

# PROBLEMS

**P 7-1** The voltage across a  $40\text{-}\mu\text{F}$  capacitor is 25 volts at  $t = 0$ . If the current through the capacitor as a function of time is given by  $i(t) = 6e^{-6t}$  mA for  $t > 0$ , find  $v(t)$  for  $t > 0$ .

**Answer:**  $v(t) = 100 - 25e^{-6t}$  V

**P 7-2** Find  $v(0^+)$  and  $dv_c(0^+)/dt$  for the circuit of Figure P 7-2 if  $v_c(0^-) = 40$  V.

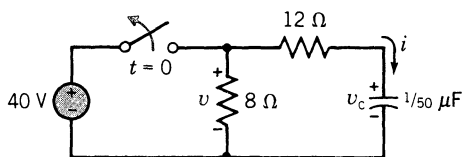


Figure P 7-2

**P 7-3** Find  $dv_c(0^+)/dt$  and  $di_L(0^+)/dt$  for the circuit of Figure P 7-3 if  $v_c(0^-) = 8$  V and  $i_L(0^-) = 4$  A.

**Answer:**  $dv_c(0^+)/dt = -20$  V/ $\mu$ s  
 $di_L(0^+)/dt = -23$  mA/ $\mu$ s

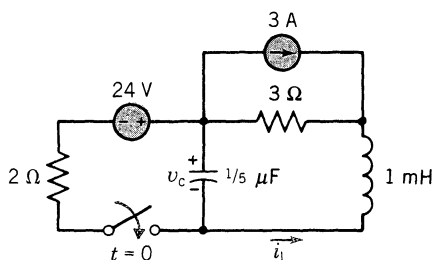


Figure P 7-3

**P 7-4** Find  $i$  for the circuit of Figure P 7-4 if  $v = 5(1 - 2e^{-2t})$  V.

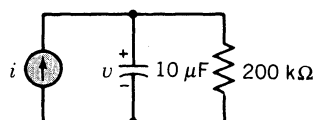


Figure P 7-4

**P 7-5** A  $15\text{-}\mu\text{F}$  capacitor has a voltage of 5 volts across it at  $t = 0$ . If a constant current of 25 mA flows through the capacitor, how long will it take for the capacitor to hold a charge of  $150\text{ }\mu\text{C}$ ?

**Answer:**  $t = 3$  ms

**P 7-6** A  $0.1\text{-mF}$  capacitor is initially charged to 10 V. Between  $t = 0$  and  $t = 10$  ms, the capacitor is discharged with the current shown in Figure P 7-6.

- What is the capacitor voltage at  $t = 6$  ms and  $t = 10$  ms?
- How much energy was drained from the capacitor between  $t = 0$  and  $t = 10$  ms?

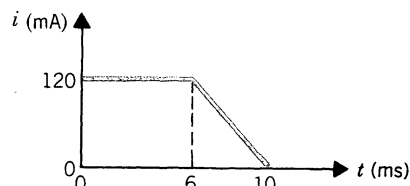


Figure P 7-6

**P 7-7** Find the equivalent capacitance  $C_{eq}$  at terminals a-b for the circuit of Figure P 7-7. All capacitances are given in mF.

**Answer:**  $C_{eq} = 16$  mF

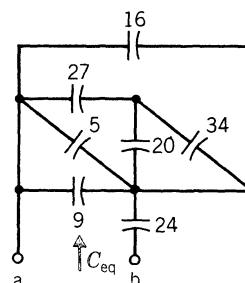


Figure P 7-7

**P 7-8** If  $v_c(t)$  is given by the waveform shown in Figure P 7-8, sketch the capacitor current for  $-1 \leq t \leq 2$  seconds. Sketch the power stored by the capacitor over the same time interval when  $C = 1$  mF.

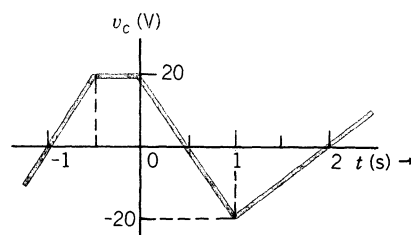
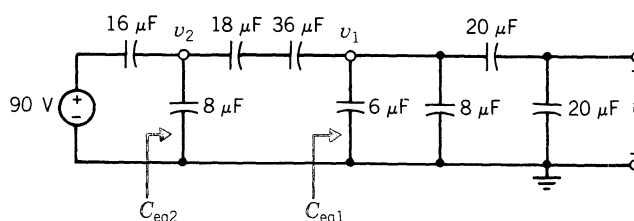


Figure P 7-8

**P 7-9** Find  $C_{eq1}$ ,  $C_{eq2}$ ,  $v_1$ ,  $v_2$ , and  $v$  for the circuit of Figure P 7-9.



**Figure P 7-9** The symbol  $C_{eq}$  represents the equivalent capacitance looking into the designated terminals.

**Answer:**  $C_{eq1} = 24 \mu\text{F}$   
 $C_{eq2} = 16 \mu\text{F}$   
 $v_2 = 45 \text{ V}$   
 $v_1 = 15 \text{ V}$   
 $v = 15/2 \text{ V}$

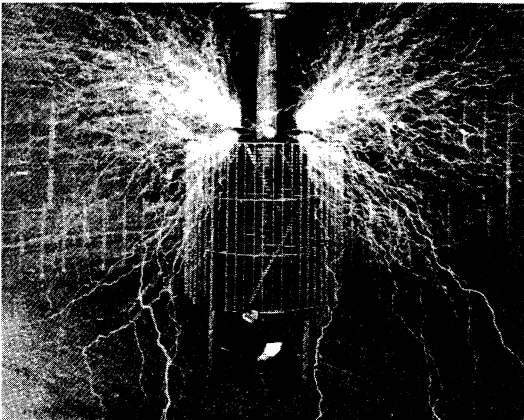
**P 7-10** The current through a  $2\text{-}\mu\text{F}$  capacitor is  $50 \cos(10t + \pi/6) \mu\text{A}$  for all time. The average voltage across the capacitor is zero. What is the maximum value of the energy stored in the capacitor? What is the first nonnegative value of  $t$  at which the maximum energy is stored?

**P 7-11** The energy stored by a  $1\text{-mF}$  capacitor used in a laser power supply is given as  $w = 4e^{-10t} \text{ J}$  for  $t \geq 0$ . Find the capacitor voltage and current at  $t = 0.1$  second.

**P 7-12** Nikola Tesla (1857–1943) was an American electrical engineer who experimented with electric induction. Tesla built a large coil with a very large inductance, shown in Figure P 7-12. The coil was connected to a source current

$$i_s = 100 \sin 400t \text{ A}$$

so that the inductor current  $i_L = i_s$ . Find the voltage across the inductor and explain the discharge in the air shown in the figure. Assume that  $L = 100 \text{ H}$ .

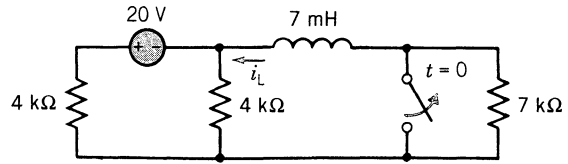


**Figure P 7-12** Nikola Tesla sits impassively as alternating current induction coils discharge millions of volts with a roar audible 10 miles away (about 1910). Courtesy of Burndy Library.

**P 7-13** The current through a  $10\text{-mH}$  inductor used in a fusion power experiment is given by  $i(t) = 2 \sin^2 t \text{ mA}$  for  $t > 0$ . Find the instantaneous power delivered to the inductor and the energy stored in the inductor at  $t = 3\pi/4$  seconds.

**Answer:**  $P(t) = 8 \times 10^{-8} (\sin^3 t) \cos t \text{ W}$   
 $w = 5 \text{ nJ}$

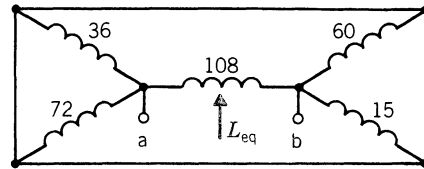
**P 7-14** Find  $di_L(0^-)/dt$  for the circuit of Figure P 7-14 if  $i_L(0^-) = 5 \text{ mA}$ .



**Figure P 7-14**

**P 7-15** Find the equivalent inductance  $L_{eq}$  for terminals a–b for the circuit of Figure P 7-15 when all the inductances are in microhenrys.

**Answer:**  $L_{eq} = 27 \mu\text{H}$



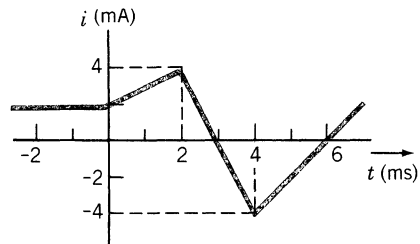
**Figure P 7-15** The symbol  $L_{eq}$  represents the equivalent inductance looking into the designated terminals.

**P 7-16** The current in a  $10\text{-mH}$  inductor is zero for  $t < 0$  and  $5te^{-t} \text{ A}$  for  $t \geq 0$ . Find the time when the maximum power is delivered to the inductor. At what time is the energy stored in the inductor a maximum?

**P 7-17** The model of an electric motor consists of a series combination of a resistor and inductor. A current  $i(t) = 4te^{-t} \text{ A}$  flows through the series combination of a  $10\text{-}\Omega$  resistor and  $0.1\text{-henry}$  inductor. Find the voltage across the combination.

**Answer:**  $v(t) = 0.4e^{-t} + 39.6te^{-t} \text{ V}$

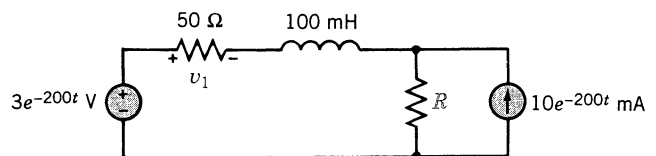
**P 7-18** The current through a  $20\text{-mH}$  inductor is shown in Figure P 7-18. Find the inductor voltage at  $t = 1 \text{ ms}$  and  $t = 6 \text{ ms}$ .



**Figure P 7-18**

**P 7-19** Find  $R$  of the circuit shown in Figure P 7-19 if  $v_1 = 1e^{-200t} \text{ volts}$  for  $t \geq 0$ .

**Answer:**  $R = 80 \Omega$



**Figure P 7-19**

**P 7-20** Find  $v_c(0^+)$  and  $dv_c(0^+)/dt$  if  $v(0^-) = 15$  V for the circuit of Figure P 7-20.

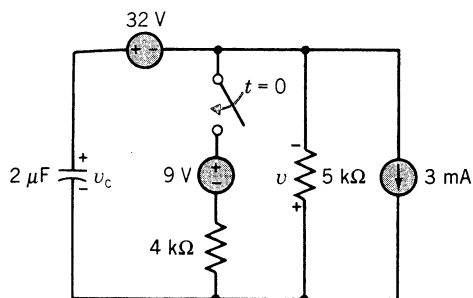


Figure P 7-20

**P 7-21** Given that  $i_1 = 0.5$  A and  $i_3 = 0.2$  A at  $t = 0$ , find  $i(t)$  for  $t \geq 0$  for the circuit of Figure P 7-21.

**Answer:**  $i(t) = 1.06 - 0.48e^{-5000t}$  A

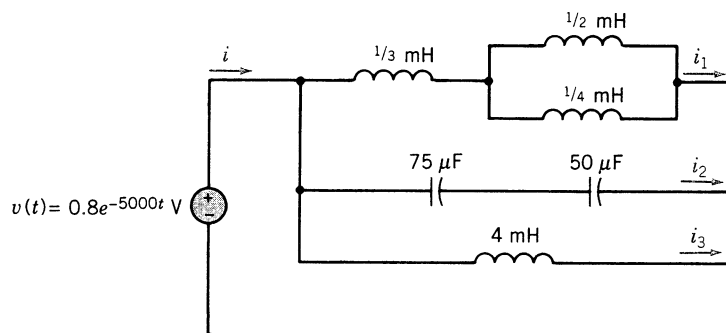


Figure P 7-21

**P 7-22** For the circuit of Figure P 7-22, determine the value of  $R$  such that the energy stored in both the capacitor and the inductor is the same at steady state (when  $t \rightarrow \infty$ ).

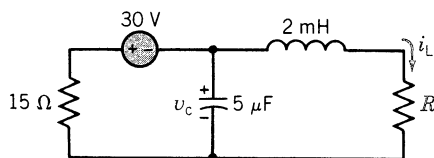


Figure P 7-22

**P 7-23** Find the equivalent inductance  $L_{eq}$  at terminals a–b of the circuit of Figure P 7-23 when the switch  $S$  is at (a) position A and (b) position B.

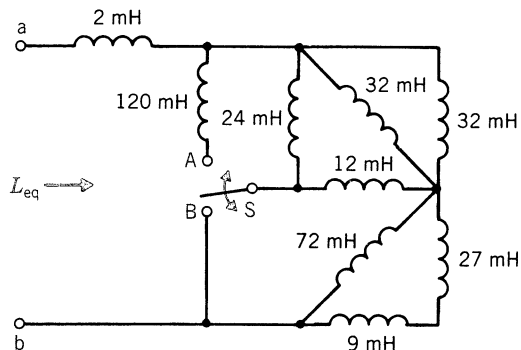


Figure P 7-23

**P 7-24** A 1.0-mH inductor connected in a telephone circuit is found to have a voltage across it as shown in Figure P 7-24. The initial inductor current is given by  $i(0) = 20$  mA. Find (a)  $i(t)$  for  $t > 0$  with  $t$  in ms and  $i$  in mA, (b) the initial energy in the inductor at  $t = 0$ , and (c) the net amount of energy that was transferred to the inductor between  $t = 0$  and  $t = 10$  ms.

**Answer:** (a)  $i(t) = 20 + 6t^2$  mA ( $0 < t < 2$  ms)  
 $i(t) = -4 + 24t$  mA ( $2 < t < 4$  ms)  
 $i(t) = -36 + 40t - 2t^2$  mA ( $4 < t < 10$ ) ms  
 $i(t) = 164$  mA ( $t > 10$  ms)  
 (b)  $w(0) = 0.2$  μJ

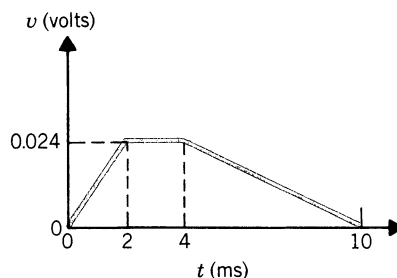


Figure P 7-24

**P 7-25** A current of  $5e^{-4t}$  mA flows through a series combination of a  $10\text{-}\Omega$  resistor and a  $0.1\text{-H}$  inductor. Find the total power absorbed by this series combination as a function of time.

**P 7-26** For the circuit shown in Figure P 7-26, find  $dv_c(0^+)/dt$ ,  $di_L(0^+)/dt$ , and  $i(0^+)$  if  $v(0^-) = 16\text{ V}$ . Assume that the switch was closed for a long time prior to  $t = 0$ .

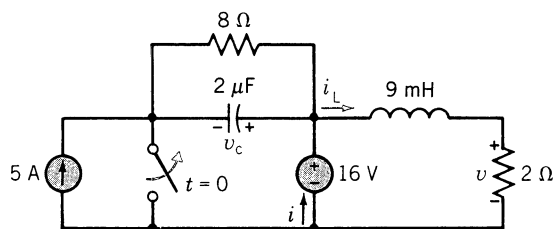


Figure P 7-26

**\*P 7-27** Find  $C_{eq}$  looking into terminals a–b for the circuit of Figure P 7-27 when all the capacitances are in microfarads.

**Answer:**  $C_{eq} = 4\text{ }\mu\text{F}$

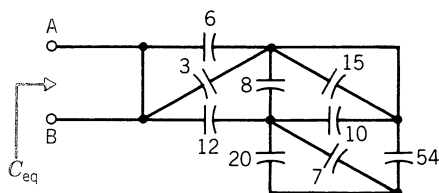
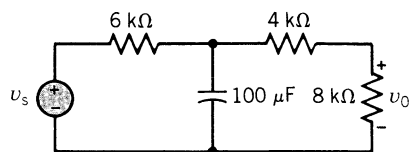
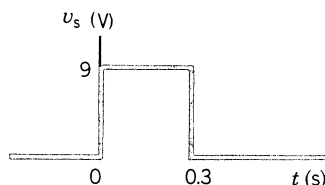


Figure P 7-27

**\*P 7-28** A circuit used in a mobile telephone is shown in Figure P 7-28a where the input source  $v_s$  is a pulse defined as shown in Figure P 7-28b. Find the time response  $v_o(t)$  for this circuit when the initial voltage of the capacitor is zero.



(a)



(b)

Figure P 7-28 A circuit used in a mobile telephone

**\*P 7-29** If a capacitor can store energy, as does a battery, could not a capacitor be used to power an electric train? Such a capacitor (electrolytic) would be about 1 cubic centimeter per 10 microfarads, for the voltage level required. Suppose we start with a charge at 100 volts and can extract all the energy from the capacitor to drive the train for 1 hour at 30 kW. How big would the capacitor have to be? Is this practical?

**P 7-30** A capacitor is used in the electronic flash unit of a camera. A small battery with a constant voltage of 6 V is used to charge a capacitor with a constant current of  $10\text{ }\mu\text{A}$ . How long does it take to charge the capacitor when  $C = 10\text{ }\mu\text{F}$ ? What is the stored energy?

**P 7-31** Determine each capacitor voltage in the circuit of Figure P 7-31. Assume the circuit is at steady state.

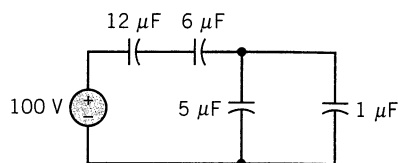


Figure P 7-31

**P 7-32** In a pulse power circuit the voltage of a  $10\text{-}\mu\text{F}$  capacitor is zero for  $t < 0$  and

$$v = 5(1 - e^{-4000t})\text{ V} \quad t \geq 0$$

Determine  $i_c$  and the energy stored in the capacitor at  $t = 0$  and  $t = 10\text{ ms}$ .

**P 7-33** For the series connection of  $N$  capacitors, as shown in Figure 7-17, determine the voltage divider relationship  $v_N/v$  where  $N$  is the capacitor for which we seek the output voltage. When we have three equal capacitors, determine the ratio  $v_N/v$ . Assume the initial voltage of each capacitor is zero.

**P 7-34** Determine

$$\left. \frac{dv_c}{dt} \right|_{t=0^+} \quad \text{and} \quad \left. \frac{di}{dt} \right|_{t=0^+}$$

for the  $RLC$  circuit shown in Figure P 7-34 when  $v_c(0^-) = -1\text{ V}$  and  $i_s = 2e^{-t}$  for  $t \geq 0$  and  $i_s = 0$  for  $t < 0$ .

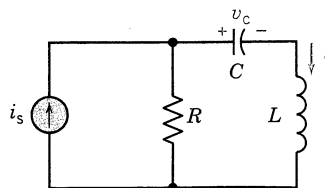


Figure P 7-34

\* Note: An asterisk denotes a challenging problem.

**P 7-35** For the circuit of Figure P 7-35, determine the current and voltage of each passive element at  $t = 0^-$  and  $t = 0^+$ . The current source is  $i_s = 0$  for  $t < 0$  and  $i_s = 4$  A for  $t \geq 0$ .

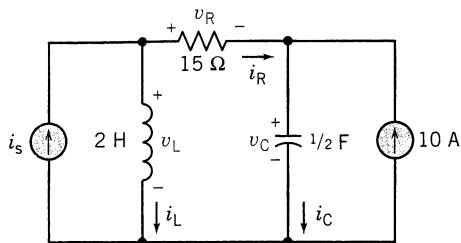


Figure P 7-35

**P 7-36** Determine  $i(0^+)$ ,  $v(0^+)$ ,  $dv(0^+)/dt$ , and  $di(0^+)/dt$  for the circuit of Figure P 7-36 when  $C = 1/5$  F and  $L = 1/2$  H.

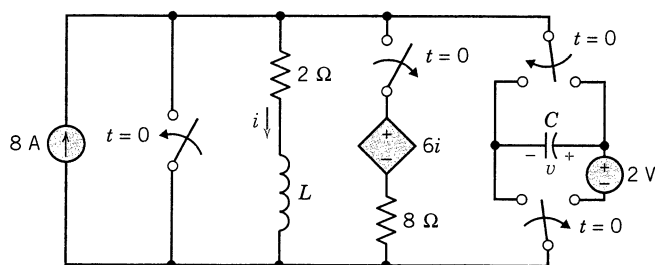


Figure P 7-36

**P 7-37** The current,  $i$ , through a capacitor is shown in Figure P 7-37. When  $v(0) = 0$  and  $C = 0.5$  F, determine and plot  $v(t)$ ,  $p(t)$ , and  $w(t)$  for  $0 < t < 6$  s.

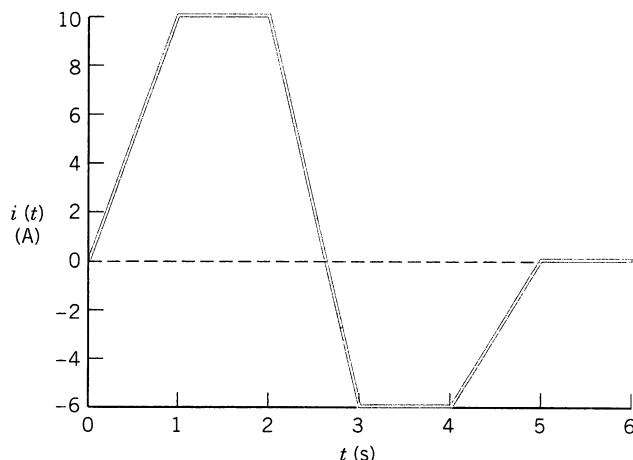


Figure P 7-37

**P 7-38** Consider the circuit shown in Figure 7-12a. (a) Find  $v_o(t)$  and show that the circuit performs as a differentiator. (b) Find values for  $R$  and  $C$  so that  $v_o = -(1/10) dv_s/dt$ . Assume  $A > 10^5$ .

## ADVANCED PROBLEMS

**AP 7-1** A current source,  $i_s$ , as shown in Figure AP 7-1, is connected to an uncharged capacitor at  $t = 0$ . Determine the voltage waveform from  $t = 0$  to  $t = 2.5$  seconds, and sketch the waveform when  $C = 1$  mF.

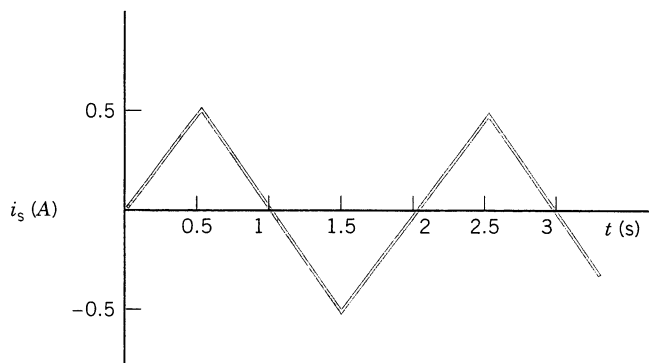


Figure AP 7-1

**AP 7-2** A *gyrator* consists of the two voltage-controlled sources shown in Figure AP 7-2. It is shown connected to a capacitor at the terminals c-d. Show that the  $i$ - $v$  relationship at terminals a-b is that of an inductor. Assume that the capacitor is initially uncharged.

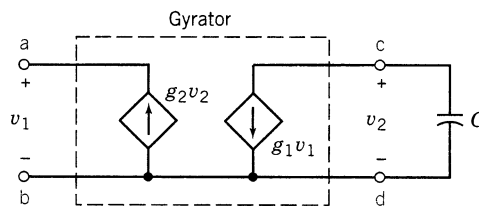


Figure AP 7-2 A model of a gyrator connected to a capacitor.

**AP 7-3** If a capacitor can store energy, as does a bat-

tery, could not a capacitor be used to power an electric car? Such a capacitor (electrolytic) would be about 1 cubic centimeter per 100 microfarads, for the voltage level required. Suppose we start with an initial voltage

of 500 volts and can extract all the energy from the capacitor to drive the car for 1 hour. The auto drive train requires 1.5 kW. How big would the capacitor have to be? Is this practical?

## DESIGN PROBLEMS

**DP 7-1** Select the resistance  $R$  for the circuit shown in Figure DP 7-1 so that  $v(0) = 20$  V and  $i(0) = 5$  A. Assume that the switch has been closed for a long time.

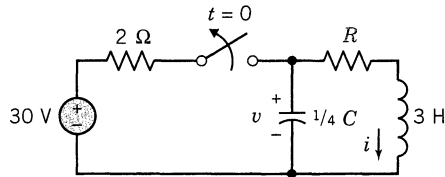


Figure DP 7-1

**DP 7-2** A laser pulse power circuit is shown in Figure DP 7-2. It is required that  $v(0) = 7.4$  V and  $i(0) = 3.7$  A. Determine the required resistance  $R$ . Assume that the switch has been closed for a long time before it is opened at  $t = 0$ .

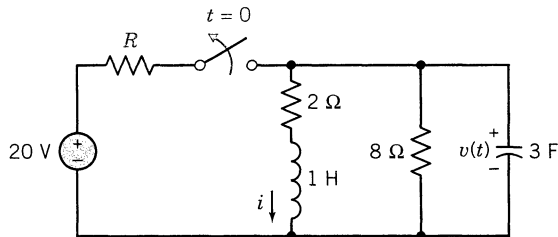


Figure DP 7-2 Laser pulse power circuit.

**DP 7-3** A high-speed flash unit for sports photography requires a flash voltage  $v(0^+) = 3$  V and

$$\left. \frac{dv(t)}{dt} \right|_{t=0^+} = 24 \text{ V/s}$$

The flash unit uses the circuit shown in Figure DP 7-3. Switch 1 has been closed a long time, and switch 2 has been open a long time at  $t = 0$ . Actually, the long time in this case is 3 seconds. Determine the required battery voltage,  $V_B$ , when  $C = 1/8$  F.

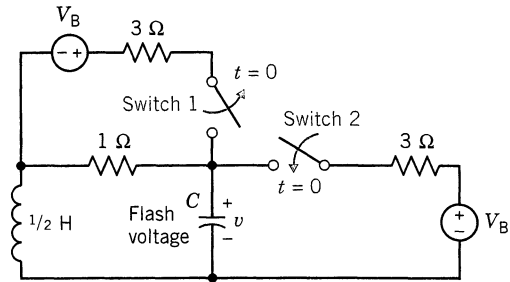


Figure DP 7-3 High-speed flash unit circuit.

**DP 7-4** For the circuit shown in Figure DP 7-4, select a value of  $R$  so that the energy stored in the inductor is equal to the energy stored in the capacitor at steady state.

