

ENGR 210 Lab 10

Frequency Response of Active RC Filters

You have studied some simple filters which used capacitors and resistors. However, these filters have several limitations. One is that the frequency roll-off of a passive filter does not occur very rapidly. Another, which was not mentioned in class, is that the frequency response depends upon the load that is attached to the filter—in general you don't want the frequency response to change when you connect different instruments or equipment. In this lab you will examine several types of filters that incorporate op amps. You will also use a band pass filter to remove interference (hum) from a signal.

A. BACKGROUND

1. Band Pass Filter with an Op Amp Buffer

You should have built and measured the performance of a band pass filter that consisted of a high pass filter in series with a low pass filter, as shown in Figure 1, in the previous lab. The frequency response for this filter is significantly different from a simple sum of the separate filter responses since the second circuit loads the first. One of the great applications of the operational amplifier is to use an op amp buffer amplifier to isolate two circuits so that the second is not loaded by the first. In this lab you will examine a bandpass filter where the two filter sections are isolated by inserting an op amp buffer, as shown in Figure 2.

2. Active Filters

An active filter is a circuit that includes an op amp as well as having the characteristics of a filter. Recall the circuits for passive high pass and low pass filters. Their circuit diagrams are shown in Figure 3. Note that in each case a load resistor, R_L , has been added to the circuit to make it describe a more realistic situation (there will always be a load on the output of a filter circuit). The frequency characteristics of the filter are dependent upon the load resistor, which is hardly a desirable feature. In this lab you will study a variation of the low pass filter, namely, an active low pass filter as shown in Figure 4. Active filters include an op amp as an integral part of the circuit. The effect of the op amp is two-fold. First, it serves as the source of current for the load that follows the filter, thus eliminating the effect of the load resistance on the characteristics of the filter and, second, it permits adding positive gain to the filter.

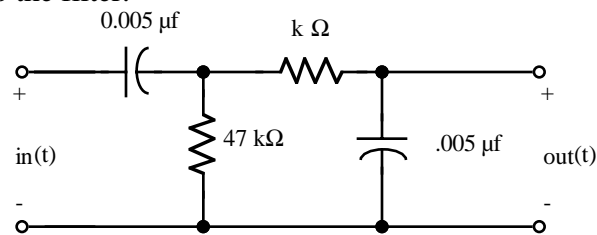


Figure 1. Band pass filter.

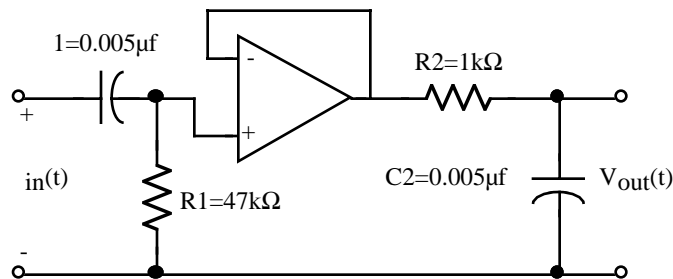


Figure 2. Buffered Band pass filter

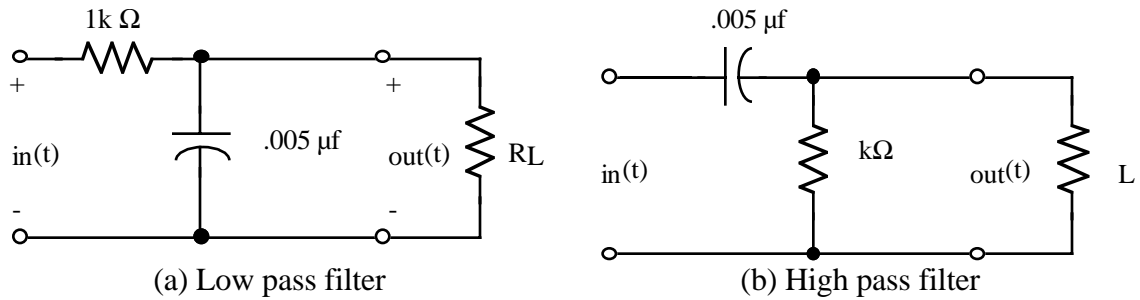


Figure 3. Passive filters.

