Chapter 5

dynamic circuits – voltages and current vary as functions of time

- models for time-varying signals (Chap. 5)
- models for devices that describe the effects of time-varying signals in circuits (Chap. 6 7

5.1 waveform – equation or graph defining a signal as a function of time

NOTATION

up to now we have considered dc (direct current) signals which are constants, i.e.

$$v(t) \quad V_0 \qquad -\infty < t < \infty$$

usually use uppercase for dc quantities

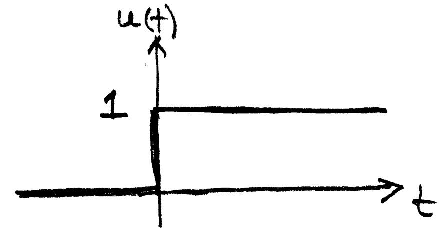
Time-varying quantities are usually represented by lower case

$$v(t)$$

Reference marks (+, −) for voltage and (→) for current are not THE polarity or direction but the reference for what positive means

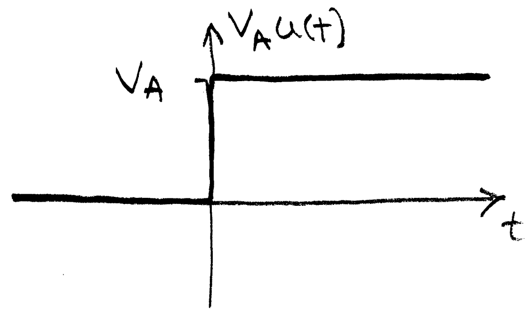
5-2 The Unit Step

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$



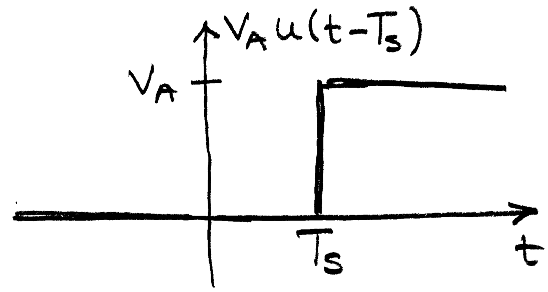
Signals can be scaled

$$V_A u(t) = \begin{cases} 0 & t<0 \\ V_A & t \geq 0 \end{cases}$$



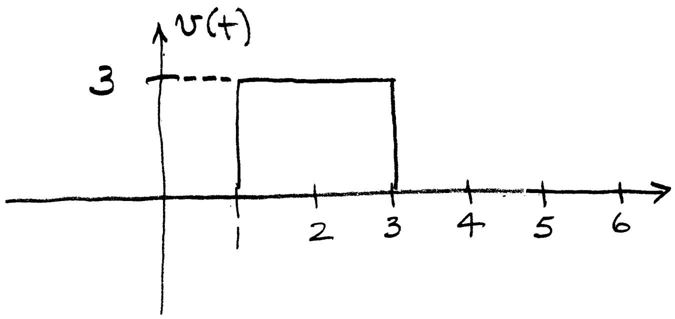
Signals can also be delayed.

$$V_A u(t-T_s) = \begin{cases} 0 & t<T_s \\ V_A & t \geq T_s \end{cases}$$

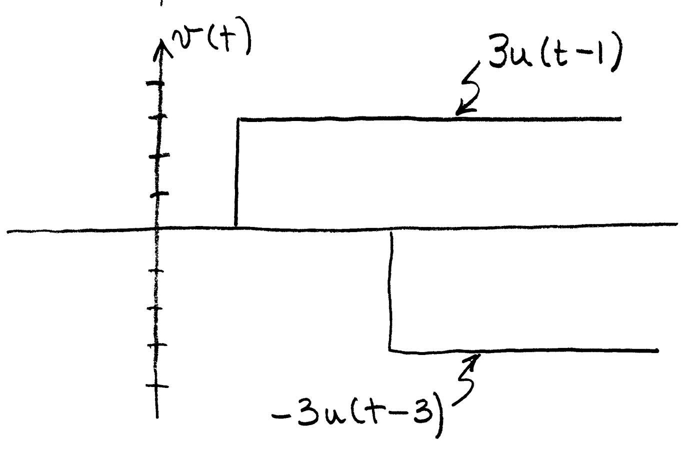


Example 5-1

Express the given pulse waveform in terms of step functions.