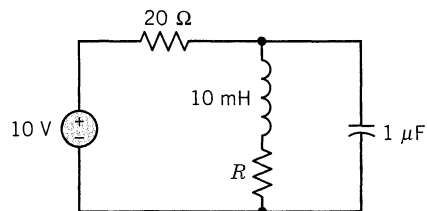


Figure DP 7-4

## TERMS AND CONCEPTS

**Exponential Solution** Assumed solution of the form  $x = Ae^{st}$  where  $A$  and  $s$  are constants to be determined.

**First-Order Linear Differential Equation** Differential equation where the highest-order derivative term is the first derivative, i.e.,  $dx/dt$ .

**Natural Response** Response of an  $RL$  or  $RC$  circuit that depends only on the nature of the circuit and not on external sources.

**Operator** A symbol that represents a mathematical operation. Differential operator  $s$  such that  $s^n x = d^n x/dt^n$ .

**Radio Transmission** Transmission of communication messages by means of radiated electromagnetic waves other than heat or light waves.

**Sequential Switching** Action of two or more switches activated at different instants of time in a circuit.

**Steady-State Response** Response that exists after a long time following any switch activation.

**Telegraph** Device for communication of messages using the activation of a switch or key to send a code.

**Time Constant** Value in seconds,  $\tau$ , in an exponential response  $Ae^{-t/\tau}$ . The constant  $\tau$  is the time to complete 63 percent of the decay.

**Transient Response** Time response  $x(t)$  of a circuit to a stimulus or to a response resulting from a switch activation.

## REFERENCES Chapter 8

- Orlando, T. *Foundations of Applied Superconductivity*, Addison-Wesley, Reading, MA, 1991.  
 Quint, David W. "Measurement of R, L and C Parameters in VLSI Packages," *Hewlett-Packard Journal*, October 1990, pp. 73–77.  
 Scott, David "Electronic Pistol," *Popular Science*, August 1991, p. 70.

## PROBLEMS

**P 8-1** A complex circuit used in a telegraph transmitter is shown in Figure P 8-1. For the circuit of Figure P 8-1, find  $i(t)$  for  $t > 0$ . The switch has been in position a for a long time at  $t = 0^-$ .

**Answer:**  $i(t) = -3e^{-4000t}$  mA

**P 8-2** For the circuit of Figure P 8-2, find  $i(t)$  for  $t > 0$ . Assume that the switches have been in their respective positions for a long time at  $t = 0^-$ .

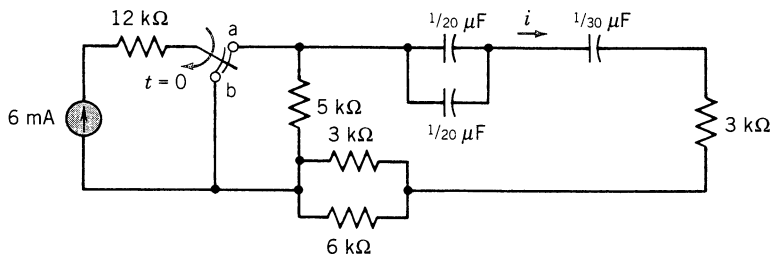


Figure P 8-1 A telegraph transmitter circuit.

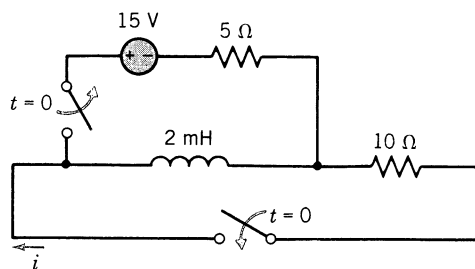


Figure P 8-2

**P 8-3** For the circuit of Figure P 8-3, find  $i(t)$  for  $t > 0$ . Assume that the switch has been closed a long time at  $t = 0^-$ .

**Answer:**  $i(t) = \frac{2}{5} e^{-10t}$  A

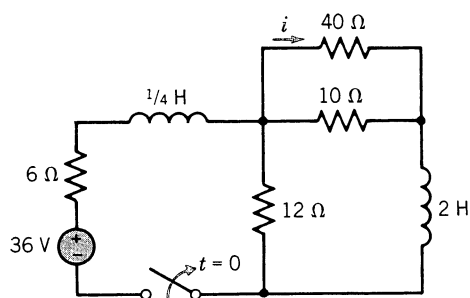


Figure P 8-3

**P 8-4** For the circuit of Figure P 8-4, find  $v_c(t)$  for  $t > 0$  if  $v_c(0) = 6$  V.

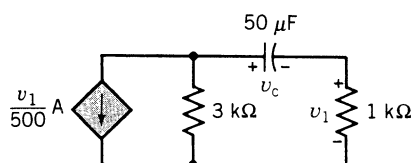
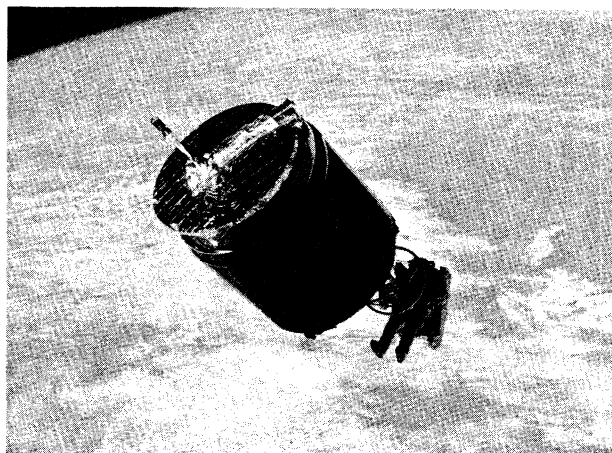


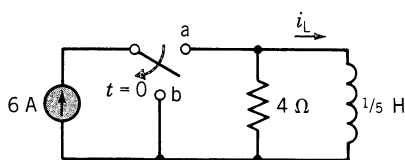
Figure P 8-4

**P 8-5** Figure P 8-5a shows astronaut Dale Gardner using the manned maneuvering unit to dock with the spinning Westar VI satellite on November 14, 1984. Gardner used a large tool called the apogee capture device (ACD) to stabilize the satellite and capture it for recovery, as shown in Figure P 8-5a. The ACD can be modeled by the circuit of Figure P 8-5b. Find the inductor current  $i_L$  for  $t > 0$ .

**Answer:**  $i_L(t) = 6e^{-20t}$  A



(a)



(b)

**Figure P 8-5** (a) Astronaut Dale Gardner using the manned maneuvering unit to dock with the Westar VI satellite. Courtesy of NASA. (b) Model of the apogee capture device. Assume that the switch has been in position a for a long time at  $t = 0^-$ .

**P 8-6** A laser fusion experiment uses the circuit of Figure P 8-6. Find  $v(t)$  for  $t > 0$  for the circuit. Assume that the switch has been closed a long time at  $t = 0^-$ .

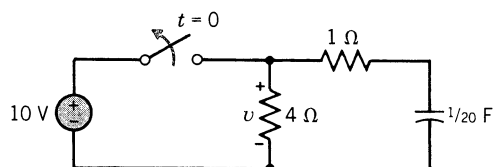


Figure P 8-6

**P 8-7** The switch of the circuit shown in Figure P 8-7 is moved from A to B at  $t = 0$ , after being at A for a long time. Find  $v(t)$  for  $t > 0$ .

**Answer:**  $v(t) = -0.18e^{-100t}$  V

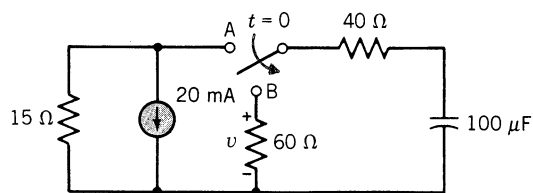


Figure P 8-7



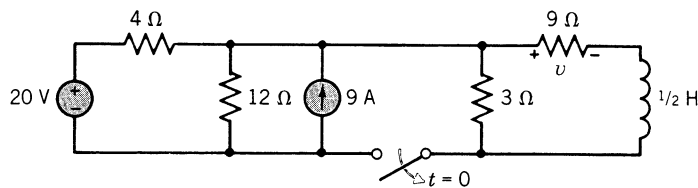


Figure P 8-8 A security alarm circuit.

**P 8-8** A security alarm for an office building door is modelled by the circuit of Figure P 8-8. The switch represents the door interlock and  $v$  is the alarm indicator voltage. Find  $v(t)$  for  $t > 0$  for the circuit of Figure P 8-8. The switch has been closed for a long time at  $t = 0^-$ .

**P 8-9** Find  $i(t)$  for  $t > 0$  for the circuit of Figure P 8-9. The circuit is in steady state at  $t = 0^-$ .

**Answer:**  $i(t) = -14.3e^{-3.25t}$  A

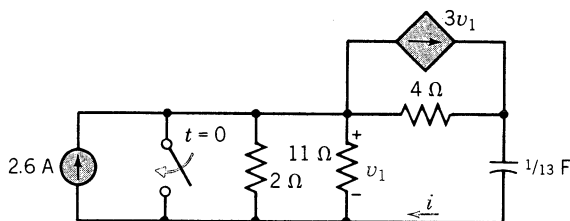


Figure P 8-9

**P 8-10** A complex electronic circuit for a communication satellite is represented by Figure P 8-10. Find  $i(t)$  for  $t > 0$  for the circuit shown in Figure P 8-10. The circuit is in steady state at  $t = 0^-$ .

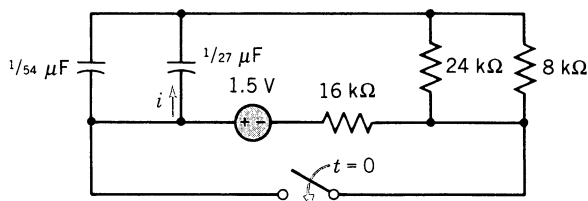


Figure P 8-10 A circuit for a communication satellite.

**P 8-11** Find  $v(t)$  for  $t > 0$  for the circuit shown in Figure P 8-11. The circuit is in steady state at  $t = 0^-$ .

**Answer:**  $v(t) = -216e^{-96t}$  V

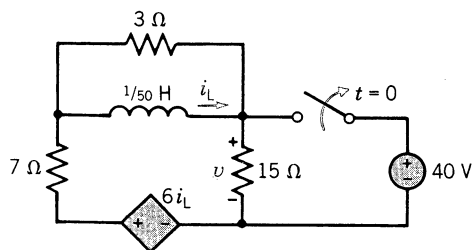


Figure P 8-11

**P 8-12** In the circuit shown in Figure P 8-12, the switch is flipped from A to B at  $t = 0$ , after being at A for a long time. Find  $v_c(t)$  and  $i_L(t)$  for  $t > 0$ .

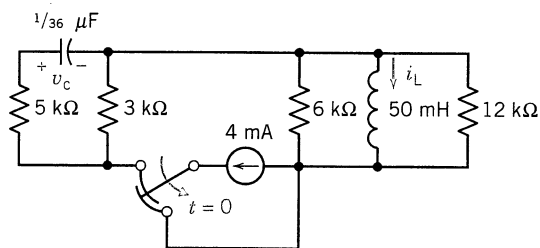


Figure P 8-12

**P 8-13** Find  $i(t)$  for  $t > 0$  for the circuit shown in Figure P 8-13. The circuit is in steady state at  $t = 0^-$ .

**Answer:**  $i(t) = \frac{2}{3}e^{-6t}$  A for  $0 \leq t \leq 51$  ms  
 $i(t) = 1.47e^{-14(t-0.051)}$  A for  $t > 51$  ms

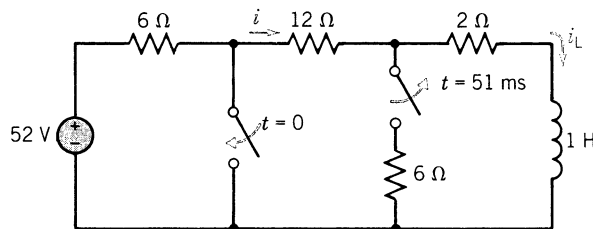


Figure P 8-13

**P 8-14** Efficient use of energy can be achieved by load management programs of electric utilities. The utility hooks up electronically activated controllers to appliances such as water heaters, air conditioners, and swimming-pool pumps at the homes of volunteer customers (they receive a rebate for participating). During peak hours, signals go out over the power lines ordering controllers at selected houses to turn off for periods of typically 15–30 minutes; the shutdown call then rotates to another group of houses. The effect is barely noticeable to the consumer (water stays fairly hot in the tank, for example), but the cumulative effect should be significant.

A model of a local switch and the energy load from a home is shown in Figure P 8-14. Find the energy  $w$  dissipated in the two load resistors for the first minute after the switch is opened at  $t = 0$  after having been closed for a long time.

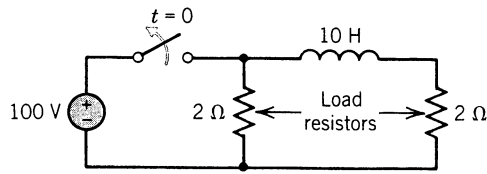


Figure P 8-14

**P 8-15** For the circuit shown in Figure P 8-15, switch 1 has been closed for a long time, and switch 2 has been in position a for a long time. At  $t = 0$  switch 1 is opened, and at  $t = 4$  ms switch 2 is changed to position b. Find  $i$  for  $t \geq 0$  when (a)  $k = 10$  and (b)  $k = -11$ .

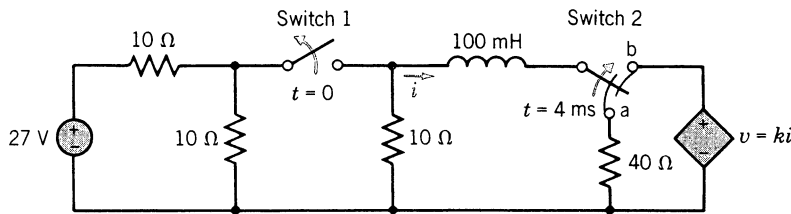


Figure P 8-15

**P 8-16** Sequential switching is used repetitively to generate communication signals. For the circuit shown in Figure P 8-16, switch a has been in position 1 and switch b has been open for a long time. At  $t = 0$ , switch a moves to position 2. Then, 100 ms after switch a moves, switch b closes. Find the capacitor voltage  $v$  for  $t \geq 0$ .

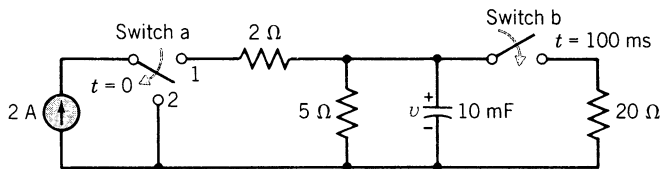


Figure P 8-16

**P 8-17** A circuit is shown in Figure P 8-17 with two capacitors where  $C_1 = C_2 = 1$  F. At  $t = 0$  the switch is thrown from position a to position b. (a) Find the initial voltage and energy stored in the left-hand capacitor,  $C_1$ , if the switch has been in position a for a long time. The voltage on the right-hand capacitor,  $C_2$ , at  $t = 0^-$  is zero. (b) Find the energy stored by the two capacitors and the energy dissipated by the resistor for  $t \geq 0$ .

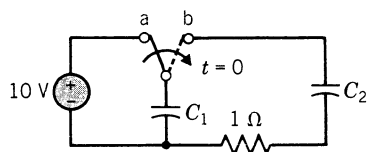


Figure P 8-17

**\*P 8-18** Cardiac pacemakers are used by people to maintain regular heart rhythm when they have a damaged heart. The circuit of a pacemaker can be represented as shown in Figure P 8-18. The resistance of the wires,  $R$ , can be neglected since  $R < 1$  m $\Omega$ . The heart's load resistance,  $R_L$ , is 1 k $\Omega$ . The first switch is activated at  $t = t_0$  and the second switch is activated at  $t_1 = t_0 + 10$  ms. This cycle is repeated every second. Find  $v(t)$  for  $t_0 \leq t \leq 1$ . Note that it is easiest to consider  $t_0 = 0$  for this calculation. The cycle repeats by switch 1 returning to position a and switch 2 returning to its open position.

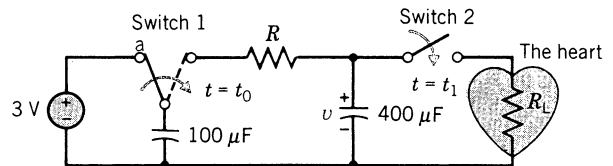


Figure P 8-18

**\*P 8-19** A home handyman decides to fix his own TV set, despite the warning on the back. He switches it off, pulls the plug, removes the back cover, reaches inside, and receives a large electrical shock. The dc source that drives the set's picture tube is 20 kV. The capacitance and resistance that form its load are 1 microfarad and 100 megaohms and are connected in parallel. Why did he get a shock, and what was the voltage? If his body resistance was 10 kilohms, how much energy did his body absorb? Assume that it took 30 seconds between switching off the set and placing his hands across the picture tube circuit. Assume he touched the circuit for  $\frac{1}{2}$  second.

\* Note: An asterisk denotes a challenging problem.

**P 8-20** A circuit is shown in Figure P 8-20. It is desired to select the capacitance  $C$  so that  $v(t) = 189$  V at  $t = 0.25$  s. Assume that the switch was connected to terminal 1 for a long time.

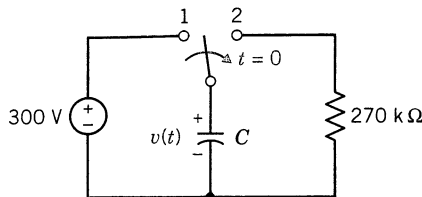


Figure P 8-20

**P 8-21** From a practical viewpoint, the crucial difference between copper wire and superconductors is their respective current-carrying capabilities. Standard copper wire used in homes, for example, will carry about  $10^7$  A/m<sup>2</sup>. Practical superconductors used in wire can carry current densities of  $10^{10}$  A/m<sup>2</sup> or higher. The magnetic field produced by an electromagnet is proportional to the current in the windings. We can increase the strength of the field created by using the higher current-carrying capabilities of superconductors. Thus, we can generate the same field strength with a *smaller* superconducting electromagnet as compared to the standard copper-wound equivalent. This is important in electric generator design; it is possible to scale down the size of the generator while still producing the same power. A power plant could therefore be shrunk in size, becoming more efficient (Orlando, 1991).

Consider the loop of metallic wire shown in Figure P 8-21a. The loop is initially placed in a static magnetic field such that flux threads the hole. After the system reaches the steady state, the field is suddenly turned off (at time  $t = 0$ ). A current  $i(t)$  will therefore be induced in the wire. One model of the loop is the circuit shown in Figure P 8-21b with  $i(0) = 1$  A and  $L = 0.1$  mH. Since the loop is a superconducting wire, we can estimate  $R = 10^{-10}$  Ω. Determine the percentage reduction in the current  $i(t)$  after one day.

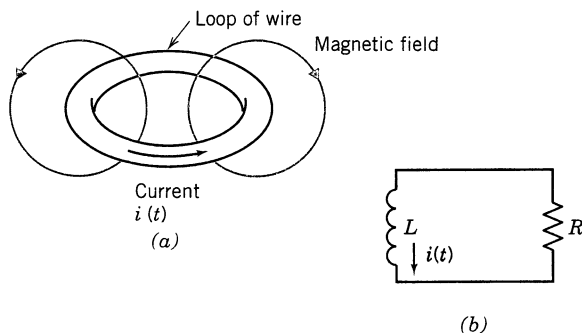


Figure P 8-21 A superconducting loop of wire.

**P 8-22** Determine  $i(t)$  for the circuit shown in Figure P 8-22. Find the initial rate of change of current and plot  $i(t)$  for four time constants.

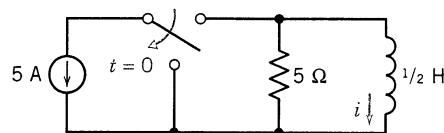


Figure P 8-22

**P 8-23** The switch of the circuit of Figure P 8-23a is in position 1 for a long time and is moved to position 2 at  $t = 0$ . The current  $i(t)$  is measured and plotted as shown in Figure P 8-23b. Determine the value of  $C$ .

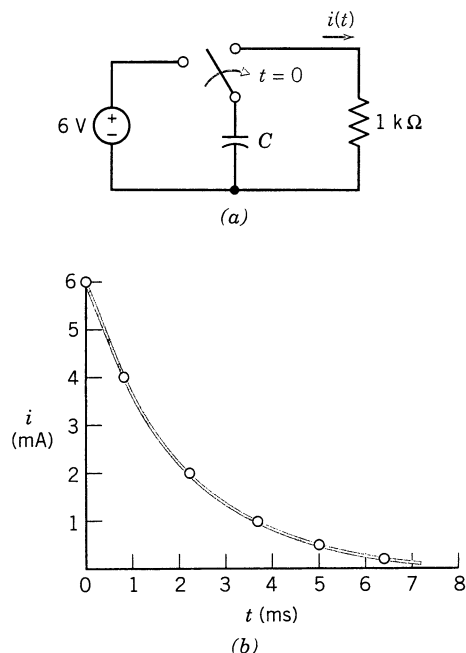


Figure P 8-23

**P 8-24** For the circuit shown in Figure P 8-24, find the voltage  $v(t)$  and the current  $i_L(t)$  for  $t > 0$ . Assume that the circuit is in steady state at  $t = 0^-$ .

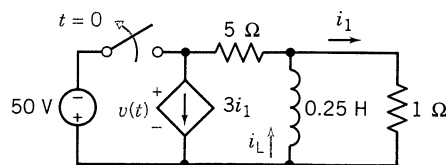


Figure P 8-24

**P 8-25** For the circuit of Figure P 8-25, determine  $v_c(t)$  and  $v_x(t)$  for  $t > 0$  when  $C = 0.2$  F. Assume that the circuit is in steady state when  $t = 0^-$ .

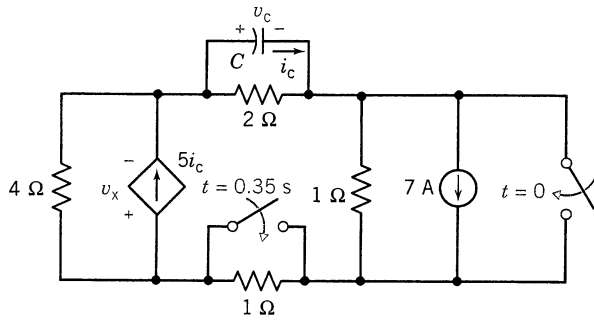


Figure P 8-25

## ADVANCED PROBLEMS

**AP 8-1** The switch in the circuit of Figure AP 8-1 is opened at  $t = 0$  after having been closed for a long time. Determine and plot  $i(t)$  for  $0 \leq t \leq 0.5$  s.

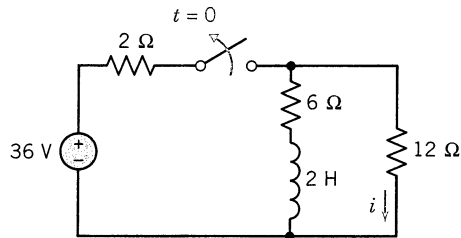


Figure AP 8-1

**AP 8-2** The switch shown in Figure AP 8-2 is opened at  $t = 0$  after having been closed for a long time. Determine  $v_o(t)$  and plot  $v_o(t)$  for  $0 \leq t \leq 3$  s.

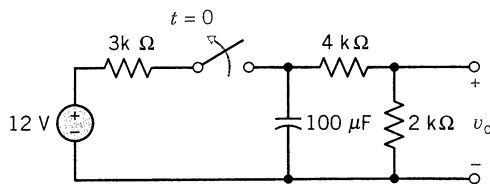


Figure AP 8-2

**AP 8-3** Inductance is measured by applying a ramp current  $i_s = At$ ,  $t \geq 0$ , as shown in Figure AP 8-3 (Quint, 1990). Determine  $v_o(t)$  and discuss how  $R_x$  and  $L_x$  can be estimated, when an inductor is represented by  $R_x$  and  $L_x$  in series.

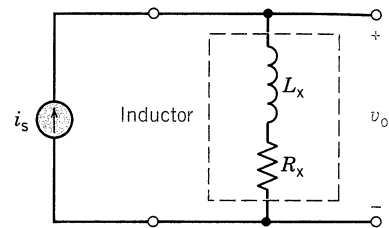


Figure AP 8-3 Circuit for measuring inductance.

**AP 8-4** The most important safety feature of new automobiles is the airbag. The airbag deploys when a pendulum sensor detects a sudden deceleration of greater than  $10g$  and closes a switch. An equivalent circuit of the airbag deployment device is shown in Figure AP 8-4. Determine the time required before the energy absorbed by the resistor reaches 25 mJ and triggers the explosive deployment of the bag. The capacitor is precharged to 24 V,  $R = 100 \Omega$ , and  $C = 0.1$  mF.

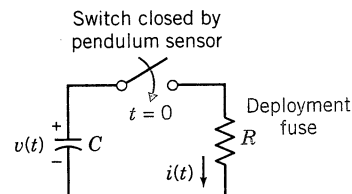


Figure AP 8-4 Airbag deployment circuit.

**AP 8-5** Determine the current  $i(t)$  and the voltage  $v(t)$  for the circuit shown in Figure AP 8-5 when  $L = 0.2$  H and  $C = 0.2$  F. Sketch the current and voltage versus time. Assume that the circuit is in steady state at  $t = 0^-$ .

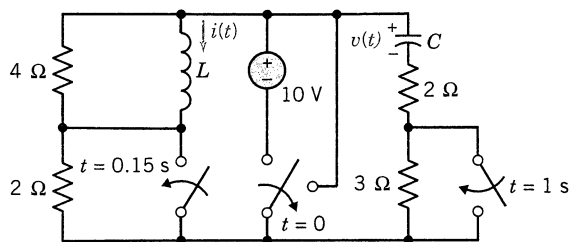


Figure AP 8-5

**AP 8-6** A home handyman decides to fix his own TV set, despite the warning on the back. He switches it off, pulls the plug, removes the back cover, reaches inside, and receives a large electrical shock. The dc source that drives the set's picture tube is 20 kV. The capacitance and resistance that form its load are 1 microfarad and 1 megaohms, as shown in Figure AP 8-6. Why did he get a shock, and what was the voltage? If his body resistance was 2 megaohms, how much energy did his body absorb if he held on for 8 seconds?

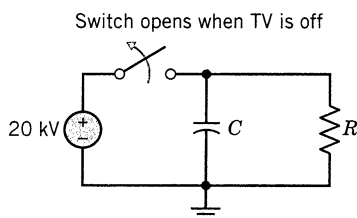


Figure AP 8-6 TV set picture tube circuit.