Note that in the circuit of Figure 4 there is a capacitor in the feedback loop. This circuit will behave similar to the low pass filter of Figure 3, but with a different cutoff frequency. (Note that the component values in the circuit of Figure 4 are not the same as those in Figure 3.) This is primarily because of non-ideal characteristics of the 741 op amp.

The filter circuits of Figure 3 and Figure 4 are called **first order filters** and they have similar abilities to reject frequencies past the cutoff frequency, i.e., their frequency response rolls off at the same rate. There will be occasions when you would like to have better frequency rejection than is provided by a first order filter. (Perhaps the signal that you are interested in is corrupted by lots of noise.) A filter such as that shown in Figure 5 can provide this increased attenuation of the high frequencies. Note that this filter includes a capacitor in the input network as well as in the feedback network. Such a filter is called a **second order filter**.

3. Active "Notch" Filter

As you will quickly learn - probably the first time that you are required to use a sensor to obtain information from a system - a common problem that one encounters is the presence of 60 Hz "hum" on the output of the sensor. (You have probably already experienced this problem on your AM car radio when driving under a high voltage transmission line.) The origin of the hum is the 60 Hz power lines that supply all electrical power in the U. S. An example of the problem is shown in Figure 6. In Figure 6(a), a 500 mV, 5 Hz signal with 250 mV of hum superimposed is shown. In Figure 6(b) the hum has been removed with some attenuation of the desired signal.

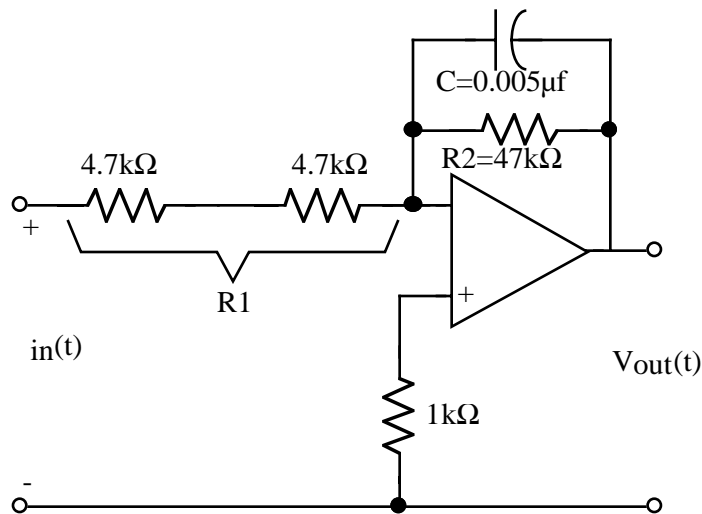


Figure 4. First Order Active low pass filter.

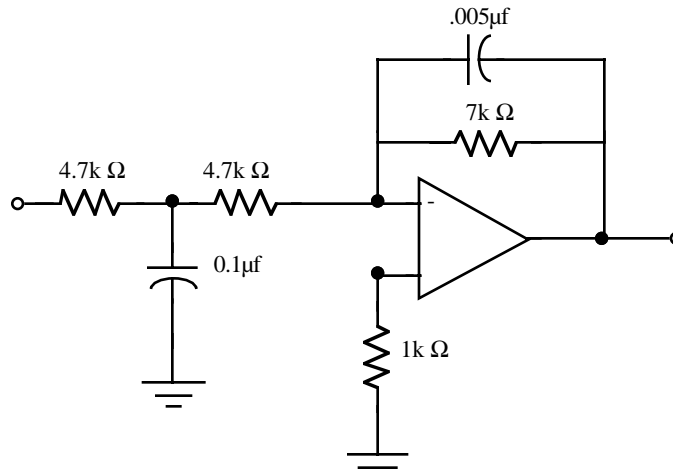
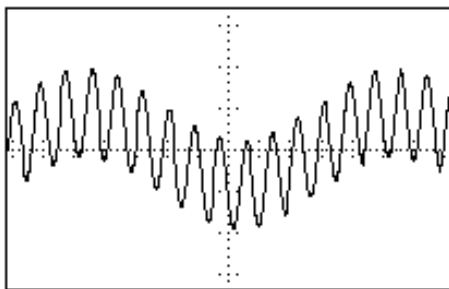
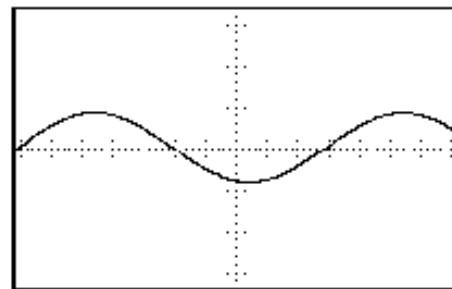


Figure 5. Second order Active low pass filter.

