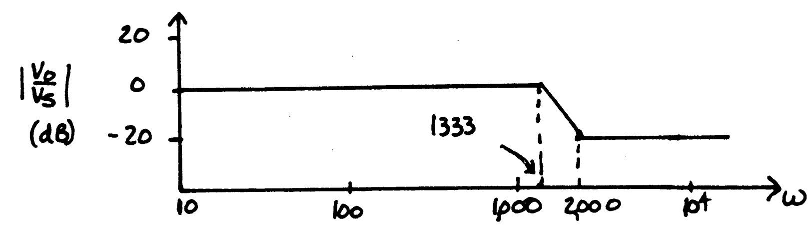


P. 13-28 continued

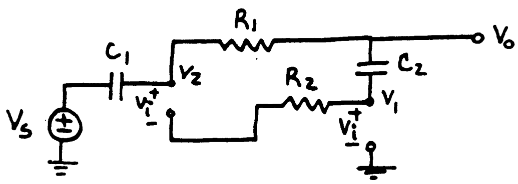
$\underline{C_1 = 0.1\mu F}$, $\frac{V_o}{V_s} = 1/2 = \text{constant}$

$\therefore \; 20\log\left|\frac{V_o}{V_s}\right| \underline{= -3\,dB \; (\text{constant})}$

$\underline{C_1 = 0.05\mu F}$, $\frac{2V_o}{V_s} = \frac{1+j\omega(5\times10^{-4})}{1+j\omega(.75\times10^{-3})}$



P. 13-29



assume $A \to \infty$

$V_i \approx 0$

use $s = j\omega$

(a) KCL at V_2 : $(V_2 - V)sC_1 + (V_2 - V_o)/R_1 = 0$ (1)

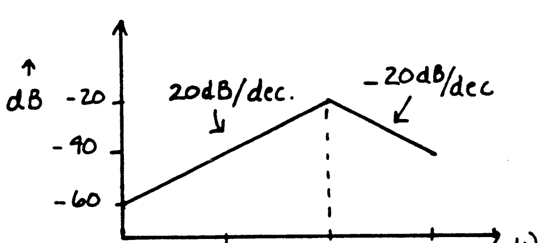
KCL at V_1 : $V_2/R_2 + V_o s C_2 = 0$

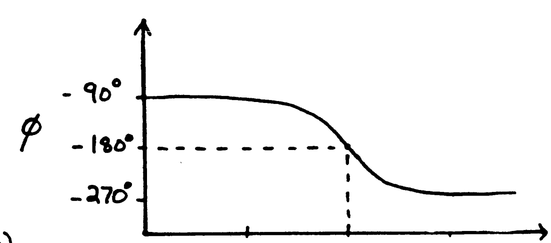
$\Rightarrow V_2 = -R_2 C_2 s V_o$ (2)

(2) into (1) yields

$$\underline{H = V_o/V_s = -\frac{R_1 C_1 s}{R_1 R_2 C_1 C_2 s^2 + R_2 C_2 s + 1} = \frac{-Q\omega_o s}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}}$$

(b) $H = (-10^{-3}j\omega)/(1+(j\omega/10^4)^2 + j\omega/10^5)$

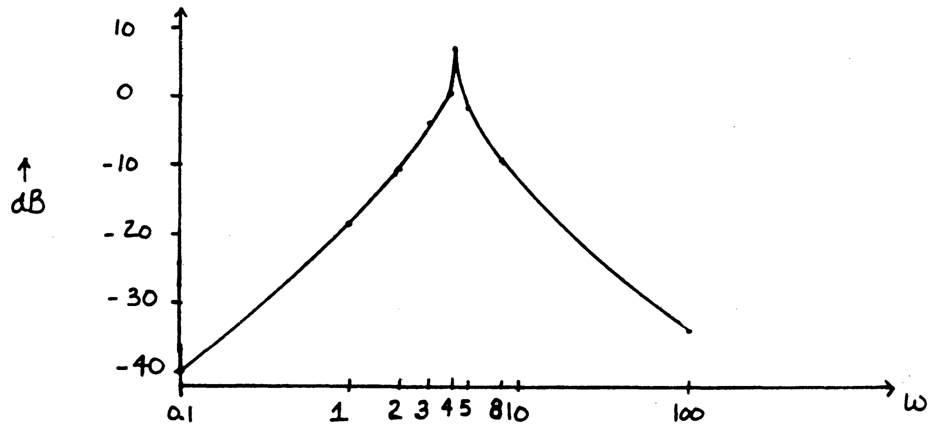




(c) $\left.\begin{aligned} R_1 C_1 &= Q/\omega_o \\ R_2 C_2 &= 1/Q\omega_o \end{aligned}\right\} \Rightarrow \underline{Q = 10}$ and $\underline{\omega_o = 10^4}$

(d) $\underline{B = \omega_o/Q = 10^3}$

P. 13-30 plot the data points



an approximate $H(j\omega)$ is $\Rightarrow$ $H(j\omega) = \dfrac{0.1\,j\omega}{1 + j\frac{\omega}{\omega_0 Q} + \left(j\frac{\omega}{\omega_0}\right)^2}$

with $\omega_0 = 4.1$

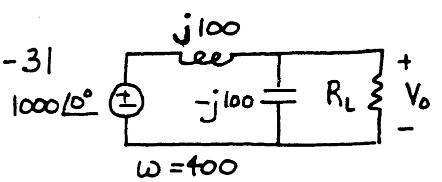
Notice that $H(j\omega)$ satisfies both $|H|$ and ϕ at $\omega = 0$ exactly.
For $\omega = 100$, $|H| \approx \dfrac{10}{(100/4.1)^2} = 0.017$

$\therefore$ $20\log|H| = -35\,\text{dB}$ compared to -34 dB for given data and $\phi \to -90°$ as $\omega \to \infty$. To estimate Q, let $20\log|H| = 0\,\text{dB}$ at ω_0. Then at $\omega = \omega_0$:

$$1 = \frac{(0.1)(j4.1)}{j\,1/Q} \quad \Rightarrow Q = 2.4$$

$$\therefore H(j\omega) = \frac{0.1\,j\omega}{1 + j\frac{\omega}{2.4\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

P. 13-31



(a) using voltage divider

$$V_o = (1000\angle 0°)\,\frac{\dfrac{(100)(-j100)}{100 - j100}}{\dfrac{(100)(-j100)}{100 - j100} + j100} = (1000\angle 0°)\,\frac{100/\sqrt{2}\,\angle -135°}{100/\sqrt{2}\,\angle -135° + j100}$$

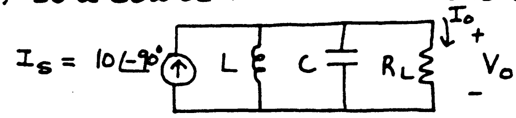
$$= \frac{10^5/\sqrt{2}\,\angle -135°}{50\sqrt{2}\,\angle 135°}$$

$$V_o = 1000\angle 90°$$

$\therefore |V_o| = 1000\,\text{v}$

P. 13-31 continued

(b) Do a source transformation to obtain



So have an RLC resonant circuit with $\omega_o = 1/\sqrt{LC} = 400$. Therefore the circuit is operated at resonance. This means that the L-C parallel combination has an overall $Z = \infty$ and hence $I_s = I_o$. When R_L is suddenly changed from 100Ω to $1k\Omega$, due to the resonance condition, $V_o = R_L I_o$ suddenly increases by a factor of 10. This in turn causes very large (equal & opposite) currents in the L & C as the capacitor voltage is forced to abruptly change. Thus very large currents radiate electromagnetic radiation and thus see sparks. Clearly we need a variable capacitor that varies as R_L varies such that when $R_L \neq 100\Omega$, $\omega \neq \omega_o$ and thus have $V_o = R_L I_o$ constant while both R_L and I_o change abruptly.

