ENGR 210 Lab 10
Sampling and Aliasing

In the previous lab you examined how A/D converters actually work. In this lab we will consider some of the consequences of how fast you sample and of the signal processing required to get valid data from samples.

A. BACKGROUND

1. Sampling

Sampling actually refers to several processes that occur when you collect data and convert it to analog form. Sampling includes:

(1) the nature of the input signal,

(2) when you actually take the samples (that is how often samples are taken),

(3) how fast the A/D converter works, and

(4) how many bits there are in the result.

You considered (3) to some extent in your last lab when you studied A/D converters. We will defer any additional study of how A/D converters work and how fast they convert to a later course.

2. Circuits as signal processors

All circuits may be thought of as signal processors. Even purely resistive circuits process a signal in the sense that the amplitude may be changed. If the circuit contains capacitors, inductors or other elements, the signal will be changed significantly by the circuit. First, our primary interest may be in the characteristics of the input signal, e.g., it may be from a sensor that is monitoring a parameter of some system in which we are interested. Since we only have the output of an electronic interface to the sensor available to us, we must remove the processing effect of that interface in order to see the sensor's output signal. (This situation represents unintentional processing of the desired signal.) In a second scenario, the circuit may be intentionally used for the purpose of signal processing. Here the signal processor is a filter with prescribed amplitude and phase characteristics. In this case it is important that we understand the effect of the circuit on signals of the type expected at the input.

In general, signals of interest are not simple sinusoids, so we must develop an expansion of our phasor analysis technique to enable us to evaluate the effect of circuits on these signals. As an example of such a signal, consider the one shown in Figure 1 : Musical Signal Produced as Sum of Sinusoids

(a), representing a musical signal composed of three sinusoidal signals.

(a) Original component sinusoids (b) Resultant complex musical signal

Figure 1 : Musical Signal Produced as Sum of Sinusoids

Figure 2 (a) and (b) show the signal after passing through a "low pass" filter and a "high pass" filter, respectively. Since the original signal is not a sinusoid, we can't represent it by a simple phasor. The signal, however, was generated by adding the three sinusoidal signals shown in Figure 1 : Musical Signal Produced as Sum of Sinusoids

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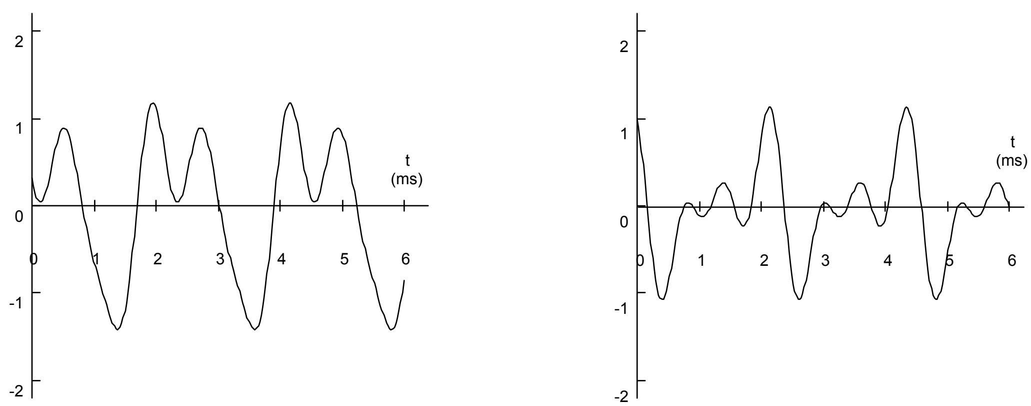
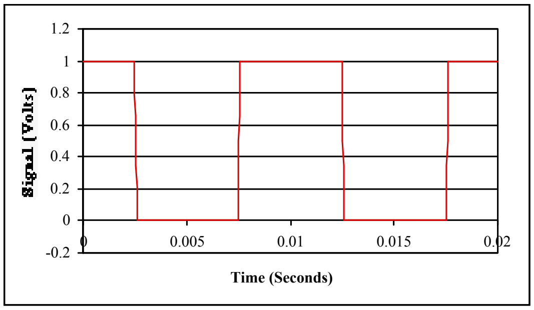


