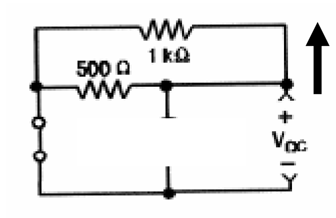
Homework Solutions 6

(3-52) Find the maximum power available to the load resistance in the Figure P3-52. What is the value of RL will extract maximum power?

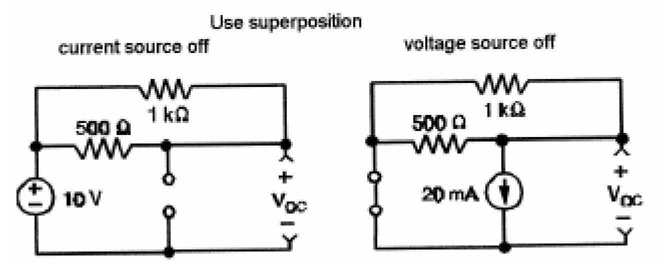
> ***Step (1)*** *Find the Thevenin ("lookback") resistance, open the current source and short the voltage source, then find the resistance by looking back into the circuit from the load.*



R_T "lookback" resistance
1kΩ || 500Ω resistor

$R_T := \frac{1000 \cdot 500}{1000 + 500}$ $\quad R_L := R_T \quad R_L = 333.333$

> ***Step (2)****Using superposition to find the too find two different Isc(short circuit) currents. Therefore, open the current source and use the voltage source to find the first short circuit current. Next short the voltage source and use the current source to find the second short circuit current.*



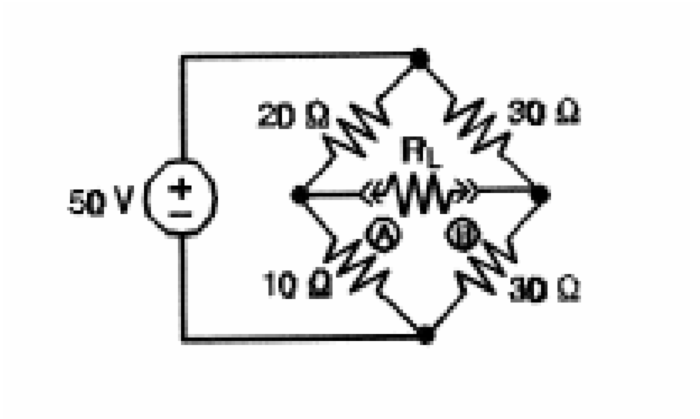
$i_{SC1} := \frac{10}{\left(\frac{1000 \cdot 500}{1000 + 500}\right)}$ $\quad i_{SC1} = 0.03 \quad i_{SC2} := -0.02 \quad i_N := i_{SC1} + i_{SC2} \quad i_N = 0.01$

Simply add i_{sc1} to i_{sc2} to find the total i_{sc} ➔ i_N

Using the equation Pmax $=(0.25)\cdot i_N^2 \cdot R_T = (0.25)\cdot(0.01)^2 \cdot 333.33 = 8.333$mA
Therefore, Pmax = **8.333mA.**

(3-53) Find the maximum power available to the load resistance in Figure P3-53. What is the value of Rl will extract the maximum power?

> *First find the "lookback" resistance in the wheatstone bridge, open the load and short the voltage source. Now find the resistance in the path from terminal A to terminal B.*



*Finding the equivalent "lookback" resistance or Thevenin resistance is quite hard to visualize but if you rearrange the circuit drawing enough you can clearly see that **10Ω || 20Ω** which is in series with the **30Ω || 30Ω. Therefore,***

$$R_T := \frac{10\cdot 20}{10+20} + \frac{30\cdot 30}{30+30}$$

<u>R_T = 21.67Ω</u>

Next, consider finding the Thevenin voltage V_T which is the voltage across terminal A and terminal B:

$$\mathbf{V_T = V_A - V_B}$$

Find the voltage at A and B using voltage division;

$$v_{AB} := \frac{10}{10+20}\cdot 50 - \frac{30}{30+30}\cdot 50 \qquad v_{AB} = -8.333$$

Therefore, **<u>$V_T = V_{AB} = -8.3333$</u>**

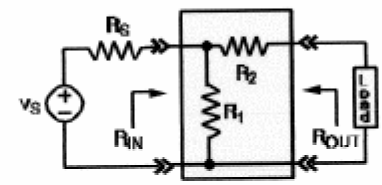
Now, solve for $i_N = V_T / R_T = -0.3846A$

And,

Pmax = (0.25)·(-0.3846A)^2 · 21.67Ω = **<u>0.801W</u>**

(3-64) In Figure P3-61 the load is a 500Ω resistor and the Rs = 75Ω. Design an interface circuit so that the input resistance of the two port is 75Ω +/-10% and the output resistance seen by the load is 500Ω +/-10%.

