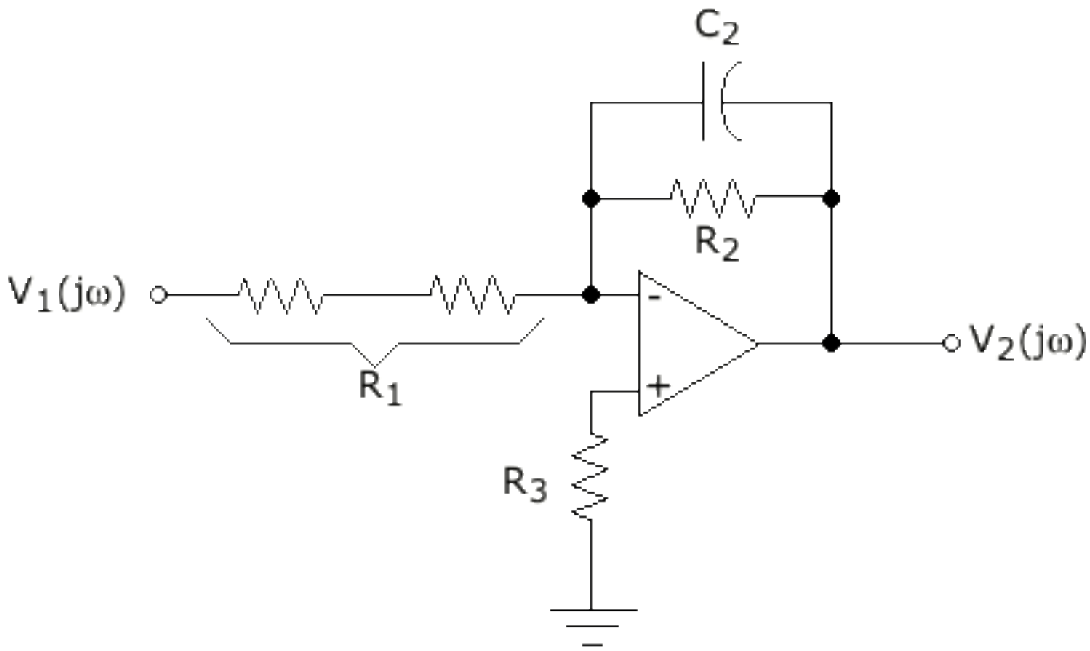
You know how to evaluate the first-order active low-pass filter of Figure 4 (Lab 12) because it is essentially identical to the circuit of HW12, problem 12-4.



R1 = 2 4.7kΩ resistors in series
R2 = 47kΩ
R3 = 1kΩ
C2 = 0.0047µf

We can evaluate the transfer function for this circuit by simply using the formula for the gain of an OP AMP amplifier and recognizing that R2-C are in parallel and form a feedback impedance $Z_2 = R_2 \parallel \frac{1}{j\omega C_2}$

$$T_V(j\omega) = -\frac{Z_2}{Z_1} = -\frac{\frac{1}{j\omega C_2} \parallel R_2}{R_1} = -\frac{\frac{\frac{1}{j\omega(0.0047\times10^{-6})}47000}{\frac{1}{j\omega(0.0047\times10^{-6})}+47000}}{4700+4700}$$

$$T_V(j\omega) = -\frac{47000}{9400}\frac{\frac{1}{j\omega(0.0047\times10^{-6})}}{\frac{1}{j\omega(0.0047\times10^{-6})}+47000} = -5\frac{1}{1+j\omega(0.0047\times10^{-6})47000}$$

