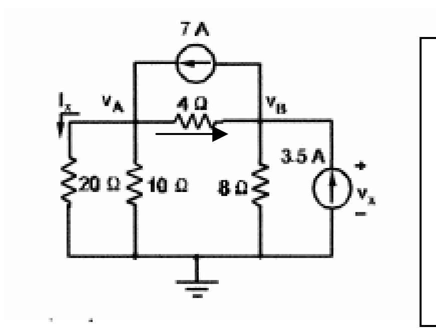
**Homework Solutions 4**

**(3-2) (a)** Formulate node-voltage equations for the circuit in Figure P3-2

**(b)** Use these equations to find Vx and Ix




Current through 4Ω resistor (choose direction of current). I chose Va ⟶ Vb. Then, current through the resistor is:

$$I_{4\Omega} = \frac{Va - Vb}{4\Omega} \text{; and } \quad Ix = \frac{Va}{20\Omega}$$

Node voltage equations

A : -Va/20 + -Va/10 + -(Va-Vb)/4 = 7A

B: (Va-Vb)/4 + -Vb/8 = -7 + 3.5

**3-2 (a)**
$$\begin{pmatrix} \frac{1}{20}+\frac{1}{10}+\frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4}+\frac{1}{8} \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 7 \\ -7+3.5 \end{pmatrix}$$

**(b)**
$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} := \begin{pmatrix} \frac{1}{20}+\frac{1}{10}+\frac{1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{1}{4}+\frac{1}{8} \end{pmatrix}^{-1} \cdot \begin{pmatrix} 7 \\ -7+3.5 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 20 \\ 4 \end{pmatrix} \qquad v_x := v_B \qquad i_x := \frac{v_A}{20} \qquad v_x = 4$$

Therefore, Vx is simply the voltage across the 8Ω resistor since Vb = 4 the only voltage drop will occur across the 8Ω resistor, hence Vx must equal Vb.

**(3-4)** (a) Formulate a set of node-voltages for the circuit in Figure P3-4
(b) Use these equation to find Vx and Ix

First draw your current directions, then use your KCL equations to simplify the problem then, find the nodal voltages through each resistor.
KCL at nodes A, B

KCL@A : 2mA – $i_{2k\Omega} + i_{4k\Omega} = 0$
KCL@B : -2mA – $i_{3k\Omega \text{ (left of node B)}} + i_{3k\Omega \text{ (up arrow into node B)}} = 0$
KCL@Center-Node : $i_{2k\Omega} + i_{3k\Omega \text{ (left of node B)}} = Ix$

**3-4 (a)**

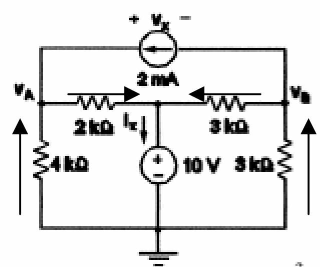
$$\left(\frac{1}{2000}+\frac{1}{4000}\right)\cdot v_A - \frac{10}{2000} = 2\cdot 10^{-3}$$

$$\left(\frac{1}{3000}+\frac{1}{3000}\right)\cdot v_B - \frac{10}{3000} = -2\cdot 10^{-3}$$

**(b) Solving the 1st equation for $v_A$** $\quad v_A := \frac{28}{3}$

**Solving the 2nd equation for $v_B$** $\quad v_B := 2$

$v_x := v_A - v_B \qquad i_x := \left(\frac{v_A - 10}{2000}\right) + \frac{v_B - 10}{3000}$




$v_x = 7.333 \qquad i_x = -3 \times 10^{-3}$

After all the simplifications, <u>**Vx = 7.333 and Ix = -3mA**</u>

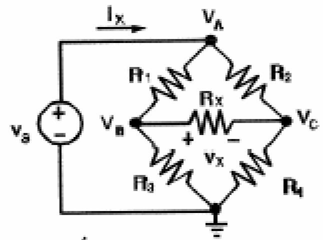
**(3-8) (**a) Formulate the node-voltage equations for the circuit in Figure P3-8
(b) Solve for Vx and Ix using R1 = 10kΩ, R2 = 20kΩ, R3 = 60kΩ, R4 = 20kΩ Rx = 3kΩ, and Vs = 15V.
(c) Find the power absorbed by the resistor R2.