(c) from the parallel circuit, $V_o\left[1/R_L + j(\omega C - 1/\omega L)\right] = 10\angle -90°$

considering magnitudes only & $|V_o| = 1000$,

$$1000\sqrt{(1/R_L)^2 + (400C - 1/100)^2} = 10$$

$$\left(\frac{1}{R_L}\right)^2 + \left(400C - \frac{1}{100}\right)^2 = \left(\frac{1}{100}\right)^2$$

$$400C - \frac{1}{100} = \pm\left[\frac{1}{(100)^2} - \left(\frac{1}{R_L}\right)^2\right]$$

$$\Rightarrow \underline{C = \frac{1}{4\times10^4}\left[\frac{1}{100} \pm \left\{\frac{1}{(100)^2} - \frac{1}{(R_L)^2}\right\}\right]}$$

both $\pm$ are okay for $100 \leq R_L \leq 1000\Omega$

P. 13-32 (a) $H_1 = V_1/V_s = \frac{R_1}{R_1 + 1/sC} = \frac{R_1 sC}{1 + R_1 sC}$

$$\Rightarrow |H_1(j\omega)| = \frac{\omega R_1 C}{\sqrt{1+(\omega R_1 C)^2}} = \frac{\omega/\omega_1}{\sqrt{1+(\omega/\omega_1)^2}} \qquad \omega_1 = 1/R_1 C$$

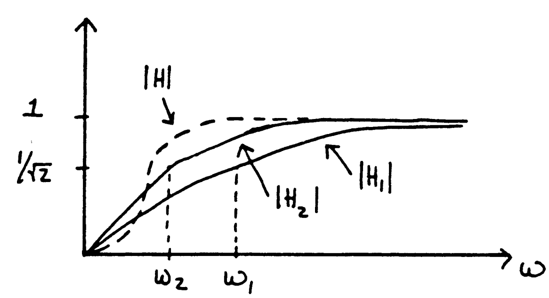
(b) $H_2 = V_2/V_1 = \frac{sL}{R_2 + sL} \Rightarrow |H_2(j\omega)| = \frac{\omega/\omega_2}{\sqrt{1+(\omega/\omega_2)^2}} \qquad \omega_2 = R_2/L$

(c) $H(j\omega) = H_1(j\omega)\, H_2(j\omega) = \frac{(sR_1C)}{(1+sR_1C)} \frac{(s\,L/R_2)}{(1+sL/R_2)}$

$$= \frac{s^2 R_1 LC/R_2}{1 + s(R_1C + L/R_2) + s^2(R_1LC/R_2)}$$

$$H(j\omega) = \frac{(s/\omega_0)^2}{1 + 1/Q\,(s/\omega_0) + (s/\omega_0)^2}$$

where $\omega_0^2 = R_2/R_1LC$, $\omega_0 Q = \frac{1}{R_1C + L/R_2}$



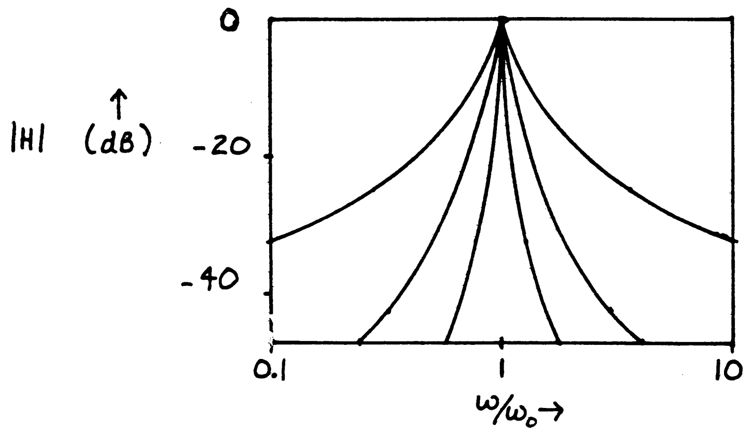
(d) The circuit of figure P 13-32d is not the same as $H = H_1H_2$ since the first filter (looking from V_s) consists of C and $R_1 \| (R_2 + sL)$. Thus effectively H_1 is altered and we won't get the response as sketched in part (c). So need to place a buffer to the right of R_1 to isolate each first order filter, then $H = H_1 H_2$ will be obtained.

P. 13-33 $\omega_0 = 1/\sqrt{LC} = \frac{1}{\sqrt{(10^{-3})(10^{-5})}} = \underline{10^4 \text{ rad/s}}$

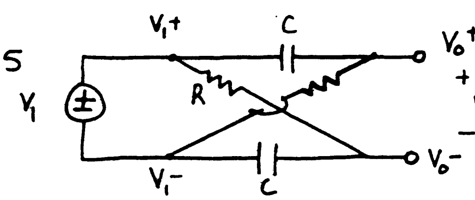
$Q = \frac{\omega_0 L}{R} = (10^4)(10^{-3})/1 = \underline{10}$

$B = \frac{\omega_0}{Q} = \frac{10^4}{10} = \underline{10^3}$

P. 13-34 $\quad H = \dfrac{j(\omega/\omega_0 Q)}{(j\omega/\omega_0)^2 + j\,\omega/\omega_0 Q + 1} \quad \Rightarrow |H| = \dfrac{u/Q}{\sqrt{(1-u^2)^2 + (u/Q)^2}} \qquad u = \omega/\omega_0$



P. 13-35



using voltage divider

$$V_{o^+} - V_{1^-} = \frac{R}{R + 1/sC}\, V_1 \qquad (1)$$

$$V_{o^-} - V_{1^-} = \frac{1/sC}{R + 1/sC}\, V_1 \qquad (2)$$

subtracting (2) from (1) yields

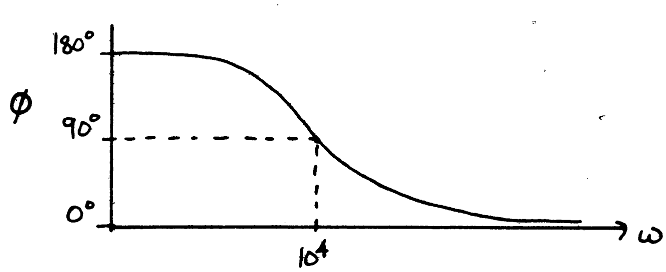
$$V_o = V_{o^+} - V_{o^-} = \frac{R - 1/sC}{R + 1/sC}\, V_1$$

$$\Rightarrow H(j\omega) = V_o/V_1 = \frac{sRC - 1}{sRC + 1} = -\frac{(1 - j\omega RC)}{1 + j\omega RC}$$

$\therefore\ |H(j\omega)| = 1 \ \Rightarrow$ sketch = 0dB constant line

$$\phi = 180^\circ - 2\tan^{-1}\omega RC$$

$$\phi = 180^\circ - 2\tan^{-1}\left[\omega(10^{-4})\right]$$



P. 13-36 have a pole at $s=-2$ and a zero at $s=-8$

(a) $Z(s) = \dfrac{k(s/8+1)}{(s/2+1)}$

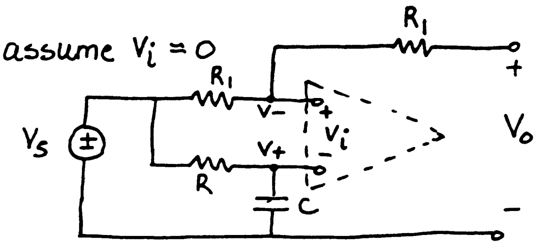
low frequency gain $= 20\log k = 17\,dB \Rightarrow k = 7.1$

$\therefore \; \underline{Z(s) = 7.1\dfrac{(s/8+1)}{(s/2+1)}}$

(b) We drop from 17dB, 6dB per octave for 2 octaves or a total of 12dB

$\therefore \; \lim_{\omega\to\infty} |Z(j\omega)| = 17-12 = \underline{5\,dB}$

P. 13-37 assume $V_i = 0$



$s = j\omega$

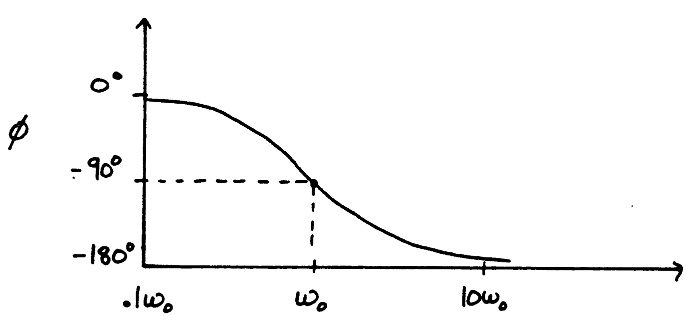
KCL at V_+: $sCV_+ + \dfrac{(V_+ - V_s)}{R} = 0 \Rightarrow V_+ = \dfrac{V_s}{1+sRC}$ (1)

KCL at V_-: $\dfrac{(V_- - V_s)}{R_1} + \dfrac{(V_- - V_o)}{R_1} = 0 \Rightarrow V_- = 1/2(V_o+V_s)$ (2)

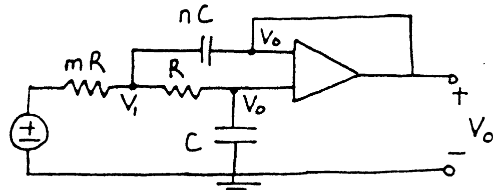
now $V_+ \simeq V_-$, so equating (1) and (2) yields

$\underline{H(j\omega) = V_o/V_s = \dfrac{1-j\omega/\omega_o}{1+j\omega/\omega_o}}$ where $\omega_o = 1/RC$

now $|H(j\omega)|$ is just a flat constant gain $= 0\,dB$



P. 13-38



Assume $V_i \simeq 0$, then $V_+ = V_- = V_o$

use $s = j\omega$

Voltage divider yields : $V_o = V_1 \dfrac{1/sC}{R + 1/sC}$, $\Rightarrow V_1 = (1 + sRC)V_o$