**AP 8-7** An electronic flash on a camera uses the circuit shown in Figure AP 8-7. Harold E. Edgerton invented the electronic flash in 1930. A capacitor builds a steady-state voltage and then discharges it as the shutter switch is pressed. The discharge produces a very brief light discharge. Determine the elapsed time  $t_1$  to reduce the capacitor voltage to one-half of its initial voltage. Find the current at  $t = t_1$ .

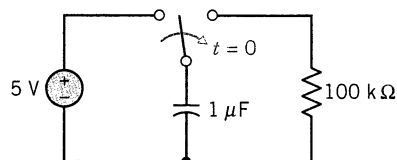


Figure AP 8-7 Electronic flash circuit.

## DESIGN PROBLEMS

**DP 8-1** The response of the circuit shown in Figure DP 8-1 is represented by the capacitor voltage  $v(t)$ . It is specified that the voltage  $v(t)$  decays to less than 1 percent of its initial value,  $v(0)$ , within 600 ms (5 time constants). Determine the required capacitance  $C$  to achieve the specified response. Assume that the switch has been in the initial position for a long time.

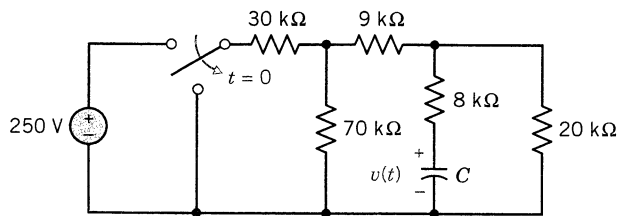


Figure DP 8-1

**DP 8-2** The switch in Figure DP 8-2 has been closed for a long time and is opened at  $t = 0$ . Determine the inductance  $L$  required for  $i(t) = 1.14$  A at  $t = 0.1$  s.

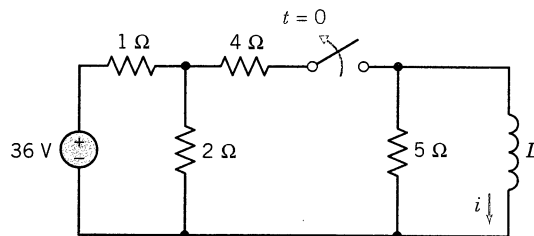


Figure DP 8-2

**DP 8-3** In order to obtain energy conservation in offices, a designer has developed a sensor that turns on the lights after determining that a person has entered a room. However, if the person does not push the continue button, the lights will again switch off in a predetermined time equal to 120 seconds. Design a pre-charged  $RC$  circuit that will hold the lights on for 120 seconds and then turn them off again unless a continue switch is pressed.

**DP 8-4** An electronically controlled target pistol has been designed (Scott, 1991). The firing pin is tripped by a battery-powered solenoid. The control circuit is shown in Figure DP 8-4. Select the inductance  $L$  so that the solenoid voltage exceeds 20 V and the magnitude of the inductor current exceeds 0.5 A during the first 25 ms following  $t = 0$ . Assume that the circuit is in steady state at  $t = 0^-$ . The trigger is pulled at  $t = 0$ .

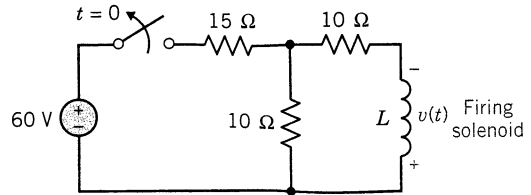


Figure DP 8-4 Target pistol trigger circuit.

**DP 8-5** For the summary design example on page 301, find the maximum value of  $L_2$  at which the current  $i_2$  will change from positive to negative at exactly 0.4 second.



## PSpice PROBLEMS

- SP 8-1** Determine and plot  $i(t)$  for Problem P 8-2.
- SP 8-2** Determine and plot  $i_L(t)$  for Problem P 8-5.
- SP 8-3** Determine and plot  $i(t)$  for Problem P 8-9.
- SP 8-4** Find  $v_c(t)$  and  $i_L(t)$  for Problem P 8-12.
- SP 8-5** Find  $v_o(t)$  for Problem AP 7-2 for  $0 \leq t \leq 3$  s.

**SP 8-6** An  $RC$  circuit is shown in Figure SP 8-6 with  $R = 1 \text{ k}\Omega$  and  $C = 20 \mu\text{F}$ . Determine the time,  $t_1$ , when  $v(t_1) = 4.00 \text{ V}$ , given  $v(0^-) = 12 \text{ V}$ .

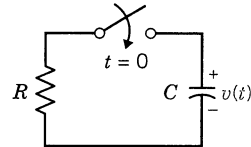


Figure SP 8-6

## TERMS AND CONCEPTS

**Electric Signal** Voltage or current varying in time in a manner that conveys information.

**Forced Response** Response of a circuit to an independent source.

**Integrating Factor Method** Method for obtaining the solution of a differential equation by multiplying the equation by an exponential factor that makes one side of the equation a perfect derivative and then integrating both sides of the equation.

**Pulse** Function of time that is zero for  $t < t_0$ , has magnitude  $M$  for  $t_0 < t < t_1$ , and is equal to zero for  $t > t_1$ .

**Signal** Real-valued function of time; waveform that conveys information.

**Sinusoidal Signal** A waveform that varies in accordance with a sine or cosine function of time.

**Step Response** Response of a circuit to the sudden application of a constant source when all the initial conditions of the circuit are equal to zero.

**Step Voltage Source** Voltage source,  $v$ , represented by  $v = Vu(t - t_0)$ .

**Unit Step Forcing Function** Function of time that is zero for  $t < t_0$  and unity for  $t > t_0$ . At  $t = t_0$  the magnitude changes from zero to one. The unit step is dimensionless.

## REFERENCES Chapter 9

Gould, Larry A. "Electronic Valve Timing," *Automotive Engineering*, April 1991, pp. 19–24.

Jurgen, Ronald "Electronic Handgun Trigger Proposed," *IEEE Institute*, February 1989, p. 5.

Pierce, John R., and A.M. Noll, "Signals: The Science of Telecommunications," *Scientific American Library*, W.H. Freeman, San Francisco, 1990.

Wright, A. "Construction and Application of Electric Fuses," *Power Engineering Journal*, Vol. 4, No. 3, 1990, pp. 141–148.

## PROBLEMS

**P 9-1** Using the superposition principle, find the current  $i$  of the circuit of Figure P 9-1.

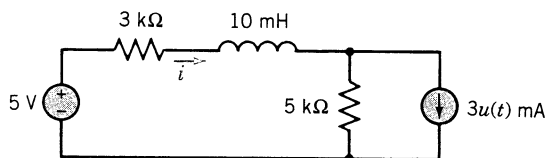


Figure P 9-1

**P 9-2** Find  $i(t)$  for  $t > 0$  for the circuit of Figure P 9-2. Assume the circuit is in steady state at  $t = 0^-$ .

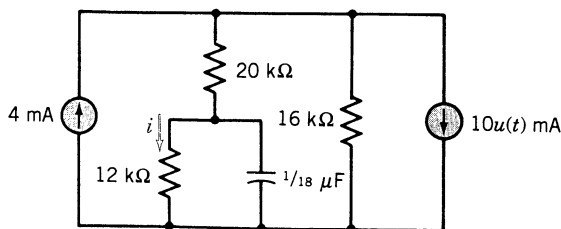


Figure P 9-2

**P 9-3** Find  $i(t)$  for  $t > 0$  for the circuit shown in Figure P 9-3. Assume that the circuit is in steady state at  $t = 0^-$ .

**Answer:**  $i(t) = -\frac{1}{4}e^{-25t} + 2 \text{ A}$

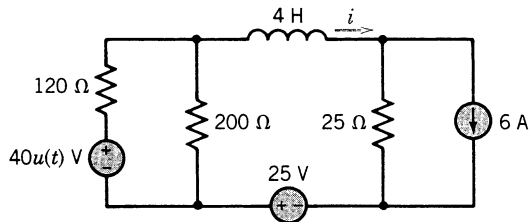


Figure P 9-3

**P 9-4** Find  $v$  and  $i$  for  $t > 0$  if the circuit in Figure P 9-4 is in steady state at  $t = 0^-$ . All resistances are in ohms.

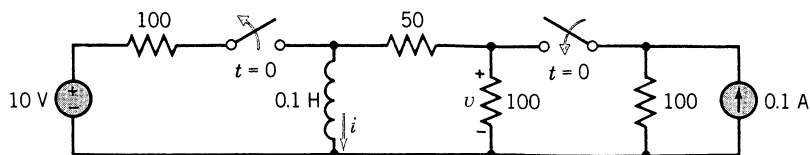


Figure P 9-4

**P 9-5** The uncharged capacitor of the circuit shown in Figure P 9-5 is switched from position a to position b at  $t = 0$  and remains there for 200 ms before being switched to position c, where it remains indefinitely. Find  $v$  and plot  $v$  for  $0 < t < 500$  ms.

**Answer:**  $v = \begin{cases} 200(1 - e^{-10t}), & 0 < t < 200 \text{ ms} \\ 173e^{-20(t-0.2)}, & t > 200 \text{ ms} \end{cases}$

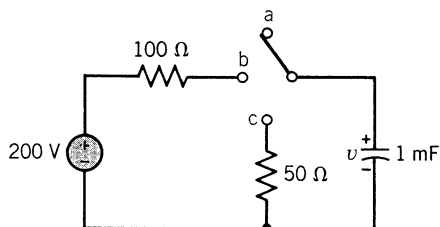


Figure P 9-5

**P 9-6** Find  $i(t)$  for  $t > 0$  if  $v_c(0) = 12$  V for the circuit of Figure P 9-6.

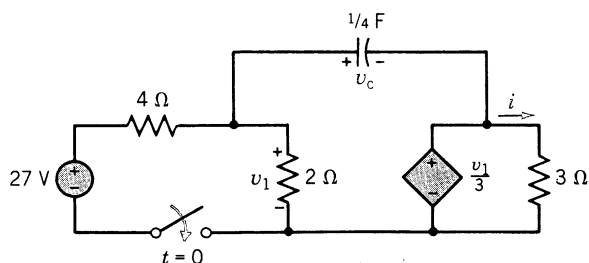


Figure P 9-6

**P 9-7** Find  $v_c(t)$  for  $t > 0$  for the circuit shown in Figure P 9-7 when  $v_1 = 8e^{-5t}u(t)$  V. Assume the circuit is in steady state at  $t = 0^-$ .

**Answer:**  $v_c(t) = 4e^{-9t} + 18e^{-5t}$  V

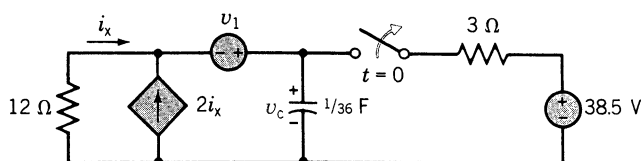


Figure P 9-7

**P 9-8** Find  $v(t)$  for  $t > 0$  for the circuit shown in Figure P 9-8 when  $v_1 = (25 \sin 4000t)u(t)$  V. Assume steady state at  $t = 0^-$ .

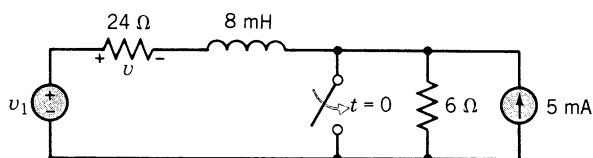


Figure P 9-8

**P 9-9** Find  $i(t)$  for  $t > 0$  for the circuit shown in Figure P 9-9. Assume the circuit reached steady state at  $t = 0^-$ .

**Answer:**  $i(t) = \frac{3}{2}e^{-6000t} + \frac{1}{2}$  mA

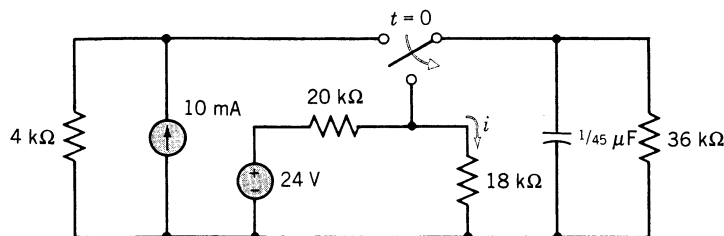


Figure P 9-9



P 9-10 For the circuit shown in Figure P 9-10, find  $v(t)$  for  $t > 0$ . Assume steady state at  $t = 0^-$ .

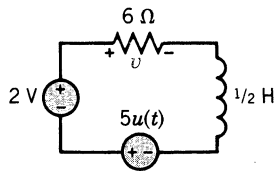


Figure P 9-10

P 9-11 For the circuit of Figure P 9-11, find  $v_c(t)$  for  $t > 0$ . Note that the current source on the left is  $3u(-t)$  A and will be zero for  $t > 0$ . Assume steady state at  $t = 0^-$ .

**Answer:**  $v_c(t) = 18e^{-2t} + 6$  V

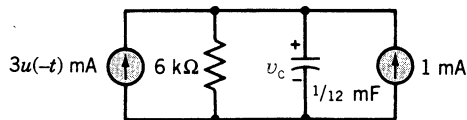


Figure P 9-11

P 9-12 Find  $v(t)$  for  $t > 0$  for the circuit shown in Figure P 9-12 when  $v_s = 15e^{-t}[u(t) - u(t-1)]$  V.

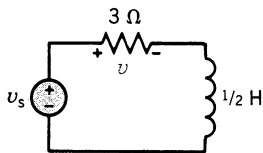


Figure P 9-12

P 9-14 Find  $v_c(t)$  for  $t > 0$  for the circuit shown in Figure P 9-14 when  $i_s = [2 \cos 2t]u(t)$  mA.

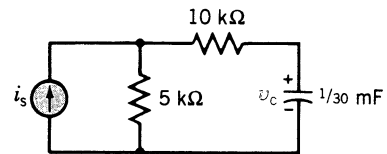


Figure P 9-14

P 9-15 Find  $v_1(t)$ ,  $v_2(t)$ , and  $i(t)$  for  $t > 0$  for the circuit of Figure P 9-15.

**Answer:**  $v_1(t) = -2e^{-5t} + 2$  V

$v_2(t) = -6e^{-4t} + 6$  V

$i(t) = \frac{1}{3}(2te^{-5t})$  A

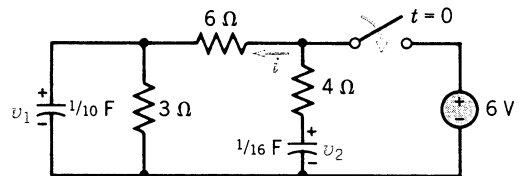


Figure P 9-15

P 9-16 Find  $i(t)$  for  $t > 0$  for the circuit shown in Figure P 9-16. Assume steady-state conditions at  $t = 0^-$ .

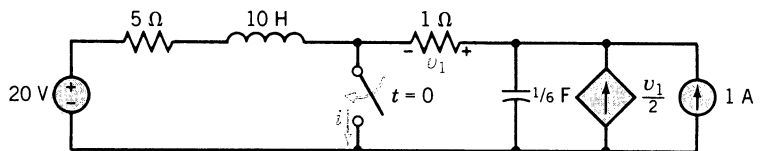


Figure P 9-16

P 9-13 Find  $v(t)$  for  $t > 0$  for the circuit shown in Figure P 9-13. Assume steady state at  $t = 0^-$ .

**Answer:**  $v(t) = 20e^{-10t/3} - 12e^{-2t}$  V

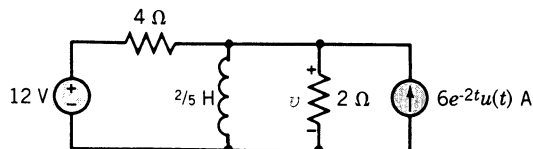


Figure P 9-13

P 9-17 Find  $v_c(t)$  for  $t > 0$  for the circuit shown in Figure P 9-17. Assume steady state at  $t = 0^-$ .

**Answer:**  $v_c(t) = -3e^{-4t} - 9$  V

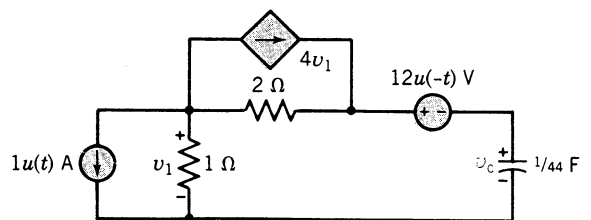


Figure P 9-17

**P 9-18** For the circuit of Figure P 9-18 find  $i(t)$  for  $t > 0$ . Assume steady state at  $t = 0^-$ .

**Answer:**  $i(t) = 3e^{-5000t} - e^{-2000t} - 6 \text{ mA}$

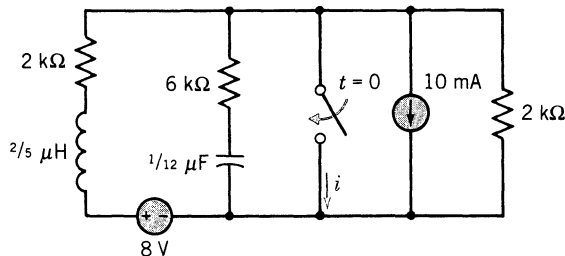


Figure P 9-18

**P 9-19** Many have witnessed the use of an electrical megaphone for amplification of speech to a crowd. A model of a microphone and speaker is shown in Figure P 9-19a, and the circuit model is shown in Figure P 9-19b. Find  $v(t)$  for  $v_s = 10(\sin 100t)u(t)$ , which could represent a person whistling or singing a pure tone.

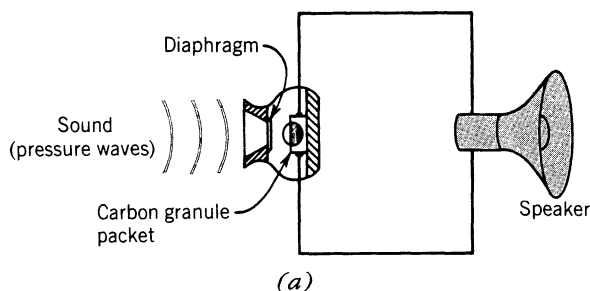


Figure P 9-19 Megaphone circuit.

**P 9-20** For the circuit shown in Figure P 9-20, (a) find  $i(t)$  for  $t > 0$  when  $i(0) = I_0$ ; (b) for  $\alpha > 0$ , draw the equivalent circuit with only passive elements; (c) for  $\alpha < 0$ , determine whether it is possible to obtain an equivalent passive circuit and state your reasons.

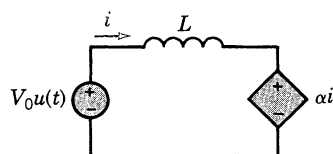


Figure P 9-20

**P 9-21** Find the step response  $v_c(t)$  of the circuit shown in Figure P 9-21 when  $v_s = 20u(t) \text{ V}$ . The initial voltage  $v_c(0)$  is zero.

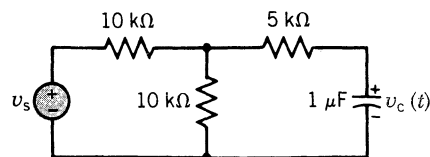


Figure P 9-21

**\*P 9-22** An electronic flash of a camera uses a small battery to charge a capacitor. Then, when the flash is activated, the capacitor is switched across the flashbulb. Assume that the battery is a 6-V battery that should not be operated with a current above  $100 \mu\text{A}$ . The capacitor is to be selected. (a) Draw a circuit model that will represent the charging and discharging action. (b) It is desired to charge the capacitor within 5 seconds and to discharge it within  $1/2$  second. Select the appropriate values for the elements in the circuit. Assume the value of the bulb resistance is  $10 \text{ k}\Omega$ . Assume that the capacitor is charged or discharged in five time constants.

**\*P 9-23** Consider the circuit shown in Figure P 9-23. The switch is moved from A to B at  $t = 0$  after being at A for a long time. Let  $v_2(0^-) = 0 \text{ V}$ . Find  $v_1(0^-)$ ,  $v_1(0)$ ,  $v_2(0)$ ,  $v_R(0)$ ,  $v_R(t)$ ,  $i(t)$ ,  $v_1(t)$ , and  $v_2(t)$  for  $t > 0$ .

**Answer:**  $v_1(0^-) = 40 \text{ V}$   
 $v_1(0) = 40 \text{ V}$   
 $v_2(0) = 0 \text{ V}$   
 $v_R(0) = 40 \text{ V}$   
 $v_R(t) = 40e^{-18t} \text{ V}$   
 $i(t) = 8e^{-18t} \text{ A}$   
 $v_1(t) = \frac{80}{3} + \frac{40}{3}e^{-18t} \text{ V}$   
 $v_2(t) = \frac{80}{3} - \frac{80}{3}e^{-18t} \text{ V}$

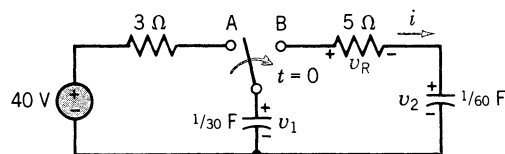


Figure P 9-23

**\*P 9-24** A neon bulb flashes on and off depending on the current passing through it. The neon bulb shown in the circuit of Figure P 9-24 has the following behavior. The bulb remains off and acts as an open circuit until the bulb voltage  $v$  reaches a threshold value of  $V_T =$

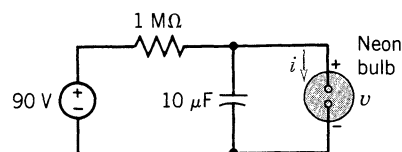


Figure P 9-24 Neon bulb flasher circuit.

\_\_\_\_\_ \* Note: An asterisk denotes a challenging problem.

65 V. Once  $v$  reaches  $V_T$ , a discharge occurs and the bulb acts like a simple resistor of value  $R_N = 1 \text{ k}\Omega$ ; the discharge is maintained as long as the bulb current  $i$  remains above the value  $I_s = 10 \text{ mA}$  needed to sustain the discharge (even if the voltage  $v$  drops below  $V_T$ ). As soon as  $i$  drops below 10 mA, the bulb again becomes an open circuit.

- Find  $v(t)$  and  $i(t)$  for one period of operation.
- Estimate the flashing rate for the bulb.

**P 9-25** Determine and sketch  $i(t)$  for the circuit shown in Figure P 9-25. Calculate the time required for  $i(t)$  to reach 99 percent of its final value.

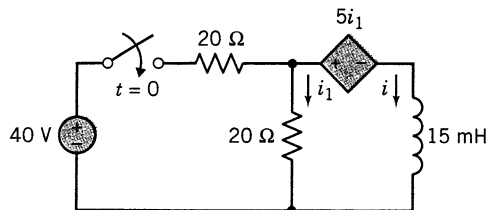


Figure P 9-25

**P 9-26** A voltage signal is the input of the circuit of Figure P 9-26 and it is desired to determine  $v(t)$ . The pulse is

$$v_s(t) = 4 - 4u(t)$$

Find  $v(t)$ , assuming an ideal op amp.

**Answer:**  $v = -10 + 10(1 - e^{-4t})u(t) \text{ V}$

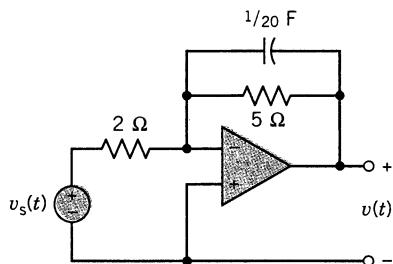


Figure P 9-26

**P 9-27** A photoflash for a camera has a circuit shown in Figure P 9-27. The photoflash tube will conduct when the voltage across it reaches 240 V and the conduction continues until the voltage across it drops to 30 V. Assume that the resistance of the tube is negligibly small when it is conducting and infinite when it is

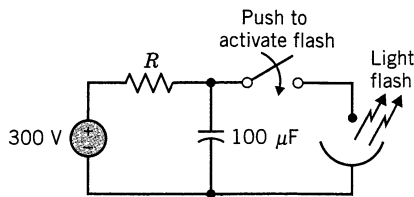


Figure P 9-27 Photoflash circuit.

not. Select a value of  $R$  so that the minimum time between flashes will be 10 seconds. For the same value of  $R$ , what percentage increase in energy delivered to the tube is achieved by waiting 20 seconds between flashes instead of 10 seconds?

**P 9-28** Determine  $v(t)$  and  $i(t)$  for the circuit shown in Figure P 9-28. Assume a steady-state condition at  $t = 0^-$ ,  $v(0^-) = 2.14 \text{ V}$ , and let  $C = 2 \text{ mF}$ .

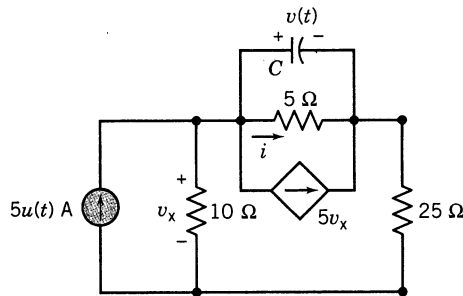


Figure P 9-28

**P 9-29** An electronic circuit can be used to replace the springs and levers normally used to detonate a shell in a handgun (Jurgen, 1989). The electric trigger would eliminate the clicking sensation, which may cause a person to misaim. The proposed trigger uses a magnet and a solenoid with a trigger switch. The circuit of Figure P 9-29 represents the trigger circuit with  $i_s(t) = 40[u(t) - u(t - t_0)] \text{ A}$  where  $t_0 = 1 \text{ ms}$ . Determine and plot  $v(t)$  for  $0 < t < 0.3 \text{ s}$ .

**Answer:**

$$v = \begin{cases} 480(1 - e^{-1000t}) & 0 < t < 1 \text{ ms} \\ 480(1 - e^{-1})e^{-(t-t_0)} & t > 1 \text{ ms}, \quad t_0 = 1 \text{ ms} \end{cases}$$

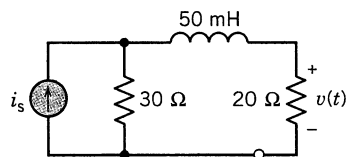


Figure P 9-29 Electric trigger circuit for handgun.

**P 9-30** Determine  $v(t)$  for the circuit shown in Figure P 9-30.

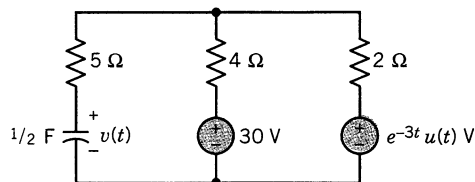


Figure P 9-30

**P 9-31** Determine and sketch  $v(t)$  for the circuit shown in Figure P 9-31 when  $v_s = -30 + 60u(t) \text{ V}$  and  $i_s = 6 - 3u(t) \text{ A}$ .