**THE WHEATSTONE BRIDGE!!!**

The trick to this problem is rather simple, do not think too much into the problem you cannot reduce this circuit, the resistors are neither in parallel or series! Start by labeling your nodes, then draw your current directions. Next, use your KCL's and solve the current through each resistor with nodal analysis. Simplify. Find Ix, Vx, $P_{R2}$.

**3-8 (a)**

$$\begin{pmatrix} 1 & 0 & 0 \\ -G_1 & G_1 + G_X + G_3 & -G_X \\ -G_2 & -G_X & G_2 + G_X + G_4 \end{pmatrix} \cdot \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$




**(b)** $G_1 := \left(10^4\right)^{-1}$ $G_2 := \left(2 \cdot 10^4\right)^{-1}$ $G_3 := \left(6 \cdot 10^4\right)^{-1}$

$G_4 := \left(2 \cdot 10^4\right)^{-1}$ $G_X := \left(3 \cdot 10^3\right)^{-1}$ $v_S := 15$ $G_X = 3.333 \times 10^{-4}$

$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} := \begin{pmatrix} 1 & 0 & 0 \\ -G_1 & G_1 + G_X + G_3 & -G_X \\ -G_2 & -G_X & G_2 + G_X + G_4 \end{pmatrix}^{-1} \cdot \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 15 \\ 10.728 \\ 9.983 \end{pmatrix} \quad v_X := v_B - v_C \quad v_X = 0.745 \quad i_X := (v_A - v_B) \cdot G_1 + (v_A - v_C) \cdot G_2 \quad i_X = 6.78 \times 10^{-4}$$

**(c)** $P_{R2} := (v_A - v_C)^2 \cdot G_2$ $P_{R2} = 1.2583 \times 10^{-3}$ W

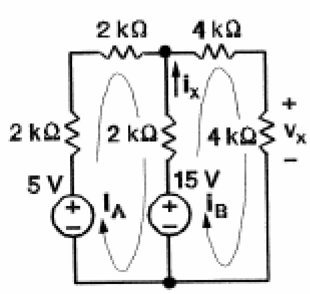
Therefore, **<u>Va = 15; Vb = 10.72; Vc = 9.98; and $P_{R2}$ = 1.26mW.</u>**

**(3-10) (**a) Formulate mesh current equations for the circuit in Figure P3-10.
(b) Use these equations to find Vx and Ix.

These problems end to being a lot of algebra and reducing but they get the job done. The main key that many students forget in these type of mesh problems is where the center 2kΩ resistor when finding the mesh equation for A, the voltage through the 2kΩ resistor is (Ia – Ib)* 2kΩ. For finding mesh equation for B, the voltage through the 2kΩ resistor is (Ib –Ia)* 2kΩ

**3-10 (a)**

$$\begin{pmatrix} 2000 + 2000 + 2000 & -2000 \\ -2000 & 2000 + 4000 + 4000 \end{pmatrix} \cdot \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 5 - 15 \\ 15 \end{pmatrix}$$




**(b)**

$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} := \begin{pmatrix} 2000 + 2000 + 2000 & -2000 \\ -2000 & 2000 + 4000 + 4000 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -10 \\ 15 \end{pmatrix}$$

$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} -1.25 \times 10^{-3} \\ 1.25 \times 10^{-3} \end{pmatrix}$$