KCL at V_1 : $(V_1 - V_s)/mR + (V_1 - V_o)/R + (V_1 - V_o)snC = 0$

plugging V_1 into above yields

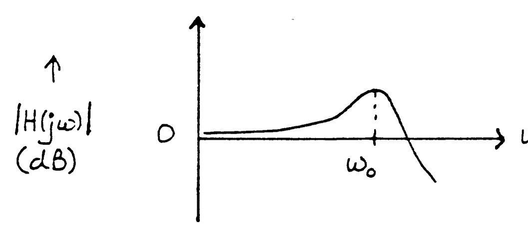
$$V_o\left[\frac{1}{mR} + sC + \frac{sC}{m} + s^2 nRC^2\right] = \frac{V_s}{mR}$$

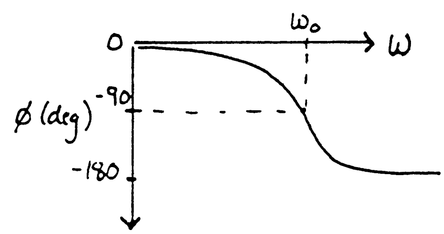
$$\therefore \frac{V_o}{V_s} = \frac{1}{1 + s(m+1)RC + nmR^2C^2s^2}$$

$$\Rightarrow H(j\omega) = V_o/V_s = \frac{1}{1 - (\omega/\omega_o)^2 + j(\omega/Q\omega_o)}$$

where $\omega_o = \dfrac{1}{\sqrt{mn}\,RC}$

$Q = \dfrac{\sqrt{mn}}{m+1}$



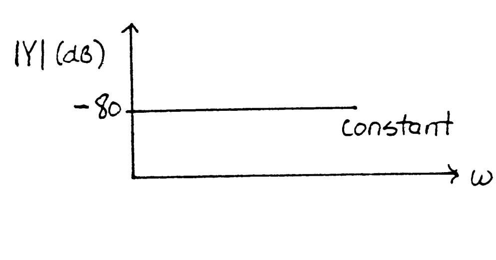


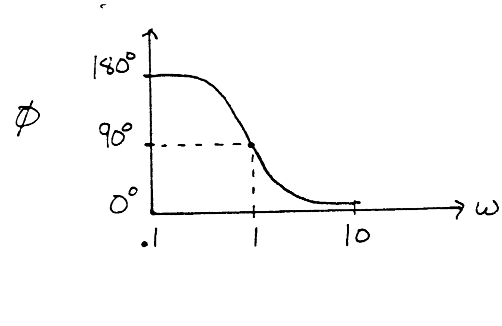
P. 13-39 (a) $Z_{ab} = 10^4 + Z_C R/Z_C + R$ where $Z_C = \dfrac{1}{j\omega C} = -j10^4/\omega$

$$= 10^4 + \frac{(-j10^4/\omega)(-2\times10^4)}{-j10^4/\omega - 2\times10^4}$$

$$Z_{ab} = 10^4\left(\frac{j\omega+1}{j\omega-1}\right) \Rightarrow Y_{ab} = \left(\frac{j\omega-1}{j\omega+1}\right)10^{-4} \Rightarrow |Y(j\omega)| = 10^{-4}$$

(b) $|Y| = 10^{-4}$, $\phi = 180° - 2\tan^{-1}\omega$





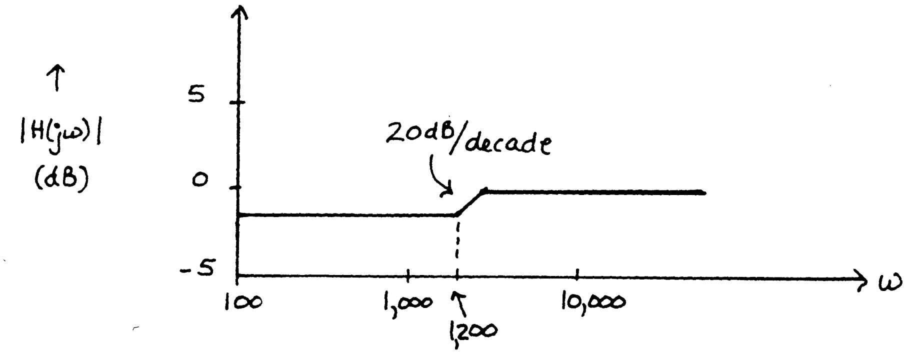
P. 13-40 $\quad \dfrac{V_o}{V_s} = \dfrac{R_t}{R_t + \dfrac{R_1/sC}{R_1 + 1/sC}} = \dfrac{R_t(1+sR_1C)}{R_1 + R_t(1+sR_1C)} = \dfrac{\dfrac{R_t}{R_1+R_t}(1+sR_1C)}{1 + s\dfrac{R_1R_t}{R_1+R_t}C}$

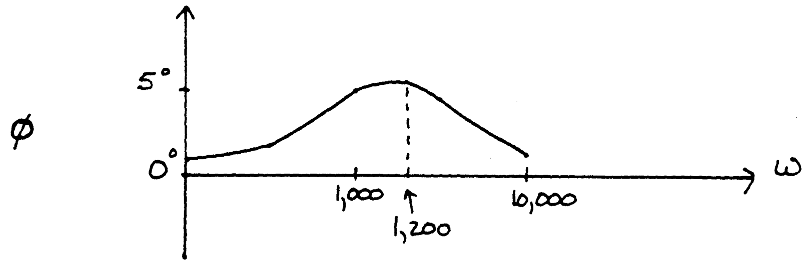
now let $R = R_1R_t/R_1+R_t$

$\Rightarrow \dfrac{V_o}{V_s} = \dfrac{R/R_1\,(1+sR_1C)}{1+sR_1C}$

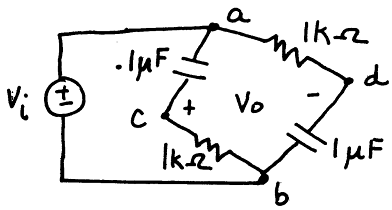
when $R_1 = 1k\Omega$, $C = 1\mu F$ and $R_t = 5k\Omega \Rightarrow R = 833\,\Omega = 5/6\,k\Omega$

$\therefore\; H(j\omega) = \dfrac{V_o}{V_s} = \dfrac{5/6\,[1 + j\omega/1000]}{1 + j\omega/1200}$





P 13-41



$$V_d = \frac{1/sC}{R + 1/sC} V_s = \frac{1}{\tau s + 1} V_s$$

where $\tau = RC = 10^{-4}$s

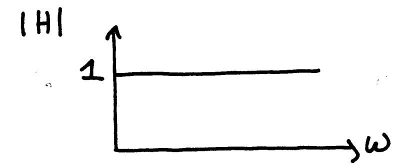
$$V_c = \frac{R}{R + \frac{1}{sC}} V_s = \frac{s\tau}{s\tau + 1} V_s$$

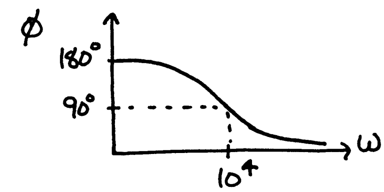
$$V_o = V_c - V_d = \frac{s\tau - 1}{\tau s + 1}$$

$$H(j\omega) = \frac{j\omega RC - 1}{j\omega RC + 1} = -\frac{(1 - j\omega RC)}{(1 + j\omega RC)}$$

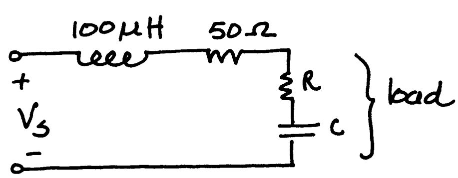
so $\left|\frac{V_o}{V_s}\right| = \frac{\sqrt{1+\omega^2R^2C^2}}{\sqrt{1+\omega^2R^2C^2}} = 1$

$\angle\frac{V_o}{V_s} = 180° - 2\tan^{-1}\omega RC$





P 13-42



a) set R = 50Ω
and $\omega L = 1/\omega C$ or $C = 1/\omega^2 L$

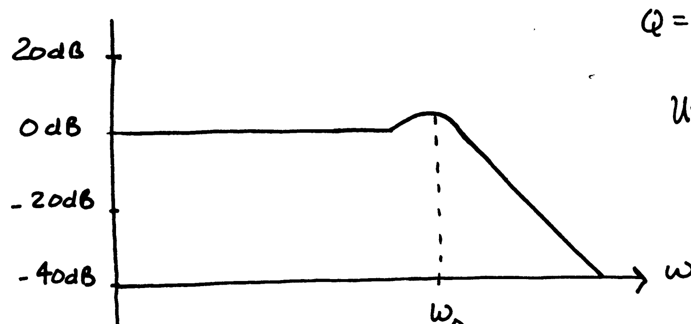
$\therefore C = \frac{1}{(2\pi\times10^6)^2(10^{-4})} = \underline{253\,pF}$

b) $Q = \frac{\omega L}{R_{series}} = \frac{(2\pi\times10^6)(10^{-4})}{50+50} = 2\pi = \underline{6.28}$