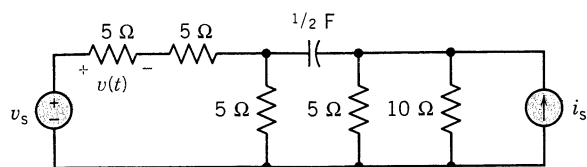


Figure P 9-31

**P 9-32** Determine and sketch  $v(t)$  for the circuit shown in Figure P 9-32 when  $v_s = 4[u(t) - u(t - 1.8)]$  V. Assume that the circuit is in steady state at  $t = 0^-$ .

**Answer:**

$$v = \begin{cases} 4 - \frac{20}{9} e^{-t/0.45} \text{ V} & 0 \leq t < 1.8 \text{ s} \\ 2.18 e^{-(t-1.8)/0.45} \text{ V} & t \geq 1.8 \text{ s} \end{cases}$$

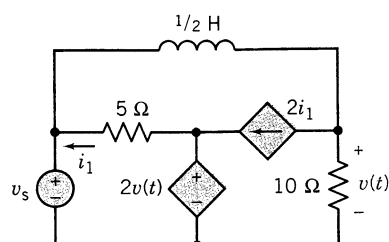


Figure P 9-32

**P 9-33** A car battery is connected to an ignition coil as shown in Figure P 9-33. A short circuit as represented by the switch occurs at  $t = 0$ . Determine  $i(t)$  for  $t > 0$ .

**Answer:**  $i = 3 - 1e^{-8t}$  A,  $t \geq 0$

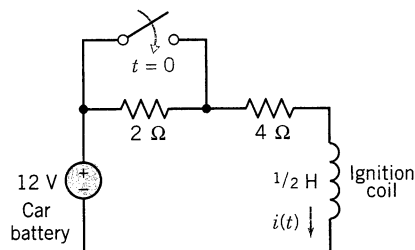


Figure P 9-33 Automobile ignition circuit.

**P 9-34** A capacitor is connected to a voltage source at position 1 as shown in Figure P 9-34. The switch is

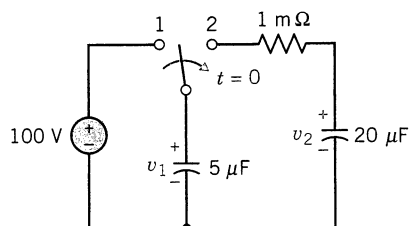


Figure P 9-34

moved instantaneously to position 2 at  $t = 0$ . Determine the voltage appearing across each capacitor,  $v_1(t)$  and  $v_2(t)$ , for  $t > 0$ .

**P 9-35** The initial voltage of the capacitor of the circuit shown in Figure P 9-35 is zero. Determine the voltage  $v(t)$  when the source is a pulse, so that

$$v_s = \begin{cases} 0 & t < 1 \text{ s} \\ 4 \text{ V} & 1 < t < 2 \text{ s} \\ 0 & t > 2 \text{ s} \end{cases}$$

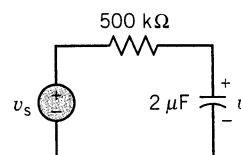


Figure P 9-35

**P 9-36** Determine  $v(t)$  for  $t > 0$  for the circuit shown in Figure P 9-36. Assume that the circuit is in steady state at  $t = 0^-$ .

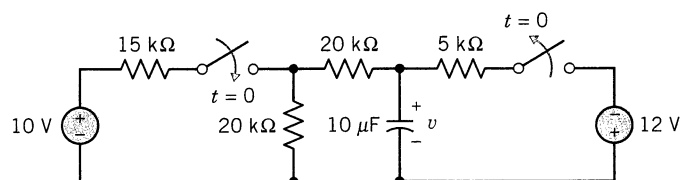


Figure P 9-36

**P 9-37** A lossy integrator is shown in Figure P 9-37. It can be seen that the lossless capacitor of the ideal integrator circuit has been replaced with a model for the lossy capacitor, namely, a lossless capacitor in parallel with a  $1\text{-}\Omega$  resistor. If  $v_s = 15e^{-2t}u(t)$  volts and  $v_o(0) = 10$  V, find  $v_o(t)$  for  $t > 0$ . Assume  $A > 10^5$ .

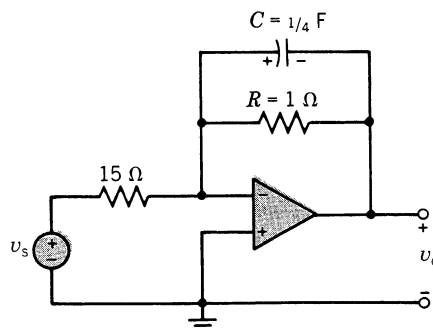


Figure P 9-37 AU resistances in k ohm.

**P 9-38** (a) In the circuit of Figure P 9-38, given  $v_{c1}(0) = 10$  V,  $v_{c2}(0) = 20$  V,  $C_1 = 4$  F, find  $v_o(t)$  in terms of  $v_1(t)$  and  $v_2(t)$  for  $t > 0$ . (b) If  $v_1(t) = 10e^{-2t}$  V and  $v_2(t) = 20e^{-t}$  V, find  $v_o(t)$  for  $t > 0$ .

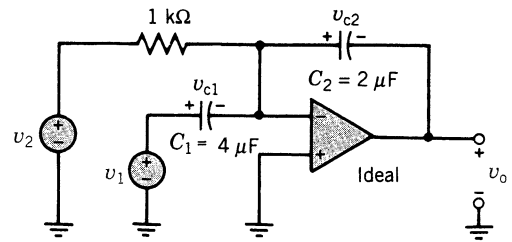


Figure P9-38

## ADVANCED PROBLEMS

**AP 9-1** Switch 1 has been open and switch 2 has been closed for a long time at  $t = 0^-$  in the circuit of Figure AP 9-1. At  $t = 0$ , switch 1 is closed and then switch 2 is opened at  $t = 3$  s. Determine  $i(t)$  and plot  $i(t)$  for  $0 \leq t \leq 8$  seconds.

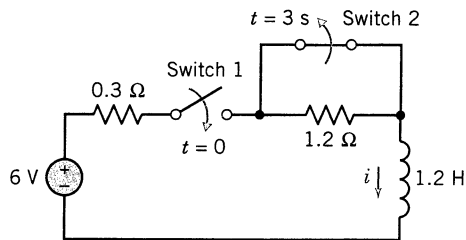


Figure AP 9-1

**AP 9-2** The switch of the circuit of Figure AP 9-2 has been connected to terminal 1 for a long time and is switched to terminal 2 at  $t = 0$  for a duration of 30 s. At  $t = 30$  s, the switch is returned to terminal 1. Determine the voltage  $v(t)$  and plot  $v(t)$  for  $0 \leq t \leq 80$  s.

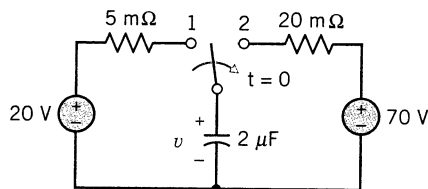
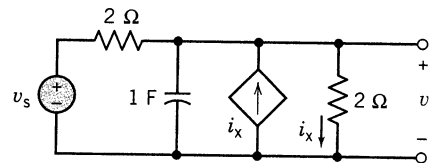
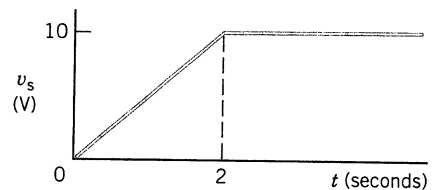


Figure AP 9-2

**AP 9-3** Determine  $v(t)$  for the circuit shown in Figure AP 9-3a when  $v_s$  varies as shown in Figure AP 9-3b. The initial capacitor voltage is  $v_c(0) = 0$ .



(a)



(b)

Figure AP 9-3

**AP 9-4** The electron beam, which is used to “draw” signals on an oscilloscope, is moved across the face of a cathode-ray tube (CRT) by a force exerted on electrons in the beam. The basic system is shown in Figure AP 9-4a. The force is created from a time-varying, ramp-type voltage applied across the vertical or the horizontal plates. As an example, consider the simple circuit of Figure AP 9-4b for horizontal deflection where the capacitance between the plates is  $C$ .

- Derive an expression for the voltage across the capacitance. If  $v(t) = kt$  and  $R_s = 625$  kΩ,  $k = 1000$ , and  $C = 2000$  pF, compute  $v_c$  as a function of time. Sketch  $v(t)$  and  $v_c(t)$  on the same graph for time less than 10 milliseconds. Does the voltage across the plates track the input voltage?
- Describe the deflection of the electron beam if the force is  $F = qE$ , where  $q$  is the charge and  $E$  is the electric field;  $E$  is the ratio of the voltage across the plates to the spacing of the plates ( $E = v/S$ ). Assume a zero initial condition for the capacitor.

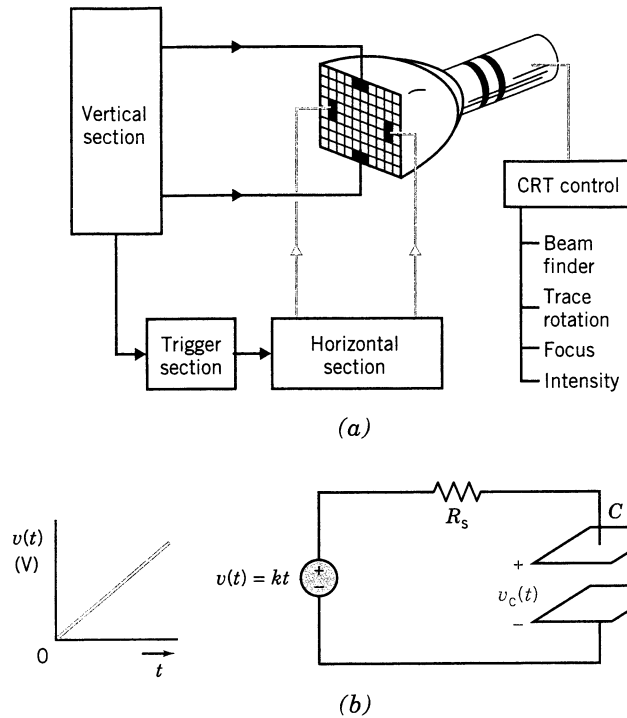


Figure AP 9-4 Cathode-ray tube beam circuit.

**AP 9-5** An experimental electronic valve timing system for an automobile engine uses pneumatic (compressed air) actuators to lift the valves, actuated by electromagnetically operated pneumatic valves (Gould, 1991). The engine valve must open within 2 ms after being commanded to open by the engine-control computer.

The pneumatic valve will begin to open when the current in the valve solenoid coil exceeds 300 mA, its guaranteed “on” current. The valve will begin to close when the solenoid coil current drops below 225 mA, its guaranteed “off” current. To ensure that the engine valve opens on time, only 0.1 ms has been allowed for the current in the solenoid to reach its on current after the applied voltage changes from 0 to 12 V, and 0.2 ms is allowed for the current to decay below its off current after the supply voltage is removed and the coil is shorted (the applied voltage changes from 12 to 0 V). Either response may be faster. The solenoid coil can be modeled as an inductance of 3 mH with a series resistance that is determined by the diameter (gauge) of wire chosen for the coil. Smaller-gauge wire will cause higher resistance, lower power dissipation, and allow a smaller solenoid package. What is the maximum value of solenoid resistance that will meet both timing specifications? With this resistance, what is the maxi-

imum current in the solenoid coil? Assume that the solenoid is to be energized 25 percent of the time. What is the *approximate* average power dissipation in this solenoid coil? What simplifying assumptions did you make?

In practice, a solenoid is not directly shorted to turn it off, but is shorted through a device called a diode with a voltage drop of 0.7 V, as this is easier to implement. Why would you not simply open-circuit the coil to achieve zero current?

**AP 9-6** Human cell membranes resemble capacitors with a capacitance of  $1 \mu\text{F}$ . The pulse stimulus  $i_s(t)$  has a magnitude of  $I_m$  and a duration  $d$ , so that

$$i_s = I_m \quad 0 < t < d \\ = 0 \quad \text{elsewhere}$$

where  $I_m = 1 \text{ mA}$  and  $d = 5 \text{ ms}$ . Determine and plot  $v(t)$ . The resistance  $R$  is equal to  $1 \text{ k}\Omega$  for the heart membrane, as shown in Figure AP 9-6.

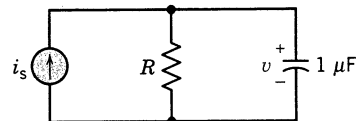


Figure AP 9-6 Model for cell membrane circuit.

**AP 9-7** Determine  $v(t)$  and  $i(t)$  for the circuit shown in Figure AP 9-7. Assume that the switch has been connected to position 1 for a long time before it is moved to position 2 at  $t = 0$ . Sketch  $i(t)$  and  $v(t)$  for  $-1 < t < 6$  seconds.

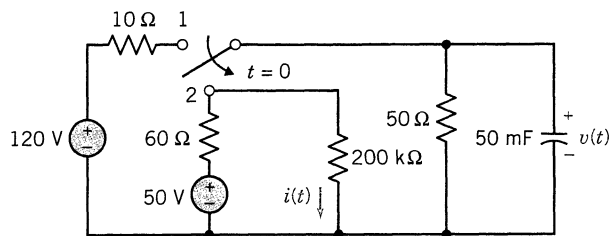


Figure AP 9-7

## DESIGN PROBLEMS

**DP 9-1** For the circuit shown in Figure DP 9-1, it is desired that  $i(t) = 2.5$  A at  $t = 47$  ms. Determine the resistance  $R$  that meets this specification when  $v_s = 300$  V. The initial voltage on the capacitor is  $v_c(0) = 100$  V.

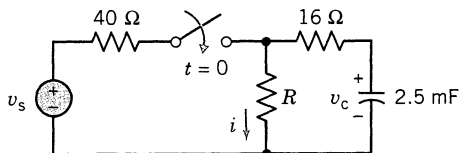


Figure DP 9-1

**DP 9-2** For the circuit shown in Figure DP 9-2, specify the inductance  $L$  so that the current in the inductor reaches its steady-state value (five time constants) in  $3.1 \mu\text{s}$  after the switch is closed at  $t = 0$ . Determine the energy stored in the inductor after  $t > 3.1 \mu\text{s}$ . Assume that the circuit was unexcited prior to  $t = 0$ .

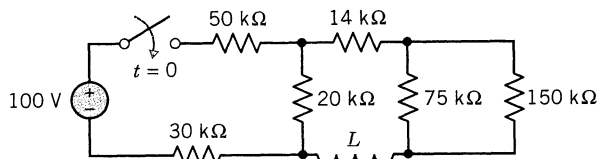


Figure DP 9-2

**DP 9-3** For the circuit shown in Figure DP 9-3, select the inductance  $L$  so that the inductor current  $i(t) = 3$  A at  $t = 14$  ms. What is  $i(t)$  when  $t = 1$  s?

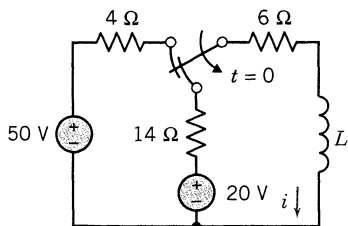


Figure DP 9-3

**DP 9-4** Solenoids are electromechanical components that convert electrical energy into mechanical work and move a plunger. Current passing through a helical coil winding of closely spaced turns of copper magnet wire produces a magnetic field that surrounds the coil. All solenoids develop magnetizing force, which has a relationship to the current and number of turns in the coil. Solenoid size determines the amount of work a solenoid can perform. A large unit will develop more force at a given stroke than a small solenoid (with the same coil current) because its greater physical volume will accommodate more turns of wire on the coil. It is important that a solenoid selected for a particular application have a rated force as close to the load requirements as possible. Too much force reduces solenoid life because the unit must absorb the excess energy. If the force is too low, the result will be unsatisfactory performance because the plunger will not pull in or seat properly.

The circuit model of a simple solenoid is shown in Figure DP 9-4. The source is a 12-V dc battery with a resistance  $R_s = 1 \Omega$ . Select the required  $R$  and  $L$  of the solenoid and  $v_s = V_0 u(t)$  so that the solenoid will close in 100 ms while restricting the force to less than 0.5 N. The force-developed relationship is measured as  $f = i(t)$  N.

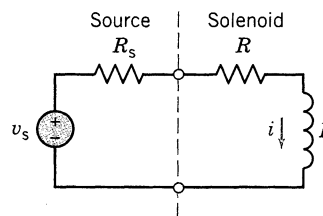


Figure DP 9-4 Model of solenoid circuit.

**DP 9-5** The switch in the circuit shown in Figure DP 9-5 is at position 1 for a long time and is switched to position 2 at  $t = 0$ . The output voltage,  $v(t)$ , is to be changed from the initial voltage  $v(0)$  to within 1 per-

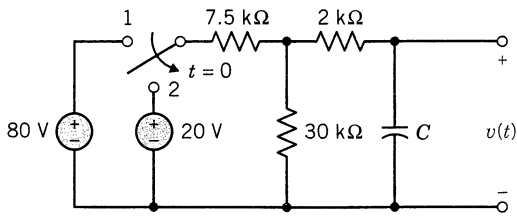


Figure DP 9-5

cent of its final value within 0.2 second. Determine the required  $C$  and sketch  $v(t)$  for  $t \geq 0$ .

**DP 9-6** A laser trigger circuit is shown in Figure DP 9-6. In order to trigger the laser, we require  $60 \text{ mA} < |i| < 180 \text{ mA}$  for  $0 < t < 200 \mu\text{s}$ . Determine a suitable value for  $R_1$  and  $R_2$ .

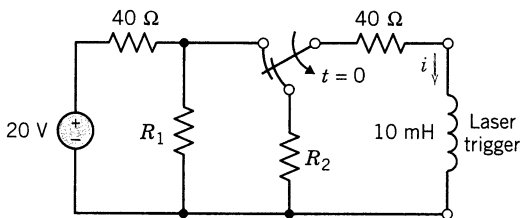


Figure DP 9-6 Laser trigger circuit.

**DP 9-7** The initial voltage of the capacitor in the circuit of Figure DP 9-7 is  $-10 \text{ V}$ . Select  $C$  so that  $v(t) = 0$  at  $t = 4.0 \text{ s}$  when  $v_s = 20 \text{ V}$ . Sketch  $v(t)$  for  $0 < t < 5 \text{ s}$ .

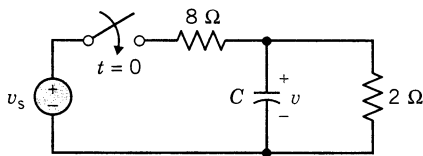


Figure DP 9-7

**DP 9-8** Fuses are used to open a circuit when excessive current flows (Wright, 1990). One fuse is designed to open when the power absorbed by  $R$  exceeds  $10 \text{ W}$  for  $0.5 \text{ second}$ . The source represents the turn-on condition for the load where  $v_s = A[u(t) - u(t - 0.75)] \text{ V}$ . Assume that  $i_L(0^-) = 0$ . The goal is to achieve the maximum current while not opening the fuse. Determine an appropriate value of  $A$  and sketch the current waveform. The circuit is shown in Figure DP 9-8.

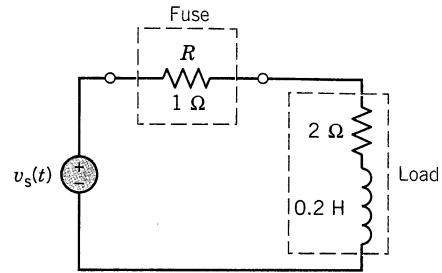


Figure DP 9-8 Fuse circuit.

**DP 9-9** Repeat Design Problem 9-7 when  $v_s(t) = 10e^{-t/2}u(t) \text{ V}$ .

**DP 9-10** An  $RL$  circuit as shown in Figure DP 9-10 is used to provide an actuating pulse for a power laser. The circuit is at steady state at  $t = 0^-$  and  $i(0^-) = 0$ . The voltage source  $v_s = V_0e^{-bt} \text{ V}$  is connected at  $t = 0$ . Select  $V_0$  and  $b$  so that the peak magnitude of the current pulse is greater than  $0.6 \text{ A}$ . Determine  $i(t)$  and plot the current pulse.

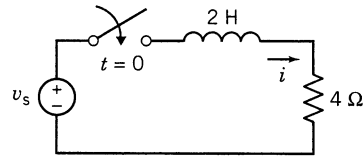


Figure DP 9-10



## PSpice PROBLEMS

**SP 9-1** Determine the current  $i$  of the circuit shown in Figure 9-10 using PSpice. Plot  $i(t)$  for five time constants and find  $i(t)$  at  $t = 10 \text{ ms}$  assuming  $i(0) = 3 \text{ A}$ .

**SP 9-2** Determine  $i(t)$  for Problem 8-25.

**SP 9-3** Determine and plot  $v_c(t)$  for AP 9-4.

**SP 9-4** Determine and plot  $v(t)$  for P 9-13.

**SP 9-5** Determine and plot  $v(t)$  for AP 9-3.

**SP 9-6** Determine and plot  $v_c(t)$  for P 9-17.

**SP 9-7** Determine and plot  $i(t)$  of the circuit of Figure 9-14 with  $v(0) = 5 \text{ V}$ .

**SP 9-8** Determine and plot  $v(t)$  for P 9-26.

**SP 9-9** An  $RC$  circuit as shown in Figure SP 9-9 has  $v(0) = 0$ . It is desired to plot the response of the circuit for four time constants: 2, 4, 8, and 16 ms. Select the appropriate value of  $R$  and use a PSpice program to plot the step response for the four time constants on one graphical plot.

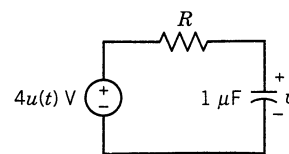


Figure SP 9-9



**Conductance** Real part of admittance, denoted as  $G$ . The units are siemens.

**Frequency** Radian frequency  $\omega$  in the sinusoidal waveform  $x = \cos \omega t$ ; also ordinary frequency  $f$ , where  $\omega = 2\pi f$  ( $f$  has units of hertz).

**Frequency Domain** Mathematical domain where the set of possible values of a variable is expressed in terms of frequency.

**Impedance** Ratio of the phasor voltage  $\mathbf{V}$  to the phasor current  $\mathbf{I}$  for a circuit element or set of elements so that  $\mathbf{Z} = \mathbf{V}/\mathbf{I}$ . The units are ohms.

**Periodic Function** Function defined by the property  $x(t + T) = x(t)$  so that it repeats every  $T$  seconds.

**Period of Oscillation**  $T$  is the period of oscillation of the periodic function, the time between two identical maximum points in the waveform.

**Phase Shift** Phase angle  $\phi$  associated with a variable  $x$  so that  $x = V_m \sin(\omega t + \phi)$ , or phase angle between two sinusoidal waveforms.

**Phasor** Complex number associated with a circuit variable—for example, the phasor voltage  $\mathbf{V}$ . The transform of a sinusoidal voltage or current that contains the magnitude and the phase angle information.

**Phasor Diagram** Relationship of phasors on the complex plane.

**Reactance** Imaginary part of impedance, denoted as  $X$ . The units are ohms.

**Resistance** Real part of impedance, denoted as  $R$ . The units are ohms.

**Susceptance** Imaginary part of admittance, denoted as  $B$ . The units are siemens.

**Transform** Change in the mathematical description of a physical variable to facilitate computation.

## REFERENCES Chapter 11

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Lyle H. McCarty, "Catheter Clears Coronary Arteries," *Design News*, September 23, 1991, pp. 88–92.

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## PROBLEMS

**P 11-1** For the following voltage and current expressions, indicate whether the element is capacitive, inductive, or resistive and find the element value.

- (a)  $v = 15 \cos(400t + 30^\circ)$ ;  $i = 3 \sin(400t + 30^\circ)$
- (b)  $v = 8 \sin(900t + 50^\circ)$ ;  $i = 2 \sin(900t + 140^\circ)$
- (c)  $v = 20 \cos(250t + 60^\circ)$ ;  $i = 5 \sin(250t + 150^\circ)$

**Answer:** (a)  $L = 12.5 \text{ mH}$   
 (b)  $C = 277.77 \text{ } \mu\text{F}$   
 (c)  $R = 4 \text{ } \Omega$

**P 11-2** Express the following summations of sinusoids in the general form  $A \sin(\omega t + \theta)$  by using trigonometric identities.

- (a)  $i(t) = 2 \cos(6t + 120^\circ) = 4 \sin(6t - 60^\circ)$
- (b)  $v(t) = 5\sqrt{2} \cos 8t + 10 \sin(8t + 45^\circ)$

**P 11-3** Given the phasor  $\mathbf{I} = 6 + j8$ , rotate it in the phasor plane by the following amounts and express the final result in rectangular coordinates: (a)  $-45^\circ$ ; (b)  $90^\circ$ .

**Answer:** (a)  $\mathbf{I} = 7\sqrt{2} + j\sqrt{2}$   
 (b)  $\mathbf{I} = -8 + j6$

**P 11-4** Add the following voltage waveforms using phasors, obtaining the resultant voltage in the form  $v_1 + v_2 = A \cos(\omega t + \phi)$ .

- (a)  $v_1 = 3 \cos(2t + 60^\circ)$ ;  $v_2 = 8 \cos(2t - 22.5^\circ)$
- (b)  $v_1 = 2\sqrt{2} \sin 4t$ ;  $v_2 = 10 \cos(4t + 30^\circ)$

P 11-5 Determine the polar form of the quantity

$$\frac{(5/36.9^\circ)(10/-53.1^\circ)}{(4+j3) + (6-j8)}$$

**Answer:**  $2\sqrt{5} / 10.36^\circ$

\*P 11-6 Find the two phasors **A** and **B** so that  $|A| = 5\sqrt{2}$ ,  $|B| = 4$  and so that  $2\mathbf{A} + 5\mathbf{B} = j10(1 + \sqrt{3})$  and **B** leads **A** by  $75^\circ$ .

P 11-7 Determine the polar and rectangular form of the expression

$$5/81.87^\circ \left( 4 - j3 + \frac{3\sqrt{2}/-45^\circ}{7 - j1} \right)$$

**Answer:**  $28/\underline{45^\circ} = 14\sqrt{2} + j14\sqrt{2}$

P 11-8 Consider the signal

$$i(t) = 72\sqrt{3} \cos 8t + 36\sqrt{3} \sin(8t + 140^\circ) + 144 \cos(8t + 210^\circ) + 25 \cos(8t + \phi)$$

Using the phasor plane, for what value of  $\phi$  does the  $|I|$  attain its maximum?

P 11-9 For the circuit shown in Figure P 11-9, find (a) the impedances  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$  in polar form, (b) the total combined impedance in polar form, and (c) the steady-state current  $i(t)$ .

