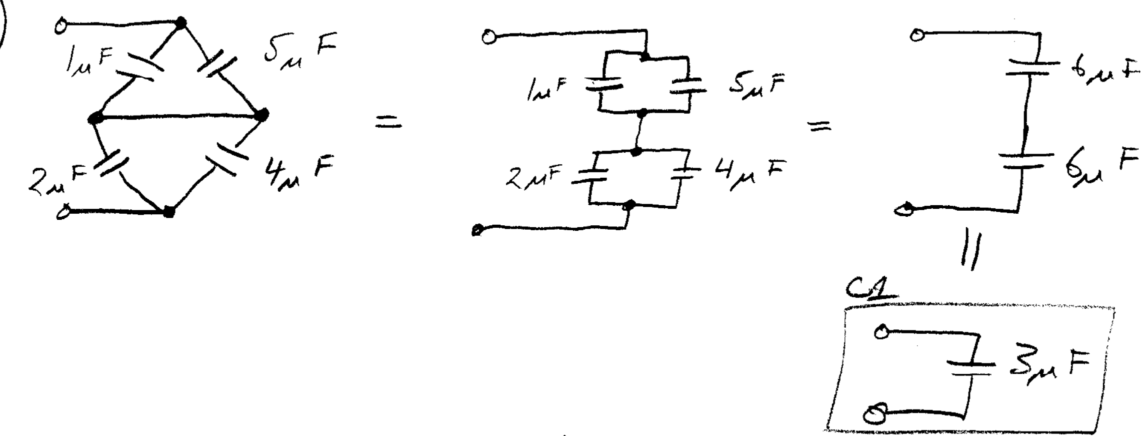
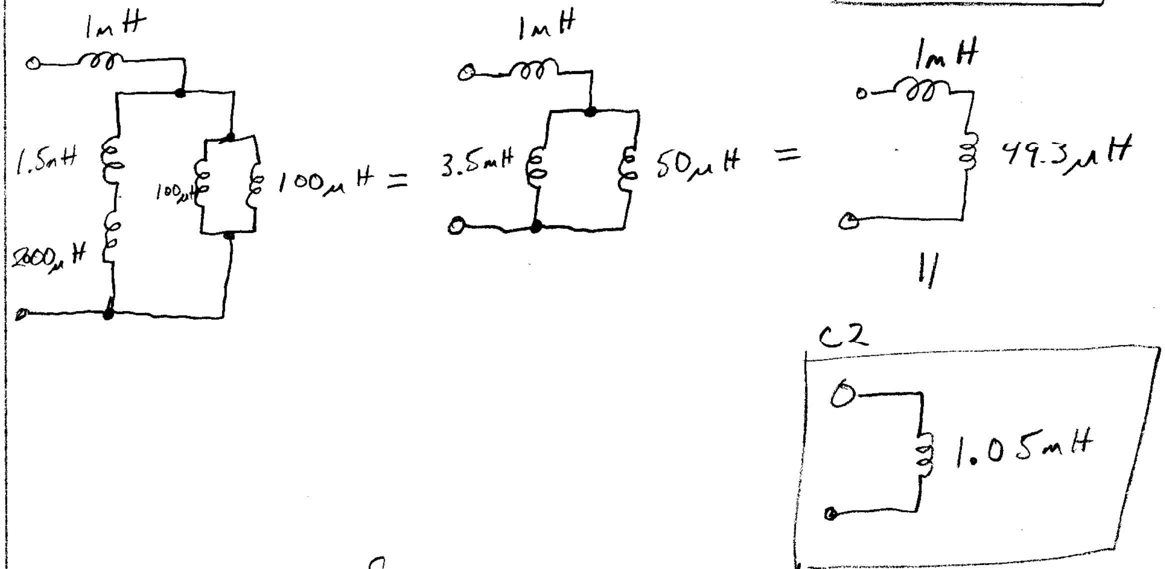
ENGR 210
F03 HW10

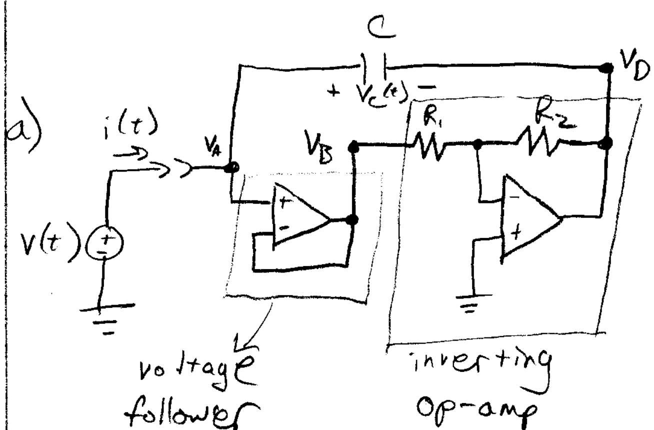
SOLUTIONS

1) 6-36)