P 13-43 from given H(s), $\omega_o = 10$, $Q = 1$ ⇒ $B = \frac{\omega_o}{Q} = 10$ rad/s

$Q = 1 \quad \zeta = \frac{1}{2Q} = \frac{1}{2}$

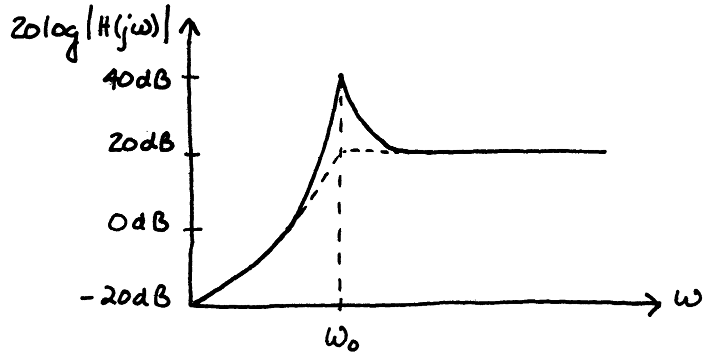
Use Fig 13-26



P 13-44 from given $H(s)$, $Q=10$, $\omega_0=10^4$ $\quad \zeta = \frac{1}{2Q} = \frac{1}{20}$

maximum in $H(j\omega)$ occurs at $\omega=\omega_0$

$$|H(j\omega_0)| = \left| \frac{10(j\omega_0)^2}{1000(j\omega_0)} \right| = \left| \frac{j\omega_0}{100} \right| = 100 \Rightarrow 20\log(100) = 40\,dB$$



P 13-45 with $A=10^6$, $R_i = 500\,k\Omega$ and $R_o = 1\,k\Omega$

the op-amp will be very close to ideal

Then $\dfrac{V_o(s)}{V_s(s)} = -\dfrac{Z_2(s)}{Z_1(s)}$

$$Z_1(s) = R_1 + \frac{1}{C_1 s} = 100 + \frac{1}{C_1 s} = \frac{R_1 C_1 s + 1}{C_1 s}$$

$$Z_2(s) = \frac{R_2\left(\frac{1}{C_2 s}\right)}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

$$H(s) = \frac{V_o(s)}{V_s(s)} = -\frac{R_2 C_1 s}{(R_2 C_2 s + 1)(R_1 C_1 s + 1)} = \frac{-R_2 C_1 s}{R_2 C_2 R_1 C_1 s^2 + (R_1 C_1 + R_2 C_2)s + 1}$$

P.13-45 (Continued)

a bandpass filter

center frequency $\omega_o^2 = \dfrac{1}{R_2 C_2 R_1 C_1} = \dfrac{1}{(2\times10^5 \times 5\times10^{-11})(10^2 \times .2\times10^{-6})}$

$= \dfrac{5}{(10^{-5})(10^{-4})} = 50\times10^8$

then $\omega_o = 70{,}711$ rad/s

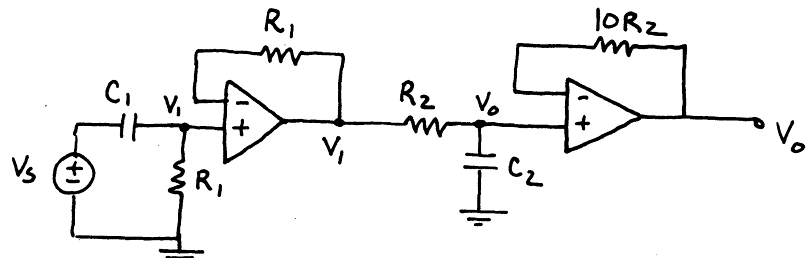
$= 70.7$ k rad/s

then $f_o = 11.25$ kHz

Bandwidth $\cong 23.9$ kHz

Gain $\cong 62$ dB at ω_o

P 13-46



$V_1 = \dfrac{R_1}{R_1 + \frac{1}{sC_1}} V_s = \dfrac{sR_1C_1}{1+sR_1C_1} V_s$

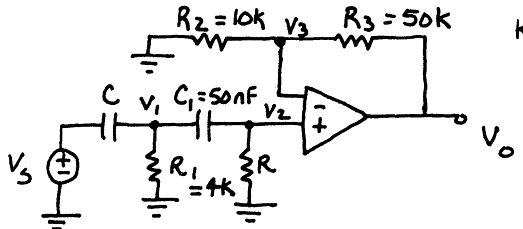
$V_o = \dfrac{1/sC_2}{R_2 + 1/sC_2} V_1 = \dfrac{1}{1+sR_2C_2} \cdot \dfrac{sR_1C_1}{1+sR_1C_1} V_s$

$\dfrac{V_o}{V_s} = \dfrac{j\omega/\omega_L}{(1+j\omega/\omega_L)(1+j\omega/\omega_H)}$ where $\omega_L = \dfrac{1}{R_1C_1} = 250$ rad/s

$\omega_H = \dfrac{1}{R_2C_2} = 5\times10^4$ rad/s

$\therefore B = \omega_H - \omega_L = \underline{49.75 \text{ k rad/s}}$

P 13-47



KCL at V_1 :

$$\frac{V_1 - V_s}{1/sC} + \frac{V_1}{R_1} + \frac{V_1 - V_2}{1/sC_1} = 0 \qquad (1)$$

at V_2 :

$$\frac{V_2 - V_1}{1/sC_1} + \frac{V_2}{R} = 0 \qquad (2)$$

at $V_3 = V_2$:

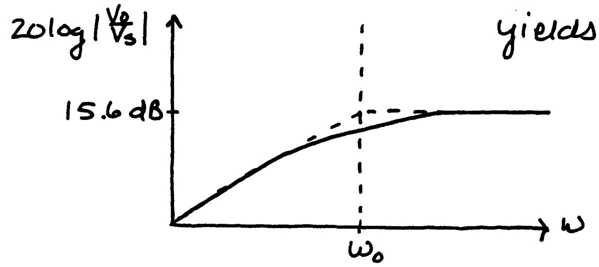
$$\frac{V_2}{R_2} + \frac{V_2 - V_o}{R_3} = 0 \qquad (3)$$

Solving (1) – (3) yields

$$\frac{V_o}{V_s} = \frac{(1 + R_3/R_2)\,s^2}{s^2 + \left[\frac{1}{C}\left(\frac{1}{R} + \frac{1}{R_1}\right) + \frac{1}{RC_1}\right]s + \frac{1}{RR_1C^2}}$$

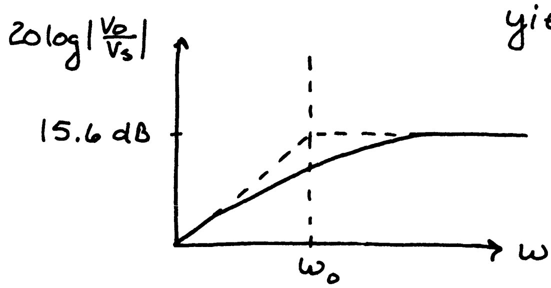
a) $C = 50nF$, $R = 4k\Omega$ $\Rightarrow$ $\frac{V_o}{V_s} = \frac{6s^2}{s^2 + 15000s + 2.5\times10^7}$

yields $\omega_o = 5000$, $Q = 1/3$



b) $C = 100nF$, $R = 2k\Omega$ $\Rightarrow$ $\frac{V_o}{V_s} = \frac{6s^2}{s^2 + 17500s + 1.25\times10^7}$

yields $\omega_o = 3536$, $Q = 0.20$



P 13-48 at low frequency $H(s) = ks$ or $H(j\omega) = kj\omega$

odb at $\omega = .7$ so $k = 1/.7 = 1.43$

a pole at $s = -.7$

a zero at $s = -10$

a pole at $s = -100$

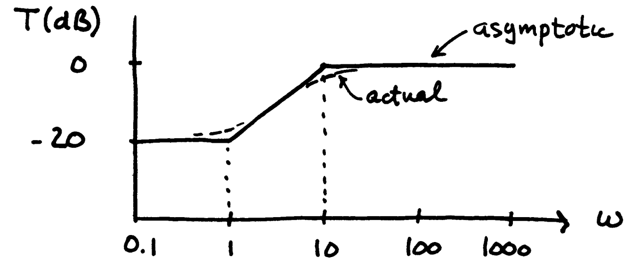
two poles at $\omega_o = 600$

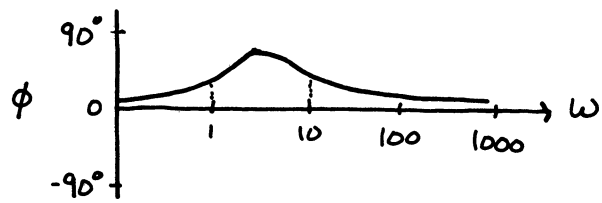
ζ = unknown

$$H(s) = \frac{1.43\,s\,(s+10)}{(s+0.7)(s+100)(s^2 + 2\zeta\omega_o s + \omega_o^2)}$$