The hum in Figure 6(a) has been removed by a “notch” filter. A notch filter is a circuit having a very high attenuation for a selected frequency and very little attenuation for other frequencies. A notch filter is the inverse of a bandpass filter.



(a) Signal with 250 mV hum



(b) Signal with hum removed

Figure 6. Example of effect of 60 Hz hum on 500 mV, 5 Hz signal.

There are many types of notch filters and they are typically named after their inventor. One such filter, called a Wein bridge circuit, is shown in Figure 7. You will study the Wien Bridge circuit in this lab. You may wish to determine the transfer characteristic of this circuit to test your understanding of phasor analysis, although this is not required.

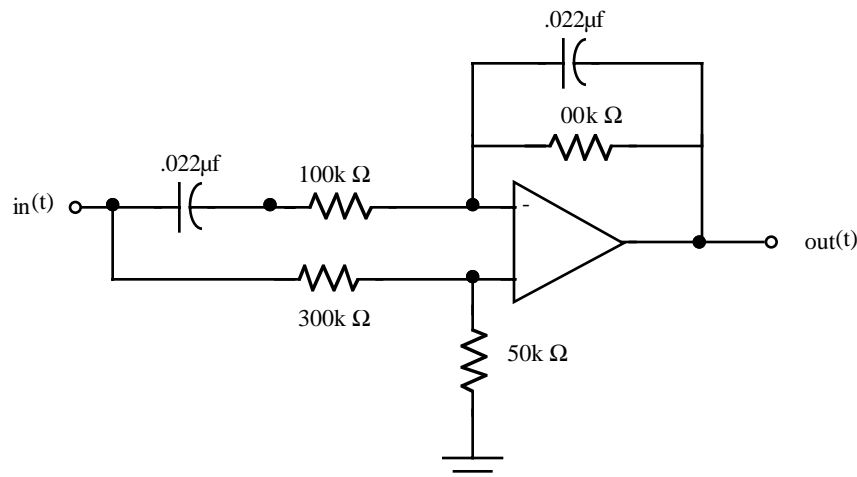


Figure 7. Wein Bridge notch filter.

B. LAB PREPARATION

The transfer function, $H(j\omega)$, for the active low-pass filter circuit in Figure 4 can be mathematically shown to be

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \cdot \frac{1 - j\omega R_2 C}{1 + (\omega R_2 C)^2} \quad (1)$$

where R_1 is the sum of the input resistances and R_2 is in parallel with the capacitor C across the op-amp. The resultant expressions for the magnitude and angle of $H(j\omega)$ are given in

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Data Table 1.

NOTE: The inverting input of the op-amp introduces a 180° phase shift at the output in addition to the phase shift due to the filter.

The transfer function, $H(j\omega)$, for the buffered band pass filter shown in Figure 2 can be mathematically shown to be

$$\frac{V_{out}}{V_{in}} = \frac{\omega^2 R_1 C_1 (R_1 C_1 + R_2 C_2) + j\omega R_1 C_1 (1 - \omega^2 R_1 C_1 R_2 C_2)}{(1 - \omega^2 R_1 C_1 R_2 C_2)^2 + \omega^2 (R_1 C_1 + R_2 C_2)^2} \quad (2)$$

where R_1 and C_1 are the input components; R_2 and C_2 are the output components. The phase angle goes from $+90$ degrees for very low frequencies to -90 degrees at very high frequencies. The peak output occurs at about 4650 Hz. The expressions for the magnitude and angle of $H(j\omega)$ are given in

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Data Table 1.

The phase of the output wave relative to the input wave can be determined using a procedure similar to that used for measuring the period of a wave. Figure 8 shows two waveforms which represent the input to and output from a circuit displayed on an oscilloscope. Note first that the two signals have the same period and, therefore, the same frequency; however, they are shifted in time relative to each other. You will measure this time shift between the two signals, and then calculate the corresponding phase shift.