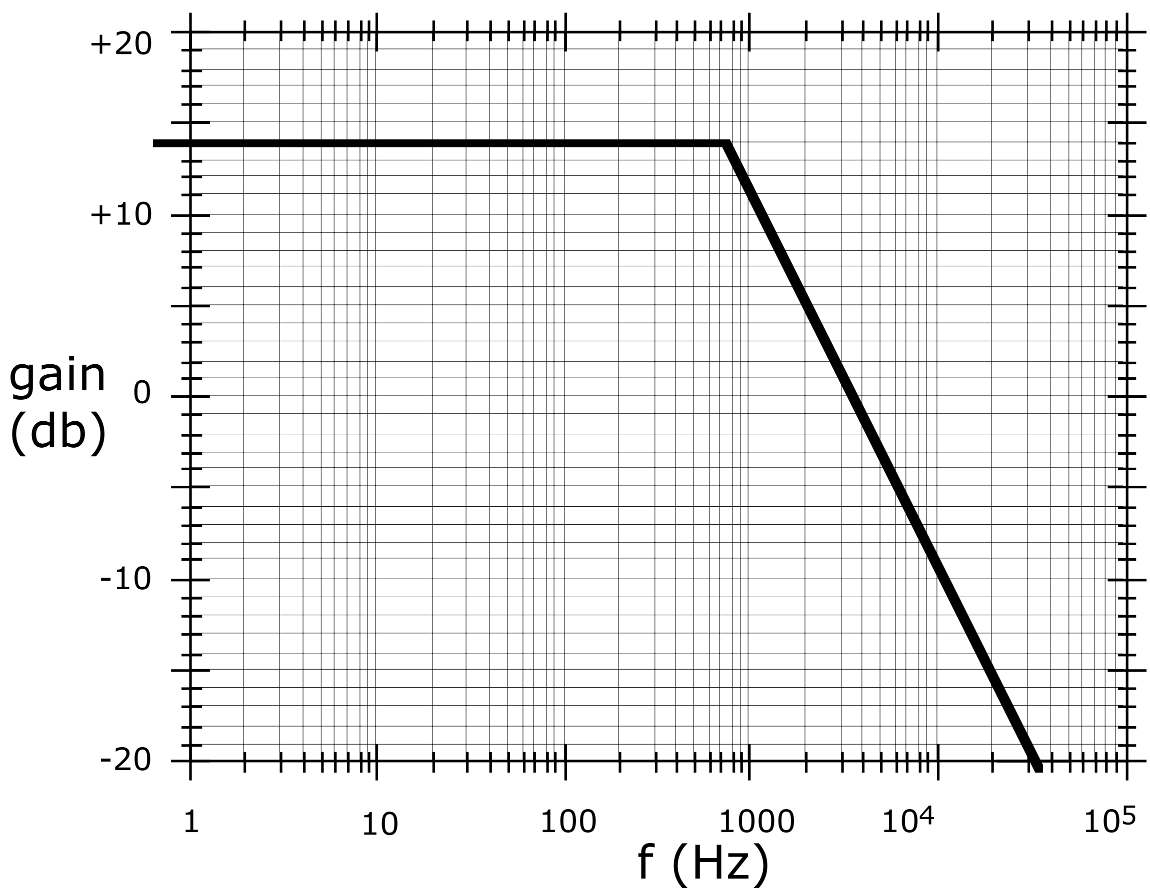
$$T_V(j\omega) = \frac{-5}{1+j\omega(2.209\times10^{-4})}$$