$\omega_o = 600$

P 13-50





$$\phi = -\tan^{-1}\omega - \tan^{-1}(\omega/10)$$

peak phase occurs at $\omega = \sqrt{10}$

P 13-51

$$T(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

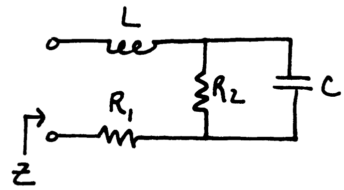
$Q = 1.25 \qquad \zeta = .80$

A second-order low pass circuit

the Bode diagram is shown in Fig 13-26 with $\zeta = .80$

Advanced Problems

AP 13-1



$$\underset{\sim}{Z} = R_1 + j\omega L + \frac{1}{G_2 + j\omega C} \quad ; \quad G_2 = 1/R_2$$

$$= \frac{(R_1 G_2 + 1 - \omega^2 LC) + j(\omega L G_2 + \omega C R_1)}{G_2 + j\omega C}$$

at resonance $\underset{\sim}{Z} = Z\angle 0^\circ$

or $\tan^{-1} \dfrac{\omega L G_2 + \omega C R_1}{(R_1 G_2 + 1 - \omega^2 LC)} = \tan^{-1} \dfrac{\omega C}{G_2}$

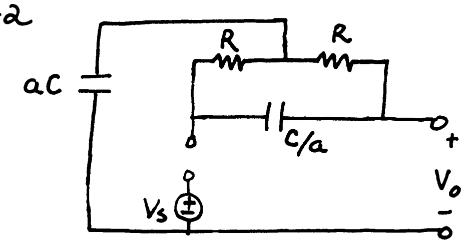
thus $\dfrac{\omega L G_2 + \omega C R_1}{(R_1 G_2 + 1 - \omega^2 LC)} = \dfrac{\omega C}{G_2} \Rightarrow \omega^2 = \dfrac{C - LG_2^2}{LC^2}$; $C > G_2^2 L$

with $R_1 = R_2 = 1\,\Omega$ and $\omega_0 = 100\,\text{rad/s}$

$\omega_0^2 = 10^4 = \dfrac{C-L}{LC^2}$, if $\underline{C = 10\,\text{mf}} \Rightarrow \underline{L = 5\,\text{mH}}$

and $C > G_2^2 L$ checks

AP 13-2



$$H(s) = \frac{(RCs)^2 + (\frac{2}{a} + a)RCs + 1}{(RCs)^2 + \frac{2}{a}RCs + 1}$$

$\omega_0^2 = \dfrac{1}{RC}$ or $10^{10} = \dfrac{1}{RC}$

if $R = 100\,\Omega$ then $C = 10^{-12}\,F$

let $RC = \tau$

$$H(j\omega) = \frac{-\tau^2\omega^2 + (\frac{2}{a} + a)\tau j\omega + 1}{-\tau^2\omega^2 + \frac{2}{a}\tau j\omega + 1}$$

at resonance $|H| \cong 1 + \frac{a^2}{2} = 201 \Rightarrow \underline{a = 20}$

AP 13-3 assume $Q >> 1$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 12.909 \text{ M rad/s} \quad \text{or} \quad \underline{f_0 = 2.055 \text{ MHz}}$$

$$\therefore \omega_0 L = 129.1\ \Omega$$

$$L + 1.8\ \Omega : \quad Y = \frac{1}{1.8 + j129.1} = 0.00775\angle{-89.2^\circ}$$

$$Re\{Y\} = 1.08 \times 10^{-4}, \quad G = 4.545 \times 10^{-5}$$

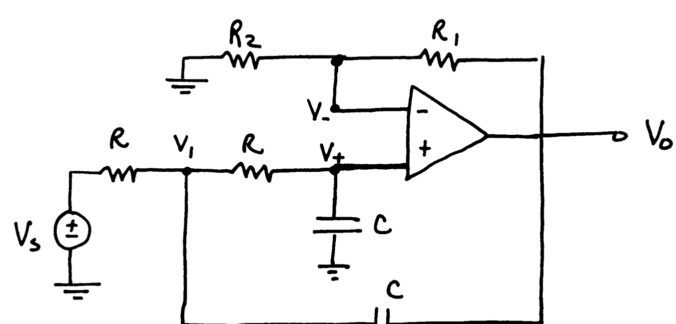
effective R in parallel with L and C is:

$$R = \frac{1}{Re[Y] + G} = 6.51 \times 10^3\ \Omega$$

$$Q = \omega_0 CR = (12.909 \times 10^6)(600 \times 10^{-12})(6.51 \times 10^3) = \underline{50.4}$$

$$B = \frac{12.909 \times 10^6}{50.4} = 256 \text{ krad/s} \Rightarrow \underline{40.8 \text{ kHz}}$$

AP 13-4 a) ideal op amp
with $V_- = V_+$
$I_{in} = 0$



KCL at V_1:

$$\left(\frac{1}{R} + \frac{1}{R} + sC\right)V_1 - \frac{1}{R}V_+ - sCV_o = \frac{1}{R}V_s \qquad (1)$$

KCL at V_+:

$$-\frac{1}{R}V_1 + \left(\frac{1}{R} + sC\right)V_+ = 0 \qquad (2)$$

KCL at V_-:

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_+ - \frac{1}{R_1}V_o = 0 \qquad (3)$$

eliminating V_+ and V_1 from above eqns. yields

$$\frac{V_o}{V_s} = \frac{1 + R_1/R_2}{1 + s[RC(2 - R_1/R_2)] + s^2R^2C^2}$$

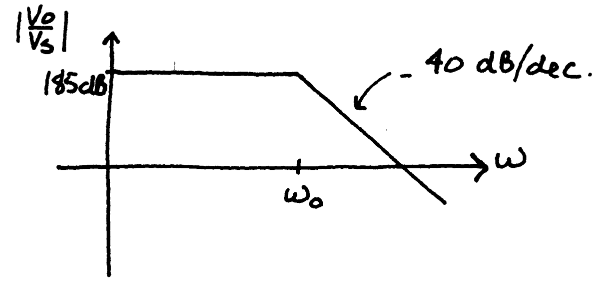
AP.13-34 (Continued)

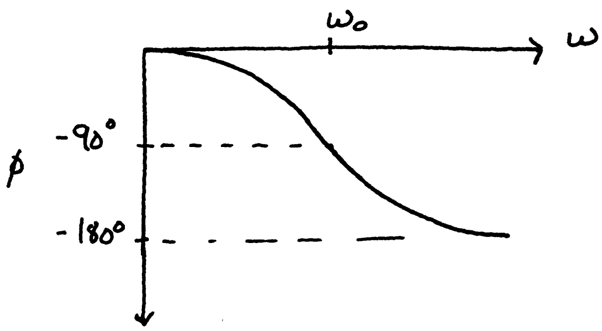
b) if $R_1/R_2 \simeq 0$

$$\frac{V_o}{V_s} = \frac{1}{(sRC+1)^2} = \frac{(1/RC)^2}{(s+1/RC)^2}$$

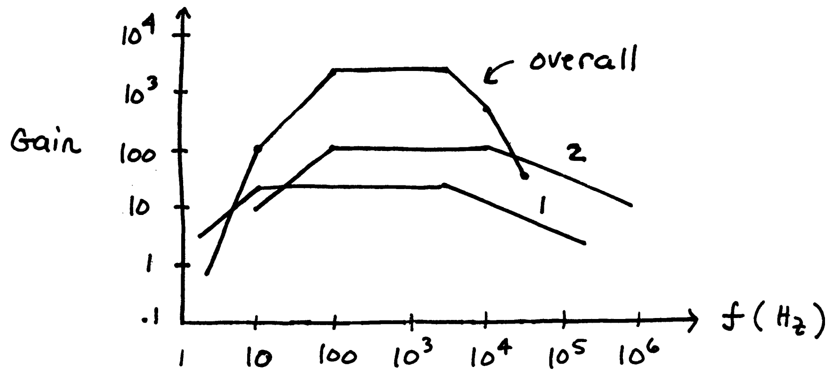
$R = 1.2\,k\Omega$, $C = 20\,nF \Rightarrow RC = 2.4\times10^{-5}$

$$V_o/V_s = \frac{\omega_o^2}{(s+\omega_o)^2} \Rightarrow \omega_o = \frac{1}{RC} = \underline{4.167\times10^4 \text{ rad/s}}$$