(a) Musical signal filtered with a LPF (b) Musical signal filtered with a HPF

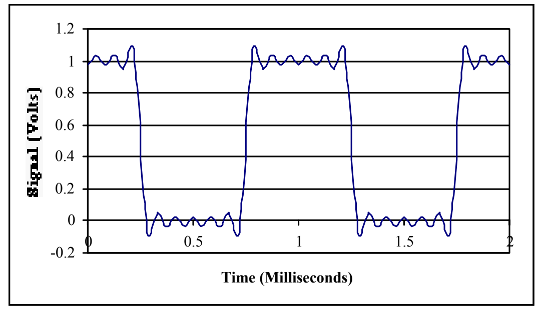
Figure 2. Musical Signal Processed by a Filter

2. Frequency Content of Signals

The nature of the input signal is actually very critical to data acquisition. As described in class complex signals are typically composed of many frequencies. For the purpose of this lab we will be concerned only with the distribution of these frequencies. This will be studied in later courses under various names such as spectrum analysis and Fourier analysis. It will be very rare that you actually have to sample a sinusoidal signal of a single frequency, i.e., a sine wave. More common are pulsed and digital waveforms such as the square wave. Let us consider representing a 100 Hz square wave with amplitude of 1 volt by a sum of sinusoids. Technically, this is called Fourier analysis and, using mathematical techniques which will we will not cover in this course, you can actually calculate what the amplitudes of the various sinusoids should be to represent some known function.



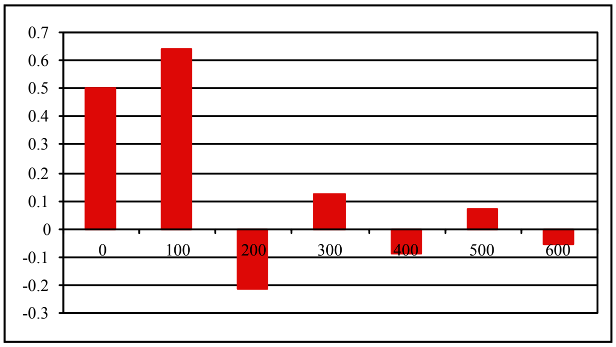
(a) f(t) = 100 Hz square wave

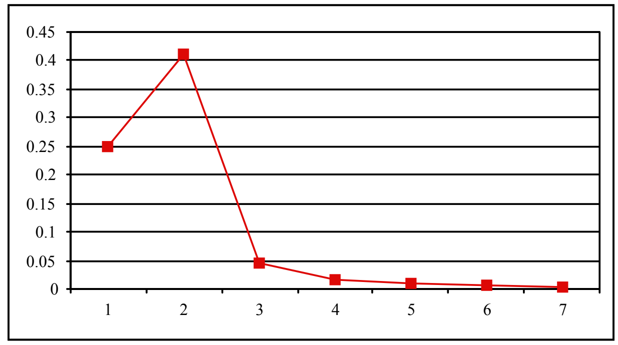


(b) $f(t) = 0.5 + 0.64\cos\omega t - 0.212\cos 3\omega t + 0.127\cos 5\omega t - 0.0909\cos 7\omega t + 0.0707\cos 9\omega t - 0.0579\cos 11\omega t$