Computing the gain in decibels gives

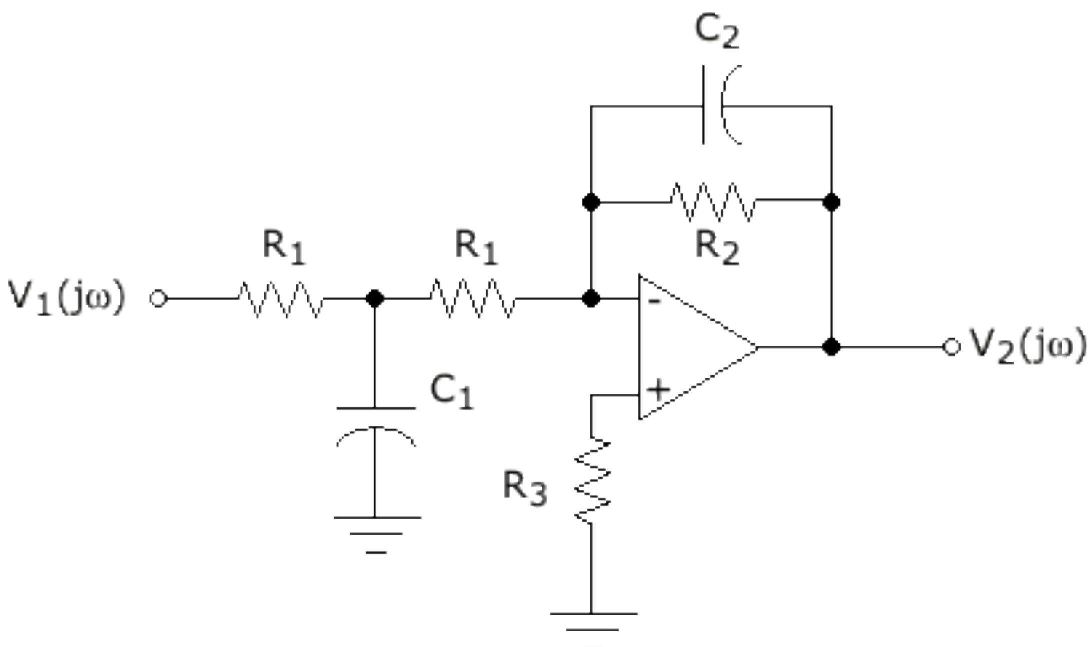
$$20\log_{10}\left|T_V\left(j\omega\right)\right| = 20\log_{10}\left|-5\right| - 20\log_{10}\left|1+j\omega\left(2.209\times10^{-4}\right)\right|$$

By inspection this is a low pass filter with a passband gain of –5 since $T_V\left(0\right)=-5$ and $\lim_{\omega\to\infty} T_V\left(j\omega\right)=0$. This can also be evaluated as $\omega\to\infty$ to give $T_V\left(j\infty\right)=0$. The cutoff frequency is given by $\omega_C\left(2.209\times10^{-4}\right)=1$ or $\omega_C=\dfrac{1}{2.209\times10^{-4}}$ = 4526.94 radians/sec. This can be converted to a linear frequency $f_C=\dfrac{4526.94}{2\pi}$ = 720.5 Hz.

We can plot this filter response as shown below.

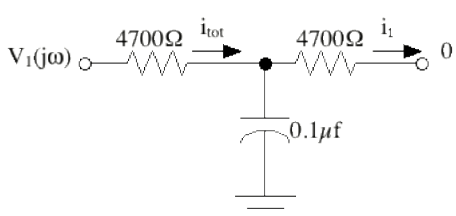


The second order filter is more complicated to analyze because there is no simple impedance for Z_1 and we need to analyze the circuit from basic principles.



R1 = 4.7kΩ
R2 = 47kΩ
R3 = 1kΩ
C1= 0.1µf
C2 = 0.0047µf

The voltage at the non-inverting input of the OP AMP is zero because there is no current into the OP AMP and consequently no voltage drop across R3 to ground. This requires the voltage at the inverting input to be zero. We then proceed to apply KCL at the inverting input. To do this we need to determine the current entering the node through the 4.7kΩ resistor.