AP 13-5



total midband gain = 2000

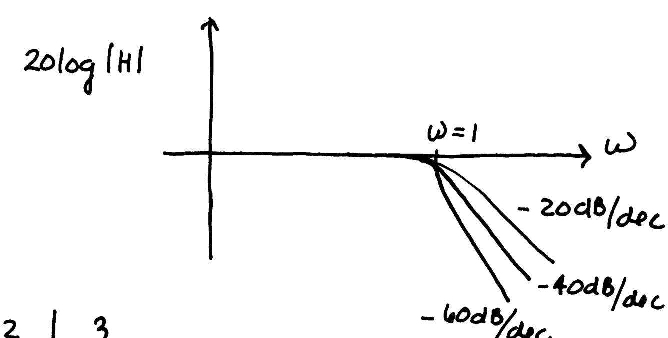
f(Hz)	$\lvert G_1 \rvert$	$\lvert G_2 \rvert$	$\lvert V_o/V_s \rvert$
1	2	1	2
10	20	10	200
100	20	100	2000
2000	20	100	2000
10,000	4	100	400
100,000	0.4	10	4

AP 13-6

$n=1 \quad \dfrac{1}{s+1}$

$n=2 \quad \dfrac{1}{s^2+\sqrt{2}s+1}$

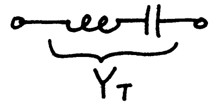
$n=3 \quad \dfrac{1}{s^3+2s^2+2s+1}$



at $\omega = 2$

n	1	2	3
$\lvert H \rvert$	.45	.25	.15

AP 13-7 to reject 30 kHz $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi \times 30{,}000 \rightarrow \underline{L \simeq 28.15\,\mu H}$

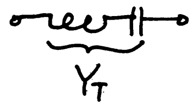
at 20 kHz $\Rightarrow$ 

$Y_T \simeq j\,0.226$

$\therefore$ X must be an inductor

$$\omega L = \frac{1}{0.226} \rightarrow L = \frac{1}{2\pi \times 20{,}000 \times 0.226} \simeq \underline{35.2\,\mu H}$$

AP 13-8 a) $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi \times 30{,}000 \rightarrow \underline{L \simeq 28.15\,\mu H}$

b) at 40 kHz $\Rightarrow$ 

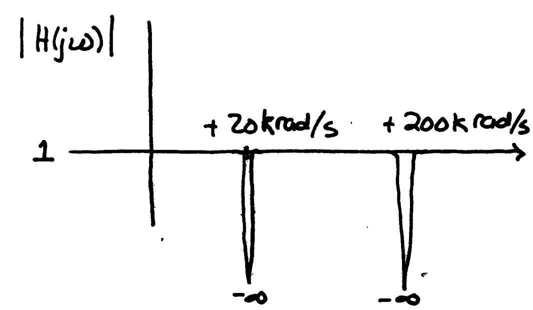
$Y_T \simeq -j\,0.323$

$\therefore$ X must be a capacitor $\omega C = 0.323 \rightarrow C = \frac{0.323}{2\pi \times 40{,}000} \simeq \underline{1.29\,\mu F}$

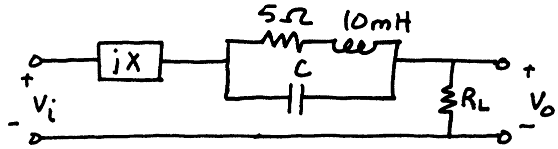
AP 13-9 for parallel L and C : $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25 \times 10^{-10}}} = 20\text{ k rad/s}$

Voltage divider with all voltage across parallel L and C

for series L and C : $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25 \times 10^{-12}}} = 200\text{ krad/s}$



AP 13-10

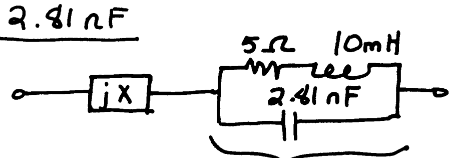


$$\omega_0^2 = \frac{1}{LC} - \left(\frac{R}{L}\right)^2 = [2\pi \times 30{,}000]^2$$

$$\Rightarrow \frac{1}{LC} = (2\pi \times 30{,}000)^2 + \left(\frac{5 \times 10^3}{10}\right)^2 \simeq 3.55 \times 10^{10}$$

$$\therefore \underline{C \simeq 2.81\,nF}$$

at 60 kHz have



$$Z_T = 0.548 - j\,1256.92$$

so $jX = j\,1256.92$ (thus inductive)

The value of the inductor can be found from

$$(2\pi \times 60{,}000)\,L = 1256.92 \quad \text{or} \quad \underline{L = 3.33\,mH}$$

Design Problems

DP 13-1

woofer: $X_{L_1} = R_{woofer}$ at lower corner freq. f_{cL}

$2\pi f_{cL} L_1 = 8\,\Omega \Rightarrow f_{cL} = \dfrac{8}{2\pi \times 2.5\times10^{-3}} = \underline{509\text{ Hz}}$

tweeter: $X_{C_3} = R_{tweeter}$ at corner freq f_{cu}

$\dfrac{1}{2\pi f_{cu} C_3} = 8\,\Omega \Rightarrow f_{cu} = \dfrac{1}{(2\pi \times 5\times10^{-6})8} = 3978\text{ Hz}$

mid-range: $f_{res} = \dfrac{1}{2\pi\sqrt{L_2 C_2}} = \dfrac{1}{2\pi\sqrt{0.364\times10^{-3}\times 34.8\times10^{-6}}} = \underline{1414\text{ Hz}}$

$Q = \dfrac{\omega L_2}{R} = \dfrac{2\pi(1414)}{8} = \underline{0.4}$

$\text{Bandwidth} = \dfrac{f_{res}}{Q} = \dfrac{1414}{0.4} = \underline{3535\text{ Hz}}$

for $Q < 10$ (Q here $= 0.4$)

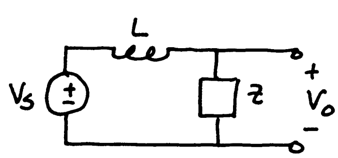
$$\left.\begin{aligned} f_{res} &= \sqrt{f_u \times f_L} \\ BW &= f_u - f_L \end{aligned}\right\} \quad \begin{aligned} &f_L^2 + BW f_L - (f_{res})^2 = 0 \\ \text{or}\quad &f_L^2 + 3535 f_L - (1414)^2 = 0 \end{aligned}$$

solving for $f_L \Rightarrow f_L = \dfrac{992}{2}$ or $-\dfrac{8062}{2}$

discarding the "$-$" solution get $\underline{f_L = 496\text{ Hz}}$

$\therefore f_u = 3535 + 496 = \underline{4031\text{ Hz}}$

DP 13-2



$H(j\omega) = \dfrac{Z}{sL + Z}$ $\qquad G = \dfrac{1}{1\text{ k}\Omega} = 10^{-3}\text{ S}$

$Z = \dfrac{1}{G + sC}$

so $H(j\omega) = \dfrac{1/(G+sC)}{sL + 1/(G+sC)} = \dfrac{1}{sL(G+sC)+1} = \dfrac{1/LC}{s^2 + \frac{G}{C}s + 1/LC}$

(2nd order filter)

to get -3dB at $\omega_1 = 2\pi(100\times10^3\text{ Hz}) = 6.283\times10^5$

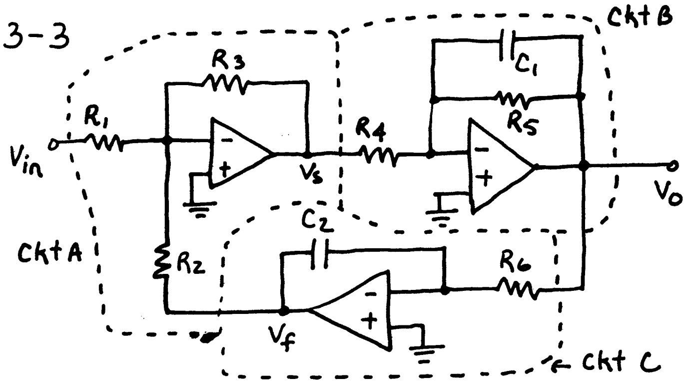
use two poles at $\omega_1 \Rightarrow (s+\omega_1)^2 = s^2 + 2\omega_1 s + \omega_1^2$

set $\omega_1^2 = \dfrac{1}{LC}$ and $2\omega_1 = \dfrac{G}{C}$

then $2(6.283\times10^5) = 10^{-3}/C \Rightarrow \underline{C = 79.6\text{ nF}}$

$L = \dfrac{1}{\omega_1^2 C} = \dfrac{1}{(39.48\times10^{10})(7.96\times10^{-10})} = \underline{3.18\text{ mH}}$

DP 13-3



$R_1 = 10k\Omega$
$R_2 = 866k\Omega$
$R_3 = 8.06k\Omega$
$R_4 = 1M\Omega$
$R_5 = 2.37M\Omega$
$R_6 = 499k\Omega$

$C_1 = 0.47\mu F$
$C_2 = 0.1\mu F$

ckt A is inverting summer $\Rightarrow V_s = -R_3\left[\frac{V_{in}}{R_1} + \frac{V_f}{R_2}\right]$ (1)

