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Department of Electrical Engineering and Computer Science
ENGR 210. Introduction to Circuits and Instruments (4)
ENGR 210 - SPRING 2004

SOLUTIONS

FINAL EXAMINATION- given 5/4/03

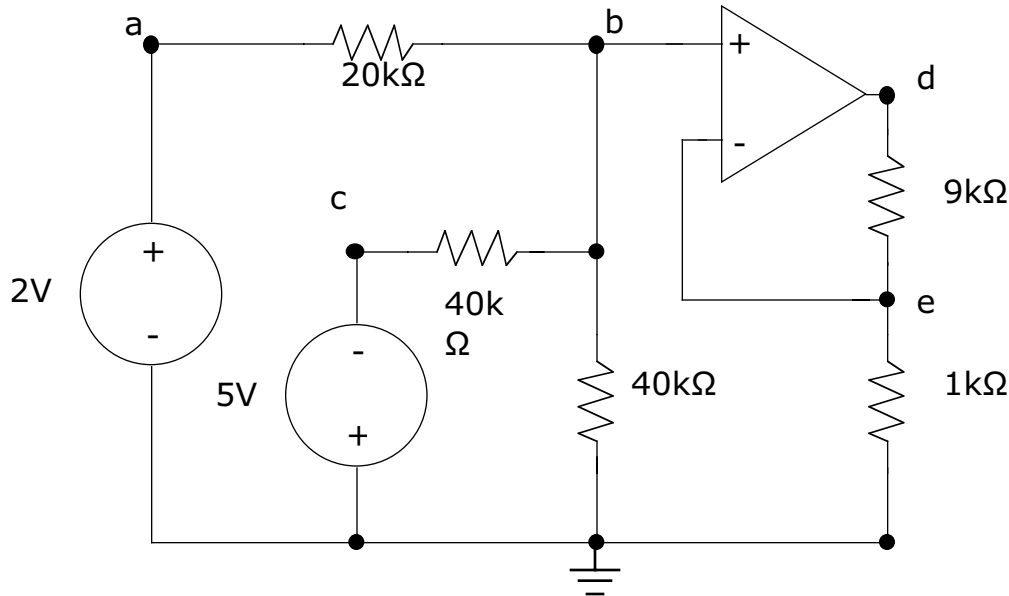
		Possible	
1.	<input type="text"/>	10	DC OP AMPS
2.	<input type="text"/>	10	Controlled sources
3.	<input type="text"/>	10	Equivalent circuits
4.	<input type="text"/>	10	Maximum power transfer
5.	<input type="text"/>	10	Capacitors and inductors
6.	<input type="text"/>	10	Phasor circuit analysis
7.	<input type="text"/>	10	Transfer function (OP AMP)
8.	<input type="text"/>	10	Passive transfer function/Bode plot
9.	<input type="text"/>	10	Complex transient
10.	<input type="text"/>	10	Switched transient
SCORE	<input type="text"/>	100	

ACADEMIC DISHONESTY

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Problem 1 DC OP AMPS (10 points)

Determine the node voltages V_a , V_b , V_c , V_d , and V_e (with respect to ground) for the following OP AMP circuit. Assume the OP AMP is ideal.

**SOLUTIONS:**

The first place to start is to assign voltages to the nodes which are directly connected to the voltage sources: $V_a = +2$ volts and $V_c = -5$ volts.

We determine the voltage at node B using KCL, i.e., $\sum_{+in} i = 0$. Assuming the current

through the $20k\Omega$ resistor is from a to b, the current through the horizontal $40k\Omega$ resistor is from c to b, and the current through the vertical $40k\Omega$ resistor is from b

to c we can write $\frac{V_a - V_b}{20k} + \frac{V_c - V_b}{40k} - \frac{V_b - 0}{40k} = 0$. Note that there is no current into the

non-inverting input of the OP AMP. Solving this equation gives

$2V_a - 2V_b + V_c - V_b - V_b = 0$, or $2V_a - 4V_b + V_c = 0$. Using our previous values for V_a and

V_c gives $2(2) - 4V_b + (-5) = 0$, or $V_b = -\frac{1}{4}$ volt.

With V_b determined we use the property of the ideal OP AMP that $V_+ = V_-$ to get $V_e = V_b = -0.25$ volts.

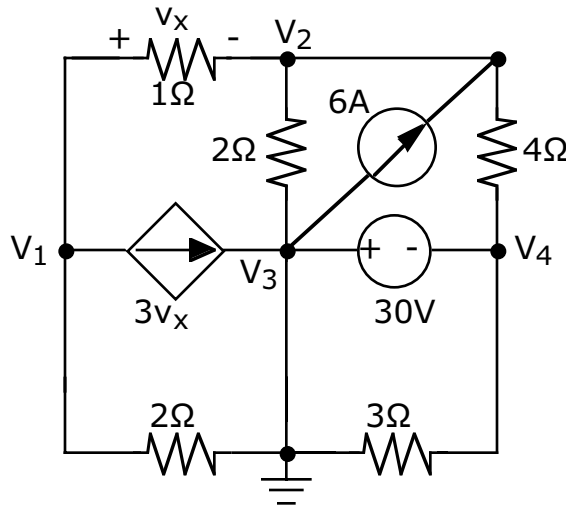
With V_e determined we can compute the current through the vertical $1k\Omega$ resistor as

$i_{1k\Omega} = \frac{V_e}{1000} = \frac{-0.25}{1000} = -0.25$ mA. This same current flows through the $9k\Omega$ resistor so

the voltage drop across the $9k\Omega$ resistor is then

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$V_{9k\Omega} = i_{1k\Omega} 9000 = (-0.25 \times 10^{-3}) 9000 = -2.25$ volts. The voltage at node d is then given
as $V_d = V_e + V_{9k\Omega} = (-0.25) + (-2.25) = -2.5$ volts.

Problem 2 Circuits with controlled sources (10 points)

Find the node voltages V_1 , V_2 , V_3 and V_4 .