Figure 3. Square Wave and Its Approximation By Sinusoids

The approximation in Figure 3 is an approximation because the square wave is represented by a sum of only seven sinusoids; an exact representation would require an infinite sum of sinusoids. Notice that these sinusoids are all multiples, also called harmonics, of the fundamental frequency of the square wave. This will always be true for periodic signals. Note also that the amplitudes of the harmonics decreases with frequency. This is generally true but not always true for adjacent harmonics. You can plot the magnitudes of these harmonics to get Figure 4 (a). You can also plot the square of these amplitudes to get the power spectrum shown in Figure 4(b). In general, you use an amplitude spectrum when you are computing the spectrum. In the laboratory you will typically use a special instrument called a spectrum analyzer to measure the power spectrum and display it. There are also special computer programs which compute the Discrete Fourier Transform (DFT) or the Fast Fourier Transform (FFT) of a signal. While similar to the spectra shown in Figure 4, the DFT and FFT have special mathematical meanings, which will be covered in later courses in signal processing.





(a) Amplitude spectrum (b) Power spectrum

Figure 4. Spectrum of 100 Hz Square Wave

3. Sampling of Signals

When you previously used LabView you simply sampled the data without any thought about how fast you were sampling the changing data. In reality, any engineer who wants to acquire meaningful data cannot ignore the relationship between the sampling rate (or frequency) and the frequency spectrum of the data. This relationship can best be illustrated by sampling sinusoids. For example, let us sample a 100 Hz sine wave at 500 Hz, i.e. 5 samples per period of the sine wave. As shown in Figure 5 one can draw a smooth sinusoid through the sample points.

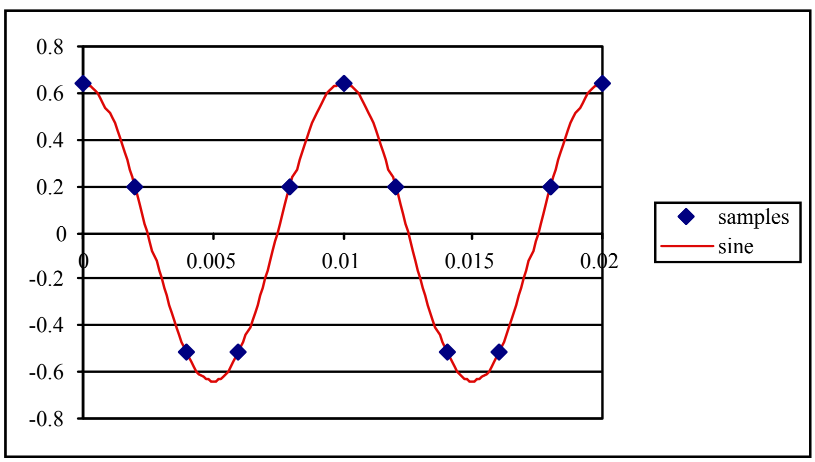


Figure 5. Sampling of 100 Hz sine wave at 500 Hz

The ability to draw a sinusoid of the correct, original frequency through these sample points also holds true if we sample the sine wave at 200 Hz as shown in Figure 6 and Figure 7.