3-64 $R_S := 75$ $R_L := 500$



First guess let: $R_1 := 75$ $R_2 := 500$

$$R_{IN} := \frac{R_1 \cdot (R_2 + R_L)}{R_1 + R_2 + R_L} \qquad R_{IN} = 69.767$$

$$R_{OUT} := R_2 + \frac{R_1 \cdot R_S}{R_1 + R_S} \qquad R_{OUT} = 537.5$$

Both answers are with ± 10%

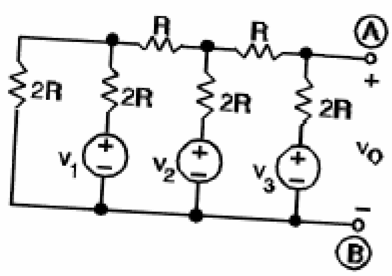
$$\text{Given} \quad \frac{R_1 \cdot (R_2 + 500)}{R_1 + R_2 + 500} = 75 \quad R_2 + \frac{R_1 \cdot 75}{R_1 + 75} = 500 \quad \text{Find}(R_1, R_2) = \begin{pmatrix} 81.349 \\ 460.977 \end{pmatrix}$$ <---exact solutions

The best way to solve these design problems is to try each Interface Circuit equivalent shown in Figure 3-52 on pg. 118 of the text. Since this is a design problem you can guess some values and use logic into which is the best interface circuit to use for specifications required.

(3-71) Comparison of Analysis Methods for (a), (b), (c); problem statement located in the text

The maximum power available is less than 1 mW. Hence no interface circuit is required.

3-71 (a) Writing node-voltage equations:



$$\begin{pmatrix} \frac{2}{R} & -\frac{1}{R} & 0 \\ -\frac{1}{R} & \frac{5}{2 \cdot R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{3}{2 \cdot R} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix}$$

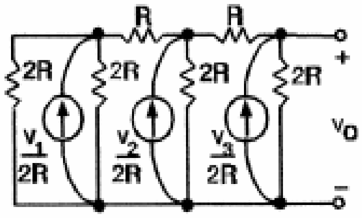
Solving for the node-voltage vector

$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{2}{R} & -\frac{1}{R} & 0 \\ -\frac{1}{R} & \frac{5}{2 \cdot R} & -\frac{1}{R} \\ 0 & -\frac{1}{R} & \frac{3}{2 \cdot R} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix} = \begin{pmatrix} \frac{11}{16} \cdot R & \frac{3}{8} \cdot R & \frac{1}{4} \cdot R \\ \frac{3}{8} \cdot R & \frac{3}{4} \cdot R & \frac{1}{2} \cdot R \\ \frac{1}{4} \cdot R & \frac{1}{2} \cdot R & R \end{pmatrix} \cdot \begin{pmatrix} \frac{v_1}{2 \cdot R} \\ \frac{v_2}{2 \cdot R} \\ \frac{v_3}{2 \cdot R} \end{pmatrix} = \begin{pmatrix} \frac{11}{32} \cdot v_1 + \frac{3}{16} \cdot v_2 + \frac{1}{8} \cdot v_3 \\ \frac{3}{16} \cdot v_1 + \frac{3}{8} \cdot v_2 + \frac{1}{4} \cdot v_3 \\ \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3 \end{pmatrix}$$ <---output

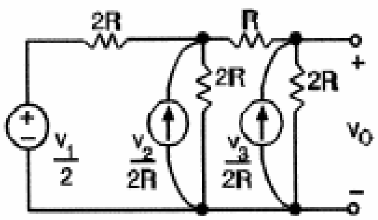
Hence $v_O = v_C = \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3$

3-71 (b) Using circuit reduction methods

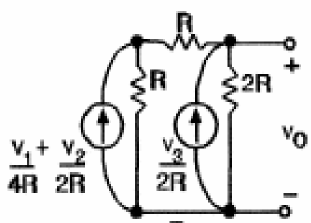
Step 1: Convert the voltage sources in series with 2R to equivalent current sources in parallel



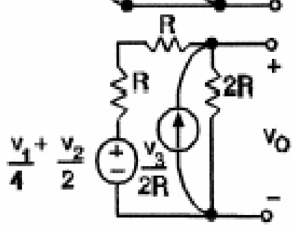
Step 2: Replace left-most 2R||2R by a single resistor R and then convert the left-most current source to an equivalent voltage sources $v_1/2$ in series with R + R = 2R.