Pulses are often called gating functions and are used to turn things on or off.

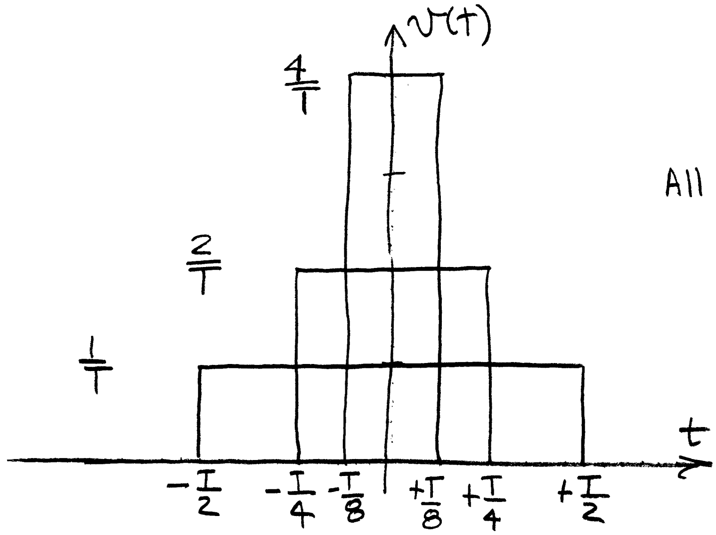


$$v(t) = 3u(t-1) - 3u(t-3)$$

The impulse function

Consider the pulse $v(t)$ of unit area centered at $t=0$

$$v(t) = \frac{1}{T}\left[u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)\right]$$



All three functions have unit area.

$$\frac{4}{T} \times \frac{T}{4} = 1$$

$$\frac{2}{T} \times \frac{T}{2} = 1$$

$$\frac{1}{T} \times T = 1$$

Consider the limit as $T \to 0$ [the amplitude goes to ∞]

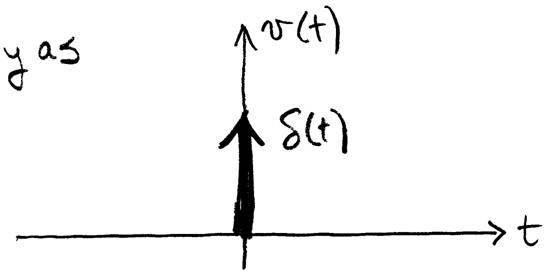
mathematically for the unit impulse

$$\delta(t) = 0 \quad \text{for } t \neq 0$$

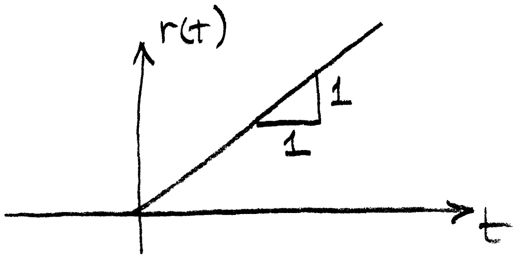
$$\int_{-\infty}^{+\infty} \delta(x)\, dx = \delta(t)$$

The definition is a derivative $\quad \lim_{T \to 0} \dfrac{u(t+\frac{T}{2}) - u(t-\frac{T}{2})}{T} = \dfrac{du(t)}{dt}$