As shown in Figure 8, set the t_1 time cursor at a location where the input signal crosses the time axis with a positive slope. Then set the t_2 time cursor to the first following point at which the output signal crosses the time axis with a positive slope. Since, in one period the phase of a periodic wave changes by 2π radians or 360° , the phase shift, $\Delta\phi$, is given by

$$\Delta\phi(\text{radians}) = \frac{t_1 - t_2}{T} \times 2\pi \quad (3)$$

or

$$\Delta\phi(\text{degrees}) = \frac{t_1 - t_2}{T} \times 360^\circ. \quad (4)$$

In this lab you will always measure t_2 for the output waveform, and t_1 for the input waveform. The resulting phase shift is often called the delay of a filter.

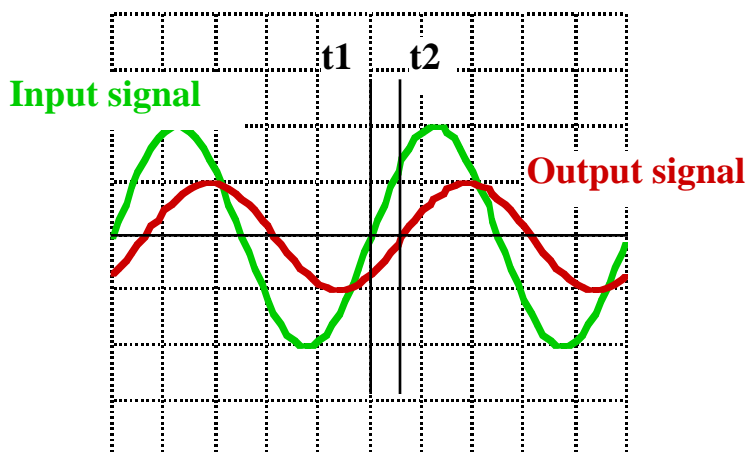


Figure 8. Measurement of phase shift.

VERY IMPORTANT: Be careful of the sign of the time delay in calculating delay. Note that the scope will calculate t_2-t_1 , which is the opposite sign of the expression in equations (3) and (4). Also, many times you will get angles between -270° and -360° when the actual angle should be their complement, i.e., $-270^\circ+360^\circ=+90^\circ$ and $-360^\circ+360^\circ=0^\circ$ respectively. When in doubt as to whether you need to take a complement look at the results you derived for these filters.

C. LAB INSTRUCTIONS

Part 1: Active low pass filter.

1. Build the active first order low pass filter circuit shown in Figure 9.

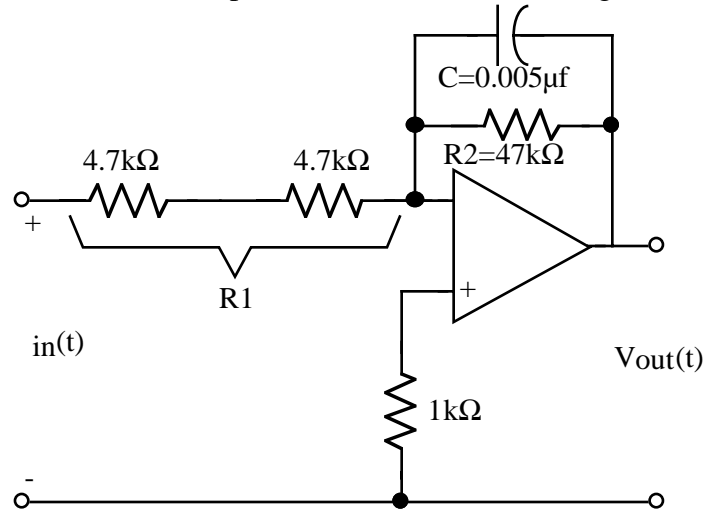


Figure 9. First Order Active low pass filter.

Keep the power supply output turned off while assembling the circuit. After completing a set of measurements on this circuit you will be adding a capacitor between the two $4.7\text{k}\Omega$ resistors as shown in Figure 5, so be sure to leave adequate space. (The pinout diagram for the 741 is shown in Figure 10.)