SOLUTION:

This problem looks more intimidating than it really is.

First, we note that $V_3=0$ volts since it is directly connected to ground. V_4 is then given as $V_4=-30+V_3=-30$ volts. All that remains is to solve for V_1 and V_2 .

Since V_1 and V_2 involve current sources and we want the voltage at the nodes we choose to use KCL.

Using KCL we write for node 1 that $-\frac{V_1-V_2}{1}-3v_x-\frac{V_1-0}{2}=0$ which can be simplified to $-3V_1+2V_2-6v_x=0$.

The corresponding equation for node 2 is $+\frac{V_1-V_2}{1}-\frac{V_2-V_3}{2}+6-\frac{V_2-V_4}{4}=0$. Using

the known values for V_3 and V_4 allows this to be simplified to

$$4V_1-4V_2-2V_2+2V_3+24-V_2+V_4=4V_1-4V_2-2V_2+2(0)+24-V_2+(-30)=0, \text{ or}$$

$$4V_1-7V_2-6=0$$

We now have two equations which we put into standard form as

$$-3V_1+2V_2=6v_x$$

$$4V_1-7V_2=6$$

Getting to this point in the solution used a knowledge of circuits. The remainder of the problem is algebra.

We substitute $v_x = \frac{V_1-V_2}{1} = V_1-V_2$ to get the equations

$$-3V_1+2V_2=6(V_1-V_2)=6V_1-6V_2$$

$$4V_1-7V_2=6$$

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which can be further reduced to

$$-9V_1 + 8V_2 = 0$$

$$4V_1 - 7V_2 = 6$$

and solved simultaneously

$$4(-9V_1 + 8V_2 = 0)$$

$$9(4V_1 - 7V_2 = 6)$$

giving

$$-36V_1 + 32V_2 = 0$$

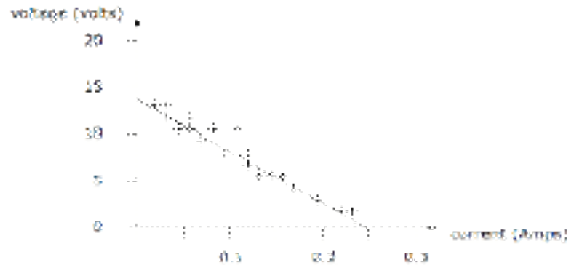
$$+36V_1 - 63V_2 = 54$$

Adding these last two equations together gives $-31V_2 = 54 \rightarrow V_2 = -1.7419$ volts. We

substitute this into $-9V_1 + 8V_2 = 0$ giving $V_1 = \frac{8}{9}V_2 = \frac{8}{9}(-1.7419) = -1.548$ volts.

Problem 3 Equivalent Circuits (10 points)

You measure the output voltage of a new fuel cell under a variety of different loads. You plot the data and fit a curve to the data getting the equation $V = 13.45 - 53.84i$.



You need an electrical model for this fuel cell to be included in a report to your supervisor.

(a) What is the Thevenin voltage V_T for this fuel cell?

SOLUTION:

This comes from the identification of the Thevenin voltage as the open circuit voltage. For an open circuit $I=0$ so we simply evaluate $V_{oc} = 13.45 - 53.84(0) = 13.45$ volts. This could also have been estimated from the graph as the y-intercept.

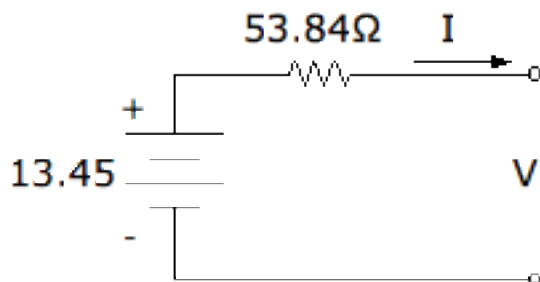
(b) What is the Thevenin resistance R_T for this fuel cell?

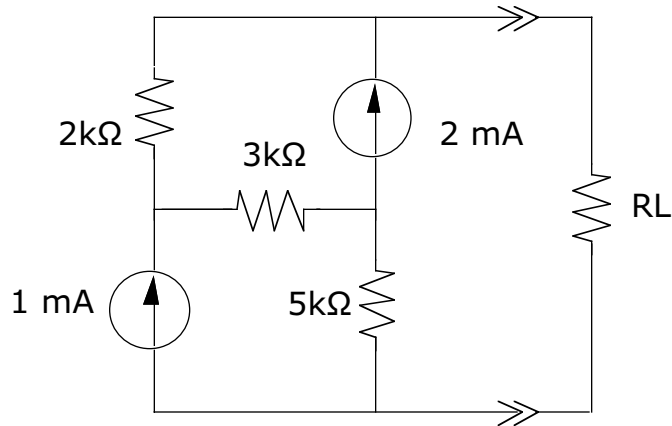
SOLUTION:

We solve for the short circuit current. This is the point on the curve for which $V=0$. Solving gives $0 = 13.45 - 53.84I_{sc}$ or $I_{sc} = \frac{13.45}{53.84} = 0.2498$ amps.

The Thevenin resistance is simply the slope of the curve which is given by (ignoring the sign) $R_T = \frac{V_{oc}}{I_{sc}} = \frac{13.45}{0.2498} = 53.84$ ohms.

(c) Draw a Thevenin equivalent circuit for this fuel cell.

SOLUTION:

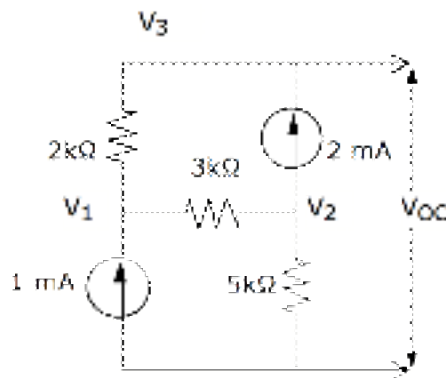
Problem 4 Maximum power transfer (10 points)

Consider the above circuit.