**Answer:** (a)  $\mathbf{Z}_1 = 5/53.1^\circ$ ;  $\mathbf{Z}_2 = 8\sqrt{2}/-45^\circ$   
 (b)  $\mathbf{Z}_1 + \mathbf{Z}_2 = 11.7/-20^\circ$   
 (c)  $i(t) = (8.55) \cos(1250t + 20^\circ) \text{ A}$

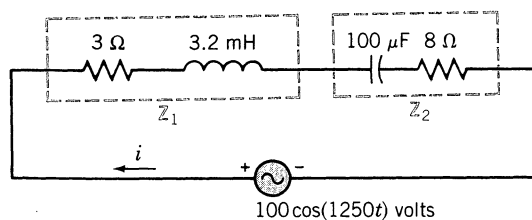


Figure P 11-9

P 11-10 Given the phasor circuit shown in Figure P 11-10, find (a)  $\mathbf{Z}_{eq}$ , (b)  $\mathbf{I}$ , (c)  $\mathbf{V}_{ef}$ , (d)  $\mathbf{Z}_{fg}$ , and (e)  $\mathbf{I}_{cd}$ . The  $\mathbf{Z}_{eq}$  is the equivalent impedance found by disconnecting the source and determining the impedance at terminals e-g.

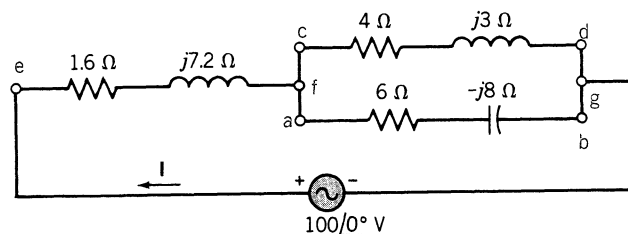


Figure P 11-10

P 11-11 Find the steady-state response  $v_x$  for the circuit in Figure P 11-11 using nodal analysis.

**Answer:**  $v_x(t) = 1168 \cos(500t - 66.3^\circ) \text{ V}$

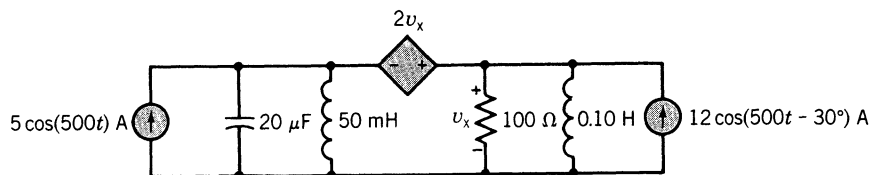


Figure P 11-11

P 11-12 Using a supermesh and mesh analysis, find the steady-state response  $i_x$  for the circuit shown in Figure P 11-12.

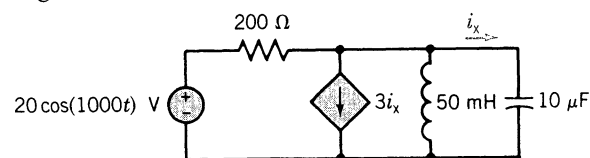


Figure P 11-12

P 11-13 Find  $v_x(t)$  by first replacing the circuit to the left of terminals a-b in Figure P 11-13 with its Thévenin equivalent circuit when  $C = 10 \text{ mF}$ .

**Answer:**  $v_x(t) = 33.13 \cos(20t - 83.66^\circ) \text{ V}$

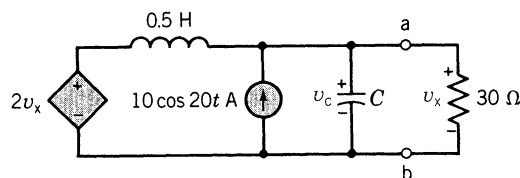


Figure P 11-13

P 11-14 Find the phasor voltage  $\mathbf{V}_c$  for the circuit shown in Figure P 11-14.

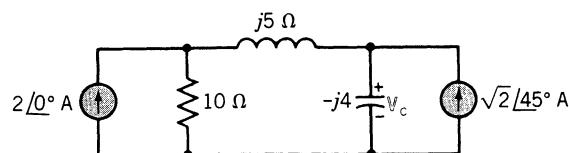


Figure P 11-14

\* Note: An asterisk denotes a challenging problem.

**P 11-15** Find the equivalent input impedance  $\mathbf{Z}_{in}$  for the circuit shown in Figure P 11-15.

**Answer:**  $6\sqrt{3}/26.6^\circ \Omega$

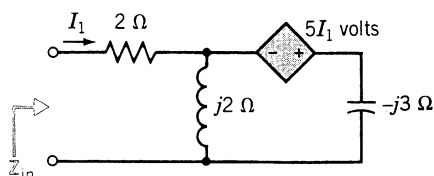


Figure P 11-15

**P 11-16** Spinal cord injuries result in paralysis of the lower body and can cause loss of bladder control. Numerous electrical devices have been proposed to replace the normal nerve pathway stimulus for bladder control. Figure P 11-16 shows the model of a bladder control system where  $v_s = 20 \cos \omega t$  V and  $\omega = 100$  rad/s. Find the steady-state voltage across the  $10\text{-}\Omega$  load resistor.

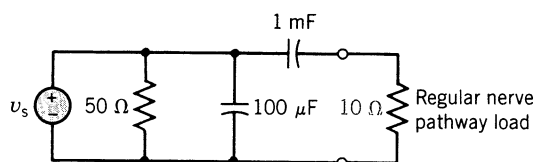


Figure P 11-16

**P 11-17** For the circuit shown in Figure P 11-17, find the capacitance,  $C$ , if  $v(t) = 20 \cos 100t$  V.

**Answer:**  $C = 1/800$  F

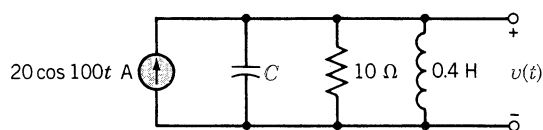


Figure P 11-17

**P 11-18** There are 500 to 1000 deaths each year in the United States from electric shock. If a person makes a good contact with his hands, the circuit can be represented by Figure P 11-18, where  $v_s = 160 \cos \omega t$  volts and  $\omega = 2\pi f$ . Find the steady-state current  $i$  flowing through the body when (a)  $f = 60$  Hz and (b)  $f = 400$  Hz.

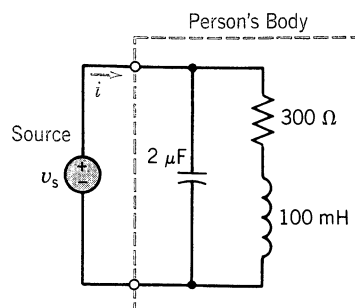


Figure P 11-18

**P 11-19** For the circuit shown in Figure P 11-19, determine the phasor currents  $\mathbf{I}_s$ ,  $\mathbf{I}_C$ ,  $\mathbf{I}_L$ , and  $\mathbf{I}_R$  if  $\omega = 1000$  rad/s.

**Answer:**  $\mathbf{I}_s = 0.347/-25.5^\circ$  A

$\mathbf{I}_C = 0.461/112.9^\circ$  A

$\mathbf{I}_L = 0.720/-67.1^\circ$  A

$\mathbf{I}_R = 0.230/22.9^\circ$  A

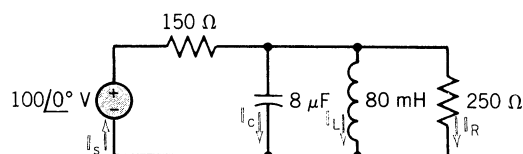


Figure P 11-19

**P 11-20** The model of a high-frequency transistor amplifier is shown in Figure P 11-20 with a source voltage and a load resistor. The source voltage is  $v_s = 10 \cos \omega t$  where  $\omega = 10^8$  rad/s. Find the voltage across the load resistor.

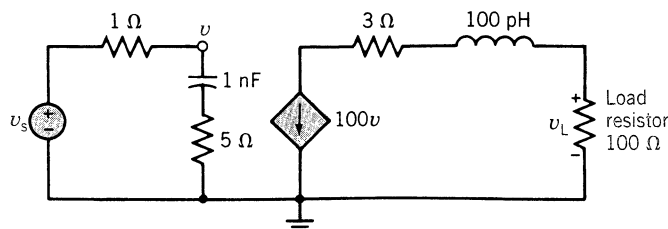


Figure P 11-20 Model of a high-frequency transistor amplifier.

**P 11-21** Determine the Thévenin equivalent circuit for the circuit shown in Figure P 11-21 when  $v_s = 5 \cos(4000t - 30^\circ)$ .

**Answer:**  $V_t = 5.7 \angle -21.9^\circ$  V  
 $Z_t = 23 \angle -81.9^\circ \Omega$

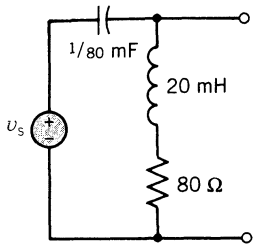


Figure P 11-21

**P 11-22** Find the Thévenin equivalent circuit for the circuit shown in Figure P 11-22 using the mesh current method.

**Answer:**  $V_t = 3.71 \angle -16^\circ$  V  
 $Z_t = 247 \angle -16^\circ \Omega$

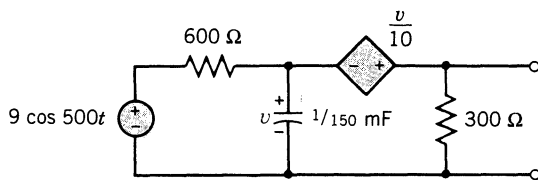


Figure P 11-22

**P 11-23** Determine  $I_1$ ,  $I_2$ ,  $V_L$ , and  $V_c$  for the circuit of Figure P 11-23 using KVL and mesh analysis.

**Answer:**  $I_1 = 2.5 \angle 29.0^\circ$  A  
 $I_2 = 1.8 \angle 105^\circ$  A  
 $V_L = 16.3 \angle 78.7^\circ$  V  
 $V_c = 7.2 \angle 15^\circ$  V

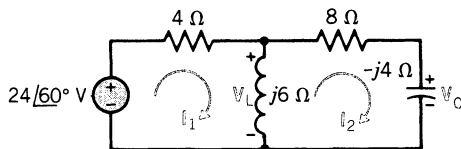


Figure P 11-23

**P 11-24** Consider the circuit of Figure P 11-24, where we wish to determine the current  $I$ . Use a series of source transformations to find a current source in parallel with an equivalent impedance, and then find the current in the  $2\text{-}\Omega$  resistor by current division.

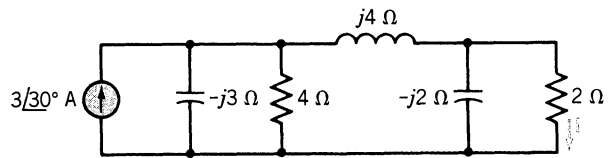


Figure P 11-24

**\*P 11-25** Consider the circuit of Figure P 11-25. Using nodal analysis, find  $V_s$  when  $I$  in the  $4\text{-}\Omega$  resistor is  $3 \angle 45^\circ$  A.

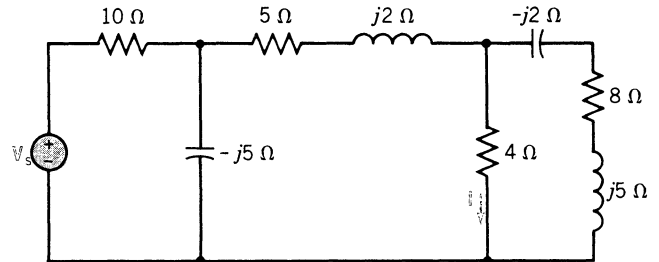


Figure P 11-25

**P 11-26** Find the node voltages  $v_1(t)$  and  $v_2(t)$  for the circuit shown in Figure P 11-26 when (a)  $i_1 = 1 \cos 100t$  A and (b)  $i_1 = 1$  A (a constant). The second current source is  $i_2 = 0.5 \sin 100t$ .

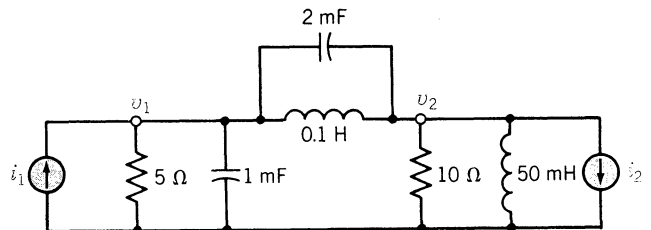


Figure P 11-26

**P 11-27** Find the two node voltages  $v_a(t)$  and  $v_b(t)$  for the circuit of Figure P 11-27 when  $v_s(t) = 1.2 \cos 4000t$ .

**Answer:**  $v_a(t) = 1.97 \cos(4000t - 171^\circ)$  V  
 $v_b(t) = 2.21 \cos(4000t - 144^\circ)$  V

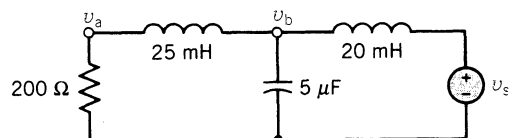


Figure P 11-27

**\*P 11-28** Find the current in the inductor for the circuit of Figure P 11-28 when  $i_1 = 10 \cos 100t$  A and  $v_1 = 100 \cos 1000t$  V.

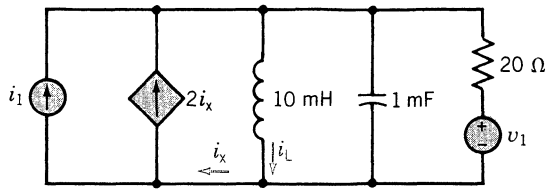


Figure P 11-28

**P 11-29** Find the current in the inductor,  $i(t)$ , as shown in Figure P 11-29. Use mesh current analysis when  $v_1 = 10 \cos \omega t$  V,  $v_2 = 10 \sin \omega t$  V, and  $\omega = 1000$ .

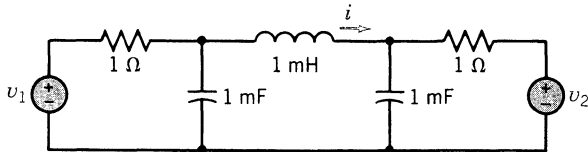


Figure P 11-29

**P 11-30** Consider the circuit shown in Figure P 11-30. Determine the required value of  $X_C$  and  $X_L$  in order to make the impedance  $Z$  seen at the terminals equal to  $aR$ , where  $0 \leq a \leq 1$ . Find  $X_C$  and  $X_L$  in terms of  $a$  and  $R$ .

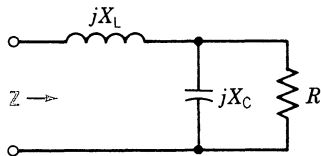


Figure P 11-30

**P 11-31** The circuit of Figure P 11-31 contains a voltage source operating at  $\omega = 100$ . Find the current  $i(t)$ .

**Answer:**  $i(t) = (1/\sqrt{2}) \cos(100t - 45^\circ)$

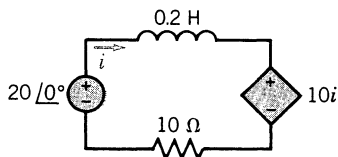


Figure P 11-31

**P 11-32** For  $t > 0$ , find the total capacitor voltage,  $v(t)$ , for the circuit shown in Figure P 11-32 when

$$v_s = \begin{cases} 50 \cos 3t \text{ V} & t < 0 \\ 20 \text{ V} & t > 0 \end{cases}$$

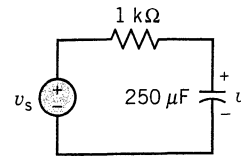


Figure P 11-32

**P 11-33** Determine  $i(t)$  of the  $RLC$  circuit shown in Figure P 11-33 when  $v_s = 2 \cos(4t + 30^\circ)$  V.

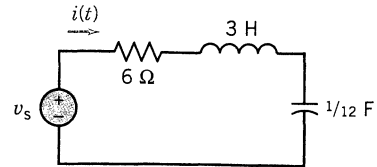


Figure P 11-33

**P 11-34** An amplifier circuit is shown in Figure P 11-34 with an input  $v_s = 5 \cos 200t$  V. Determine the output voltage  $v(t)$ .

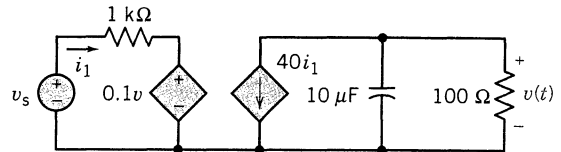


Figure P 11-34 Amplifier circuit.

**P 11-35** Determine the current  $i(t)$  for the circuit of Figure P 11-35 using mesh currents when  $\omega = 1000$  rad/s.

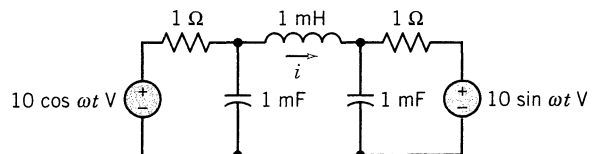


Figure P 11-35

**P 11-36** Determine the Thévenin equivalent at terminals a-b for the circuit shown in Figure P 11-36 when  $\omega = 100$  rad/s.

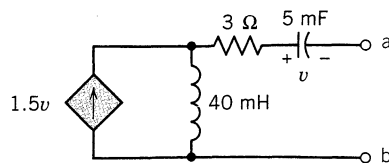


Figure P 11-36

**P 11-37** The circuit shown in Figure P 11-37 has two sources:  $v_s = 20 \cos(\omega_0 t + 90^\circ)$  V and  $i_s = 6 \cos \omega_0 t$  A where  $\omega_0 = 10^5$  rad/s. Determine  $v_0(t)$ .

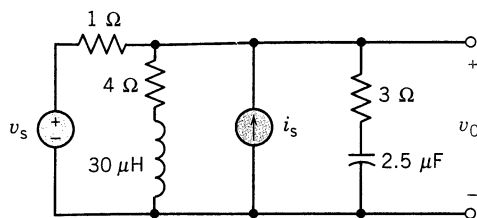


Figure P 11-37

**P 11-38** Determine the voltage  $v_a$  for the circuit in Figure P 11-38 when  $i_s = 20 \cos(\omega t + 53.13^\circ)$  A and  $\omega = 10^4$  rad/s.

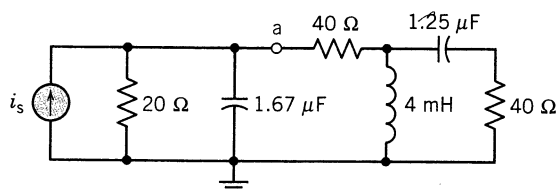


Figure P 11-38

**P 11-39** Two voltages appear in series so  $v = v_1 + v_2$ . Find  $v$  when  $v_1 = 150 \cos(377t - \pi/6)$  V and  $v_2 = 200 \angle +60^\circ$  V.

**P 11-40** Find the steady-state response  $v_0(t)$  if  $v_s(t) = \sqrt{2} \cos 1000t$  for the circuit of Figure P 11-40.

**Answer:**  $v_0(t) = 10 \cos(1000t - 225^\circ)$

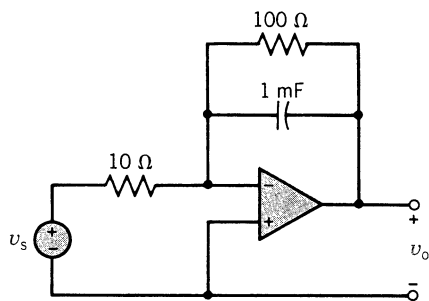


Figure P 11-40

**P 11-41** Determine  $V_0/V_s$  for the op amp circuit shown in Figure P 11-41. Assume an ideal op amp.

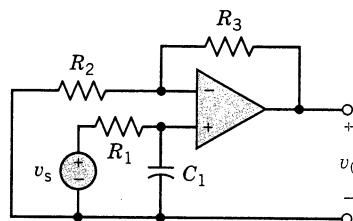


Figure P 11-41 Amplifier circuit for disk player.

**P 11-42** Determine  $V_0/V_s$  for the op amp circuit shown in Figure P 11-42. Assume an ideal op amp.

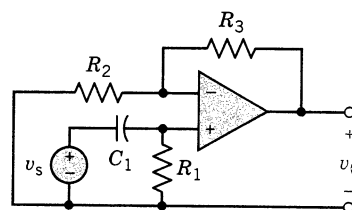


Figure P 11-42

**P 11-43** Determine  $v_0(t)$  for the op amp circuit of Figure P 11-43. Assume an ideal op amp and that  $v_s = 1 \sin(3t + 60^\circ)$  V.

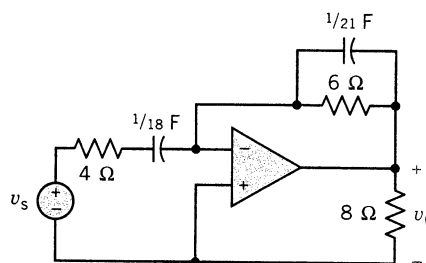


Figure P 11-43

## ADVANCED PROBLEMS

**AP 11-1** A pocket-sized mini-disk CD player system has an amplifier circuit shown in Figure AP 11-1 with a signal  $v_s = 10 \cos(\omega t + 53.1^\circ)$  at  $\omega = 10,000$  rad/s.

Determine the Thévenin equivalent at the output terminals a-b.

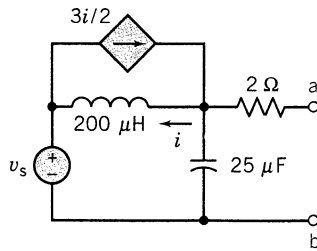


Figure AP 11-1

**AP 11-2** Determine the total inductor current  $i(t)$  for the circuit shown in Figure AP 11-2 when  $v_s = 3 \sin 2t$  V. Assume that switch 1 was closed for a long time prior to  $t = 0$ .

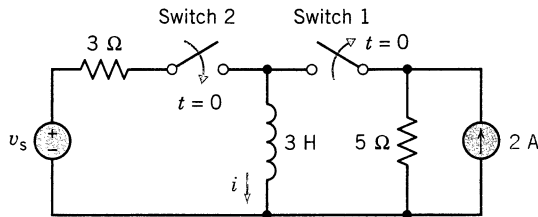


Figure AP 11-2

**AP 11-3** Determine  $v(t)$  for the circuit of Figure AP 11-3 when  $v_s = \sqrt{2} \sin(\omega t + 135^\circ)$  V,  $i_s = 4 \cos(\omega t + 30^\circ)$  A, and  $\omega = 100$  rad/s.

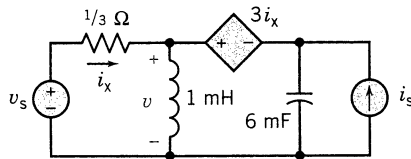


Figure AP 11-3

**AP 11-4** In the analysis of guided waves on transmission lines, circuit theory is often used. One of the simplest unit-length circuit models of a transmission line is shown in Figure AP 11-4. The transmission line is represented by  $L$  and  $C$  and the load by  $R$ . A line has  $L = 97.5$  nH and  $C = 39$  pF and operates at 100 MHz. Determine the input impedance when  $R = 25 \Omega$  and

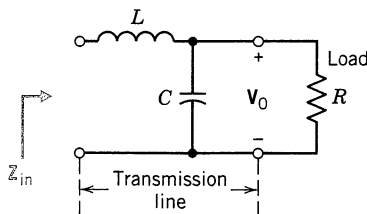


Figure AP 11-4 Circuit model of transmission line.

50  $\Omega$ . Note that this circuit translates one of the resistances almost exactly to the input impedance. This resistance is the characteristic impedance or resistance of the transmission line.

**AP 11-5** A commercial airliner has sensing devices to indicate to the cockpit crew that each door and baggage hatch is closed. A device called a search coil magnetometer, also known as a proximity sensor, provides a signal indicative of the proximity of metal or other conducting material to an inductive sense coil. The inductance of the sense coil changes as the metal gets closer to the sense coil. The sense coil inductance is compared to a reference coil inductance with a circuit called a balanced inductance bridge (see Figure AP 11-5). In the inductance bridge, a signal indicative of proximity is observed between terminals A and B by subtracting the voltage at B,  $v_B$ , from the voltage at A,  $v_A$  (Lenz, 1990).

The bridge circuit is excited by a sinusoidal voltage source  $v_s = \sin(800 \pi t)$  V. The two resistors,  $R = 100 \Omega$ , are of equal resistance. When the door is open (no metal is present), the sense coil inductance,  $L_s$ , is equal to the reference coil inductance,  $L_r = 40$  mH. In this case, what is the magnitude of the signal  $V_A - V_B$ ?

When the airliner door is completely closed,  $L_s = 60$  mH. With the door closed, what is the phasor representation of the signal  $V_A - V_B$ ?

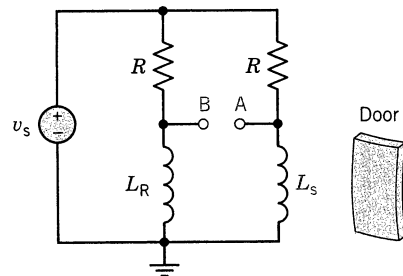


Figure AP 11-5 Airline door sensing circuit.

**AP 11-6** An AM radio receiver uses the parallel  $RLC$  circuit shown in Figure AP 11-6. Determine the frequency,  $f_0$ , at which the admittance  $\mathbf{Y}$  is a pure conductance. What is the “number” of this station on the AM radio dial?

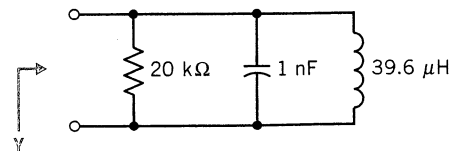


Figure AP 11-6

**AP 11-7** The idea of using an induction coil in a lamp isn't new, but applying it in a commercially available product is. An induction coil in a bulb induces a high-frequency energy flow in mercury vapor to produce light. The lamp uses about the same amount of energy as a fluorescent bulb but lasts six times longer, with 60 times the life of a conventional incandescent bulb. The circuit model of the bulb and its associated circuit is shown in Figure AP 11-7. Determine the voltage  $v(t)$  across the  $2\Omega$  resistor when  $C = 40\ \mu\text{F}$ ,  $L = 40\ \mu\text{H}$ ,  $v_s = 10 \cos(\omega_0 t + 30^\circ)$ , and  $\omega_0 = 10^5\ \text{rad/s}$ .