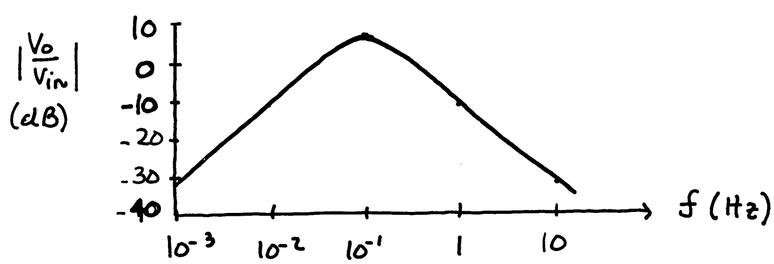
ckt B is first order LPF $\Rightarrow V_o = -V_s\frac{Z_f}{Z_i} = -V_s\frac{R_5}{R_4}\left[\frac{1}{C_1R_5s+1}\right]$ (2)

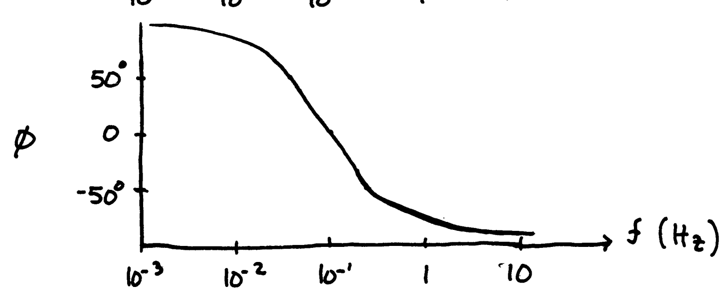
ckt C is an integrator $\Rightarrow V_f = -\frac{1}{C_2R_6s}V_o$ (3)

Solving (1) → (3) for V_o/V_{in} yields

$$V_o/V_{in} = \frac{\frac{R_3}{R_1R_4C_1}s}{s^2 + \frac{1}{R_5C_1}s + \frac{R_3}{R_2R_4R_6C_1C_2}}$$

plugging in the values for the resistors & capacitors, can draw

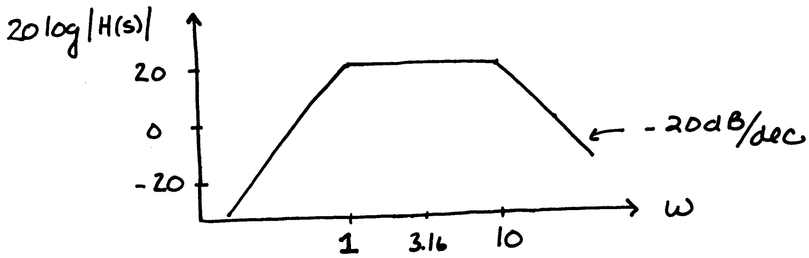


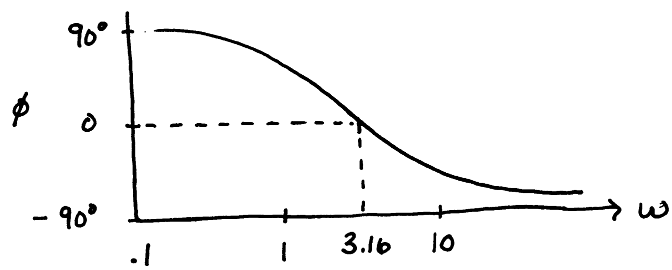


DP 13-5 $\quad H(s) = \dfrac{Ks}{(s+1)\left(\frac{s}{\omega_1}+1\right)}$

desire max = 20dB at $\omega = 3.16$ rad/s
3.16 is half way on a log chart between
1 and 10, so choose $\omega_1 = 10$ rad/s

20 dB mid band
gain $\Rightarrow K = 10$





DP 13-7 $H(s) = -\dfrac{Z_2(s)}{Z_1(s)} = \dfrac{10}{1+(s/\omega_1)^n}$ $\omega_1 = 10^4$ rad/s

n = number of stages

for equal cascaded amplifiers

n	BW (rad/s)	Gain (db)
1	1×10^4	20
2	6.23×10^3	20
3	4.98×10^3	20

$$H_i(s) = \frac{(10)^{1/n}}{(1+s/\omega_1)}$$

In order to achieve a bandwidth of 10^4 rad/s we use:

1 stage : $H_1(s) = \dfrac{10}{(1+s/\omega_1)}$ $\omega_1 = 10^4$

2 stages : $H(s) = \dfrac{(\sqrt{10})^2}{(1+s/\omega_2)^2}$ or each stage is $= \dfrac{\sqrt{10}}{1+s/\omega_2}$

$\omega_2 = 1.5\times10^4$

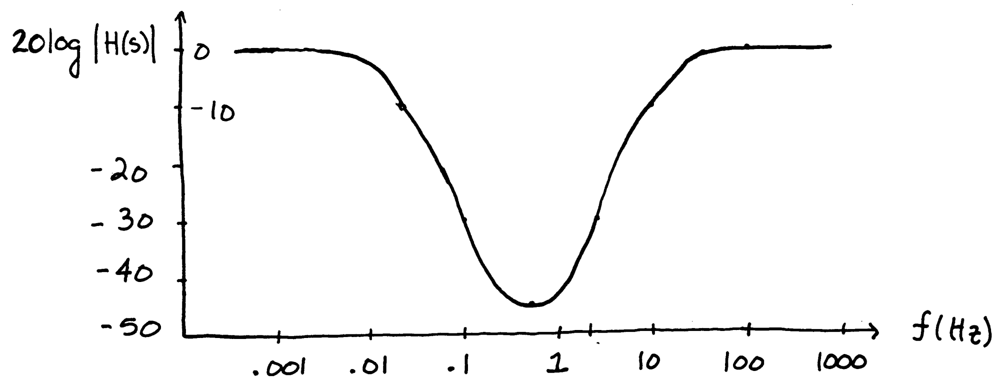
3 stages : $H(s) = \dfrac{10}{(1+s/\omega_3)^3}$ and each stage is $H_3 = \dfrac{(10)^{1/3}}{1+s/\omega_3}$

$\omega_3 = 2\times10^4$ rad/s

for this improved design we achieve the bandwidth and gain desired

DP 13-8 Choose $\omega_1 = 0.1$, $\omega_2 = 2$, $\omega_3 = 5$, $\omega_4 = 100$ rad/s

corresponding Bode magnitude plot is:

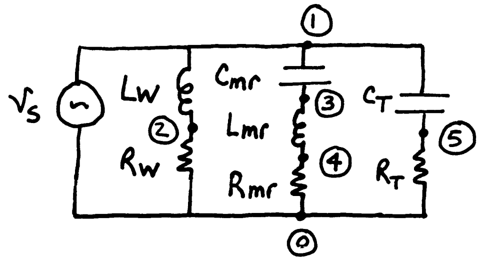


$$H(s) = \frac{(1+s/\omega_1)^2\ (1+s/\omega_3)^2}{(1+s/\omega_2)^2\ (1+s/\omega_4)^2}$$

minimum is -46.2 db at $f_{min} = 0.505$ Hz

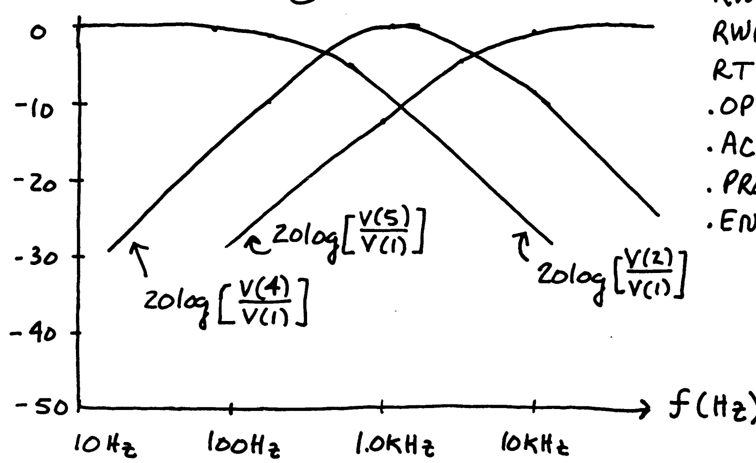
Spice Problems

SP 13-7

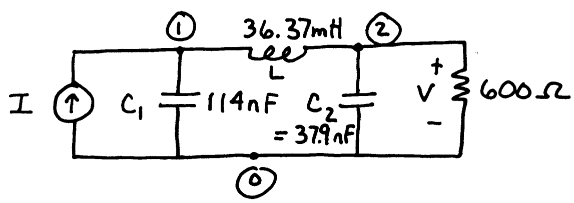


input file:

```
.AC1 1 0 AC 10
LW 1 2 2.5m
LWR 3 4 .364m
CMR 1 3 34.82u
CT 1 5 5u
RW 2 0 8
RWR 4 0 8
RT 5 0 8
.OPTIONS NOPAGE
.AC DEC 20 20 20000
.PROBE
.END
```