(a) What is the open circuit voltage, i.e., if $R_L = \infty$?

SOLUTION:

Since current sources are involved we can use the node voltage technique to find the open circuit voltage. Labeling the nodes as shown below we proceed to write KCL at nodes 1 and 2.



At node 1 we write $+1mA - \frac{V_1 - V_2}{3k} + 2mA = 0$.

NOTE that I observed that the current going through the $2k\Omega$ resistor had to be $2mA$ since there is no other place for the $2mA$ current from the current source to go.

At node 2 we write $\frac{V_1 - V_2}{3k} - 2mA - \frac{V_2}{5k} = 0$.

We set these up as simultaneous equations and solve:

$$+(3mA)(3k) - V_1 + V_2 = 0$$

$$5k(V_1 - V_2) - 30 - 3kV_2 = 0$$

$$-V_1 + V_2 = -9$$

$$5V_1 - 8V_2 = 30$$

$$-5V_1 + 5V_2 = -45$$

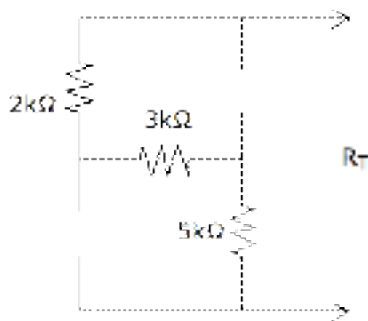
$$+5V_1 - 8V_2 = 30$$

Adding these equations gives $-3V_2 = -15$ or $V_2 = +5$ volts. Using the equation $-V_1 + V_2 = -9$ gives $V_1 = V_2 + 9 = 5 + 9 = 14$ volts. We observe that there is 2mA of current through the $2k\Omega$ resistor giving $V_3 = V_1 + (2mA)(2k\Omega) = 14 + 4 = 18$ volts.

(b) What is the Thevenin resistance of this circuit to the left of R_L ? ANS: $R_T = 10k\Omega$

SOLUTION:

We find the Thevenin resistance by turning OFF the current sources to get the

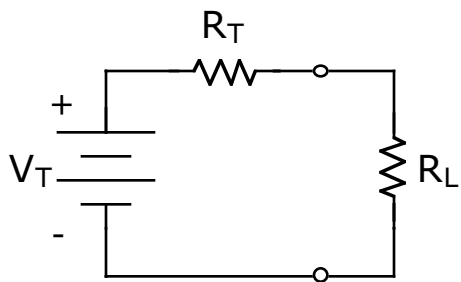


circuit shown above. We see that $R_T = 2k\Omega + 3k\Omega + 5k\Omega = 10k\Omega$.

(c) What is the maximum power that can be dissipated by R_L ? ANS: 40.5mW

SOLUTION:

The maximum power that can be dissipated by R_L occurs when R_L equals the Thevenin resistance.



When these resistances are equal exactly 1/2 of V_T appears across R_L and this is the point of maximum power being delivered to R_L . Since $V_T = 18$ volts we can

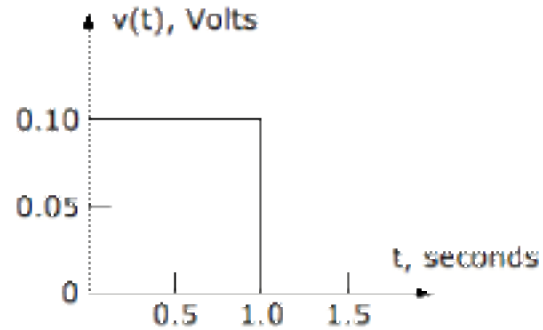
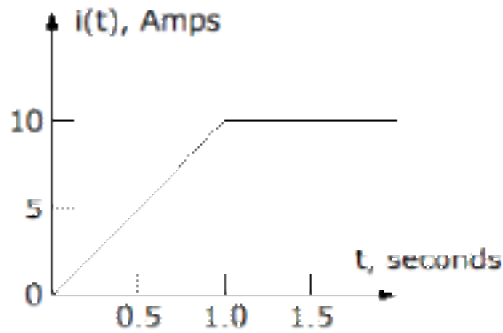
$$\text{compute this maximum power as } P_{\max} = \frac{V^2}{R} = \frac{\left(\frac{1}{2}18\right)^2}{10k} = 8.1mW$$

(d) At what value of R_L does this maximum power dissipation occur?

SOLUTION: The maximum power occurs when $R_L = R_T$.

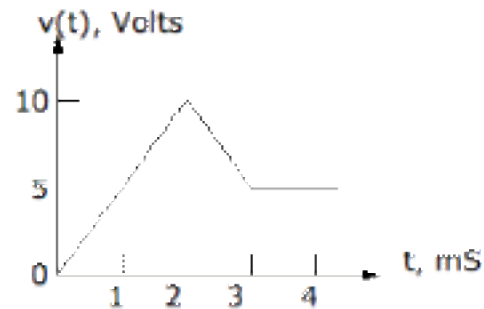
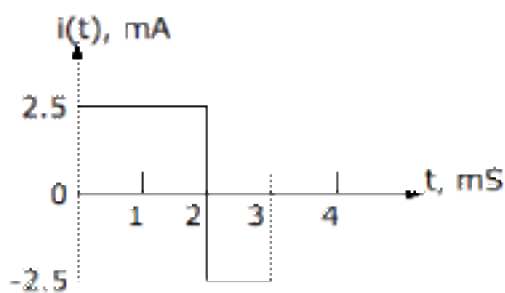
Problem 5 Capacitors and Inductors (10 points)

- (a) A particular element is tested and found to have $i(t)$ and $v(t)$ as shown here. Identify the type of element and its value.