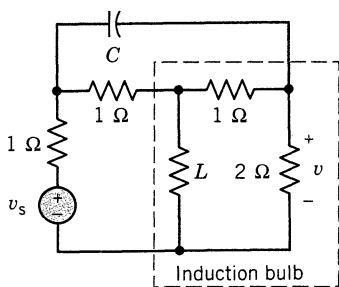
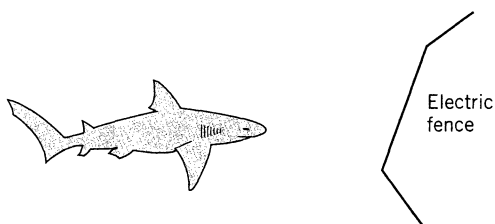
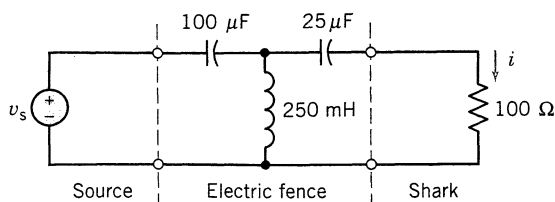


Figure AP 11-7 Induction bulb circuit.

**AP 11-8** The development of coastal hotels in various parts of the world is a rapidly growing enterprise. The need for environmentally acceptable shark protection is manifest where these developments take place alongside shark-infested waters (Smith, 1991). One concept is to use an electrified line submerged in the water in order to deter the sharks, as shown in



(a)



(b)

Figure AP 11-8 Electric fence for repelling sharks.

Figure AP 11-8a. The circuit model of the electric fence is shown in Figure AP 11-8b where the shark is represented by an equivalent resistance of  $100\ \Omega$ . Determine the current flowing through the shark's body,  $i(t)$ , when  $v_s = 375 \cos 400t\ \text{V}$ .

**AP 11-9** Using a tiny diamond-studded burr operating at 190,000 rpm, cardiologists can remove life-threatening plaque deposits in coronary arteries. The procedure is fast, uncomplicated, and relatively painless (McCarty, 1991). The Rotablator, an angioplasty system, consists of an advancer/catheter, a guide wire, a console, and a power source. The advancer/catheter contains a tiny turbine that drives the flexible shaft that rotates the catheter burr. The model of the operational and control circuit is shown in Figure AP 11-9. Determine  $v(t)$ , the voltage that drives the tip, when  $v_s = \sqrt{2} \cos(40t + 135^\circ)\ \text{V}$ .

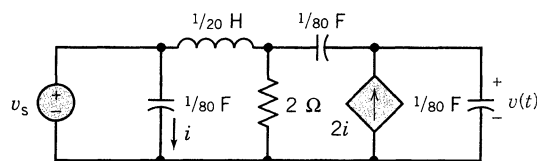


Figure AP 11-9 Control circuit for Rotablator.

**AP 11-10** The big toy from the hit movie *Big* is a child's musical fantasy come true—a sidewalk-sized piano. Like a hopscotch grid, this Christmas's hot toy invites anyone who passes to jump on, move about, and make music. The developer of the "toy" piano used a tone synthesizer and stereo speakers as shown in Figure AP 11-10 (Gardner, 1988). Determine the current  $i(t)$  for a tone at 796 Hz when  $C = 10\ \mu\text{F}$ .

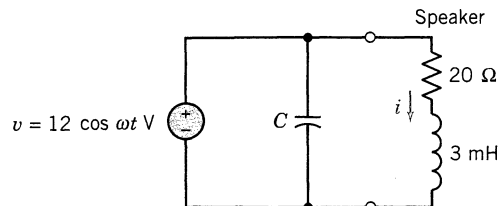


Figure AP 11-10 Tone synthesizer.



## DESIGN PROBLEMS

**DP 11-1** A circuit with an unspecified  $R$ ,  $L$ , and  $C$  is shown in Figure DP 11-1. The input source is  $i_s = 10 \cos 1000t$  A and the goal is to select the  $R$ ,  $L$ , and  $C$  so that the node voltage is  $v = 80 \cos(1000t - \theta)$  V where  $-40^\circ < \theta < 40^\circ$ .

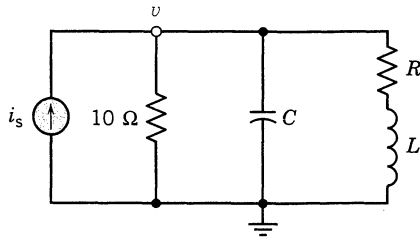


Figure DP 11-1

**DP 11-2** The source  $v_s = (15/\sqrt{2}) \cos 3t$  V in the circuit of Figure DP 11-2. Select the capacitance  $C$  so that the output voltage is  $v_o = V_o \cos(3t + \phi)$  V and  $V_o = 9$  V. Determine the resulting  $\phi$ .

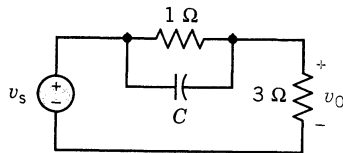


Figure DP 11-2

**DP 11-3** An  $RLC$  circuit shown in Figure DP 11-3 has a source voltage  $v_s = 10 \cos \omega_0 t$  V where  $\omega_0 = 10^4$  rad/s. It is desired to adjust the capacitor from  $0.5$  to  $3.5 \mu\text{F}$  in steps of  $0.5 \mu\text{F}$  to achieve the maximum steady-state magnitude  $|V_o|$ . Determine the required value of  $C$  and the maximum value of  $|V_o|$ .

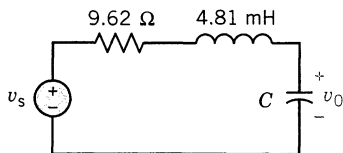


Figure DP 11-3

**DP 11-4** The input impedance of the circuit of Figure DP 11-4 can be adjusted to yield  $Z = aR$  where  $0 \leq a \leq 1$ .

- Select suitable values for  $X_L = \omega L$  and  $X_C = 1/\omega C$  in terms of  $R$  and  $a$ .
- Select  $L$  and  $C$  when  $\omega = 1000$ ,  $a = 0.2$ , and  $R = 100 \Omega$ .

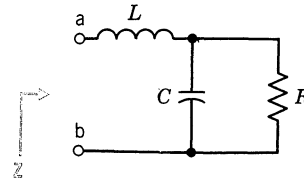


Figure DP 11-4

**DP 11-5** Select  $L$  so that the impedance  $Z$  is  $10 \angle 0^\circ$  at  $\omega = 400$  for the circuit shown in Figure DP 11-5.

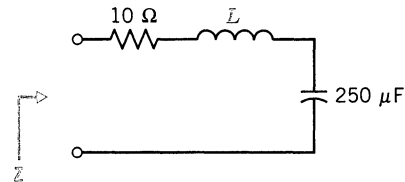


Figure DP 11-5

**DP 11-6** A power circuit shown in Figure DP 11-6 takes the input signal  $v_s = 12 \cos 1000t$  V and provides an output  $v_o = 6 \cos 1000t$  V. Determine the required value of  $b$  in order to achieve this output voltage.

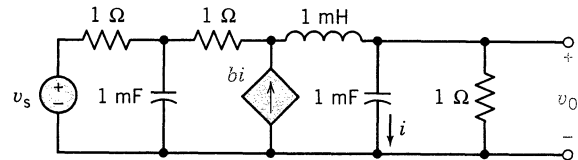


Figure DP 11-6

**DP 11-7** Determine the impedance at terminals a-b for the circuit shown in Figure DP 11-7. One resistor, one inductor, and one capacitor is available, and  $b = 2$ .

- Select suitable values for the components and place in the appropriate positions in the circuit so that  $Z_{ab} = 14.1 \angle -45^\circ \Omega$ . Assume  $\omega = 377$  rad/s.
- Determine the effect on  $Z_{ab}$  of  $b$  changing by  $\pm 10\%$ .

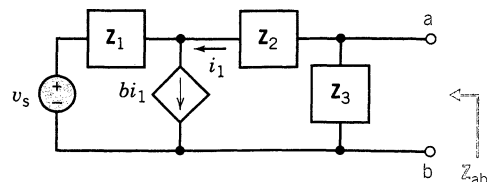


Figure DP 11-7



## PSPICE PROBLEMS

SP 11-1 Determine the current  $i(t)$  for the circuit of Figure P 11-35 when  $\omega = 1000$  rad/s.

SP 11-2 Determine  $v(t)$  for the circuit of Figure AP 11-3 when  $v_s = \sqrt{2} \sin(\omega t + 135^\circ)$  V,  $i_s = 4 \cos(\omega t + 30^\circ)$  A, and  $\omega = 100$  rad/s.

SP 11-3 Determine  $i(t)$  of the circuit of Problem P 11-33.

SP 11-4 Determine  $v(t)$  for Problem P 11-34.

SP 11-5 Determine  $V_0$  for Problem P 11-37.

SP 11-6 Determine the current  $i(t)$  for the circuit of Figure SP 11-6 when  $v_s = 10 \cos(6t + 45^\circ)$  V and  $i_s = 2 \cos(6t + 60^\circ)$  A.

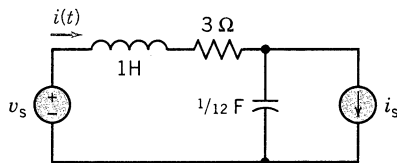


Figure SP 11-6

SP 11-7 Determine  $v_0(t)$  for the circuit of Figure SP 11-7 when  $v_s = 5 \cos(3t - 30^\circ)$  V.

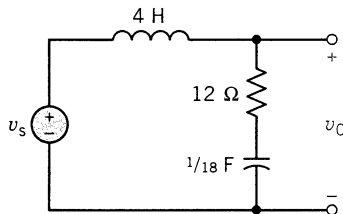


Figure SP 11-7

SP 11-8 Determine the current  $i(t)$  for the circuit shown in Figure SP 11-8 when  $v_s = 200 \cos \omega t$  V,  $i_s = 8 \cos(\omega t + 90^\circ)$  A, and  $\omega = 1000$  rad/s.

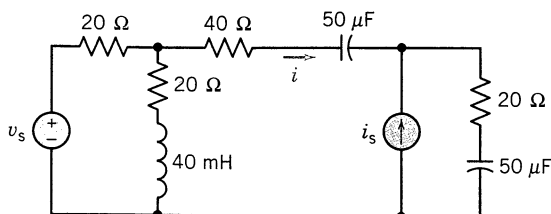


Figure SP 11-8

SP 11-9 Determine the output voltage,  $v$ , for the circuit shown in Figure SP 11-9 when  $v_s = 4 \cos \omega t$  V and  $\omega = 2\pi \times 1000$  rad/s.

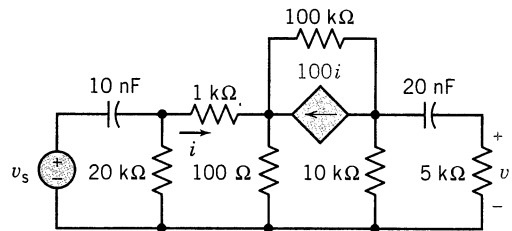


Figure SP 11-9

SP 11-10 Determine  $i(t)$  for the circuit of Figure SP 11-10 when  $i_s = 2 \cos(3t + 10^\circ)$  A and  $v_s = 3 \cos(2t + 30^\circ)$  V.

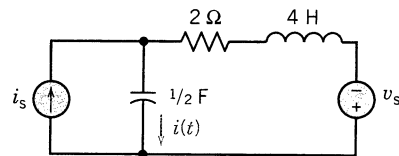


Figure SP 11-10

SP 11-11 Determine the impedance at terminals a-b for Design Problem 11-7 for a suitable design.

SP 11-12 Determine the voltage  $v_a$  for Problem P 11-38.

SP 11-13 Determine  $v_a(t)$  and  $i(t)$  for the circuit shown in Figure SP 11-13 when  $v_s = 5 \cos 2t$  V and  $i_s = 5 \cos 2t$  A.

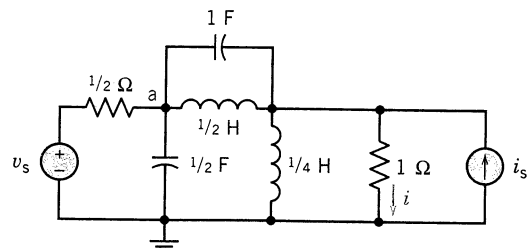


Figure SP 11-13

SP 11-14 Determine  $i(t)$  for the circuit shown in Figure SP 11-14 when  $v_s = 4 \cos 5000t$  V.

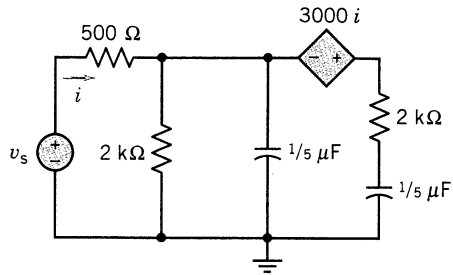


Figure SP 11-14

SP 11-15 Determine the impedance  $\mathbf{Z}$  for the circuit as shown in Figure SP 11-15 at a frequency of 60 Hz.

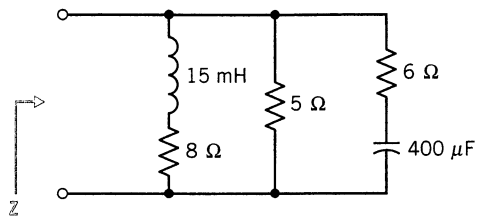


Figure SP 11-15

SP 11-16 Determine the current  $i(t)$  for the circuit shown in Figure SP 11-16 when  $v_s(t) = 120 \sin(\omega t + 30^\circ)$  V and  $f = 10$  kHz.

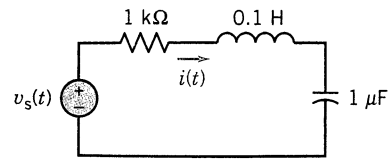


Figure SP 11-16

SP 11-17 Consider the RLC circuit of Figure 11-16a with  $v_s = 1 \sin \omega t$  at 1023 Hz. Determine and plot  $v_s(t)$  and  $v_c(t)$  when  $R = 33 \text{ k}\Omega$ , (a)  $C = 4.7 \text{ nF}$  and  $L = 0$  and (b)  $L = 1 \text{ mH}$  and  $C = 20 \text{ }\mu\text{F}$ .

**Decibels** Units of the logarithmic gain.

**Electric Filter** See Filter.

**Filter** Circuit designed to provide a magnitude gain and/or loss over a prescribed range of frequencies.

**First-Order Circuit** Circuit that contains only one energy storage element, either capacitance or inductance.

**Frequency Response** Frequency-dependent relation, in both gain and phase, between an input sinusoidal steady-state signal and an output sinusoidal signal.

**Gain Ratio** Ratio of a selected output signal to a designated input signal, written as  $\mathbf{H} = \mathbf{X}_o / \mathbf{X}_i$  where  $\mathbf{X}$  may be a phasor voltage or current.

**High-Pass Filter** Filter that will ideally pass all frequencies above the cutoff frequency  $\omega_c$  and reject all frequencies below the cutoff frequency.

**Logarithmic Gain**  $20 \log_{10} H$  with units of decibels (dB).

**Low-Pass Filter** Filter that will ideally pass all frequencies up to the cutoff frequency  $\omega_c$  and perfectly reject all frequencies above  $\omega_c$ .

**Parallel Resonant Circuit** Circuit with a resistor, capacitor, and inductor in parallel.

**Peak Frequency,  $\omega_p$**  Frequency at which a peak magnitude occurs for the pair of complex poles.

**Poles** Roots of the denominator polynomial of the gain ratio  $\mathbf{H}(s)$ .

**Quality Factor,  $Q$**  Measure of the energy storage property in relation to the energy dissipation property of a circuit. For a parallel resonant circuit  $Q = R/\omega_0 L = \omega_0 RC$ . A measure of the frequency selectivity of a filter.

**Resonance** Condition in a circuit, occurring at the resonant frequency, when  $\mathbf{H}(j\omega)$  becomes a real number (nonreactive).

**Resonant Frequency** Frequency  $\omega_0$  at which a gain ratio  $\mathbf{H}(j\omega)$  is nonreactive. For a parallel resonant circuit the resonant frequency occurs when its admittance is nonreactive.  $\omega_0 = 1/\sqrt{LC}$  for both the parallel and the series  $RLC$  circuit.

**Series Resonant Circuit** Circuit with a series connection of a resistor, capacitor, and inductor.

**Zeros** Roots of the numerator polynomial of the gain ratio  $\mathbf{H}(s)$ .

## REFERENCES Chapter 13

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Brown, S. F. "Predicting Earthquakes," *Popular Science*, June 1989, pp. 124–125.

Lewis, Raymond. "A Compensated Accelerometer," *IEEE Transactions on Vehicular Technology*, August 1988, pp. 174–178.

Loeb, Gerald E. "The Functional Replacement of the Ear," *Scientific American*, February 1985, pp. 104–108.

## PROBLEMS

**P 13-1** For a parallel  $RLC$  circuit with  $R = 10 \text{ k}\Omega$ ,  $L = 1/120 \text{ H}$ , and  $C = 1/30 \text{ }\mu\text{F}$ , find  $\omega_0$ ,  $Q$ ,  $\omega_1$ , and the bandwidth  $B$ .

**Answer:**  $\omega_0 = 60 \text{ krad/s}$   
 $Q = 20$   
 $\omega_1 = 58.519 \text{ krad/s}$   
 $B = 3 \text{ krad/s}$

**P 13-2** Determine the ratio  $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_s$  for the circuit of Figure P 13-2. Determine  $|\mathbf{H}|$  and  $\phi(\omega)$  and plot the magnitude and phase on a linear scale for  $|\mathbf{H}|$  and  $\phi$  versus a logarithmic scale for  $\omega$ .

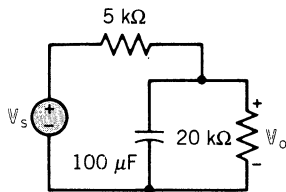


Figure P 13-2

**P 13-3** (a) Consider the series resonant circuit shown in Figure P 13-3 and find  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_s$ . (b) Sketch the Bode diagram for the circuit, assuming that  $2 < Q < 10$ . (c) Find the expression for the resonant frequency  $\omega_0$ . (d) Find the peak frequency  $\omega_p$  if  $Q = 5$  and  $\omega_0 = 1000$ .

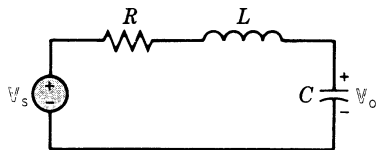


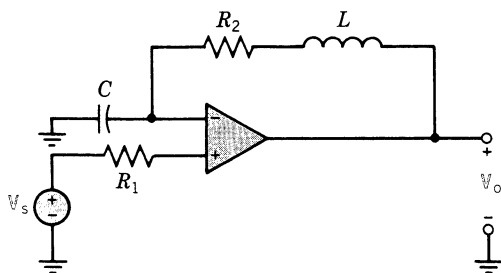
Figure P 13-3 A series resonant circuit.

**P 13-4** A parallel resonant  $RLC$  circuit is driven by a current source  $I_s = 0.2 \cos \omega t$  mA and shows a maximum response of 8 V at  $\omega = 2500$  rad/s and 4 V at 2200 rad/s. Find  $R$ ,  $L$ , and  $C$ .

**Answer:**  $R = 40$  k $\Omega$   
 $L = 237$  mH  
 $C = 6.76$  nF

**P 13-5** Consider the circuit shown in Figure P 13-5. (a) Find the gain  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_s$ . (b) Plot the Bode diagram for  $\mathbf{H}(j\omega)$  for  $R_1 = R_2 = 20$   $\Omega$ ,  $C = 2.4$   $\mu$ F, and  $L = 0.25$  mH.

**Answer:**  $\mathbf{H}(j\omega) = LC \left[ (j\omega)^2 + j \frac{R_2}{L} \omega + \frac{1}{LC} \right]$

Figure P 13-5 An  $RLC$  operational amplifier circuit.

**P 13-6** For the circuit shown in Figure P 13-6, (a) derive an expression for the magnitude response  $|\mathbf{Z}_{in}|$  versus  $\omega$ ; (b) sketch  $|\mathbf{Z}_{in}|$  versus  $\omega$ ; (c) find  $|\mathbf{Z}_{in}|$  at  $\omega = 1/\sqrt{LC}$ .

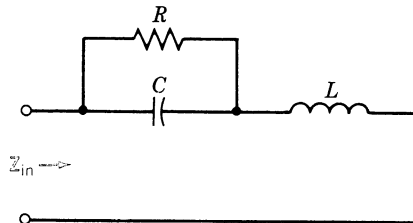
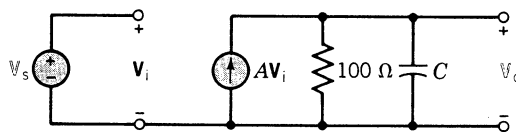


Figure P 13-6

**P 13-7** A linear model of the 741 noninverting operational amplifier that includes the effect of the output capacitance is shown in Figure P 13-7. Determine  $\mathbf{V}_o/\mathbf{V}_s$  and the bandwidth of the operational amplifier when  $C = 10$  nF and  $A = 10$ . Draw the Bode diagram for  $10^2 < \omega < 10^8$ . Determine the gain bandwidth product, which is equal to the gain at low frequencies times the radian frequency where  $|\mathbf{H}| = (1/\sqrt{2})H_{\max}$ .

**Answer:** Gain bandwidth =  $10^9$  rad/s

Figure P 13-7 A linear model of the 741 operational amplifier that includes the effect of the output capacitance  $C$ .

**P 13-8** Consider the circuit shown in Figure P 13-8, where  $\mathbf{V}_i$  and  $\mathbf{V}_o$  are phasor voltages. (a) Find expressions for  $\mathbf{V}_o/\mathbf{V}_i$  and  $|\mathbf{V}_o/\mathbf{V}_i|$ , both as a function of the radian frequency  $\omega$ . (b) Sketch  $|\mathbf{V}_o/\mathbf{V}_i|$  versus  $\omega$  for  $\omega > 0$ . Does your sketch give the correct limiting results for  $\omega \rightarrow 0$  and  $\omega \rightarrow \infty$ ? Briefly justify your answers. (c) Sketch the phase of  $\mathbf{V}_o/\mathbf{V}_i$  for  $\omega > 0$ .

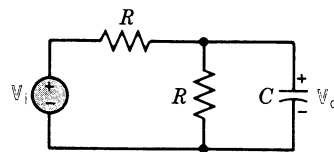


Figure P 13-8

**P 13-9** Consider the circuit shown in Figure P 13-9a, where the two-terminal circuit contained within the box is known to consist of two linear circuit elements. The magnitude plot for the Bode diagram of  $|\mathbf{Z}|$  is shown in Figure P 13-9b. Find the two-element circuit and specify the numerical value of each element.

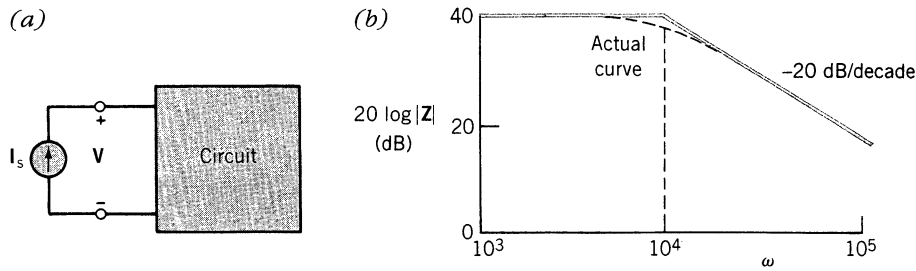


Figure P 13-9

**P 13-10** The amplitude gain of a circuit is shown in Figure P 13-10 for  $1 \leq \omega \leq 10^5$  rad/s. Find  $\mathbf{H}(j\omega)$  by estimating the asymptotic approximation for the Bode diagram of the circuit. The peak value of  $20 \log H$  is 43 dB at  $\omega = 450$ . You may assume that this circuit's gain function  $\mathbf{H}(j\omega)$  includes only first-order factors of the form  $(1 + j\omega\tau)$ .

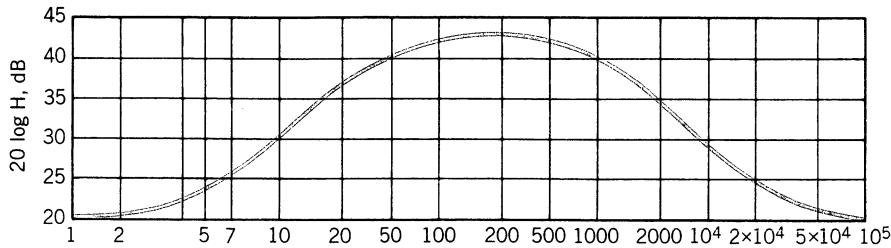


Figure P 13-10

**P 13-11** An operational amplifier can be accurately described by its frequency response with the equation

$$\mathbf{H}(j\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{K\omega_1}{j\omega + \omega_1} = \frac{K}{j\omega\tau + 1}$$

where  $K\omega_1$  is called the gain bandwidth product. The gain bandwidth product of a specific amplifier is 20 MHz and  $K = 10^5$ . Sketch the magnitude (dB) portion of the Bode diagram.

**\*P 13-12** The audio range can be considered to be between  $\omega_1 = 100$  rad/s and  $\omega_2 = 10^5$  rad/s. Using operational amplifiers and resistors and capacitors, design a bandpass filter by using a low-pass filter cascaded with a high-pass filter to obtain

$$\mathbf{H}(j\omega) = \frac{10(j\omega/\omega_1 + 1)}{(j\omega/\omega_2 + 1)}$$

**\*P 13-13** A bandstop filter can be used to eliminate unwanted signals. One common application is the removal of interference with television channel 6 due to nearby FM radio transmitters. In this problem, we wish to design a bandstop filter to eliminate the signal from an FM station broadcasting at 90 MHz.

\* Note: An asterisk denotes a challenging problem.

The circuit for the receiving antenna for the TV set is represented by  $\mathbf{V}_s$  and  $R_s$ , where  $R_s = 300 \Omega$ . The impedance of the television set is  $R_L = 300 \Omega$ , as shown in Figure P 13-13. Design a filter  $\mathbf{V}_o/\mathbf{V}_s$  so that the 90-MHz signal is removed without affecting the performance due to channel 6, which operates in the band 82 to 88 MHz. Use only an  $R$ ,  $L$ , and  $C$  network, since amplifiers that work at 90 MHz would be quite expensive.

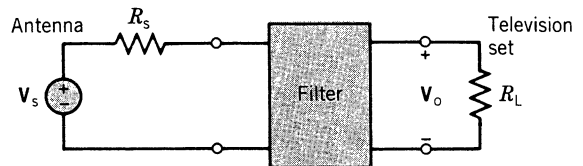


Figure P 13-13 A bandstop filter for elimination of television set interference.

**P 13-14** An audio amplifier has a gain

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1000a^2s}{(s+a)(s+1000a)}$$

where  $s = j\omega$ . The goal is to pass unimpeded all the frequencies between 20 Hz and 20 kHz. Select the pa-

parameter  $a$  and draw the Bode diagram of this amplifier for  $\omega = 1$  to  $\omega = 10^8$ , using the asymptotic approximation for the dB magnitude and an approximate curve for the phase.