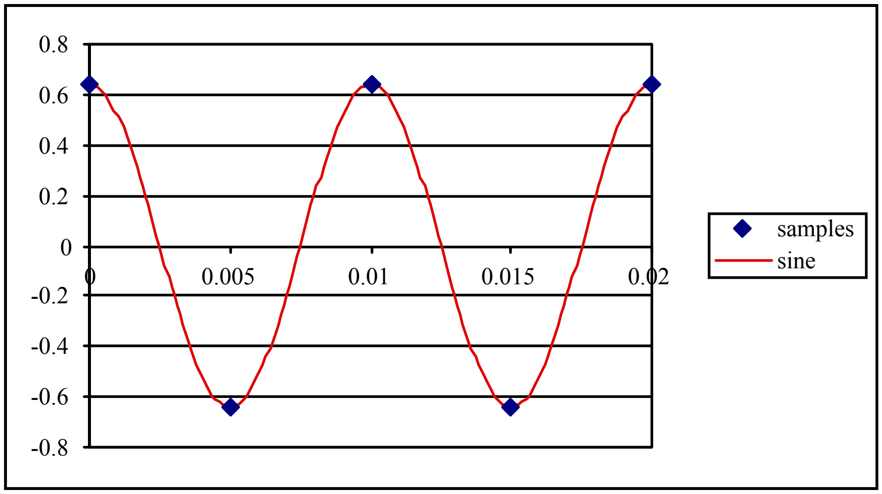


Figure 6. Sampling of 100 Hz sine wave at 200 Hz

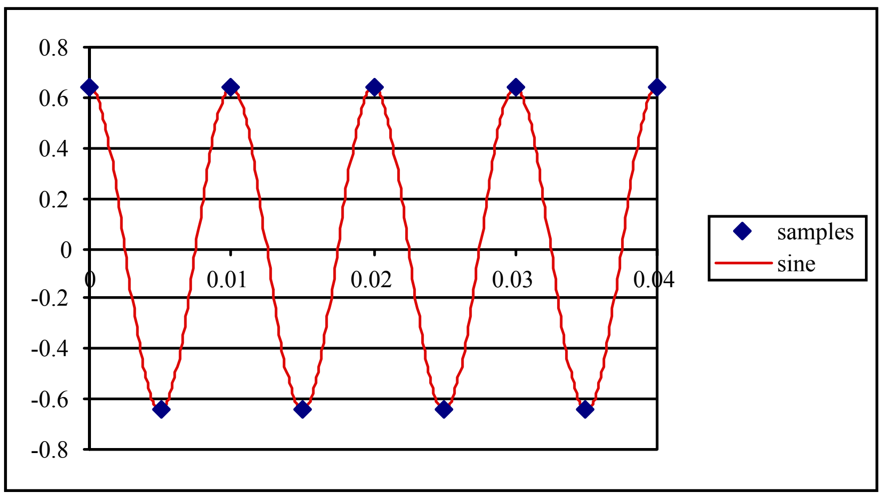


Figure 7. Sampling of 100 Hz sine wave at 200 Hz

Note that you can continue to draw a sine wave of the correct frequency through the sample points. Nyquist's Sampling Theorem (one of the most famous results in electrical engineering) says that you can reconstruct the original sine wave from the samples of the sine wave only if the sampling frequency is at least twice that of the original sine wave. We have already demonstrated that we can reconstruct the original sine wave from samples provided that the samples occur at the rate of at least two per cycle, i.e. the sampling frequency is at least twice the signal frequency. What happens if the sampling frequency is lower than the signal frequency? This is illustrated in Figure 8. The samples are still on the original sine wave but if we attempt to reconstruct a sine wave that passes through the samples we get the different sine wave shown in blue in Figure 9. Note that the frequency of the reconstructed sine wave is lower than that of the originally sampled sine wave. You will study the mathematics behind this in advanced classes but this phenomenon is generally known as aliasing. As long as the sampling frequency is lower than twice the signal frequency you cannot accurately reconstruct the signal waveform.