**SOLUTION:**

We use the relationships $v_L = L \frac{di_L}{dt}$ and $i_C = C \frac{dv_C}{dt}$ to solve this problem. Looking at the curve for $i(t)$ we see it is a ramp until $t=1$ second. During this period $v(t)$ is constant so this looks like $v(t)$ is the derivative of $i(t)$. For $t > 1$ second $i(t)$ is constant. Its derivative would then be zero and this is exactly the way $v(t)$ behaves so we conclude this is an inductor. The slope of $i(t)$ for $0 < t < 1$ second is $\frac{di}{dt} = \frac{10-0}{1-0} = 10 \frac{\text{volts}}{\text{sec}}$. Using this value in $v_L = L \frac{di_L}{dt}$ gives $0.1 \text{volts} = L \left(10 \frac{\text{volts}}{\text{sec}} \right)$ and so we conclude that $L=0.01\text{H}$.

- (b) A second element is tested and found to have $i(t)$ and $v(t)$ as shown below. Identify the type of element and its value.

**SOLUTION:**

We again use the relationships $v_L = L \frac{di_L}{dt}$ and $i_C = C \frac{dv_C}{dt}$ to solve this problem.

Looking at the curve for $v(t)$ we see it is a ramp until $t=2$ seconds. During this period $i(t)$ is constant so this looks like $i(t)$ is the derivative of $v(t)$. For $2 < t < 3$

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seconds we see exactly the same behavior. So we conclude this is a capacitor.

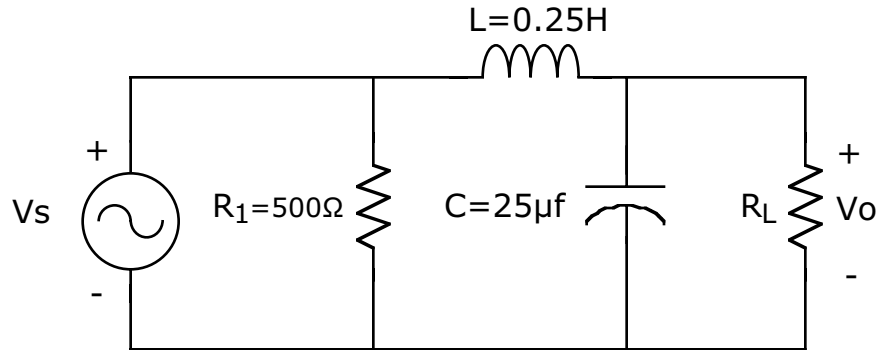
The slope of $v(t)$ for $0 < t < 2$ seconds is $\frac{dv}{dt} = \frac{10-0}{2-0} = 5 \frac{\text{volts}}{\text{sec}}$. Using this value in

$i_c = C \frac{dv_c}{dt}$ gives $2.5 \text{mA} = C \left(5 \frac{\text{volts}}{\text{sec}} \right)$ and so we conclude that

$$C = \frac{i}{\frac{dv}{dt}} = \frac{2.5 \times 10^{-3}}{5 \times 10^3} = 0.5 \times 10^{-6} \text{ or } C = 0.5 \mu\text{f}.$$

Problem 6 Phasor circuit analysis (10 points)

The following circuit is a very simple model of a power system delivering power to a small factory. If $V_s = 1000\cos(400t)$ answer the following questions.



a) What is the frequency (in Hertz) at which this power system is operating?

SOLUTION:

The power system is operating at $\omega = 400$ since $V_s = 1000\cos(400t)$. The linear frequency f is then given by $f = \frac{\omega}{2\pi} = \frac{400}{2\pi} = 63.66$ Hz.

b) What is the impedance of the inductor and the capacitor.

SOLUTION:

These values come from evaluation of the impedance formulas using $\omega = 400$ and the component values.

$$Z_L = j\omega L = j(400)(0.25) = +j100\Omega \text{ and } Z_C = \frac{1}{j\omega C} = \frac{1}{j(400)(25 \times 10^{-6})} = -j100\Omega$$

c) For $R_L = 100\Omega$, determine the complex voltage V_o appearing across R_L . Write the value of this complex voltage using $a+jb$ notation.

SOLUTION:

The voltage V_o is determined by a complex voltage divider consisting of $R_L \parallel C$ and L . Note that R_1 is in parallel with this voltage divider and does not contribute to determining V_o . The parallel combination of R_L and the $25\mu\text{f}$ capacitor has an impedance given by

$$R_L \parallel Z_C = \frac{R_L Z_C}{R_L + Z_C} = \frac{(100)(-j100)}{100 - j100} = \frac{(100)(-j100)}{100 - j100} = 50 - j50\Omega. \text{ We use this in a}$$

voltage divider with the inductor to determine V_o as

$$\underline{V}_o = \frac{Z_{\parallel}}{Z_{\parallel} + Z_L} \underline{V}_s = \frac{50 - j50}{(50 - j50) + (j100)} (1000 \angle 0^\circ) = \frac{50 - j50}{50 + j50} (1000 + j0) = -j1000 \text{ volts}$$

What is the corresponding real voltage, i.e. represented as a sinusoidal function?

SOLUTION:

To determine the real voltage corresponding to this we need to convert the previous answer from rectangular to polar form, i.e., $\underline{V}_o = -j1000 = 1000 \angle -90^\circ$. From this we can readily write the real voltage as $1000 \cos(400t - 90^\circ)$

d) At night the plant manager turns off some of the plant equipment to save money. This increases the equivalent load to $R_L = 1000 \Omega$. For $R_L = 1000 \Omega$, determine the complex voltage \underline{V}_o appearing across R_L . Write the value of this complex voltage using $a + jb$ notation.