**P 13-15** A bandpass amplifier has a frequency response for the magnitude ratio  $H$ , shown in Figure P 13-15. Find the bandwidth and the magnitude of  $H$  at the resonant frequency. Determine the  $Q$  of the circuit.

**Answer:**  $B = 0.2$  MHz

$H = 3.16$

$Q = 50$

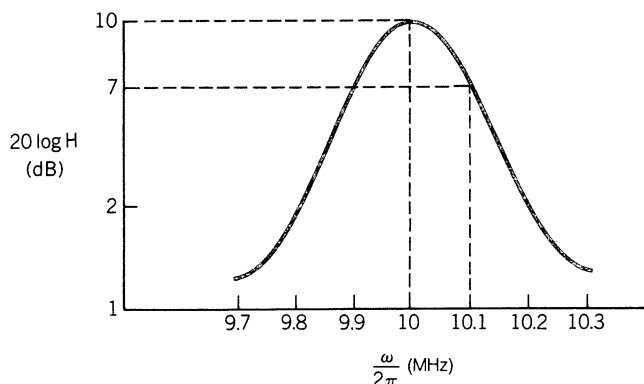


Figure P 13-15 A bandpass amplifier.

**P 13-16** For the circuit shown in Figure P 13-16, (a) show that  $\mathbf{V}_o(\omega)/\mathbf{V}_s(\omega)$  is given by

$$\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{1 + j\omega R_1 C}{1 + j\omega RC} \cdot \frac{R}{R_1}$$

where  $R$  is the equivalent resistance for  $R_1$  and  $R_2$  in parallel. (b) Sketch  $|\mathbf{V}_o(\omega)/\mathbf{V}_s(\omega)|$  and determine the filter type of the above circuit. (c) Find the frequency  $\omega_1$  when the gain ratio  $20 \log H$  is 3 dB above its low-frequency value. By choosing reasonable values for  $R_1$ , and  $C$ , design a filter with  $\omega_1 = 10^4$  rad/s.

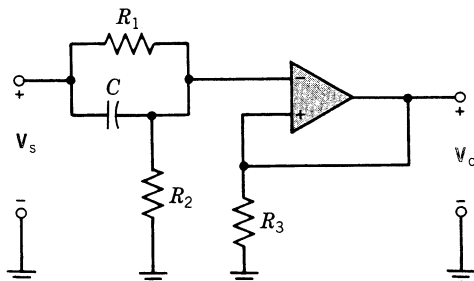


Figure P 13-16

**P 13-17** The frequency response of an unknown circuit is measured and drawn with the asymptotes for  $20 \log H$  as shown in Figure P 13-17. The phase is also

shown. Determine the equation for  $\mathbf{H}(j\omega)$ , indicating the corner frequencies.

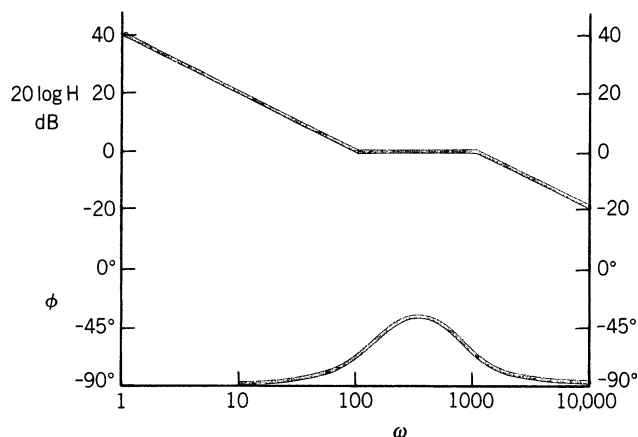


Figure P 13-17 The frequency response of an unknown circuit.

**P 13-18** A bandpass filter can be achieved using the circuit of Figure P 13-18. The advantage of this circuit is that we are not required to use inductors. Find (a) the magnitude of  $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_s$ , (b) the low- and high-frequency cutoff frequencies  $\omega_1$  and  $\omega_2$ , and (c) the peak gain in the passband.

**Answer:** (b)  $\omega_1 = \frac{1}{R_1 C_1}$

(c) midband gain =  $\frac{R_2}{R_1}$

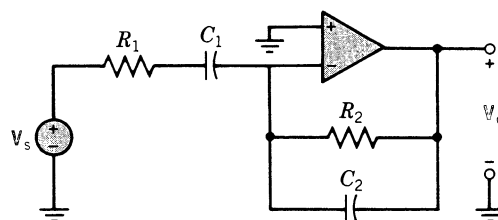


Figure P 13-18 A bandpass filter.

**P 13-19** For the  $RLC$  parallel resonant circuit, show that

$$Q = \omega_0 RC$$

by using the definition of  $Q$  expressed in terms of energy stored and dissipated.

**P 13-20** An amplifier uses two identical operational amplifier circuits as shown in Figure P 13-20. It is desired to set the cutoff or break frequency at  $\omega_c = 1000$  for this low-pass amplifier with a gain  $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_s$ . Also, it is desired that the low-frequency gain of  $\mathbf{H}$  be 0 dB. (a) Find the required  $R_1$ ,  $R_2$ , and  $C$ . (b) Find the rejection in dB for a signal at  $\omega = 10,000$ .

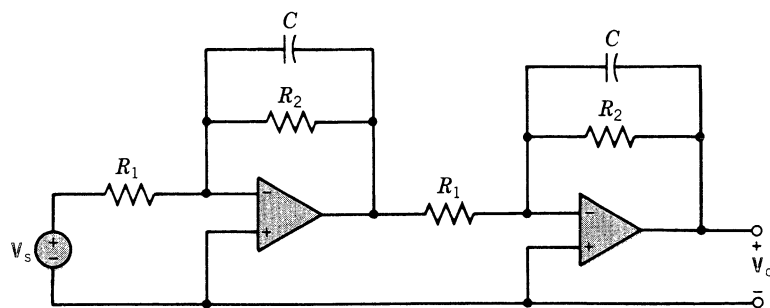


Figure P 13-20

**P 13-21** A series resonant  $RLC$  circuit has  $L = 10$  mH,  $C = .01 \mu\text{F}$ , and  $R = 100 \Omega$ . Determine  $\omega_0$ ,  $Q$ ,  $B$ , and the magnitude of the response at  $\omega = 1.1 \times 10^5$  rad/s.

**Answer:**  $\omega_0 = 10^5$ ,  $Q = 10$ ,  $B = 10^4$ ,  $H = 0.44$

**P 13-22** (a) Find  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_s$  of the phase shifter circuit shown in Figure P 13-22. (b) Sketch the Bode diagram of this circuit.

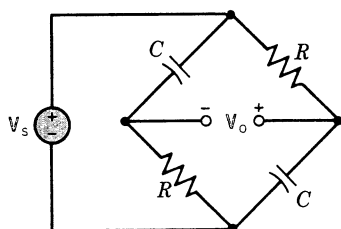


Figure P 13-22

**P 13-23** A bandpass filter can be obtained from the circuit shown in Figure P 13-23. (a) Find  $\mathbf{V}_o/\mathbf{V}_s(j\omega)$ . (b) Find the bandwidth  $B$  and resonant frequency  $\omega_0$ . Note the sign of the source. Assume an ideal operational amplifier.

**Answer:**  $B = \frac{C_1 + C_2}{R_2 C_1 C_2}$

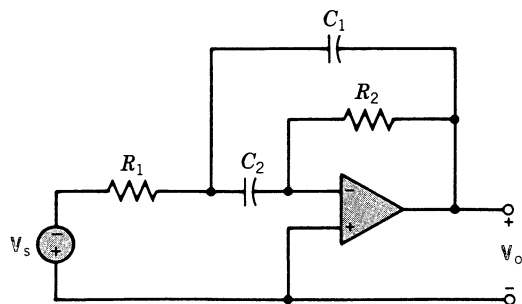


Figure P 13-23 A bandpass filter.

**P 13-24** The cochlear implant is intended for patients with deafness due to malfunction of the sensory

cells of the cochlea in the inner ear [Loeb]. These devices use a microphone for picking up the sound and a processor for converting to electrical signals, and they transmit these signals to the nervous system. A cochlear implant relies on the fact that many of the auditory nerve fibers remain intact in patients with this form of hearing loss. The overall transmission from microphone to nerve cells is represented by the gain function

$$\mathbf{H}(j\omega) = \frac{10(j\omega/50 + 1)}{(j\omega/2 + 1)(j\omega/20 + 1)(j\omega/80 + 1)}$$

Plot the Bode diagram for  $\mathbf{H}(j\omega)$  for  $1 \leq \omega \leq 100$ .

**P 13-25** A circuit called a bridged T network is shown in Figure P 13-25. The relationship for  $\mathbf{V}_o/\mathbf{V}_s$  is

$$\mathbf{H} = \frac{(j\omega/\omega_0)^2 + (j\omega/Q\omega_0) + 1}{(j\omega/\omega_1 + 1)(j\omega/\omega_2 + 1)}$$

A certain circuit is designed so that  $\omega_0 = 2500$ ,  $Q = 10/3$ ,  $\omega_1 = 370$ , and  $\omega_2 = 20,000$ . (a) Plot the Bode diagram of this circuit for  $\omega = 10$  to  $\omega = 10^5$ . (b) Describe the type of filter. (c) Find the band of frequencies for the circuit that result in an attenuation (gain reduction) of 20 dB or greater.

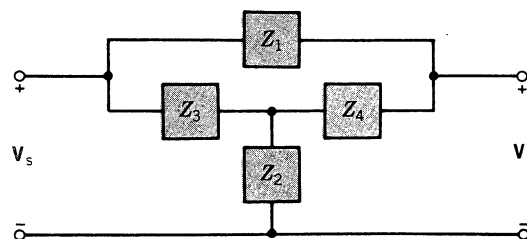


Figure P 13-25 A bridged T circuit.

**\*P 13-26** In the medical world the stethoscope remains the most practical instrument available for listening for sounds arising within the heart and lungs. A recently introduced electronic stethoscope uses 10 bandpass amplifiers to let the physician concentrate on specific sounds [Bak]. The gain characteristic of one



bandpass amplifier is represented by

$$\mathbf{H} = \frac{(j\omega/100 + 1)}{[1 + ju/10 + (ju)^2](j\omega/5000 + 1)}$$

where  $u = \omega/\omega_0$  and  $\omega_0 = 1000$ . (a) Draw the exact Bode diagram of this filter for  $100 \leq \omega \leq 10^4$ . (b) Find the bandpass bandwidth  $B$ . (c) Determine the  $Q$  of this circuit. (d) Find the resulting gain at  $\omega_0$ .

**P 13-27** An operational amplifier circuit is shown in Figure P 13-27, where  $R_2 = 5 \text{ k}\Omega$  and  $C = 0.02 \mu\text{F}$ . (a) What is the gain of the circuit,  $\mathbf{V}_o/\mathbf{V}_s$ , for  $\omega = 0$ ? (b) Find the expression for  $\mathbf{V}_o/\mathbf{V}_s(j\omega)$  and sketch the Bode diagram using asymptotic approximations. (c) At what frequency does  $|\mathbf{V}_o|$  fall to  $1/\sqrt{2}$  of its low-frequency value?

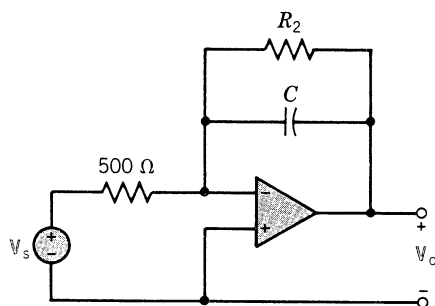


Figure P 13-27

**P 13-28** A simple voltage divider with two resistors does not work as we expect at higher frequencies because of a parasitic capacitance shunting  $R_2$ . To remedy the capacitance problem, another capacitance  $C_1$  is added in parallel with  $R_1$  as shown in Figure P 13-28. (a) Determine the relationship for  $R_1C_1$  and  $R_2C_2$  so that the output voltage is  $\mathbf{V}_o = k\mathbf{V}_s$  and is not a function of frequency. (b) Plot the gain magnitude curve  $V_o/kV_s$  (dB) for  $R_1 = R_2 = 10 \text{ k}\Omega$ ,  $C_2 = 0.1 \mu\text{F}$ , and three values of  $C_1$ :  $1 \mu\text{F}$ ,  $0.1 \mu\text{F}$ , and  $0.05 \mu\text{F}$ .

**Answer:** (a)  $R_1C_1 = R_2C_2$

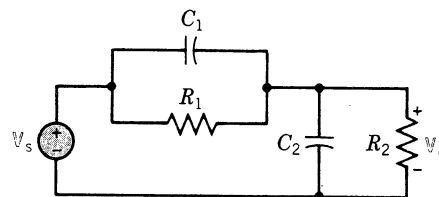


Figure P 13-28

**\*P 13-29** A bandpass filter can be achieved by using two operational amplifiers in the circuit shown in Figure P 13-29. (a) Assume the ideal model for the operational amplifiers and find  $\mathbf{H} = \mathbf{V}_o/\mathbf{V}_s$ . (b) Plot the Bode diagram for this circuit when  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 100 \Omega$ ,  $C_1 = 1 \mu\text{F}$ , and  $C_2 = 0.1 \mu\text{F}$ . (c) Find  $\omega_0$  and  $Q$  for the circuit. (d) Find the bandwidth of the circuit.

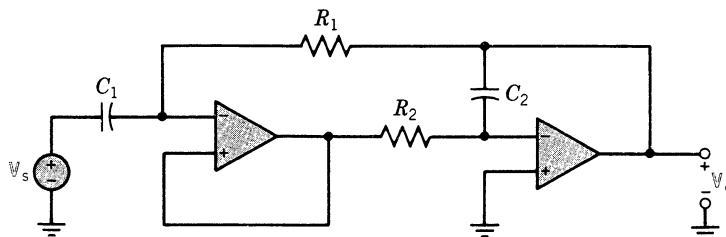


Figure P 13-29

**\*P 13-30** The magnitude and phase of a circuit are measured and the data is shown in Table P 13-30. Determine an estimate of  $\mathbf{H}(j\omega)$  when the peak magnitude occurs at  $\omega = 4.1$ .

**\*P 13-31** Consider the simple model of an electric power system as shown in Figure P 13-31. The inductance,  $L = 0.25 \text{ H}$ , represents the power line and transformer. The customer's load is  $R_L = 100 \Omega$  and the customer adds  $C = 25 \mu\text{F}$  to increase the magnitude of  $\mathbf{V}_o$ . The source is  $\mathbf{V}_s = 1000 \cos 400t \text{ V}$ , and it is desired that  $|\mathbf{V}_o|$  also be  $1000 \text{ V}$ . (a) Find  $|\mathbf{V}_o|$  for  $R_L = 100 \Omega$ . (b) When the customer leaves for the night, he turns off much of his load, making  $R_L = 1 \text{ k}\Omega$ , at which point sparks and smoke begin to appear in the equipment still connected to the power line. The customer calls you in as a consultant. Why did the sparks appear when  $R_L = 1 \text{ k}\Omega$ ? (c) If  $100 \leq R_L \leq 1000 \Omega$ , obtain a formula that will determine the appropriate  $C$  so that  $|\mathbf{V}_o|$  is always  $1000 \text{ V}$ .

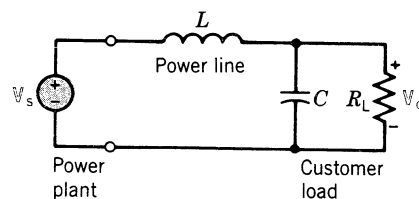
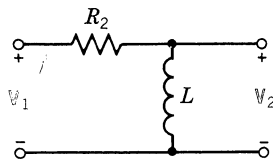
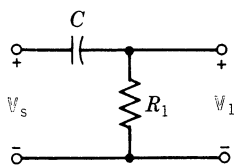
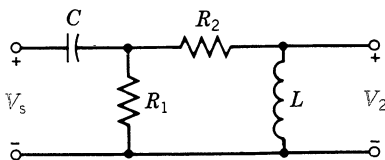


Figure P 13-31 Model of an electric power system.

Table P 13-30

$\omega$	0.1	1	2	3	4	4.1	5	8	100
$ \mathbf{H} $	0.01	0.12	0.29	0.60	1.0	2.2	0.78	0.32	0.02
$\phi$	$90^\circ$	$83^\circ$	$73^\circ$	$53^\circ$	$7^\circ$	$0^\circ$	$-39^\circ$	$-71^\circ$	$-89^\circ$

\*P 13-32 (a) For the circuit of Figure P 13-32a derive an expression for the magnitude of the gain function  $|\mathbf{H}_1(j\omega)| = |\mathbf{V}_1/\mathbf{V}_s|$  and sketch it as a function of frequency  $\omega$ . (Include values  $\omega = 0$ ,  $\omega = \omega_1$ ,  $\omega = \infty$ .) Assume the break frequency is  $\omega_1$ . (b) For the circuit of Figure P 13-32b derive an expression for the magnitude of the gain function  $|\mathbf{H}_2(j\omega)| = |\mathbf{V}_2/\mathbf{V}_1|$  and sketch it versus frequency  $\omega$  on the same graph as  $|\mathbf{H}_1(\omega)|$ . (Assume  $\omega_2 < \omega_1$  and include value of  $\omega = 0$ ,  $\omega = \omega_2$ ,  $\omega = \infty$ .) (c) Each of the above filters is known as a first-order filter. Create a second-order filter by obtaining an expression for  $|\mathbf{H}(\omega)| = |\mathbf{H}_1(\omega) \mathbf{H}_2(\omega)|$ . Draw a rough sketch of  $|\mathbf{H}(\omega)|$  versus  $\omega$ . Use the same graph page as above. Do not use your expression for  $|\mathbf{H}(\omega)|$ , but instead think about the plots from parts a and b and why the plot of  $|\mathbf{H}(\omega)|$  versus  $\omega$  for this second-order filter has a point of inflection. (d) Can you think of any reason why the circuit of Figure P 13-32d will not perform in the same way as the circuit described by  $\mathbf{H}(\omega)$  in part c? Hint: Look at the impedance to the right of the capacitor.

Figure P 13-32a Circuit for  $\mathbf{H}_1$ .Figure P 13-32b Circuit for  $\mathbf{H}_2$ .Figure P 13-32c Circuit for  $\mathbf{H}$ .

P 13-33 A quartz crystal exhibits the property that when mechanical stress is applied across its faces, a potential difference develops across opposite faces.

When an alternating voltage is applied, mechanical vibrations occur and electromechanical resonance is exhibited. A crystal can be represented by a series  $RLC$  circuit. A specific crystal has a model with  $L = 1$  mH,  $C = 10$   $\mu$ F, and  $R = 1$   $\Omega$ . Find  $\omega_0$ ,  $Q$ , and the bandwidth.

**Answer:**  $\omega_0 = 10^4$  rad/s,  $Q = 10$ ,  $B = 10^3$  rad/s

P 13-34 An  $R, L, C$  parallel resonant circuit may be represented by the equation

$$\mathbf{H} = \frac{1}{R\mathbf{Y}} = \frac{(\omega_0/Q)s}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

where  $s = j\omega$ . One can use a computer program to obtain a plot of  $H$  versus  $\omega$  as shown in Figure P 13-34 for  $Q = 10, 20$ , and  $50$ . Use a computer program to plot  $H$  versus  $\omega$  for  $Q = 5, 30$ , and  $100$ .

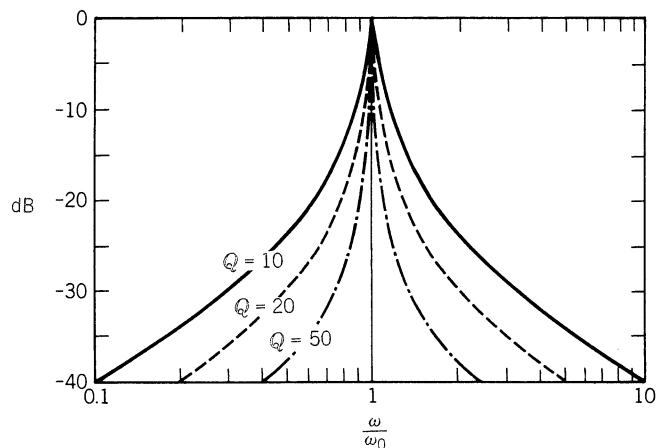


Figure P 13-34 The plot of  $\mathbf{H}$  versus  $\omega$  for  $Q = 10, 20$ , and  $50$ .

\*P 13-35 Sketch the magnitude  $|\mathbf{H}(\omega)|$  and the phase versus frequency for the circuit in Figure P 13-35 when  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_1$ ,  $C = 0.1$   $\mu$ F, and  $R = 1$  k $\Omega$ .

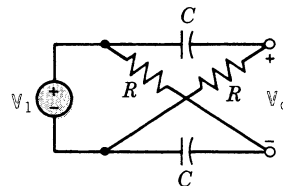


Figure P 13-35

**P 13-36** A circuit  $\mathbf{Z}(j\omega)$  has the Bode magnitude diagram shown in Figure P 13-36. (a) Find  $\mathbf{Z}(j\omega)$ . (b) What is the magnitude of  $\mathbf{Z}(j\omega)$  (in dB) as  $\omega$  approaches infinity?

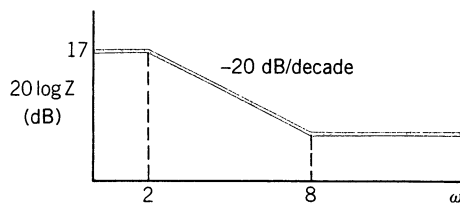


Figure P 13-36

**\*P 13-37** Find the gain function  $\mathbf{H}(j\omega)$  for the circuit of Figure P 13-37. Sketch the Bode diagram for the circuit.

**Answer:**  $\mathbf{H} = \frac{1 - j(\omega/\omega_0)}{1 + j(\omega/\omega_0)}$ ,  $\omega_0 = \frac{1}{RC}$

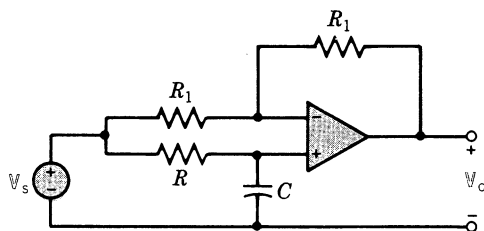


Figure P 13-37

**\*P 13-38** A unity gain, low-pass filter is obtained from the operational amplifier circuit shown in Figure P 13-38. Determine the gain function  $\mathbf{V}_o/\mathbf{V}_s = \mathbf{H}$  and sketch the Bode diagram.

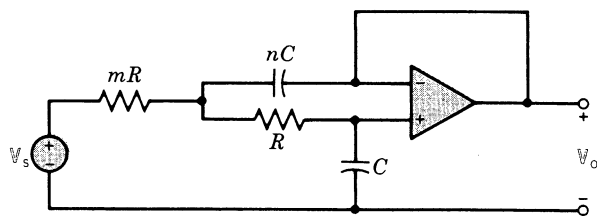


Figure P 13-38 A low-pass filter.

**\*P 13-39** The circuit shown in Figure P 13-39 has a device that can be represented by a negative resistor so that  $R = -20 \text{ k}\Omega$ . (a) Find the admittance at terminals a-b. (b) Plot the Bode diagram for the admittance of the circuit.

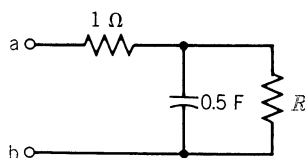


Figure P 13-39

**P 13-40** Tissue electrodes are used by physicians to form the interface that conducts current to the target tissue of the human body. The electrode in tissue can be modeled by the  $RC$  circuit shown in Figure P 13-40. The value of each element depends on the electrode material and physical construction as well as the character of the tissue being probed. Find the Bode diagram for  $\mathbf{V}_o/\mathbf{V}_s = \mathbf{H}(j\omega)$  when  $R_1 = 1 \text{ k}\Omega$ ,  $C = 1 \mu\text{F}$ , and the tissue resistance is  $R_t = 5 \text{ k}\Omega$ .

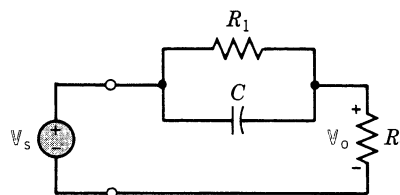


Figure P 13-40 Model of tissue electrode and target tissue.

**P 13-41** Plot the magnitude and phase of  $\mathbf{H}(j\omega)$  on a Bode diagram for the circuit shown in Figure P 13-41 where  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_1$ .

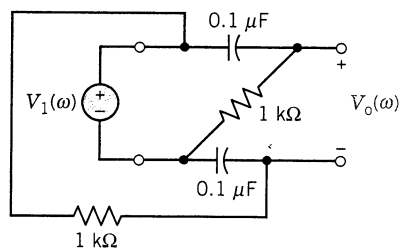


Figure P 13-41

**P 13-42** The resistance  $R$  and capacitance  $C$  of the load can be varied as shown in Figure P 13-42. The sinusoidal source operates at  $1 \text{ MHz}$  and it is desired to deliver maximum power to the load. Select  $R$  and  $C$  for maximum power transfer and calculate the  $Q$  of the total circuit.

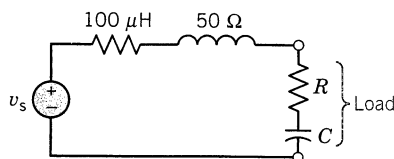


Figure P 13-42

**P 13-43** A low-pass filter has a gain function

$$H(s) = \frac{100}{s^2 + 10s + 100}$$

Plot  $20 \log |H(j\omega)|$  and determine the bandwidth of the filter.

P 13-44 A high-pass filter has a gain function

$$H(s) = \frac{10s^2}{s^2 + 1000s + 10^8}$$

Plot  $20 \log|H(j\omega)|$  and determine the maximum value of  $20 \log|H(\omega)|$  and the frequency at which it occurs.

P 13-45 An acoustic sensor operates in the range of 5 kHz and is represented by  $v_s$  in Figure P 13-45. It is specified that the bandpass filter shown in the figure pass an input signal within the specified bandwidth. Determine the bandwidth and center frequency of the circuit when the op amp has  $R_i = 500 \text{ k}\Omega$ ,  $R_o = 1 \text{ k}\Omega$ , and  $A = 10^6$ .