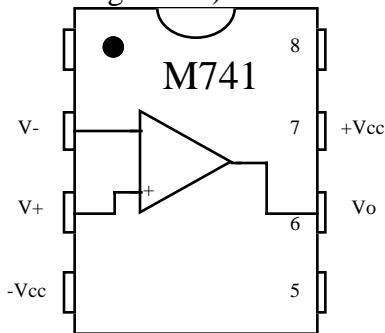


Figure 10. Pinout for 741 op amp.

2. Set up CH1 of the oscilloscope for AC coupling and a x10 probe. Set the probe itself to x10 if it has a switch. Connect the CH1 probe to the output of your circuit. Use a second x10 probe on CH2 of the oscilloscope. Connect the CH2 probe to the output of the Function Generator. Set the Waveform Generator output impedance to HIGH Z. Connect the Function Generator output to your circuit input and set the generator to produce a 10 Hz, 2 V_{pp} output.
3. Measure $V_{in,PP}$, $V_{out,PP}$, the period of the input signal that you are using, and the time shift between the output and input at the frequencies listed in Data Table 3. Record your data in Data Table 3.

- Connect a 0.1 μF capacitor into the circuit as shown in Figure 5 and repeat Steps (2) and (3) above, recording your data in Data Table 4.

Part 2: 60 Hz Notch Filter

- Build the circuit shown in Figure 11.

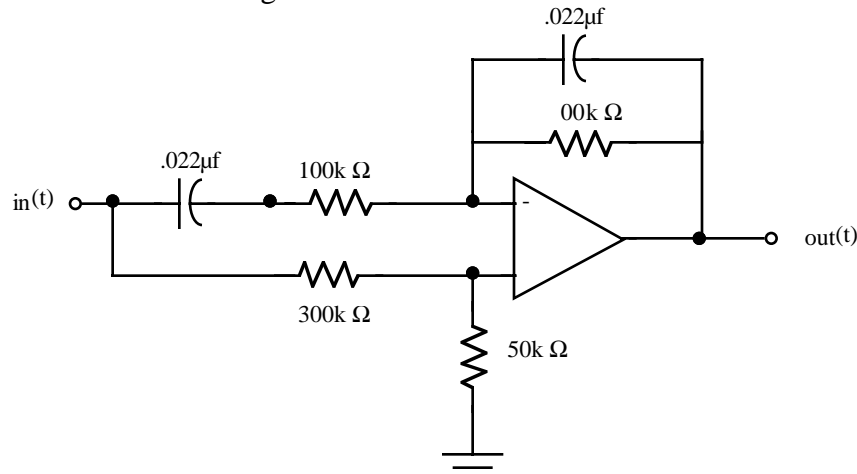


Figure 11. Wein Bridge notch filter.

- Connect the oscilloscope and Function Generator as in Part 1. Set the Function Generator to produce a 5 Hz, 5 V_{pp} output.
- Slowly** scan the output frequency of the function generator while monitoring the output of the circuit on the oscilloscope (CH2). Determine the notch frequency of your filter. (60 Hz is the nominal frequency, but it is unlikely that your circuit will have its minimum output at exactly 60 Hz. Various versions that we have built operated with notch frequencies from 49 Hz to 72 Hz!)
- Measure $V_{in,PP}$ and $V_{out,PP}$ at the frequencies listed in Data Table 5. You should modify the listed frequencies in the 40 Hz to 80 Hz range to get a good plot of the notch that your filter provides. Record your measurements in Data Table 5.
- After determining the actual notch frequency of your circuit you will test its ability to remove "hum" from a circuit. The H-P Function Generators have been pre-programmed with some signals similar to those shown in Figure 5. Determine the output of your notch filter for each of the following waveforms (see **Table 1**) from the arbitrary waveform menu of the Function Generator. Detailed instructions on how to select one of these waveforms is in **Table 2**.

EUROHUM	5 Hz signal + 50 Hz "hum"
USHUM	5 Hz signal + 60 Hz "hum"
HILOHUM	5 Hz signal + 70 Hz "hum"
HIHUM	5 Hz signal + 75 Hz "hum"

Table 1. Specifications for "Signal" + "hum" Waveforms

These have been chosen to be typical of the "hum" you might find at various locations around the world. Determine for which waveform your filter best removes the "hum." For only this waveform which best removes the "hum", print the oscilloscope display which shows the input and output of your filter circuit.