We first compute the impedance Z_{EQ} as seen by $V_1(j\omega)$:

$$Z_{EQ} = R_1 + R_1 \parallel \frac{1}{j\omega C_1} = R_1 + \frac{R_1\left(\frac{1}{j\omega C_1}\right)}{R_1 + \frac{1}{j\omega C_1}} = R_1 + \frac{R_1}{j\omega C_1 R_1 + 1} = \frac{j\omega C_1 R_1^2 + R_1 + R_1}{j\omega C_1 R_1 + 1}$$

Rearranging gives

$$Z_{EQ} = 2R_1 \frac{\left(1 + j\omega \tfrac{1}{2} R_1 C_1\right)}{1 + j\omega R_1 C_1}$$

The current i_{TOT} is given by

$$i_{TOT} = \frac{V_1(j\omega)}{Z_{EQ}} = \frac{V_1(j\omega)}{Z_{EQ}} = \frac{V_1(j\omega)}{2R_1 \frac{\left(1 + j\omega \tfrac{1}{2} R_1 C_1\right)}{1 + j\omega R_1 C_1}} = \frac{V_1(j\omega)}{2R_1} \frac{1 + j\omega R_1 C_1}{1 + j\omega \tfrac{1}{2} R_1 C_1}$$

The current i_1 entering the node at the inverting input of the OP AMP is can be determined using a current divider as

$$i_1 = \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} i_{TOT} = \frac{1}{1 + j\omega R_1 C_1} \frac{V_1(j\omega)}{2R_1} \frac{1 + j\omega R_1 C_1}{1 + j\omega \tfrac{1}{2} R_1 C_1} = \frac{V_1(j\omega)}{2R_1} \frac{1}{1 + j\omega \tfrac{1}{2} R_1 C_1}$$

The current i_1 entering the node at the inverting input of the OP AMP must be equal to the current through the feedback resistor, i.e.,

$$i_1 = \frac{V_1(j\omega)}{2R_1} \frac{1}{1 + j\omega \tfrac{1}{2} R_1 C_1} = i_f = \frac{0 - V_2(j\omega)}{\frac{R_2\left(\frac{1}{j\omega C_2}\right)}{R_2 + \frac{1}{j\omega C_2}}} = \frac{-V_2(j\omega)}{\frac{R_2}{1 + j\omega R_2 C_2}} = -V_2(j\omega)\frac{1 + j\omega R_2 C_2}{R_2}$$

Rearranging this gives

$$T_V(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{-\frac{R_2}{2R_1}}{\left(1 + j\omega \tfrac{1}{2} R_1 C_1\right)\left(1 + j\omega R_2 C_2\right)}$$

We substitute numbers to get

$$T_V(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{-\frac{47000}{2(4700)}}{\left(1 + j\omega \frac{1}{2}[4700]\left[0.1\times10^{-6}\right]\right)\left(1 + j\omega[47000]\left[0.0047\times10^{-6}\right]\right)}$$

$$T_V(j\omega) = \frac{V_2(j\omega)}{V_1(j\omega)} = \frac{-5}{\left(1 + j\omega\left[2.35\times10^{-4}\right]\right)\left(1 + j\omega\left[2.209\times10^{-4}\right]\right)}$$