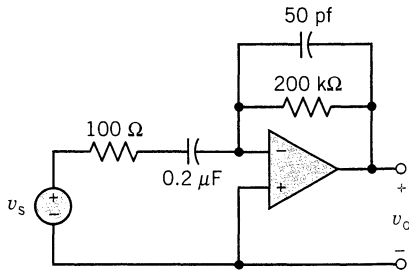


Figure P 13-45

P 13-47 Determine the voltage gain and the cutoff frequency for the high-pass filter shown in Figure P 13-47. Draw the magnitude portion of the Bode diagram for (a)  $C = 50 \text{ nF}$  and  $R = 4 \text{ k}\Omega$  and (b)  $C = 100 \text{ nF}$  and  $R = 2 \text{ k}\Omega$ .

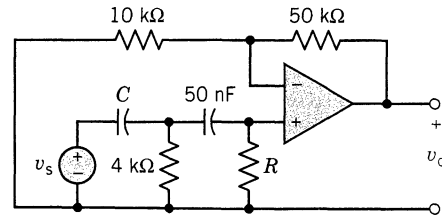


Figure P 13-47

P 13-48 Determine  $\mathbf{H}(j\omega)$  from the asymptotic Bode diagram in Figure P 13-48.

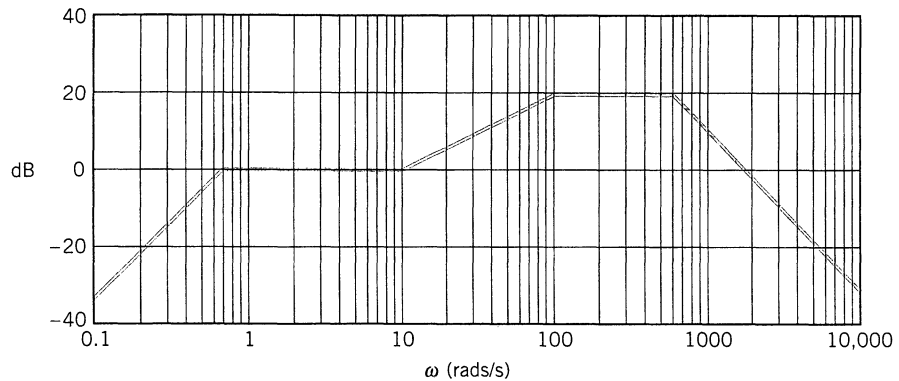


Figure P 13-48

P 13-46 Calculate the high and low cutoff frequencies for the bandpass filter shown in Figure P 13-46 when  $R_1 = R_2 = 20 \text{ k}\Omega$ ,  $C_1 = 0.2 \text{ }\mu\text{F}$ , and  $C_2 = 1 \text{ nF}$ . Determine the circuit bandwidth. Assume ideal op amps.

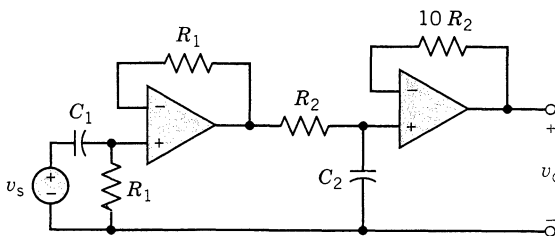


Figure P 13-46

P 13-49 A noninverting amplifier, as shown in Figure 6-18, has  $R_2 = 100 \text{ k}\Omega$ ,  $R_1 = 1 \text{ k}\Omega$ ,  $A_o = 10^5$ , and  $\omega_1 = 10 \text{ rad/sec}$ . Plot (a) the frequency response of the op amp itself and (b) the frequency response of the noninverting amplifier circuit. What is the gain-bandwidth product of this circuit?

P 13-50 A circuit is represented by

$$H(s) = \frac{0.1(1 + j\omega)}{1 + j\omega\tau}$$

where  $s = j\omega$  and  $\tau = 0.1$ . Plot the Bode diagram.

P 13-51 Determine the gain ratio  $H(s)$  for the op amp circuit shown in Figure P 13-51 and plot the Bode diagram. Assume ideal op amps.

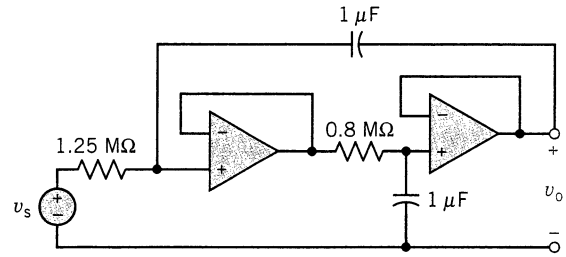


Figure P 13-51

## ADVANCED PROBLEMS

AP 13-1 Determine (a) the resonant frequency of the circuit shown in Figure AP 13-1 and select  $C$  and  $L$  to obtain  $\omega_o = 100$  rad/s when  $R_1 = R_2 = 1 \Omega$ .

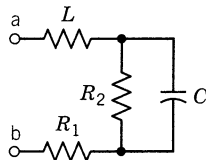


Figure AP 13-1

AP 13-2 An op amp circuit is shown in Figure AP 13-2. Determine  $H(s)$  when  $a > 10$  and the op amp is ideal. Select  $a$ ,  $R$ , and  $C$  so that the resonant gain is 201 and the resonant frequency is  $10^5$  rad/s. Draw the resulting Bode diagram.

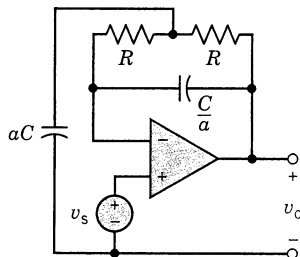


Figure AP 13-2

AP 13-3 The circuit shown in Figure AP 13-3 represents a capacitor, coil, and resistor in parallel. Calculate the resonant frequency, bandwidth, and  $Q$  for the circuit.

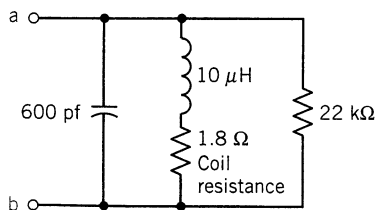


Figure AP 13-3

AP 13-4 A low-pass filter is shown in Figure AP 13-4.

- Assume an ideal op amp and find the voltage ratio  $\mathbf{V}_o/\mathbf{V}_s$ .
- Sketch the Bode plot and find the cutoff frequency when  $C = 20$  nF,  $R = 1.2$  kΩ, and  $R_1/R_2 = 1$ .

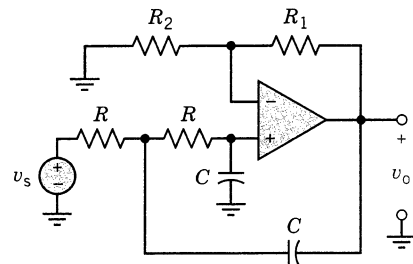


Figure AP 13-4 Low-pass filter.

AP 13-5 Two cascade amplifiers are used to obtain an output voltage as shown in Figure AP 13-5. Plot the magnitude portion of the Bode diagram of the overall gain  $\mathbf{V}_o/\mathbf{V}_s$ . Each amplifier is of the form

$$H(s) = \frac{As}{(1 + s/\omega_L)(1 + s/\omega_h)}$$

where  $s = j\omega$ .

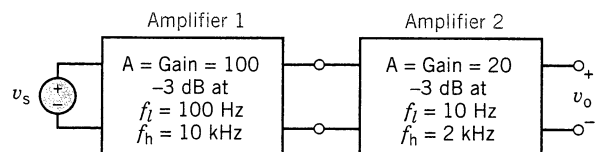


Figure AP 13-5 Two cascaded amplifiers.

AP 13-6 A filter with

$$H(s) = \frac{1}{s + 1}$$

and

$$|H(\omega)| = \frac{1}{(\omega^2 + 1)^{1/2}}$$

is called a Butterworth filter. The general  $n$ th-order Butterworth filter has

$$|H(\omega)| = \frac{1}{(\omega^{2n} + 1)^{1/2}}$$

where  $\omega = 1$  is the nominal cutoff frequency for the filter. Determine the second- and fourth-order Butterworth filters and plot  $20 \log |H|$  for  $n = 1, 2$ , and  $3$ . Describe the improvement achieved by higher  $n$  filters by determining  $|H(\omega)|$  at  $\omega = 2$ .

**AP 13-7** The filter circuit shown in Figure AP 13-7 has  $\mathbf{H}(j\omega) = \mathbf{V}_o/\mathbf{V}_s$ . The circuit is to pass a 20-kHz signal while rejecting a 30-kHz signal.

- Select an appropriate value of  $L$ .
- Choose an appropriate inductive or capacitance element and its value for the parallel admittance  $\mathbf{Y}_1$ . Assume that  $R_L > 100 \Omega$ .

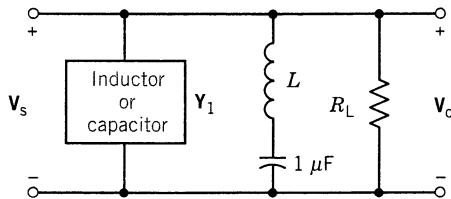


Figure AP 13-7

**AP 13-8** Repeat Advanced Problem 13-7 when the filter is to reject the 30-kHz signal (as in AP 13-7) but is to pass a 40-kHz signal.

**AP 13-9** A circuit used to reject two unwanted signal frequencies is shown in Figure AP 13-9. Sketch the magnitude of the frequency response of the circuit and determine the two frequencies rejected by the circuit.

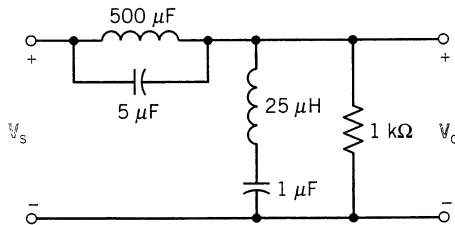


Figure AP 13-9

**AP 13-10** The circuit shown in Figure AP 13-10 has an input signal

$$v_s = 10 \sin \omega_1 t + 10 \sin \omega_2 t \text{ V}$$

where  $\omega_1 = 60,000\pi$  and  $\omega_2 = 120,000\pi$ .

- Determine the value of  $C$  and the type and component value of the element of  $\mathbf{Z}_1$  so that the circuit rejects the 30-kHz signal and transmits the 60-kHz signal.
- For the values in part (a), find the output signal.
- Obtain the frequency response of the circuit.

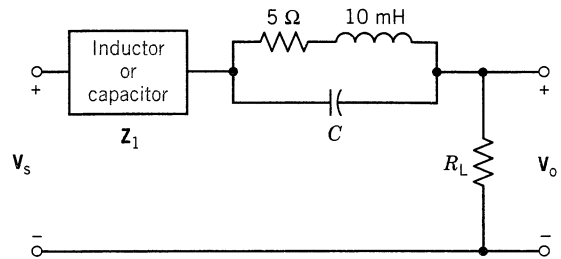


Figure AP 13-10

**AP 13-11** A filter circuit is shown in Figure AP 13-11. This filter circuit is called a double tuned circuit since it may reject a signal at one frequency and pass a signal at another frequency. Assume that the inductor  $L_s = 100 \text{ mH}$  and that the unspecified element is a capacitor or an inductor.

- Select the unknown element and design a double tuned filter to reject a signal at 50 kHz while passing a signal at 100 kHz. Determine the necessary component values.
- Repeat (a) if the rejection frequency is 100 kHz and the pass frequency is 50 kHz.

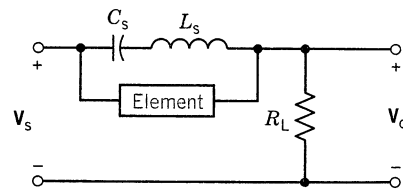


Figure AP 13-11

## DESIGN PROBLEMS

**DP 13-1** Frequently, audio systems contain two or more loudspeakers that are intended to handle different parts of the audio-frequency spectrum. In a three-way setup, one speaker, called a woofer, handles low

frequencies. A second, the tweeter, handles high frequencies, while a third, the midrange, handles the middle range of the audio spectrum.

A three-way filter, called a crossover network, is

used to split the audio signal into the three bands of frequencies suitable for each speaker. There are many and varied designs. A simple one is based on series  $LR$ ,  $CR$ , and resonant  $RLC$  circuits as shown in Figure DP 13-1. All speaker impedances are assumed resistive. The conditions are (1) woofer, at the crossover frequency  $X_{L1} = R_w$ ; (2) tweeter, at the crossover frequency  $X_{C3} = R_t$ ; (3) midrange, components  $C_2$ ,  $L_2$ , and  $R_{MR}$  form a series resonant circuit with upper and lower cutoff frequencies  $f_u$  and  $f_l$ , respectively. The resonant frequency  $= (f_u f_l)^{1/2}$ .

When all the speaker resistances are  $8\ \Omega$ , determine the frequency response and determine the cutoff frequencies. Plot the Bode diagram for the three speakers. Determine the bandwidth of the midrange speaker section.

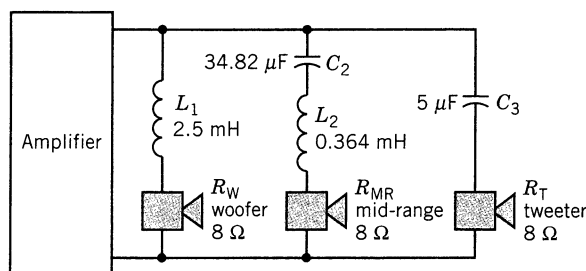


Figure DP 13-1 Three-way filter for a speaker system.

**DP 13-2** Determine  $L$  and  $C$  for the circuit of Figure DP 13-2 in order to obtain a low-pass filter with a magnitude of  $-3\text{ dB}$  at  $100\text{ kHz}$ .

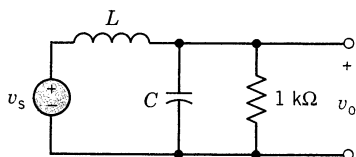


Figure DP 13-2

**DP 13-3** British Rail has constructed an instrumented rail car that can be pulled over its tracks at speeds up to  $180\text{ km/hr}$  and will measure the track-grade geometry. Using such a rail car, British Rail can monitor and track gradual degradation of the rail grade, especially the banking of curves, and permit preventive maintenance to be scheduled as needed well in advance of track-grade failure.

The instrumented rail car has numerous sensors, such as angular-rate sensors (devices that output a signal proportional to rate of rotation) and accelerometers (devices that output a signal proportional to acceleration), whose signals are filtered and combined in a fashion to create a composite sensor called a “com-

pensated accelerometer” (Lewis, 1988). A component of this composite sensor signal is obtained by integrating and high-pass filtering an accelerometer signal. A first-order low-pass filter will approximate an integrator at frequencies well above the break frequency. This can be seen by computing the phase shift of the filter-transfer function at various frequencies. At sufficiently high frequencies, the phase shift will approach  $90^\circ$ , the phase characteristic of an integrator.

A circuit has been proposed to filter the accelerometer signal, as shown in Figure DP 13-3. The circuit is comprised of three sections, labeled A, B, and C. For each section, find an expression for and name the function performed by that section. Then find an expression for the gain function of the entire circuit,  $V_o/V_s$ . For the component values, evaluate the magnitude and phase of the circuit response at  $0.01$ ,  $0.02$ ,  $0.05$ ,  $0.1$ ,  $0.2$ ,  $0.5$ ,  $1.0$ ,  $2.0$ ,  $5.0$ , and  $10.0\text{ Hz}$ . Draw a Bode diagram. At what frequency is the phase response approximately equal to  $0^\circ$ ? What is the significance of this frequency?

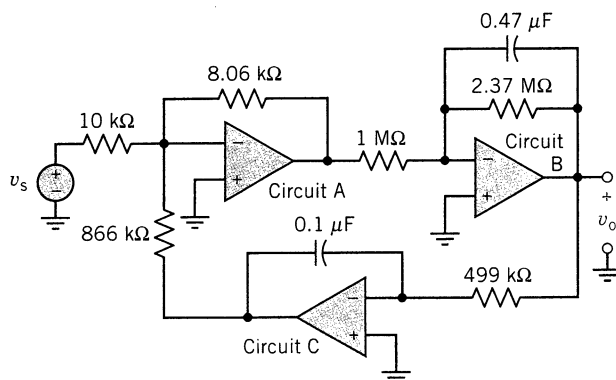


Figure DP 13-3 Accelerometer filter circuit.

**DP 13-4** Design a bandpass filter with a center frequency of  $100\text{ kHz}$  and a bandwidth of  $10\text{ kHz}$  using the circuit shown in Figure DP 13-4. Assume that  $C = 100\text{ pF}$  and find  $R$  and  $R_3$  when the nonideal op amp has  $R_i = 100\text{ k}\Omega$ ,  $R_o = 100\ \Omega$ , and  $A = 10^5$ . Use PSpice to verify the design.

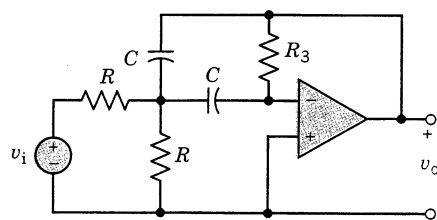


Figure DP 13-4

**DP 13-5** Strain-sensing instruments can be used to measure orientation and magnitude of strains running in more than one direction. The search for a way to predict earthquakes focuses on identifying precursors, or changes, in the ground that reliably warn of an impending event. Because so few earthquakes have occurred precisely at instrumented locations, it has been a slow and frustrating quest. Laboratory studies show that before rock actually ruptures—precipitating an earthquake—its rate of internal strain increases. The material starts to fail before it actually breaks. This prelude to outright fracture is called “tertiary creep” (Brown, 1989).

The frequency of strain signals varies from 0.1 to 100 rad/s. A bandpass filter is used to pass these frequencies, so that

$$H(s) = \frac{Ks}{(s+1) \left( \frac{s}{\omega_1} + 1 \right)}$$

where  $s = j\omega$ . Plot the Bode diagram and select  $\omega_1$  and  $K$  so that the maximum gain is 20 dB at  $\omega = 3.16$  rad/sec.

**DP 13-6** A communication transmitter requires a bandpass filter to eliminate low-frequency noise from nearby traffic. Measurements indicate that the range of traffic rumble is  $2 < \omega < 12$  rad/s. A designer proposes a filter as

$$H(s) = \frac{(1 + s/\omega_1)^2(1 + s/\omega_3)}{(1 + s/\omega_2)^3}$$

where  $s = j\omega$ .

It is desired that signals with  $\omega > 100$  rad/s pass with less than 3 dB loss while the traffic rumble be reduced by 46 dB or more. Select  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  and plot the Bode diagram.

**DP 13-7** It is desired to obtain a low-pass filter using an op amp configuration of the form shown in Figure 11-44. The goal is to achieve an overall gain of 20 dB and a bandwidth of  $10^4$  rad/sec. Thus, the desired overall response is

$$H(s) = \frac{10}{1 + (s/\omega_1)^n}$$

where  $s = j\omega$ ,  $\omega_1 = 10^4$ , and  $n$  = number of op amp cascaded circuit stages. Design the op amp circuit required for  $n = 1$ ,  $n = 2$ , and  $n = 3$  stages. Plot the magnitude plot (in dB) for the Bode diagram and determine the gain at  $\omega = 4 \times 10^4$  rad/sec. The goal is to have the three-stage circuit more closely approximate the ideal filter characteristic. Discuss the results of your design.

**DP 13-8** A communication transmitter requires a bandstop filter to eliminate low-frequency noise from nearby auto traffic. Measurements indicate that the range of traffic rumble is  $2 < \omega < 12$  rad/s. A designer proposes a filter as

$$H(s) = \frac{(1 + s/\omega_1)^2(1 + s/\omega_3)^2}{(1 + s/\omega_2)^2(1 + s/\omega_4)^2}$$

where  $s = j\omega$ . It is desired that signals above 130 rad/s pass with less than 4 dB loss while the traffic rumble be reduced by 35 dB or more. Select  $\omega_1$ ,  $\omega_2$ ,  $\omega_3$ , and  $\omega_4$  and plot the Bode diagram.



## PSpice PROBLEMS

**SP 13-1** Obtain the Bode diagram for Problem 13-3.

**SP 13-2** Obtain the Bode diagram for Problem 13-5.

**SP 13-3** Obtain the Bode diagram for Problem 13-24.

**SP 13-4** Obtain the Bode diagram for Problem 13-26.

**SP 13-5** Obtain the Bode diagram for Problem 13-41 using PSpice.

**SP 13-6** The frequency response of an actual op amp is dependent on a gain  $A(j\omega)$  which is a function of

frequency. One model of a frequency-dependent op amp circuit is shown in Figure SP 13-6, where  $R_1$  and  $R_2$  are used to establish an inverting amplifier with  $R_2 = 10R_1$ . Assume that the op amp has  $R_i = 1 \text{ M}\Omega$ ,  $R_o = 100 \Omega$ , and  $A = 10^6$  with  $R$  and  $C$  establishing the bandwidth of the op amp itself. The op amp has a gain-bandwidth product equal to  $Af = 10^9$  where  $f$  is the 3 dB bandwidth in Hz. Select the appropriate  $R$  and  $C$  to fit the model and use PSpice to obtain the Bode diagram for the circuit.



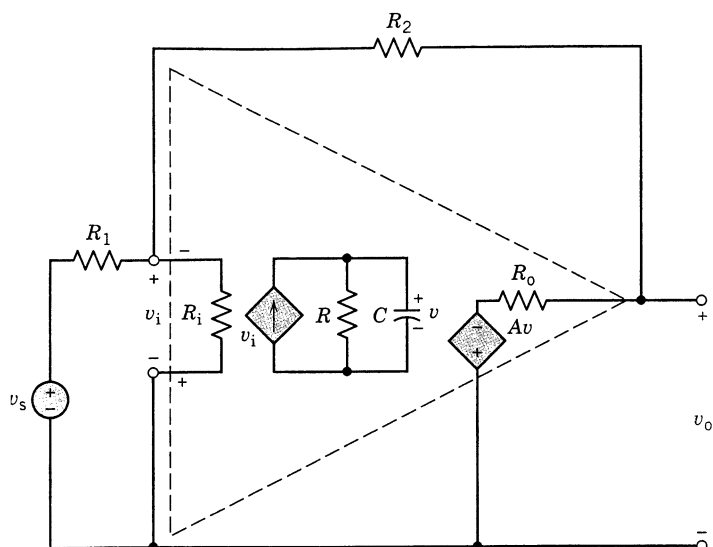


Figure SP 13-6 Bandpass filter.

**SP 13-7** Use PSpice to obtain the frequency response of the crossover network of Design Problem 13-1 when all of the speaker impedances are  $8\ \Omega$ ,  $L_1 = 2.5\ \text{mH}$ ,  $L_2 = 364\ \mu\text{H}$ ,  $C_2 = 13.76\ \mu\text{F}$ , and  $C_3 = 5\ \mu\text{F}$ . Verify that the three speakers handle different parts of the audio-system bandwidth.

**SP 13-8** Obtain the Bode diagram for  $\mathbf{V}/\mathbf{I}$  of the circuit shown in Figure SP 13-8. Determine the bandwidth of the circuit.

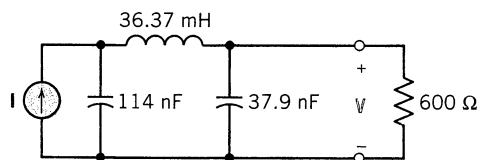


Figure SP 13-8

**SP 13-9** The model of an amplifier for a phonograph stereo is shown in Figure SP 13-9. Plot the dB magnitude portion of the Bode diagram for  $\mathbf{V}_o/\mathbf{V}_s$  for 20 Hz to 20 kHz.

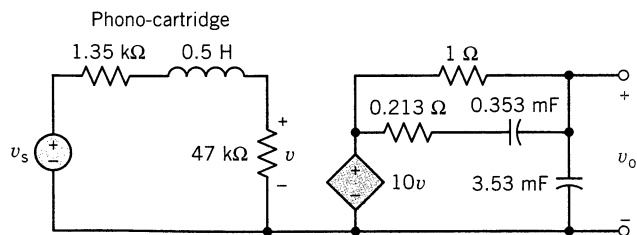


Figure SP 13-9

**SP 13-10** A low-pass filter is shown in Figure SP 13-10. The output of a two-stage filter is  $v_1$  and the output of a three-stage filter is  $v_2$ . Plot the Bode diagram of  $\mathbf{V}_1/\mathbf{V}_s$  and  $\mathbf{V}_2/\mathbf{V}_s$  and compare the results when  $L = 10\ \text{mH}$  and  $C = 1\ \mu\text{F}$ .

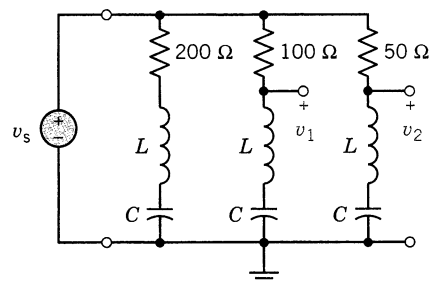


Figure SP 13-10

**SP 13-11** Use PSpice to verify the design obtained for Design Problem 13-2 by plotting the dB versus frequency for  $10\ \text{kHz} < f < 1\ \text{MHz}$ .

**SP 13-12** An acoustic sensor operates in the range of 5 kHz to 25 kHz and is represented in Figure SP 13-12 by  $v_s$ . It is specified that the bandpass filter shown in the figure pass the signal in the frequency range within 3 dB of the center frequency gain. Determine the bandwidth and center frequency of the circuit when the op amp has  $R_i = 500\ \text{k}\Omega$ ,  $R_o = 1\ \text{k}\Omega$ , and  $A = 10^6$ .

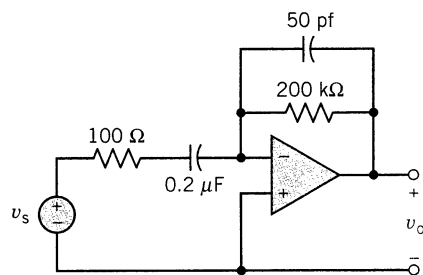


Figure SP 13-12

**SP 13-13** A circuit with a transformer is shown in Figure SP 13-13. The input current  $i$  is a 1-mA sinusoidal signal. Determine and plot the magnitude of the output signal,  $v$ , for a frequency range between 0.94 MHz and 1.06 MHz. This circuit is designed to pass a signal between 0.98 MHz and 1.02 MHz and to severely reject signals at  $(1.00 \pm 0.05)\ \text{MHz}$ . Discuss the performance of the circuit.

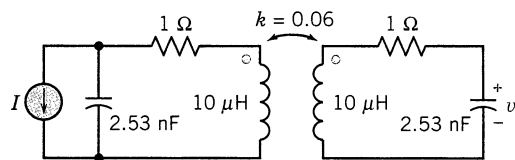


Figure SP 13-13

**SP 13-14** A series connection consists of a resistance of  $33\ \text{k}\Omega$ , a capacitance of  $4.7\ \text{nF}$  and a voltage source  $v_s = 1 \sin \omega t$ . Use PSpice to obtain the Bode plot for 10 Hz to 100 kHz.