It's shown graphically as



The unit ramp



$$r(t) = \int_{-\infty}^{t} u(x)\,dx = t\,u(t)$$

Singularity Functions

$\delta(t)$, $u(t)$ and $r(t)$ are mathematically related and are often called singularity functions

by integration

$$u(t) = \int_{-\infty}^{t} \delta(x)\,dx$$

$$r(t) = \int_{-\infty}^{t} u(x)\,dx$$

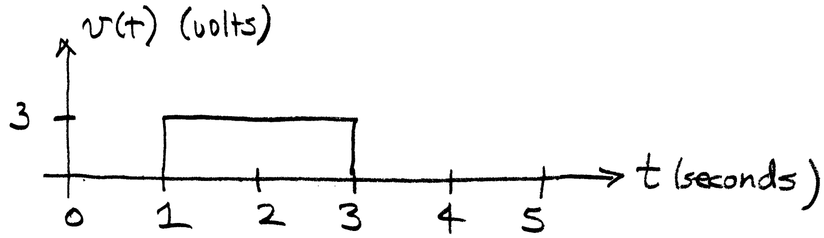
by differentiation

$$u(t) = \frac{dr(t)}{dt}$$

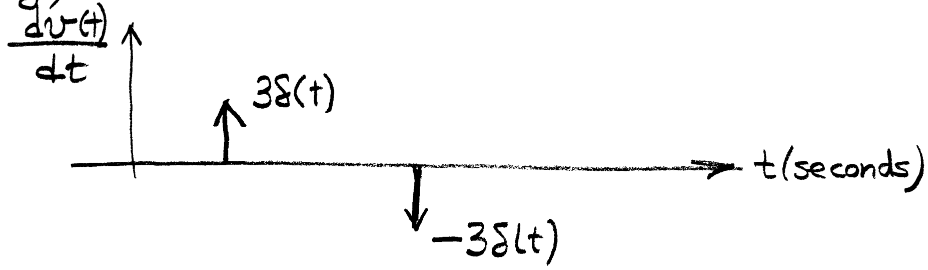
$$\delta(t) = \frac{du(t)}{dt}$$

Example 5-2

Calculate and sketch the derivative of $v(t)$.