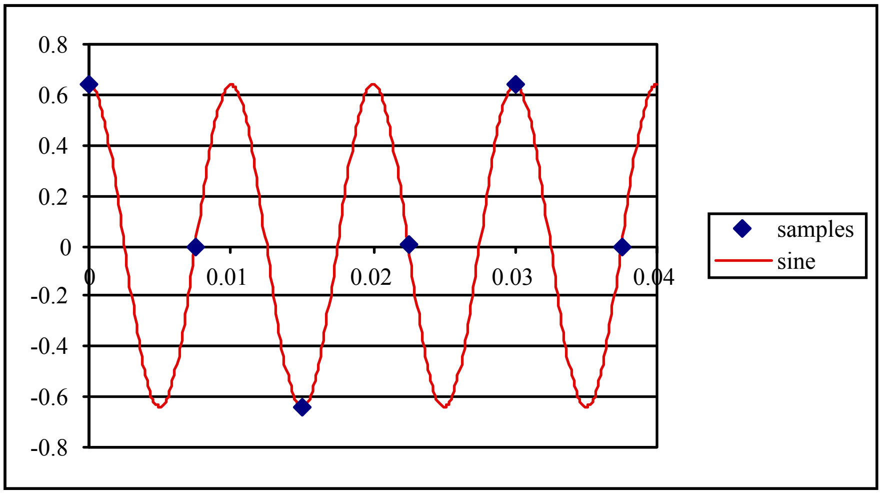


Figure 8. Sampling of 100 Hz sine wave at 50 Hz

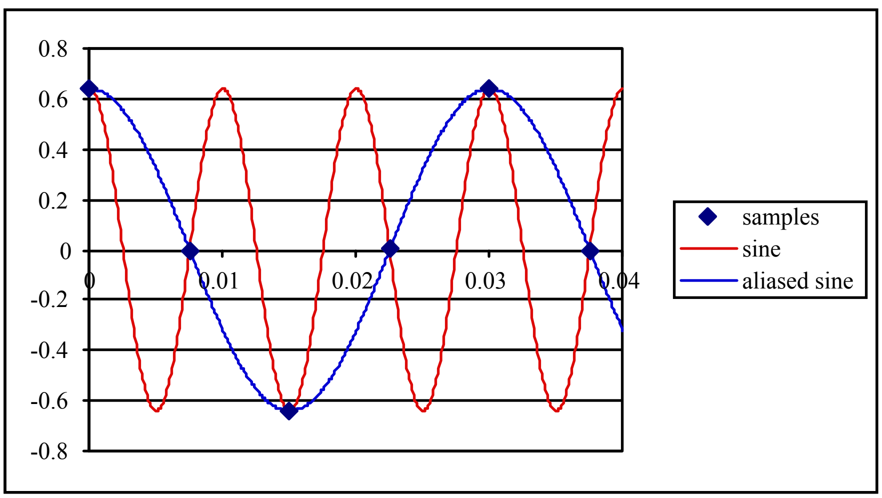


Figure 9. Aliased Reconstruction of sine wave from 50 Hz samples

4. Signal Conditioning

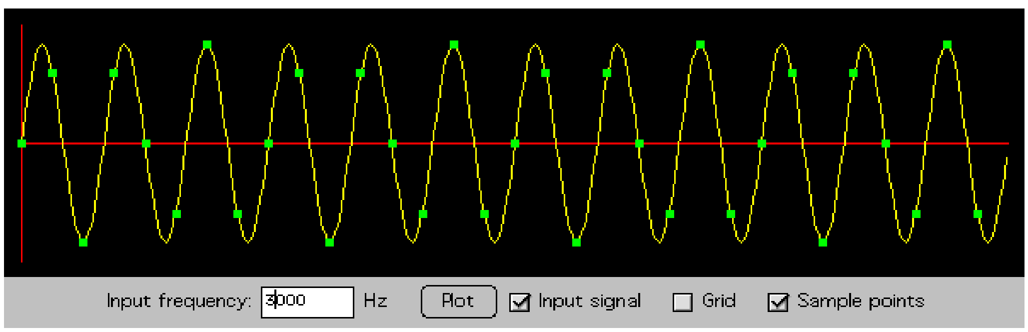
In general a signal is composed of sinusoids of many frequencies. If you want to sample a complex waveform and reconstruct it accurately the signal must be bandlimited. This means that, given the sampling frequency fs, there can be no frequency components in the spectrum of the signal which are higher in frequency than fs/2. There is only one way to guarantee this – low-pass filter the waveform to be sampled. The cut-off frequency of the low-pass filter should be chosen to be fs/2. This will result in removing all frequency components in the signal that cannot be accurately reproduced without aliasing. In practice, this is a minimum requirement. For high-quality reproduction, engineers often use active filters which will more severely attenuate frequencies above fs/2. Also, most engineers will use a higher sampling frequency, i.e., 2fs or higher. This practice is called oversampling and allows higher-quality reconstruction of the original waveform. You often find oversampling in digital recordings for compact disks. For example, audio signals typically include components up to around 20kHz. Because of this CD recording uses a sampling rate of 44.1kHz; Digital Audio Tape (DAT) uses 32, 44.1 or 48kHz sampling rates.