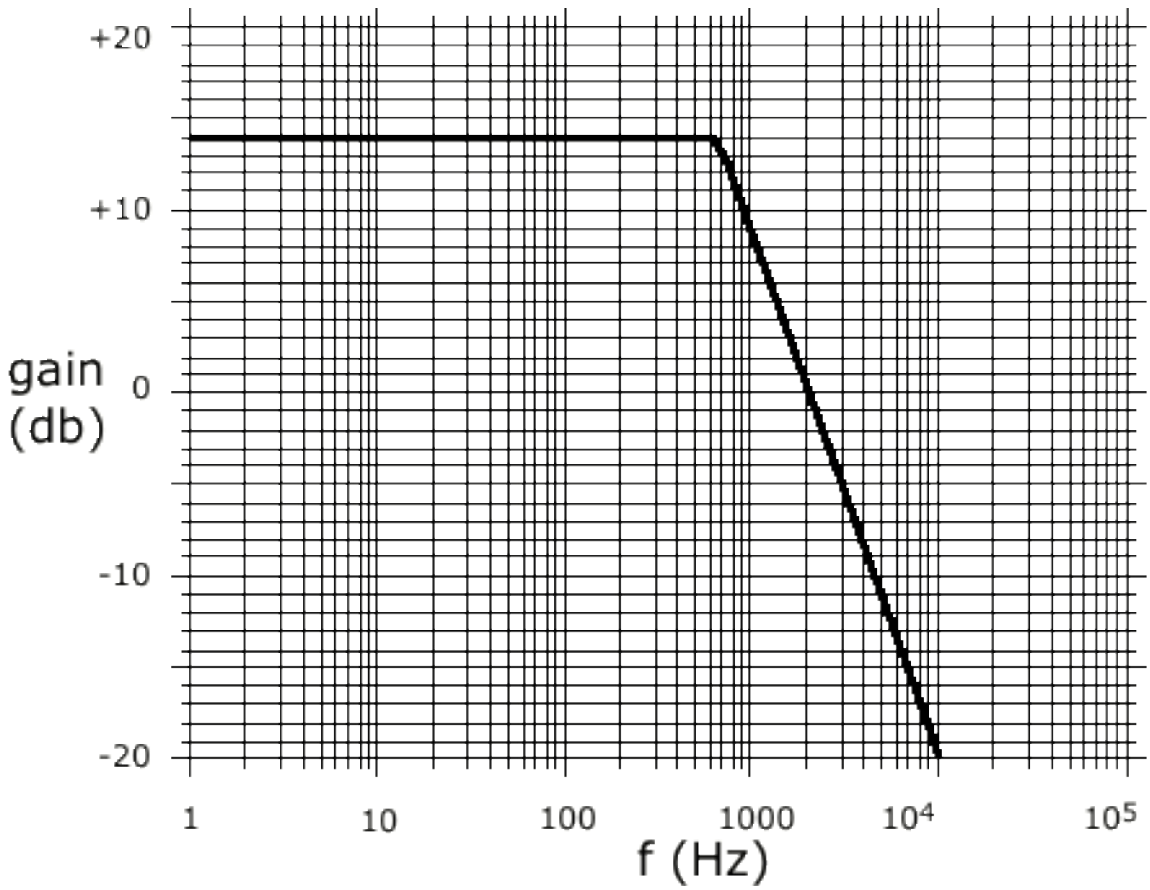
Computing the gain in decibels gives

$$20\log_{10}\left|T_V(j\omega)\right| = 20\log_{10}\left|-5\right| - 20\log_{10}\left|1 + j\omega\left(2.35\times10^{-4}\right)\right| - 20\log_{10}\left|1 + j\omega\left(2.209\times10^{-4}\right)\right|$$

$$20\log_{10}\left|T_V(j\omega)\right| = 13.97dB - 20\log_{10}\left|1 + j\omega\left(2.35\times10^{-4}\right)\right| - 20\log_{10}\left|1 + j\omega\left(2.209\times10^{-4}\right)\right|$$

This is a low pass filter with a passband gain of –5 since $T_V(0) = -20$ and $\lim_{\omega\to\infty} T_V(j\omega) = 0$. However, the slope relationship is more complicated since there are two frequencies of interest: $f_1 = \frac{1}{2\pi\left(2.35\times10^{-4}\right)}$ = 677.25 Hz and $f_2 = \frac{1}{2\pi\left(2.209\times10^{-4}\right)}$ = 720.48 Hz.

What this means is that we plot a horizontal line of 14 db until we reach 677.2 Hz. At this frequency $T_V(j\omega)$ starts decreasing at -20db/decade. This continues until we reach 720.5 Hz when we have a second term which decreases at another –20db/decade giving an overall decrease of –40 db/decade. This is plotted below.



The basic difference between these two very similar first and second order filter circuits is that the gain decreases much faster with frequency beginning at about 720 Hz. The change in slope from –20db/decade between 677 and 720 Hz to –40db/decade for frequencies above 720 Hz is hard to see on the above plot and very difficult to see in measured data.