Graphically,



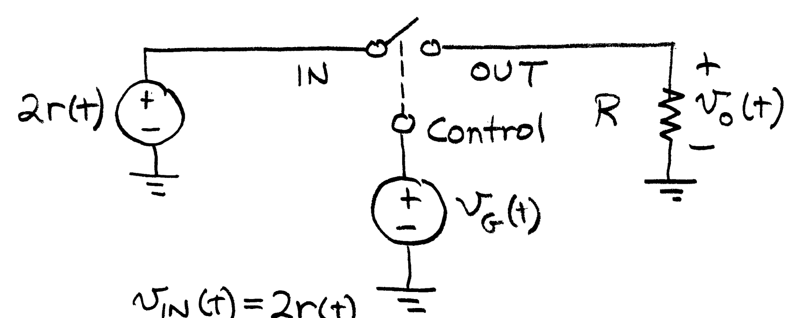
mathematically,

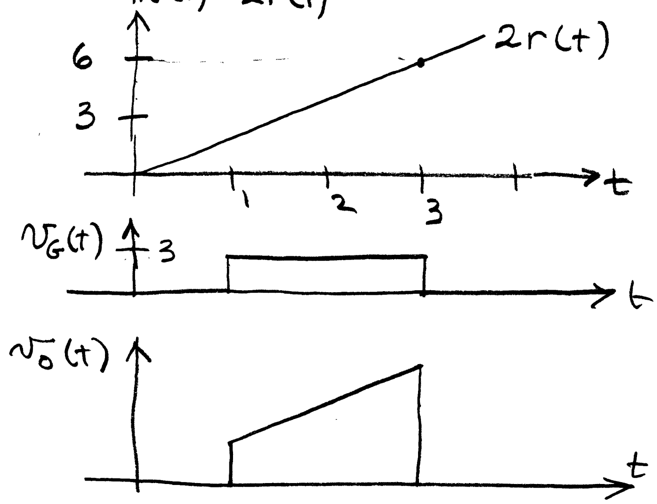
$$v(t) = 3u(t-1) - 3u(t-3)$$

$$\frac{dv(t)}{dt} = 3\delta(t-1) - 3\delta(t-3)$$

Example 5-3

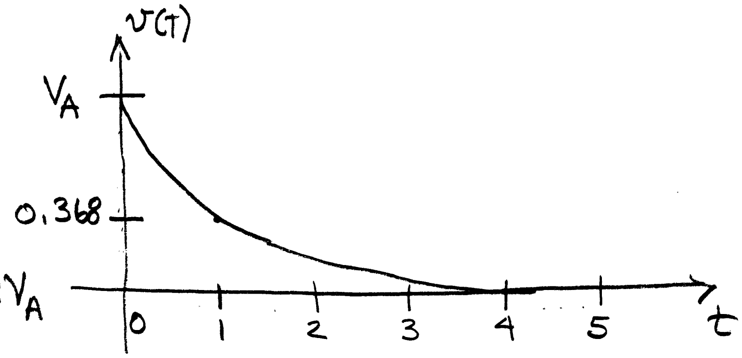
Consider an ideal electronic switch whose input is a ramp $2r(t)$. Find the switch output $v_o(t)$ for the gate function $v_G(t) = 3u(t-1) - 3u(t-3)$





5-3 The Exponential Waveform

$$v(t) = V_A e^{-\frac{t}{T_c}} u(t)$$



$$v(T_c) = V_A e^{-1} = .368 V_A$$

$$v(5T_c) = V_A e^{-5} = .00674 V_A$$

We say that after $5T_c$ $v(t) = 0$

Properties of exponential waveforms

Consider $v(t)$ for $t > 0$

$$v(t) = V_A e^{-\frac{t}{T_c}}$$

$$v(t + \Delta t) = V_A e^{-\frac{t+\Delta t}{T_c}} = V_A e^{-\frac{t}{T_c}} e^{-\frac{\Delta t}{T_c}}$$

$$\frac{v(t+\Delta t)}{v(t)} = \frac{V_A e^{-\frac{t}{T_c}} e^{-\frac{\Delta t}{T_c}}}{V_A e^{-\frac{t}{T_c}}} = e^{-\frac{\Delta t}{T_c}}$$

independent of t, V_A

This is called the decrement property.