B. LAB INSTRUCTIONS

You will use several Web sites which contain interactive Java applets which illustrate sampling and aliasing.

Part 1: Sampling and aliasing in the time domain

You will need a Java capable browser to complete this lab. Use your Internet browser to go to http://www.dsptutor.freeuk.com/aliasing/AliasingDemo.html. Depending upon your browser you may need to select either the Java 1.02 version or Java 1.1 version. You should see something like this:



You will initially see only the black background and the red axes. When you select an input frequency you can click on the Plot button to get something like that shown above, i.e. a yellow sine wave with green sampling points.

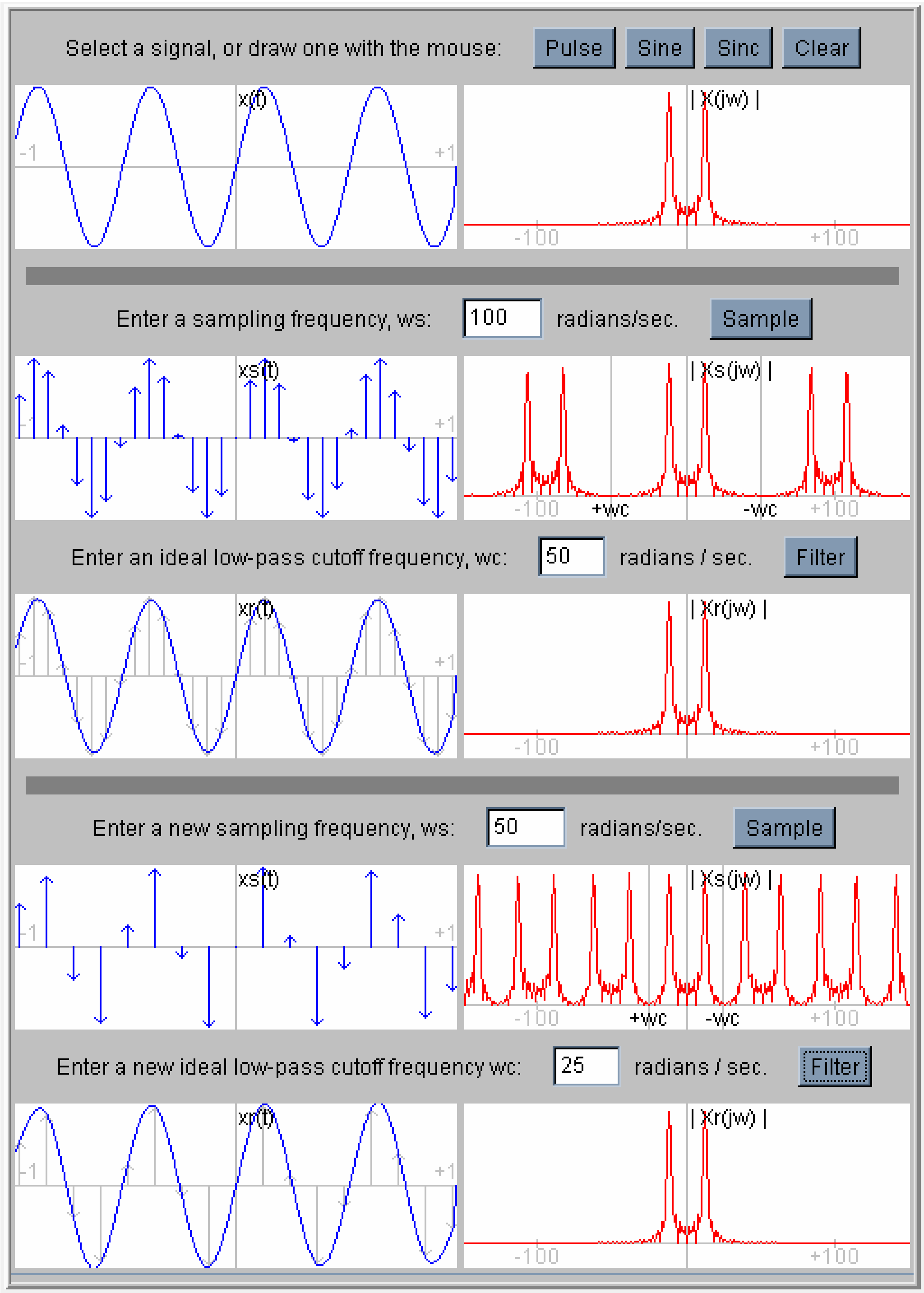
The applet is based on a fixed sampling rate of 8000 samples per second (one sample every 0.125 milliseconds). According to the sampling theorem, a sinusoidal signal (or a sinusoidal component of a complex signal) can be correctly reconstructed from values sampled at discrete, uniform intervals as long as the signal frequency is less than half the sampling frequency. This signal frequency limit is often called the folding frequency and is where aliasing occurs. Since the applet is based on a fixed sampling rate of 8000 samples per second the folding frequency is thus half of 8000 Hz or 4000 Hz.