SOLUTION:

The load voltage \underline{V}_o is again determined by the complex voltage divider consisting of $R_L \parallel C$ and L . The parallel combination of R_L and the $25 \mu\text{f}$ capacitor now has an impedance given by

$$Z_{\parallel} = R_L \parallel Z_C = \frac{R_L Z_C}{R_L + Z_C} = \frac{(1000)(-j100)}{1000 - j100} = \frac{(1000)(-j100)}{1000 - j100} = 9.901 - j99 \Omega. \text{ We can use}$$

this in a voltage divider with the inductor to determine \underline{V}_o as

$$\underline{V}_o = \frac{Z_{\parallel}}{Z_{\parallel} + Z_L} \underline{V}_s = \frac{9.901 - j99}{(9.901 - j99) + (j100)} (1000 \angle 0^\circ) = \frac{50 - j50}{9.901 + j0.99} (1000 + j0) \text{ volts}$$

$$\underline{V}_o \approx 0.1 - j9999.99 \text{ volts}$$

What is the corresponding real voltage, i.e. represented as a sinusoidal function?

SOLUTION:

To determine the real voltage corresponding to this we need to convert the previous answer from rectangular to polar form, i.e., $\underline{V}_o \approx 0.1 - j9999.99 \approx 10000 \angle -90^\circ$ volts. From this we can readily write the real voltage as $10000 \cos(400t - 90^\circ)$

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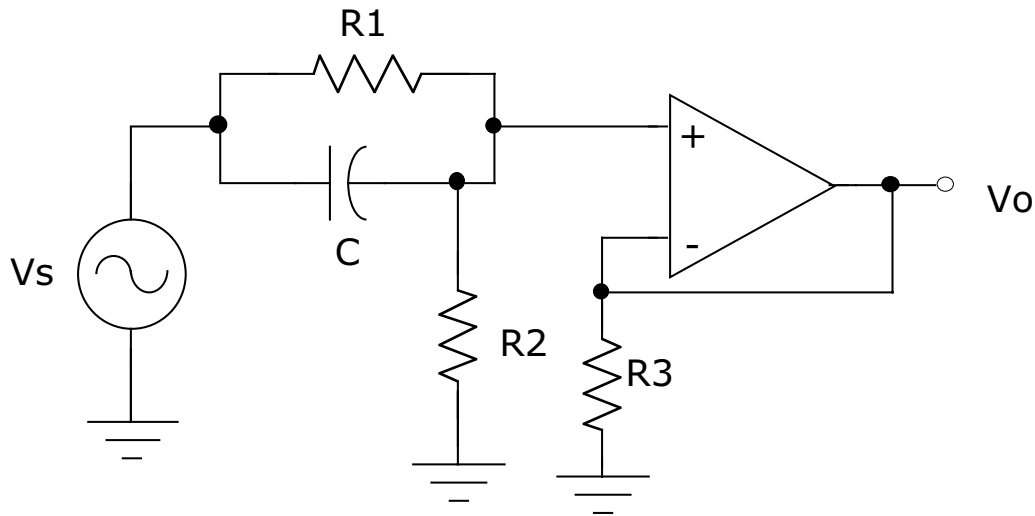
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e) (OPTIONAL) The plant equipment starts to fail at night. Based upon your answers for c) and d) can you suggest why?

SOLUTION:

In the plant manager's zeal to save money he is nightly subjecting the plant equipment to 10 times the voltage it sees during daytime operation. You do not see this type of effect in residential homes but industrial plants often use capacitors to get more favorable rates from the power company. Changing the load to save power can actually increase the power line voltage causing equipment to fail.

Problem 7 Transfer Functions (10 points)

- (a) Determine the frequency dependent system transfer function for the OP AMP circuit shown above.

SOLUTION:

Comment: There were many problems grading this question because there were no instructions as to the form of the result. We typically gave full credit if your answer looked correct.

We begin by determining the equivalent impedance of R_1 and C which are in parallel. This equivalent impedance is given by

$$Z_{EQ} = R_1 \parallel \frac{1}{j\omega C} = \frac{R_1 \frac{1}{j\omega C}}{R_1 + \frac{1}{j\omega C}} = \frac{R_1}{j\omega R_1 C + 1}. \quad \text{Since there is no current into the OP AMP the}$$

voltage at the non-inverting input is given by a voltage divider.

$$V_+ = \frac{R_2}{R_2 + Z_{EQ}} = \frac{R_2}{R_2 + \frac{R_1}{1 + j\omega R_1 C}} = \frac{R_2(1 + j\omega R_1 C)}{R_2(1 + j\omega R_1 C) + R_1}$$

This can be re-arranged into more standard form as

$$V_+ = \frac{R_2(1 + j\omega R_1 C)}{R_2(1 + j\omega R_1 C) + R_1} = \frac{R_2(1 + j\omega R_1 C)}{R_1 + R_2 + j\omega R_1 C} = \frac{R_2}{R_1 + R_2} \frac{1 + j\omega R_1 C}{1 + j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C}$$

Since this is an ideal OP AMP we note that $V_+ = V_- = V_o$ and

$$V_o(j\omega) = \frac{R_2}{R_1 + R_2} \frac{1 + j\omega R_1 C}{1 + j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C}$$

(b) What is the DC gain ($\omega \rightarrow 0$) of this circuit?

SOLUTION:

$$\lim_{\omega \rightarrow 0} V_o(j\omega) = \lim_{\omega \rightarrow 0} \left(\frac{R_2}{R_1 + R_2} \frac{1 + j\omega R_1 C}{1 + j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C} \right) = \frac{R_2}{R_1 + R_2}$$

(b) What is the high frequency gain ($\omega \rightarrow \infty$) of this circuit?

SOLUTION:

$$\lim_{\omega \rightarrow \infty} V_o(j\omega) = \lim_{\omega \rightarrow \infty} \left(\frac{R_2}{R_1 + R_2} \frac{1 + j\omega R_1 C}{1 + j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C} \right) = \frac{R_2}{R_1 + R_2} \frac{j\omega R_1 C}{j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C} = 1$$

(d) What kind of filter function (high pass, low pass, etc.) does this circuit perform?