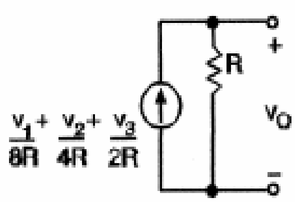
Step 3: Convert the left-most voltage source to an equivalent current sources in parallel with 2R and then combine the two left-most current sources that are connected in parallel to an equivalent current source $\frac{v_1}{4 \cdot R} + \frac{v_2}{2 \cdot R}$. connected in parallel with 2R||2R = R



Step 4: Convert the left-most current source to an equivalent voltage sources $\frac{v_1}{4} + \frac{v_2}{2}$ in parallel with R.



Step 5: Convert the left most voltage source in series with 2R to an equivalent current sources $\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R}$ in parallel with 2R and then combine with the right most current source to produce an equivalent current source $\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R} + \frac{v_3}{2 \cdot R}$. in parallel with R.



Hence $v_O = \left(\frac{v_1}{8 \cdot R} + \frac{v_2}{4 \cdot R} + \frac{v_1}{2 \cdot R} \right) \cdot R = \frac{1}{8} \cdot v_1 + \frac{1}{4} \cdot v_2 + \frac{1}{2} \cdot v_3$

(c) Either method is acceptable. Method (a) is analytical and abstract. Method (b) is more intuitive since it works directly with the circuit model.

(4-4) The circuit in Figure P4-4 is an ideal voltage amplifier with the negative feedback provided via the resistor R_F.

(a) Find the output voltage V2 and the current gain i_2 / i_1 when Vs = 10mV and $R_F = 10k\Omega$.

(b) Find the input resistance $R_{IN} = v_1 / i_1$.

4-4 $R_F := 10\cdot 10^3$ $\quad v_s := 0.01$ **(a) Writing a KCL equation at Node A yields:**

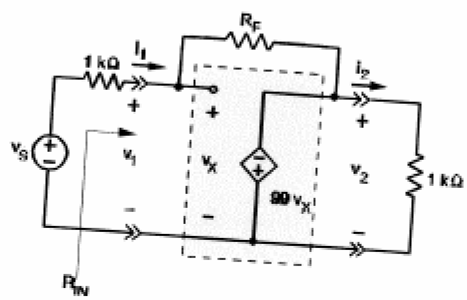
$$\frac{v_x - v_s}{1000} + \frac{v_x - (-99\cdot v_x)}{R_F} = 0$$

solving for v_x yields $v_x := \dfrac{R_F\cdot v_s}{R_F + 100000}$

$v_2 := -99\cdot v_x \quad v_2 = -9\times 10^{-2} \quad i_2 := \dfrac{v_2}{1000}$

$i_1 := \dfrac{v_s - v_x}{1000} \quad K_I := \dfrac{i_2}{i_1} \quad K_I = -9.9$

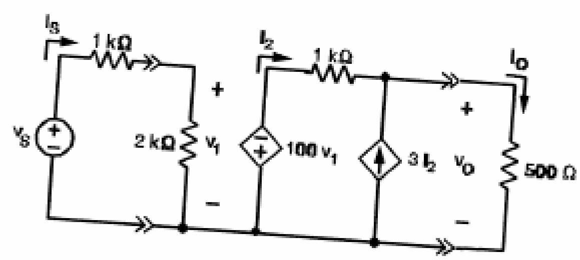
(b) $R_{IN} := \dfrac{v_x}{i_1} \quad R_{IN} = 100$



Therefore, **<u>K = -9.9</u>** for part (a); and **<u>R_{IN} = 100</u>** for part (b)

(4-6) Find the voltage gain v_O / v_S and the current gain i_O / i_S in Figure P4-6.

4-6 By voltage division $v_I = \dfrac{2000}{1000 + 2000}\cdot v_S = \dfrac{2\cdot v_S}{3}$ **(a) Writng a KCL equation at Node A yields:**



$i_2 + 3i_2 - i_O = 0 \quad i_O = 4\cdot i_2$

$i_2 = \dfrac{(-100\cdot v_I) - v_O}{1000} = \dfrac{-v_S}{15} - \dfrac{v_O}{1000}$

$i_2 = \dfrac{i_O}{4} = \dfrac{v_O}{4\cdot 500} = \dfrac{v_O}{2000}$ hence,

$\dfrac{-v_S}{15} - \dfrac{v_O}{1000} = \dfrac{v_O}{2200}$ Solve for v_O:

$v_O = \dfrac{-400}{9}\cdot v_S \quad K_V := \dfrac{-400}{9}$

(b) $i_S = \dfrac{v_S}{3000} \quad i_O = \dfrac{v_O}{500} = \dfrac{1}{500}\cdot\left(\dfrac{-400}{9}\cdot v_S\right) \quad K_I := \dfrac{-400\cdot 3000}{500\cdot 9}$

$K_V = -44.444 \quad K_I = -266.667$

Therefore, **<u>Kv = -44.44</u>** for part (a); and **<u>K_I = -266.667</u>** for part (b)