1. With the input frequency set at its initial default value of 7000 Hz, and only the Input signal checkbox selected, click the Plot button. The sinusoidal input signal, at its true frequency of 7000 Hz, is plotted.

2. Select Grid to show the instants of time at which the signal is sampled. The horizontal (time) axis in the plot covers a total of 0.004 seconds worth of the input signal. The signal is sampled at a rate of 8000 samples/s, so there are 8000*0.004 = 32 sample instants shown.
3. Select Sample points to mark the sampled values of the signal. These occur where the signal intersects the vertical sample markers. The important thing is to note that the sample points seem to trace out a sine wave of a lower frequency than the true signal frequency. You can see this more clearly if you toggle off the Input signal plot.
4. Select Alias frequency. (This feature does not always work.) This shows the sinusoidal signal at the alias frequency. A digital signal processing system to which the input signal samples are input does not know what the signal is doing between samples, and therefore cannot distinguish between sampled versions of the true input signal and the apparent alias signal. As a result it will process the signal as if it were at the lower frequency.
5. The alias frequency can be measured from the plot, just as from an oscilloscope trace. The apparent period of the sampled since wave is 8 sampling intervals, or 8*0.125 ms = 1.0 ms. The corresponding alias frequency is 1 / 1.0 ms = 1000 Hz.
6. Using this applet try input frequencies of 1000, 3000, 4000, 5000, 7000, and 9000 Hz. Indicate frequencies at which aliasing occurs and what the apparent frequency of the aliased signal is. Record your observations in Data Table 1

Part 2: Sampling and frequency in the frequency domain

1. Use your Internet browser to go to another Web site at http://www.jhu.edu/~signals/sampling/index.html. This applet will also display a frequency spectrum of the signals as the sampling proceeds.



This applet will let you control the type of input signal, the sampling frequency (rate), and the cutoff frequency of an ideal low-pass filter used to reconstruct the original signal from the sampled signal. The uppermost pane shows the input (original) signal on the left, and the frequency spectrum of the signal on the right. The different types of signals will have different frequency contents, which will be readily apparent from their frequency spectra.

The second pane lets you select the sampling rate (or frequency) in radians/sec. The plot on the left shows the sampled signal. The spectrum on the right is the spectrum of the sampled signal; notice that it is now periodic in *frequency*, with a period equal to the sample frequency. Essentially, the original signal spectrum repeats itself around multiples of the sampling frequency. This is a property of sampling and will be studied in advanced digital signal processing classes.