SOLUTION:

Since the filter response is $\frac{R_2}{R_1 + R_2}$ which is less than 1 at low frequencies and 1 at

high frequencies we conclude this is a high-pass filter. This could also be gotten by converting the transfer function to dB:

$$20 \log_{10} |V_o(j\omega)| = 20 \log_{10} \left| \frac{R_2}{R_1 + R_2} \right| + 20 \log_{10} |1 + j\omega R_1 C| - 20 \log_{10} \left| 1 + j\omega \left(\frac{R_1 R_2}{R_1 + R_2} \right) C \right|$$

The first term is a constant until it reaches the frequency $\omega_1 = \frac{1}{R_1 C}$ at which point

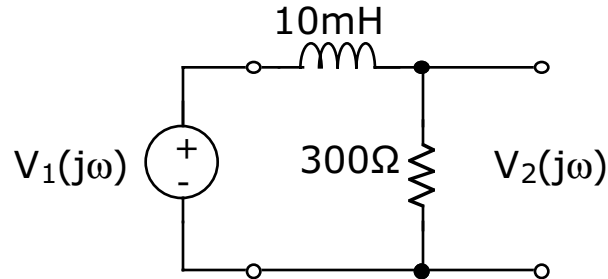
the second term begins to increase at +20dB/decade. This increase continues

until the frequency $\omega_2 = \frac{1}{\left(\frac{R_1 R_2}{R_1 + R_2} \right) C}$ at which point the third term starts to

decrease at -20db/decade canceling out the increase from the second term. For frequencies above ω_2 the gain consequently remains constant.

Problem 8 Bode Plots (10 points)

Determine the transfer function for the circuit shown below. Plot its frequency dependent gain (in db) on the graph below. Note that the horizontal axis is f.

**SOLUTION:**

The voltage divider relationship gives

$$V_2(j\omega) = \frac{R}{R + j\omega L} V_1(j\omega) = \frac{300}{300 + j\omega(0.01)} V_1(j\omega) = \frac{1}{1 + j\omega\left(\frac{0.01}{300}\right)} V_1(j\omega)$$

Re-arranging this gives

$$T_V(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{1}{1 + j\omega\left(\frac{0.01}{300}\right)} = \frac{1}{1 + j\frac{\omega}{30000}}$$

The frequency dependent gain in dB is found by taking the logarithm and multiplying by 20, i.e.,

$$20 \log_{10} |T_V(j\omega)| = 20 \log_{10} |1| - 20 \log_{10} \left| 1 + j\frac{\omega}{30000} \right|$$

The first term is a constant 0dB. The second term is a constant 0dB until the frequency $\omega_c = 30000$ at which point it decreases (because of the negative sign) at -20 dB/decade.

Because the plot is in f and not ω we need to convert the break frequency from radians/sec to Hz.

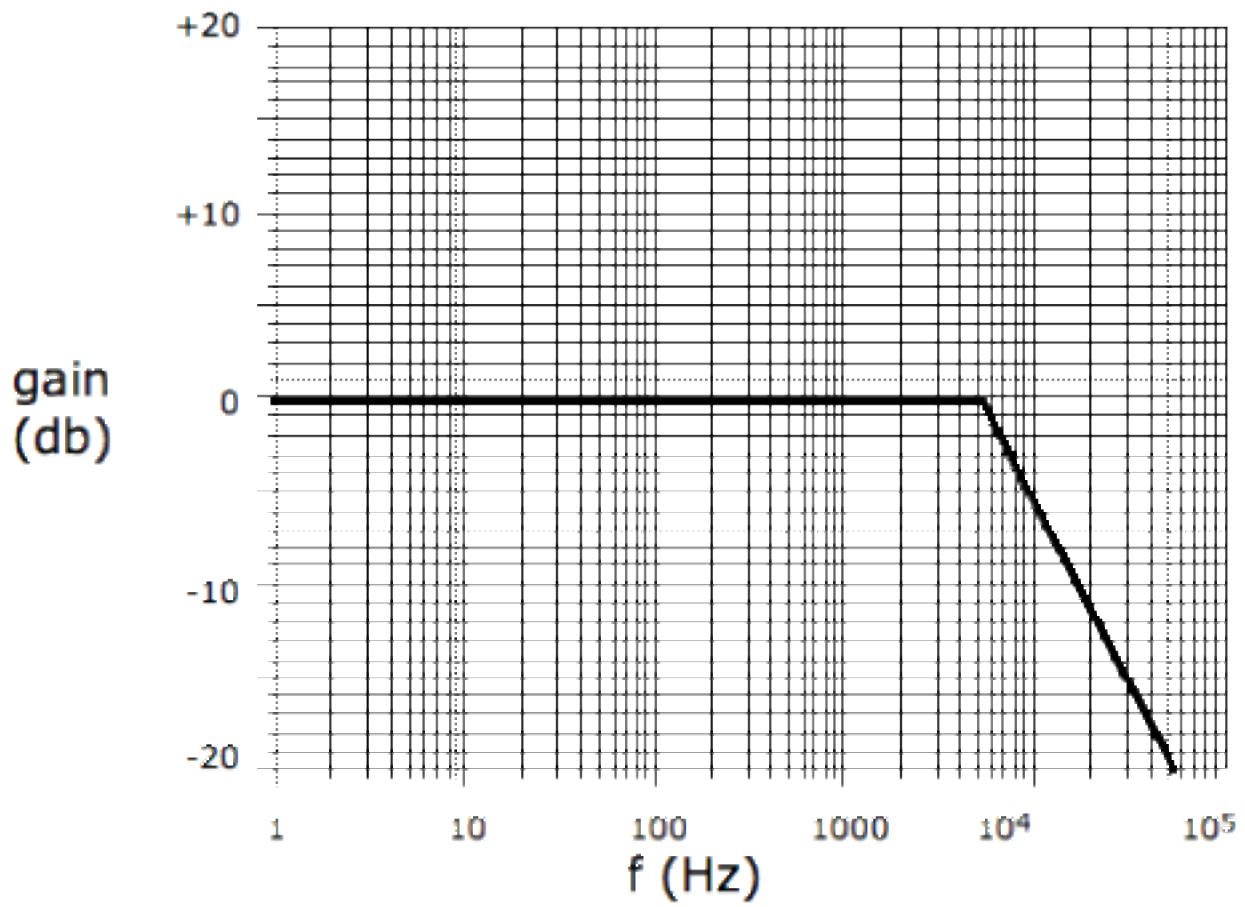
$$f_c = \frac{\omega_c}{2\pi} = \frac{30000}{2\pi} = 4775 \text{ Hz}$$