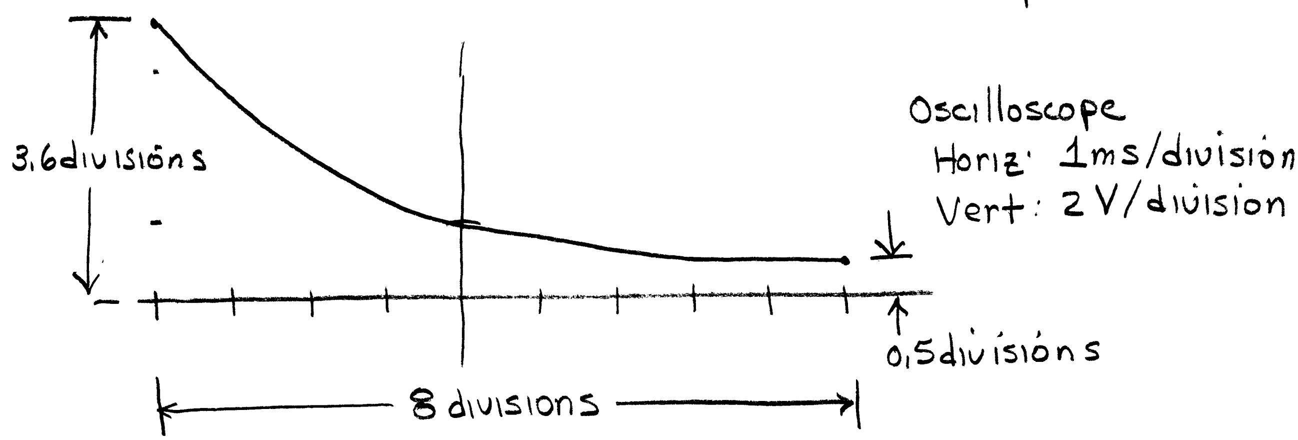
Mathematically,

$$\frac{d\,v(t)}{dt} = V_A \left(-\frac{1}{T_c}\right) e^{-\frac{t}{T_c}} = -\frac{v(t)}{T_c}$$

This is called the slope property

Example 5-6

Consider the oscilloscope measurement of an exponential waveform. Determine the time constant of the exponential.



You can readily find T_c from the decrement property of the exponential.

$$\frac{v(t+\Delta t)}{v(t)} = e^{-\frac{\Delta t}{T_c}}$$

Take the log of both sides and solve for T_c

$$\ln\left(\frac{v(t+\Delta t)}{v(t)}\right) = -\frac{\Delta t}{T_c}$$

$$T_c = \frac{-\Delta t}{\ln\left(\frac{v(t+\Delta t)}{v(t)}\right)} = \frac{\Delta t}{\ln\left[\frac{v(t)}{v(t+\Delta t)}\right]}$$

Converting the oscilloscope waveform to measured values

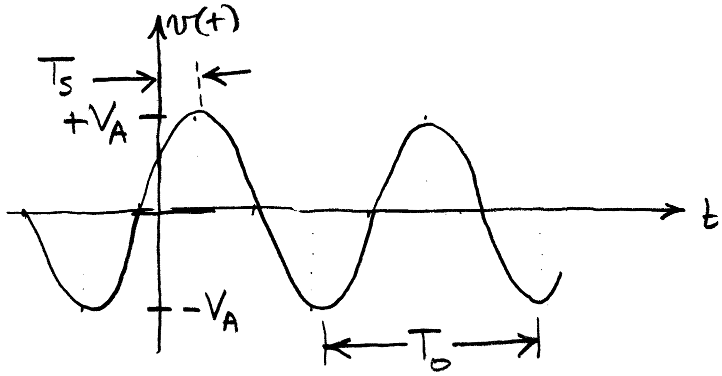
$$v(t) = 3.6 \text{ div} \times \frac{2 \text{ Volts}}{\text{division}} = 7.2 \text{ volts}$$

$$v(t+\Delta t) = 0.5 \text{ div} \times \frac{2 \text{ Volts}}{\text{division}} = 1 \text{ volt}$$

$$\Delta t = 8 \text{ div} \times \frac{1 \text{ ms}}{\text{div}} = 8 \text{ milliseconds}$$

$$T_c = \frac{\Delta t}{\ln\left[\frac{v(t)}{v(t+\Delta t)}\right]} = \frac{8\times 10^{-3}}{\ln\left[\frac{7.2}{1}\right]} = \frac{8\times 10^{-3}}{1.9741} = 4.0525\times 10^{-3} \text{ seconds}$$

5-4 The sinusoidal waveform



$$v(t) = V_A \cos\left[2\pi\left(\frac{t-T_s}{T_o}\right)\right]$$

we will typically use cosines

V_A - amplitude

Instead of T_s we often use the phase angle ϕ

$$v(t) = V_A \cos\left[2\pi\frac{t}{T_o} + \phi\right]$$

$$\therefore \phi = -2\pi\frac{T_s}{T_o}$$

cyclic frequency $f_o = \frac{1}{T_o}$

angular frequency $\omega_o = 2\pi f_o = \frac{2\pi}{T_o}$

If $v(t) = V_A \cos\left[2\pi\left(\frac{t-T_s}{T_o}\right)\right]$ Then $v(t) = V_A \cos\left[\frac{2\pi t}{T_o} \underbrace{- \frac{2\pi T_s}{T_o}}_{+\phi}\right]$