$v_X := i_B \cdot 4000$ $i_X := i_B - i_A$ $v_X = 5$ $i_X = 2.5 \times 10^{-3}$

Therefore, **Ix = Ib – Ia = <u>2.5mA</u>; Vx = Ib*4000 = <u>5V</u>**

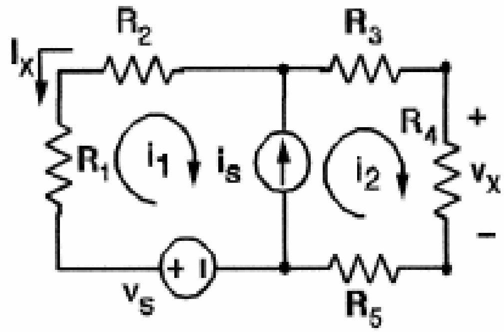
**(3-12) (**a) Formulate mesh current equations for the circuit in the Figure P3-12
(b) Solve for Vx and Ix using R1 = 200Ω, R2 = 500Ω, R3 = 60Ω, R4 = 240Ω, R5 = 200Ω, Is = 50mA,and Vs = 15V.
(c) Find the total power dissapted in the circuit

This is a good type of problem to apply the super node, but does not have to be solve that way. Simple nodal voltage equations will solve this one also.

**3-12** Assign mesh currents as shown

**(a)** by KCL $i_2 - i_1 = i_S$ Super Mesh:

$(R_1 + R_2)\cdot i_1 + (R_3 + R_4 + R_5)\cdot i_2 = v_S$




**(b)** $R_1 := 200 \quad R_2 := 500 \quad R_3 := 60$

$R_4 := 240 \quad R_5 := 200$

$v_S := 15 \quad i_S := 0.05$

$$i_1 := \frac{-(i_S \cdot R_3 + i_S \cdot R_4 + i_S \cdot R_5 - v_S)}{(R_1 + R_2 + R_3 + R_4 + R_5)} \qquad i_2 := \frac{(i_S \cdot R_1 + i_S \cdot R_2 + v_S)}{(R_1 + R_2 + R_3 + R_4 + R_5)} \qquad \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} -8.333 \times 10^{-3} \\ 4.167 \times 10^{-2} \end{pmatrix}$$

$i_x := -i_1 \quad i_x = 8.333 \times 10^{-3} \qquad v_x := R_4 \cdot (i_2) \quad v_x = 10$

**(c)** $P_{total} := (R_3 + R_4 + R_5)\cdot(i_2)^2 + (R_1 + R_2)\cdot i_1^{\,2} \qquad P_{total} = 0.917 \quad W$

Remember, P = VI ➔ P = R*I*I ➔ P = $RI^2$

Therefore, <u>**Ix = 8.33mA; Vx = 10; Ptot = 917mW**</u>

**(3-19)** Find the node voltages Va and Vb in the Figure P3-19.

This problem seems to be quite troublesome at first but consider your KCL equations first and use common sense to solve the nodal equations.

At node B, there is a 12mA supply entering the node and two current leaving the node (look at the figure). The 20V source divided by the addition of the 2kΩ and the 8kΩ is one of the current leaving node B.

At node A, the node equation can be solved simply by using voltage division.

After equations are found solve for Va and Vb.

**3-19** Writing one mesh equation

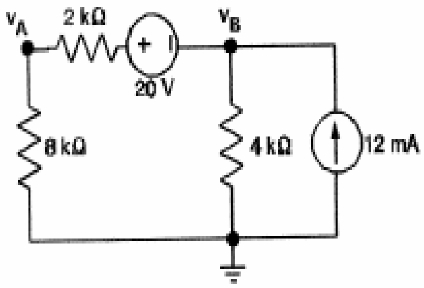
$(2000 + 4000 + 8000)\cdot i_A + 4000\cdot 12\cdot 10^{-3} = -20$

solving for $i_A$ $\quad i_A := -4.857\cdot 10^{-3}$

$v_A := -8000\cdot i_A \qquad v_A = 38.856$

$v_B := \left(i_A + 12\cdot 10^{-3}\right)\cdot 4000 \qquad v_B = 28.572$

Checking $\quad (2000 + 4000 + 8000)\cdot i_A + 4000\cdot\left(12\cdot 10^{-3}\right) = -19.998$ <---checks the mesh equation




**Therefore, <u>Va</u> = -8000*Ia = <u>38.9</u>; <u>Vb = 28.57</u>**