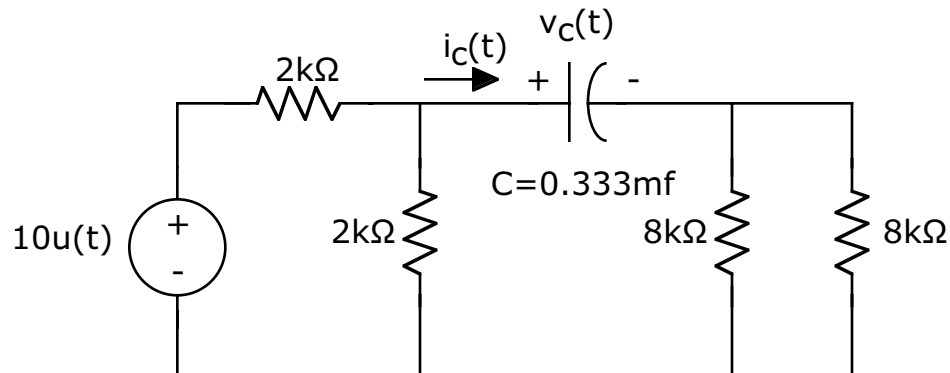
The gain is plotted below

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Problem 9 Transients (10 points)

For the above time-dependent circuit:

- a) What is the state variable in this circuit?

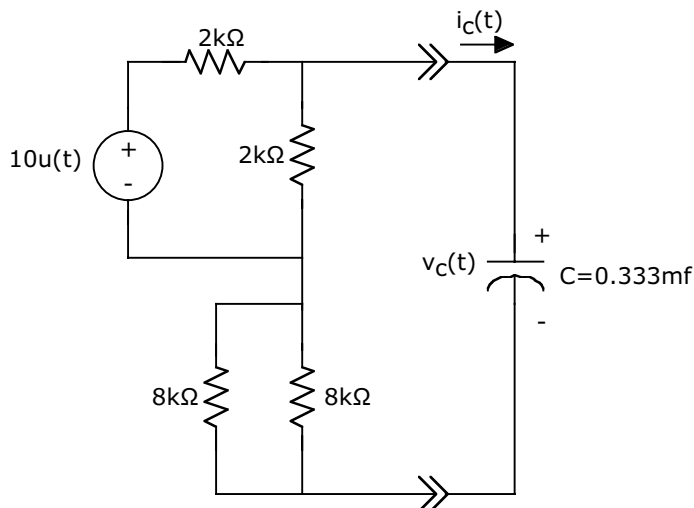
SOLUTION:

$V_c(t)$ is the state variable for a capacitive circuit.

- b) Write the differential equation that describes the response of $V_c(t)$ to the step input $v_s(t)=10u(t)$.

SOLUTION:

The key to writing the differential equation is to find the Thevenin equivalent circuit as seen by the capacitor. This is easiest seen by re-arranging the circuit as shown below.

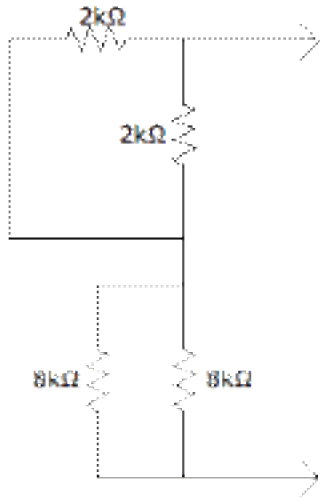


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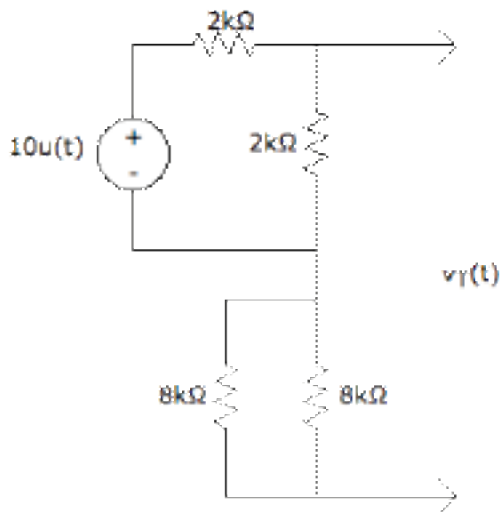
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We turn off the voltage source leaving the circuit shown below.



The equivalent resistance of this network is $2k || (2k + 8k) || 8k = 1k + 4k = 5k\Omega$.

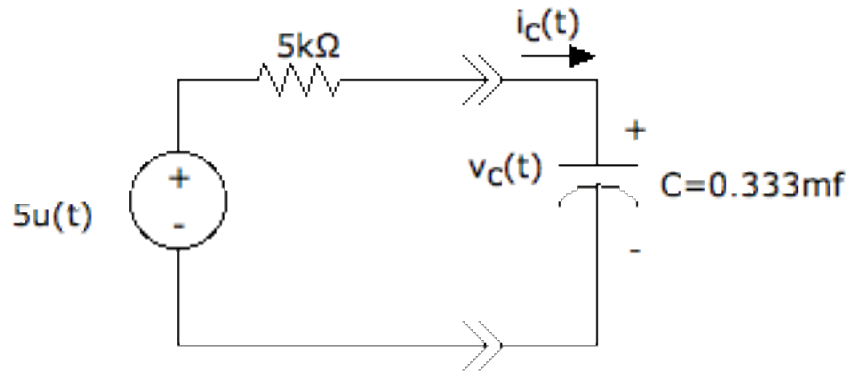
We use the circuit shown below to determine the Thevenin voltage.