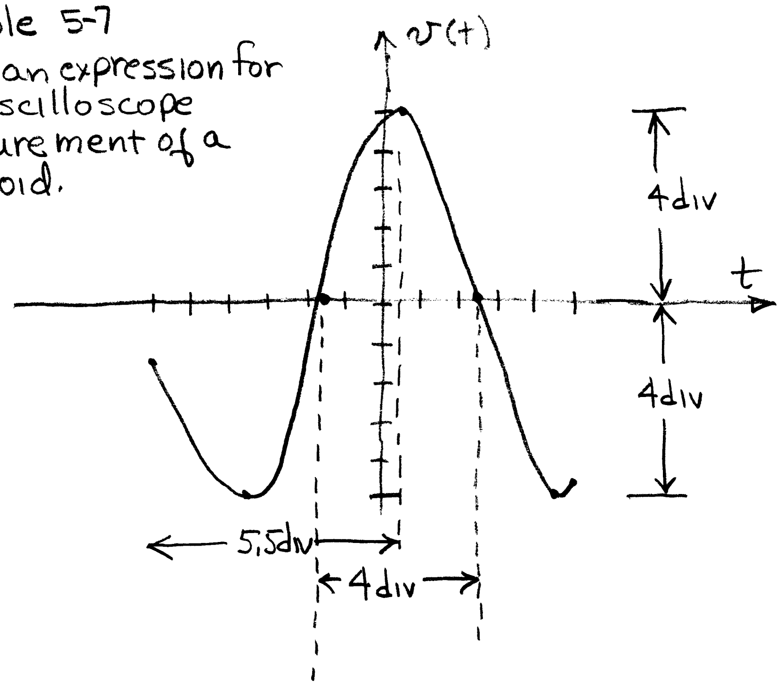
Using $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$v(t) = \underbrace{V_A \cos\phi}_{a} \cos\left(\frac{2\pi t}{T_o}\right) \underbrace{- V_A \sin\phi}_{b} \sin\left(\frac{2\pi t}{T_o}\right)$$

$v(t) = a\cos\left(\frac{2\pi t}{T_o}\right) + b\sin\left(\frac{2\pi t}{T_o}\right)$ $\quad a,b \equiv$ Fourier coefficients

$V_A = \sqrt{a^2+b^2}$ $\quad \frac{b}{a} = \frac{-V_A\sin\phi}{V_A\cos\phi} \Rightarrow \phi = \tan^{-1}\left(-\frac{b}{a}\right)$

Example 5-7
Derive an expression for this oscilloscope measurement of a sinusoid.



Oscilloscope:
Vertical: 5V/div
Horiz: 0.1 mS/div

amplitude $V_A = 4\,\text{div} \times \frac{5V}{\text{div}} = 20$ volts

period $T_0 = 8\,\text{div} \times \frac{0.1\,\text{ms}}{\text{div}} = 0.8\,\text{mS}$

oscilloscope measurement is $\frac{1}{2}$ cycle

To determine T_s you need to define the time origin.
Use the left-hand edge as our time origin.

T_s is the location of the nearest positive peak.
The closest positive peak to the origin is actually to the left of the origin. The positive peak we measured is actually one cycle AFTER the actual nearest peak.

$$\therefore\ T_0 + T_s = 5.5\,\text{div} \times 0.1\frac{\text{mS}}{\text{div}} = 0.55\,\text{mS}$$

$$T_s = 0.55\text{ms} - T_0 = 0.55\text{ms} - 0.8\text{ms} = -0.25\text{mS}$$

$$\therefore\ v(t) = V_A \cos\left[2\pi\left(\frac{t - T_s}{T_0}\right)\right] = 20\cos\left[2\pi\left(\frac{t + .00025}{.0008}\right)\right]$$

This can also be expressed in alternative forms.