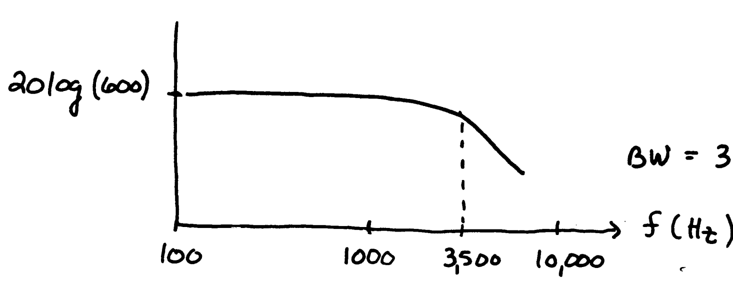


SP 13-8



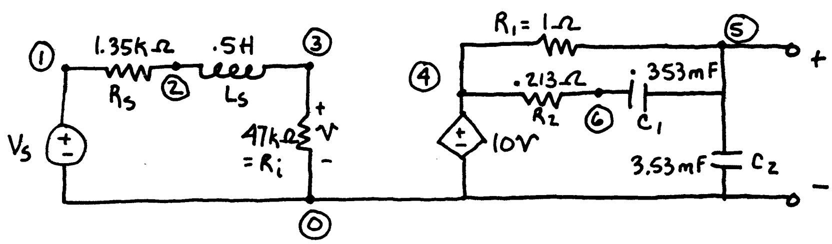
input:

```
I1 0 1 1
C1 1 0 114N
L1 1 2 36.37m
C2 2 0 37.9N
R1 2 0 600
.AC DEC 50 100 10K
.PRINT AC VDB(2)
.END
```



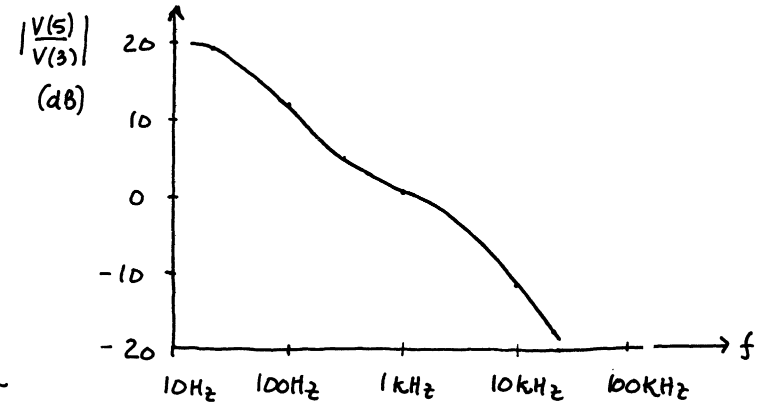
BW = 3500 Hz

SP 13-9

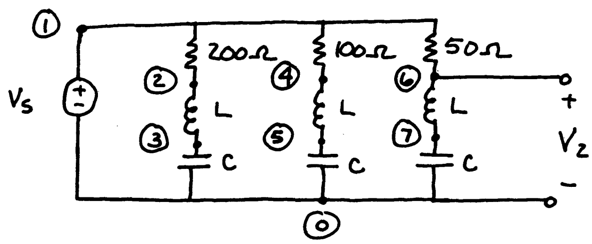


input file :

```
VS  1  0   AC  1
RS  1  2   1350
LS  2  3   0.5
RI  3  0   47K
EI  4  0   3  0  10
RI  4  5   1
CI  6  5   .353m
C2  5  0   3.53m
R2  4  6   .213
.AC DEC  100  20  20K
.PLOT  VDB(5)
.PROBE
.END
```

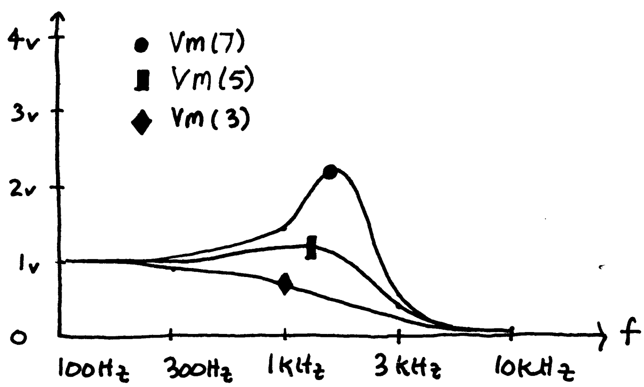


SP 13-10

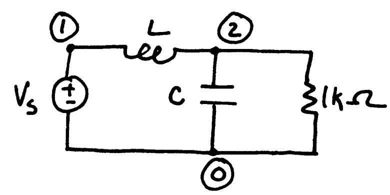


input file :

```
VS   1  0   AC  1
R1   1  2   200
L1   2  3   10mH
R2   1  4   100
L2   4  5   10m
C2   5  0   1u
C1   3  0   1u
C3   7  0   1u
R3   1  6   50
L3   6  7   10m
.AC  DEC  100  100  10K
.PROBE
.END
```



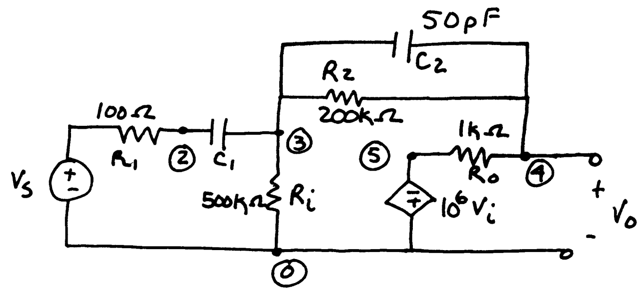
SP 13-11



input :

```
V1   1  0   AC  1
L1   1  2   318E-05
C1   2  0   79.6 E-9
R1   2  0   1K
.AC  DEC  50  1K  1MEG
.PRINT  AC  VDB(2)  VP(2)
.END
```

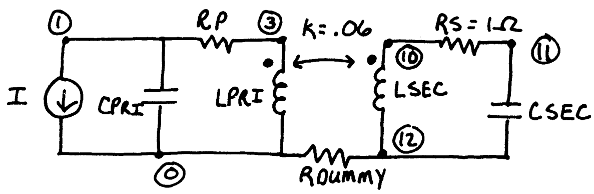
SP 13-12



input file :

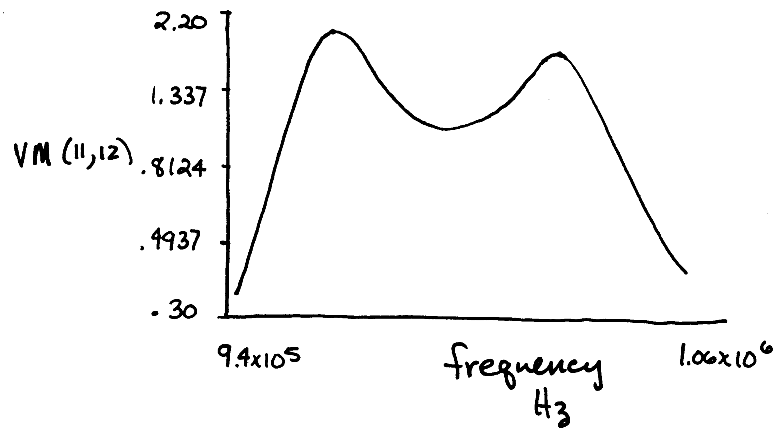
```
V1    1  0   AC  1
R1    1  2   100
RIN   3  0   500K
C1    2  3   .2U
R2    3  4   200K
C2    3  4   50E-09
RO    5  4   1K
E1    0  5   3  0  1MEG
.AC   DEC  50  3.2K  32K
.PLOT  AC  VDB(4)
.END
```

SP 13-13



input file :

```
I 1 0 AC 1m
RP 1 3 1
RS 11 10 1
Rdummy 12 0 1P
CPRI 1 0 2.53303N
CSEC 11 12 2.53303N
LPRI 3 0 10U
LSEC 10 12 10U
KPRISEC LPRI LSEC 0.06
.AC LIN 61 940K 1060K
.PLOT AC VM(11,12) (2.2 0.3)
.END
```

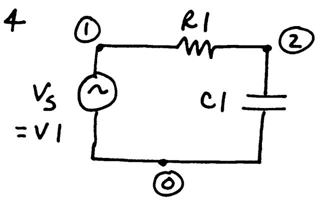


2.09 = 1st peak at 9.7×10^5 Hz

0.988 minimum at 1×10^6 Hz

1.86 = 2nd peak at 1.03×10^6 Hz

SP 13-14



input file :

```
R1 1 2 33K
C1 2 0 0.0047u
V1 1 0 AC 1
.AC DEC 101 10 100K
.PLOT VDB(2)
.PROBE
.END
```

$$\omega_1 = \frac{1}{R_1 C_1} = 6.447 \times 10^3 \text{ rad/sec}$$

$$f_1 = 1.03 \text{ kHz}$$