The third pane allows you to specify an ideal low-pass filter to use in the reconstruction of the signal from the samples, and displays the resultant (or reconstructed) signal after sampling and that signal's frequency spectrum. When you have aliasing the output waveform will not be the same as the input waveform, and it will not have the same frequency spectrum.

The fourth and fifth panes are duplicates of the second and third panes, and allow you to specify different values of sample frequency and filter frequency for comparison.

2. Using this applet you will test the various combinations of signal and sampling frequency to determine when aliasing occurs. Set the sampling frequency to the value shown in Data Table 2-4. Set the output low-pass filter to half the sample frequency. Examine the reconstructed signal for distortion due to aliasing. Record your results in Data Table 2 for the pulse function, in Data Table 3 for the sine function, and in Data Table 4 for the sinc function. Note that there is room in the data tables for you to make comments about the observed spectra.

   Now set the signal to a sinc function, and set the sampling frequency at 50 rad/s. Set the output low-pass filter frequency to the value shown in Data Table 5. Examine the output signal for distortion (in this case, watch for both the signal being significantly different from the original, and for the signal to be significantly different from the samples shown in light gray). Record the results in Data Table 5.

   Repeat the experiment for a pulse function sampled at 100 rad/s, using the filter frequencies shown in Data Table 6. Record your results in Data Table 6.

DATA AND REPORT SHEETS FOR LAB 10

Student Name (Print): ________________ Student ID: ______________________

Student Signature: ___________________ Date: ___________________________

Student Name (Print): ________________ Student ID: ______________________

Student Signature: ___________________ Date: ___________________________

Student Name (Print): ________________ Student ID: ______________________

Student Signature: ___________________ Date: ___________________________

Lab Group: __________________________

Data Table 1. Sampling and aliasing frequencies

Input frequency	Is aliasing observed? Yes or No	If so, frequency of aliased signal.
1000 Hz		
3000 Hz		
4000 Hz		
5000 Hz		
7000 Hz		
9000 Hz		

Data Table 2. Effect of sampling frequency on sampled pulse function.

Sampling frequency	Is aliasing observed? Yes or No	Notes
100 rad/s		
80 rad/s		
70 rad/s		
60 rad/s		
50 rad/s		
40 rad/s		

Data Table 3. Effect of sampling frequency on sampled sine function.

Sampling frequency	Is aliasing observed? Yes or No	Notes
80 rad/s		
60 rad/s		
50 rad/s		
40 rad/s		
20 rad/s		
10 rad/s		

Data Table 4. **Effect of sampling frequency on sampled sinc function.**

Sampling frequency	Is aliasing observed? Yes or No	Notes
80 rad/s		
60 rad/s		
50 rad/s		
40 rad/s		
20 rad/s		
10 rad/s		

Data Table 5. Effect of output low-pass filter frequency on sampled sinc function.

Output filter frequency	Is distortion observed? Yes or No	Notes
5 rad/s		
10 rad/s		
20 rad/s		
30 rad/s		
40 rad/s		
50 rad/s		

Data Table 6. Effect of output low-pass filter frequency on sampled pulse function.

Output filter frequency	Is distortion observed? Yes or No	Notes
10 rad/s		
20 rad/s		
30 rad/s		
40 rad/s		
50 rad/s		
60 rad/s		

Questions

1. Describe the required relationship between the sampling frequency and the maximum signal frequency to prevent aliasing of the sampled signal. If the signal were low-pass filtered before sampling, how would that affect your answer?

2. How does the distortion you observed in Data Table 5 and Data Table 6 relate to where the filter cutoff frequency (displayed as +/-wc on the plot.) occurs in relation to frequency spectrum of the sampled data?

3. Why can't the pulse function be exactly reconstructed from the sampled data?

4. Consider the situation of a rotating disk attached to the shaft of a motor which is visually sampled by a movie camera taking pictures at 15 frames (sample images) per second. If the disk is rotating at a rate which is exactly 15 revolutions per second, the rotating disk/shaft will appear to be motionless. However, what happens when the speed of revolution is above or below this frequency?