Note that there is no current through the $8k\Omega$ resistors so they do not contribute to the Thevenin output voltage. As a consequence the voltage across the vertical $2k\Omega$ resistor determines the output voltage. This voltage is given by a voltage divider relationship with the 10 volt ($t > 0$) source. The voltage across

the vertical $2k\Omega$ resistor is given by $V_T = V_{2k\Omega} = \frac{2k}{2k + 2k} 10 = 5$ volts.

The capacitor then sees the circuit shown below.



This is now identical to the circuits which we solved in class. We use KVL to write $-5 + 5000i_C + v_C = 0$. We then use the terminal relation $i_C = C \frac{dv_C}{dt}$ to re-write this as $-5 + 5000C \frac{dv_C}{dt} + v_C = 0$. Re-arranging gives $5000C \frac{dv_C}{dt} + v_C = 5$. Putting in the value for C gives $1.667 \frac{dv_C}{dt} + v_C = 5$.

c) Determine the time dependent behavior of $V_C(t)$ for $v_s(t) = 10u(t)$ V.

SOLUTION:

The steady state solution (when nothing is changing and the derivative is zero) is given by $v_{STEADY-STATE} = 5$

The transient solution (for the right hand side set to zero) is given by $v_{TRANSIENT} = Ke^{st}$. Putting this solution into the differential equation gives

$$1.667sKe^{st} + Ke^{st} = 0 \text{ which can be solved for } s \text{ to give } s = -\frac{1}{1.667} = -0.6$$

The total solution is given as $v_C(t) = v_{STEADY-STATE} + v_{TRANSIENT} = 5 + Ke^{-0.6t}$

To find K we need to know the voltage at $t=0$. Since no voltage was given we can assume it was zero. This gives $v_C(t=0) = 0 = 5 + Ke^{-0}$ or $K=-5$. The solution is then $v_C(t) = 5 - 5e^{-0.6t}$

d) What is $i_C(t)$?

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SOLUTION:

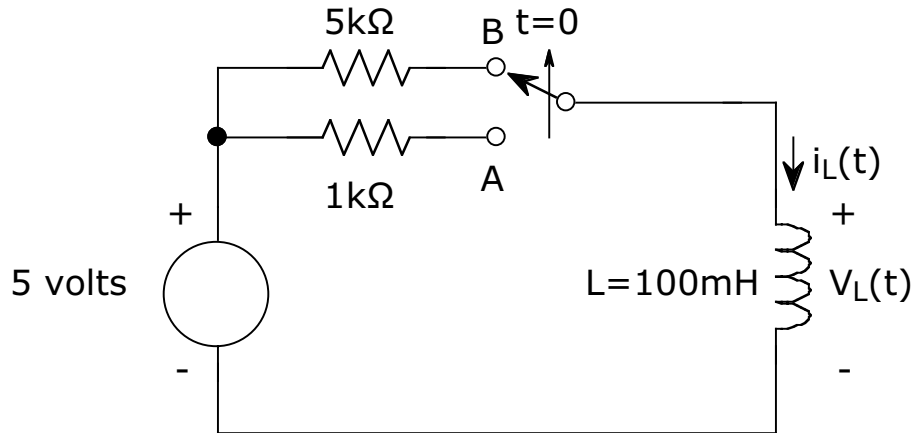
The easiest way to find the current is to simply use the terminal relationship

$i_c = C \frac{dv_c}{dt}$ to compute the current as

$$i_c = C \frac{dv_c}{dt} = (0.33 \times 10^{-3}) \frac{d(5 - 5e^{-0.6t})}{dt} = (0.33 \times 10^{-3})(-5[-0.6]e^{-0.6t}) = 0.001e^{-0.6t}$$

Problem 10 Switched circuits (10 points)

The switch in the circuit shown below has been in position A for a long time and is moved to position B at $t=0$.



(a) What is $i_L(0)$?

SOLUTION:

$i_L(t=0^-)$ is the steady-state current flowing through the inductor with the switch in position A. Since the switch has been in this position for a long time there is no voltage across the inductor ($v_L = L \frac{di_L}{dt} = 0$) and the inductor acts like a short.

The current is then given as $i_L(t=0^-) = \frac{5 \text{ volts}}{1000\Omega} = 5 \text{ mA}$. Since the current through an inductor must be continuous $i_L(t=0^+) = i_L(t=0^-)$

(b) What is $i_L(\infty)$?

SOLUTION:

This is basically asking the same question as (a) except for the switch being in position B.

Since the switch has been in this position for a long time there is no voltage across the inductor ($v_L = L \frac{di_L}{dt} = 0$) and the inductor acts like a short. The current is

then given as $i_L(t=\infty) = \frac{5 \text{ volts}}{5000\Omega} = 1 \text{ mA}$.

(c) What is the time constant of this circuit?

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This is the time constant of the circuit AFTER it has moved from position A to position B, i.e., after the switch has moved. In this position

$$T_C = \frac{L}{R} = \frac{100 \times 10^{-3}}{5000} = 20 \times 10^{-6} = 20 \mu S$$

(d) Find $i_L(t)$ for $t \geq 0$.

SOLUTION

The current is then given by the initial/final value theorem as

$$i_C(t) = (i_C(t=0^-) - i_C(t=\infty))e^{-\frac{t}{T_C}} + i_C(t=\infty) = (5-1)e^{-\frac{t}{20\mu S}} + 1 = 1 + 4e^{-\frac{t}{20\mu S}} \text{ mA}$$

(e) What part of (d) is the forced response?

SOLUTION:

$$i_{C, \text{FORCED}} = 1 \text{ mA}$$

(f) What part of (d) is the natural response?

SOLUTION:

$$i_{C, \text{NATURAL}} = 4e^{-\frac{t}{20\mu S}} \text{ mA}$$