

CASE WESTERN RESERVE UNIVERSITY
Case School of Engineering
Department of Electrical Engineering and Computer Science
ENGR 210. Introduction to Circuits and Instruments (4)

Quiz No. 9

3/26/04

PUT ANSWERS IN THE SPACE PROVIDED AND SHOW YOUR WORK IF APPROPRIATE
BE SURE TO STATE ALL ASSUMPTIONS

Problem 1 Sinusoidal Waveforms (10 points)

(a) What are the Fourier coefficients of $12 \cos(5000t + 30^\circ)$?

$a = 6\sqrt{3}$ (2 points); $b = -6$ (2 points)

Note: we are looking for numerical answers.

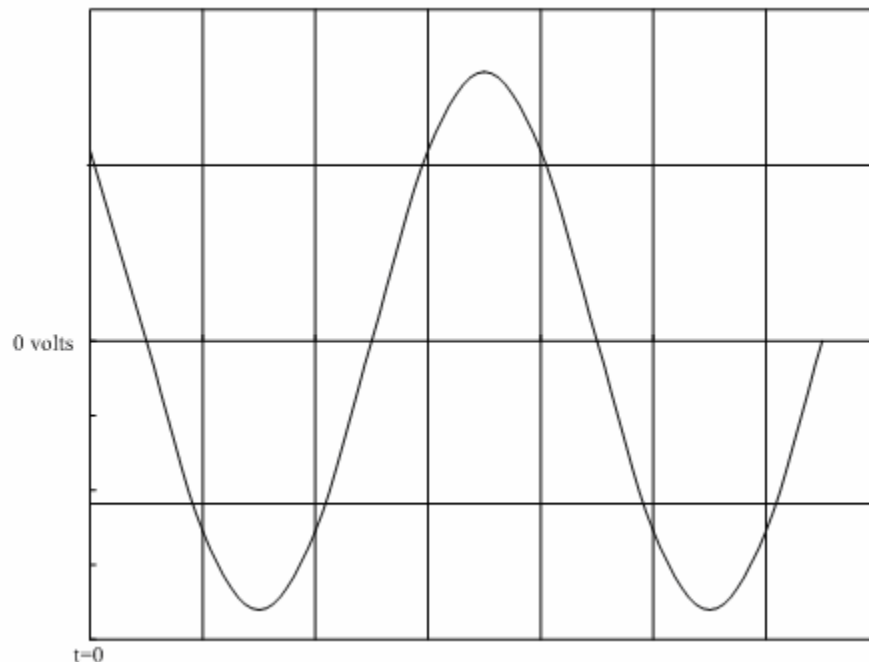
$$v(t) = V_A \cos\left[\frac{2\pi t}{T_0} + \phi\right] = 12 \cos(5000t + 30^\circ)$$

So $V_A = 12$; $\phi = 30^\circ$

And we know $a = V_A \cos \phi = 12 \cos 30^\circ = 12 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$

$$b = -V_A \sin \phi = -12 \sin 30^\circ = -12 \cdot \frac{1}{2} = -6$$

(b) You are in the laboratory and measure the waveform shown below with your oscilloscope. Assume that $t=0$ at the left of the screen and that zero volts is in the middle of the screen. The oscilloscope is set to 12 volts/vertical division, and 50 milliseconds per horizontal division. You want to express this waveform mathematically as $v(t) = A \cos[\omega(t - T_s)]$. What are the numerical values of A , ω and T_s ?



$$A = 18V \text{ (2 points);}$$

$$\omega = 31.42 \text{ rad / s (2 points);}$$

$$T_s = -25ms \text{ (2 points);}$$

The maximum amplitude of the waveform is seen to be 1.5 vertical divisions; therefore,

$$A = 1.5 * 12 = 18V;$$

There are 2 horizontal divisions between successive zero crossings, so the period of the waveform is

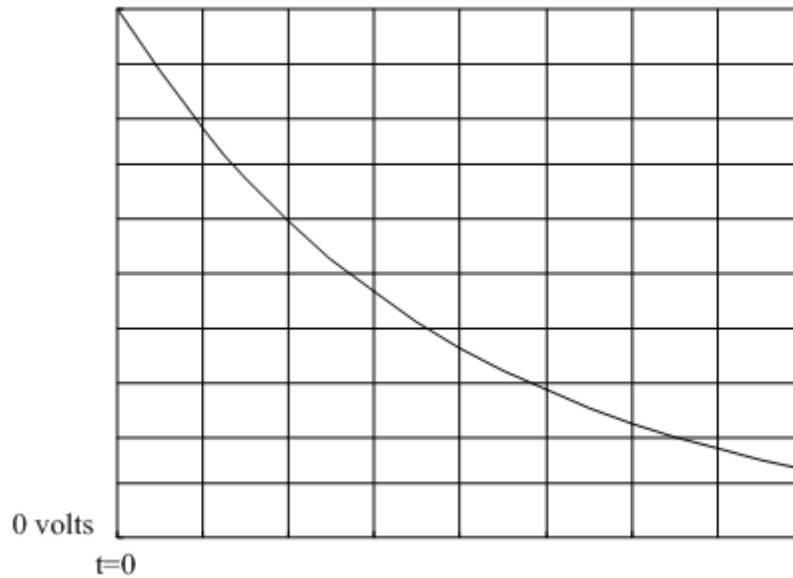
$$T_0 = 50 * 2 * 2 = 200ms;$$

$$\text{Therefore } f_0 = \frac{1}{T_0} = \frac{1}{200ms} = 5Hz; \omega_0 = 2\pi f_0 = 31.42 \text{ rad / s}$$

The positive peak shown in the display is 3.5 divisions to the right of $t=0$, so

$$T_0 + T_s = 200ms + T_s = 3.5 * 50ms;$$

$$\Rightarrow T_s = 3.5 * 50 - 200 = -25ms;$$

Problem 2 (10 points)


- (a) You are in the laboratory measuring an exponential waveform with your oscilloscope. If the units are 20 mS/div horizontal and 8 volts/div vertical determine the numerical value of the time constant T_c for the following waveform.

$$T_c = 78.3ms \text{ (5 points)}$$

$$v(t) = V_A e^{-(t-T_s)/T_c} \text{ and } T_c = \frac{\Delta t}{\ln\left[\frac{v(t)}{v(t+\Delta t)}\right]}$$

At $t=0$, $v(t=0)=10 \text{ div} \times 8 \text{ volts/div}=80 \text{ volts}$

At $t=40 \text{ ms}$, $v(t=40 \text{ ms})=6 \text{ div} \times 8 \text{ volts/div}=48 \text{ volts}$

$$\text{So } T_c = \frac{\Delta t}{\ln\left[\frac{v(t)}{v(t+\Delta t)}\right]} = \frac{40ms - 0}{\ln\left(\frac{80}{48}\right)} = 78.3ms$$

- (b) Write a mathematical expression for the above waveform. Assume that $t=0$ and $v=0$ at the lower left of the display.

$$v(t) = 80e^{-t/78.3ms} \text{ (5 points)}$$

From the graph, we can get $V_A=80V$ ($v(t=0)$);

$$\text{So } v(t) = V_A e^{-(t-T_s)/T_c} = 80e^{-t/78.3ms}$$