2) 6-48) a)



$V_B = V_A = v(t)$

$V_D = -\frac{R_2}{R_1} V_B = -\frac{R_2}{R_1} V_A = -\frac{R_2}{R_1} v(t)$

$V_c = V_A - V_D$

$= v(t) + \frac{R_2}{R_1} v(t)$

$= v(t)\left[1 + \frac{R_2}{R_1}\right]$

$$i(t) = C \frac{d}{dt} V_c(t) = C \frac{d}{dt}\left[v(t)\left(1 + \frac{R_2}{R_1}\right)\right] = C\left(1 + \frac{R_2}{R_1}\right)\frac{d}{dt} v(t)$$

$\boxed{i(t) = C_{EQ} \frac{d}{dt} v(t) \quad \text{where } C_{EQ} = \left(1 + \frac{R_2}{R_1}\right) C}$ q.e.d.

b) $v_C(0) = v(0)\left(1+\frac{R_2}{R_1}\right) = V_0$

$$\boxed{v(0) = \frac{V_0}{1+\frac{R_2}{R_1}}} = \frac{R_1 V_0}{R_1+R_2}$$

c) $w_C(0) = \frac{1}{2} C v_C^2(0) = W_0$

$$\frac{1}{2} C v^2(0)\left(1+\frac{R_2}{R_1}\right)^2 = W_0$$

want $W_{eq}(0) = \frac{1}{2} C_{EQ} v^2(0)$

$$v^2(0) = \frac{2W_0}{C\left(1+\frac{R_2}{R_1}\right)^2}$$

$$W_{EQ}(0) = \frac{1}{2} C_{EQ} v^2(0)$$

$$= \frac{1}{2}\cancel{\left(1+\frac{R_2}{R_1}\right)}\cancel{C} \cdot \frac{\cancel{2}W_0}{\cancel{C}\left(1+\frac{R_2}{R_1}\right)^{\cancel{2}}}$$

$$\boxed{W_{EQ}(0) = \frac{W_0}{1+\frac{R_2}{R_1}}} = \frac{R_1 W_0}{R_1+R_2}$$

3
7-2) $10^{-4}\frac{d}{dt} i(t) + 10^{-1} i(t) = 0 \qquad i(0) = -20\,mA$

$10^{-3}\frac{d}{dt} i(t) + i(t) = 0$

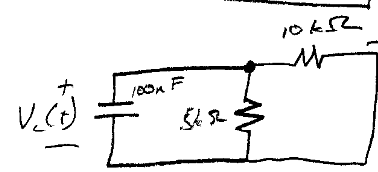
Natural response: $i_N(t) = Ke^{-t\cdot 1000}$

Forced response: $i_F(t) = 0$

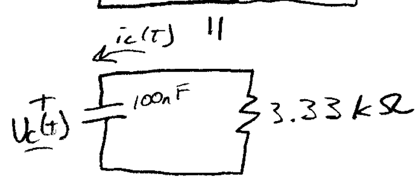
$$i(0) = Ke^0 = K = -20\,mA$$

$$\boxed{i(t) = -20\,mA \cdot e^{-1000t}}$$

4
7-5) after $t=0$:



$$i_0(t) = \frac{V_c(t)}{10{,}000\,\Omega}$$



$$i_c(t) = C\frac{d}{dt} v_c(t) = -\frac{V_c(t)}{3.33k\Omega}$$

$$CR_{eq}\frac{d}{dt} V_c(t) + V_c(t) = 0$$

$$V_c(t) = Ke^{-\frac{t}{CR_{eq}}} = Ke^{-3000t}$$

since $v_c(0) = 15V$

$$\boxed{v_c(t) = 15V \cdot e^{-3000t}}$$

$$i_o(t) = \frac{15V}{10k\Omega} e^{-3000t}$$

$$\boxed{i_o(t) = 1.5mA \cdot e^{-3000t}}$$

7-5-10) $v_s(t) = v_L(t) + v_o(t)$ $\qquad v_o(t) = Ri(t)$

since inductor, easier to solve for $i(t)$

since step function, easier to use initial and final conditions

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-\frac{t}{T_c}} \qquad T_c = \frac{L}{R}$$

initially zero-state: $i_L(0) = 0$

as $t \to \infty$ inductor acts like short, so:

$$i(\infty) = \frac{V_A}{R}$$

$$i(t) = \frac{V_A}{R} - \frac{V_A}{R} e^{-\frac{t}{L/R}}$$

$$i(t) = \frac{V_A}{R} - \frac{V_A}{R} e^{-\frac{Rt}{L}}$$

$$\boxed{v_o(t) = \underbrace{V_A}_{\text{forced}} \underbrace{- V_A e^{-\frac{Rt}{L}}}_{\text{natural}}}$$