1. Press the SHIFT button on the H-P 33120A Function Generator.
2. Press the ARB LIST button.
3. Use the RIGHT or LEFT arrow keys to find
 USHUM=60HZ+5HZ
 EUROHUM=50HZ+5HZ
 HILOHUM=70HZ+5HZ
 HIHUM=75HZ+5HZ
4. Press ENTER to select one of the above arbitrary waveforms.
5. Set frequency to 5Hz
6. Set output level to 750 mV.

Table 2. Directions for accessing arbitrary waveforms:

DATA AND REPORT SHEETS FOR LAB 10

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Data Table 1. Magnitude & Phase from Transfer Functions

	$ H(j\omega) $	$\angle H(j\omega)$
Band pass filter with op amp buffer (Figure 2.)	$\frac{\omega R_1 C_1}{\sqrt{[1 + (\omega R_1 C_1)^2][1 + (\omega R_2 C_2)^2]}}$	$Tan^{-1}\left(\frac{1 - \omega^2 R_1 C_1 R_2 C_2}{\omega R_1 C_1 + \omega R_2 C_2}\right)$
Active low pass filter (Figure 4.)	$\left(\frac{R_2}{R_1}\right) \frac{1}{\sqrt{1 + (\omega R_2 C)^2}}$	$Tan^{-1}(-\omega R_2 C)$

Data Table 3. First Order Low Pass Filter Characteristics

Frequency	V _{in}	V _{out}	20log ₁₀ H(jω)	Period	t1-t2	Δφ (deg)
30 Hz						
100						
500						
600						
700						
800						
900						
1.0 kHz						
3						
5						

10						
30						

Data Table 4. Second Order Active Low Pass Filter Characteristics

Frequency	V_{in}	V_{out}	$20\log_{10} H(j\omega) $	Period	t1-t2	$\Delta\phi$ (deg)
30 Hz						
100						
200						
500						
600						
700						
800						
900						
1.0 kHz						
3						
5						
10						
30						

Data Table 5. Notch Filter Characteristics

Frequency	V_{in}	V_{out}	$20\log_{10} H(j\omega) $
10 Hz			
25			
35			
40			
45			
55			
60			
70			
80			
100			
150			

Don't forget the Benchlink printout of your Notch filter performance!

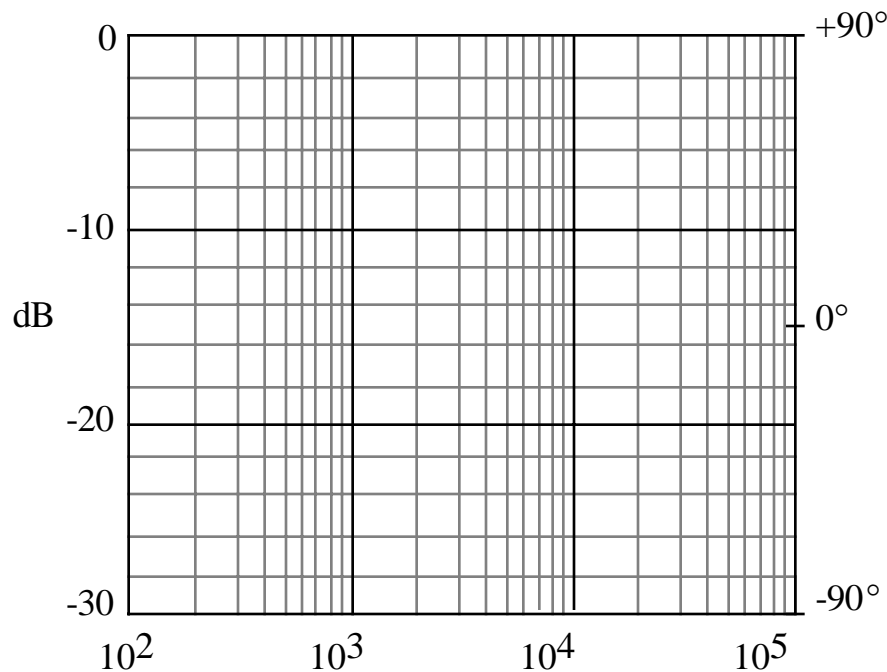
Questions:

1. For the buffered bandpass filter (data in **Error! Reference source not found.**), plot both $20\log_{10}|H(j\omega)|$ and $\Delta\phi(j\omega)$ as functions of $\log_{10}\omega$. You can use the graph below for this. Determine the cutoff* frequencies from your plot and compare them with calculated values from equation (2).

$$\omega_{LO} \text{ (calculated)} = \underline{\hspace{2cm}} \quad \omega_{LO} \text{ (measured)} = \underline{\hspace{2cm}}$$

$$\omega_{HI} \text{ (calculated)} = \underline{\hspace{2cm}} \quad \omega_{HI} \text{ (measured)} = \underline{\hspace{2cm}}$$

Discuss the differences between your calculated and measured cutoff frequencies.



* NOTE: Remember from class that this is the frequency at which the magnitude of the frequency response drops to 0.707 of its maximum value. A low-pass or high pass filter has one cutoff frequency. A notch or bandpass filter has two cutoff frequencies. Break frequency is another term for cutoff frequency.

2. Plot your data for $\log_{10}|H(j\omega)|$ and $\Delta\phi(j\omega)$ for the first and second order active low pass filters as functions of $\log_{10}\omega$. Use the graph provided below for this. Draw best fit asymptotic (straight line) approximations for each of the two gain curves. This simply means that you should use a straight edge and draw a good fit to the passband and stop band parts of your frequency response curve. From your plots determine the cutoff frequencies and the slopes (this is also called the roll-off) for the two curves. Note that we are not interested in any slight slope in the passband; we are only interested in the slope in the stop band.

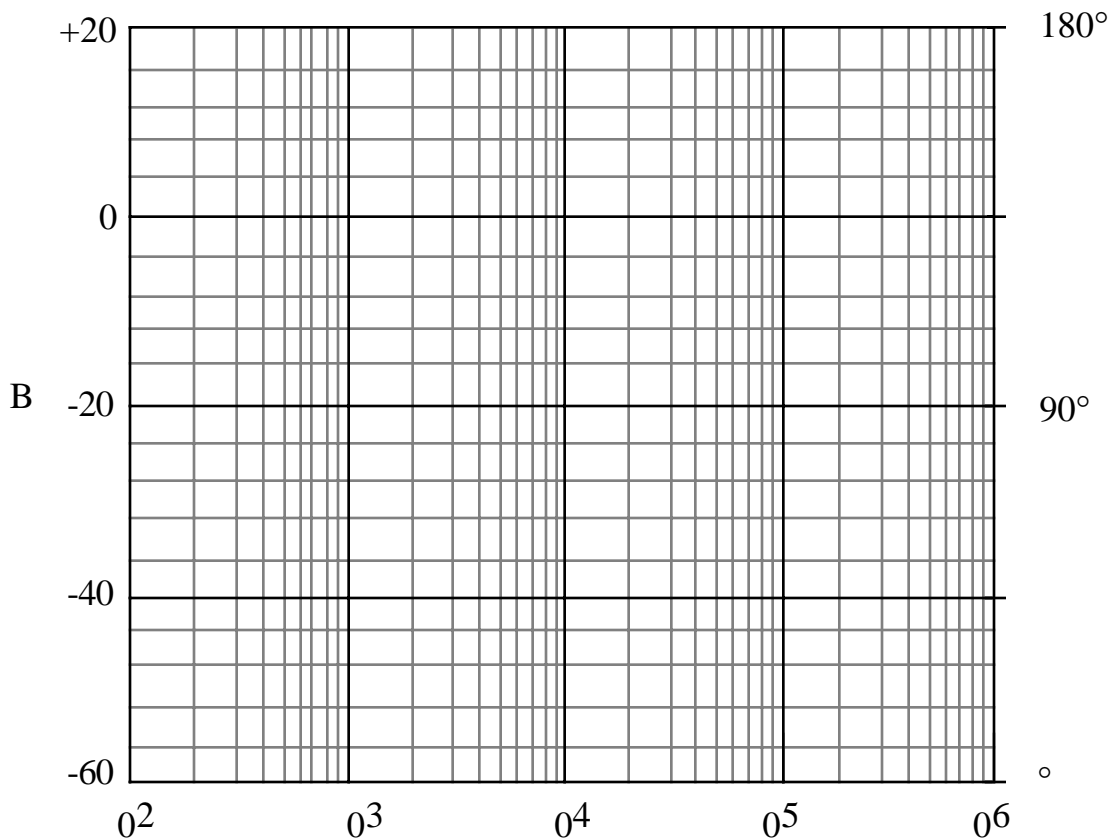
1st order low-pass:

$\omega_{\text{cutoff}} =$ _____ slope = _____ dB/decade

2nd order low-pass:

$\omega_{\text{cutoff}} =$ _____ slope = _____ dB/decade

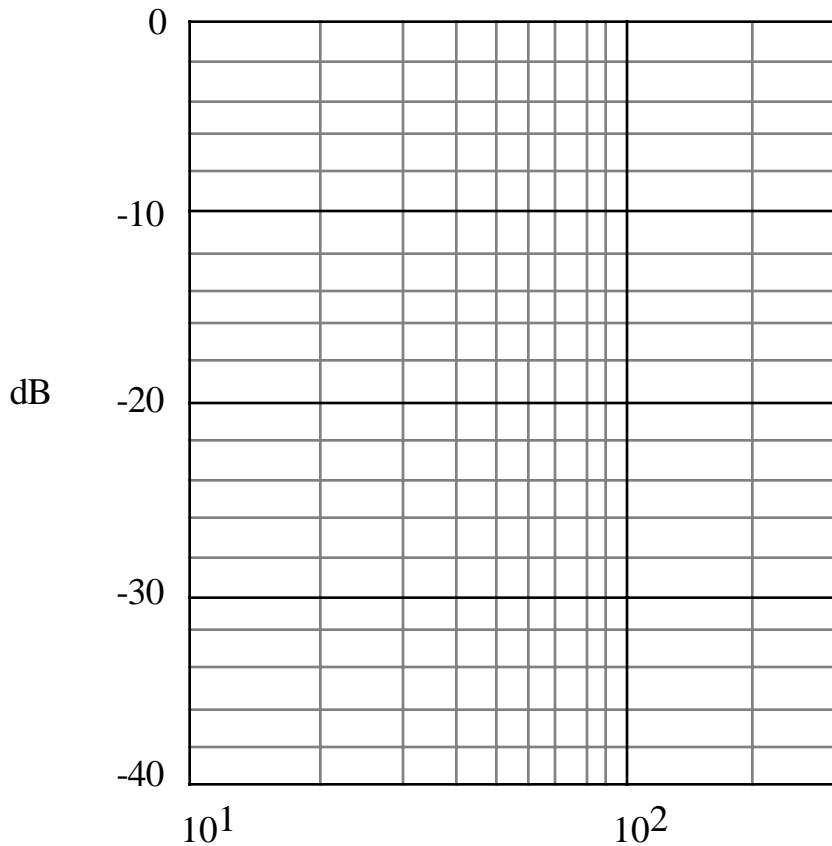
Discuss the differences between the performance of the two filters.



3. Plot $20\log_{10}|j\omega|$ as a function of $\log_{10}\omega$ for the notch filter. Use the graph provided below for this.

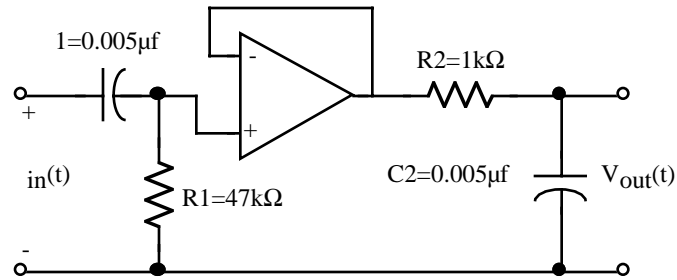
What are the notch frequency and the maximum attenuation that you obtained with your circuit? The maximum attenuation would be the ratio of the output to input voltage computed at the notch frequency. Note that this should be computed in decibels.

$f_{\text{notch}} = \underline{\hspace{2cm}}$ Hz Max attenuation = $\underline{\hspace{2cm}}$ dB



4. Recall the statement in part B that the inverting input of the op-amp introduces a 180° phase shift at the output in addition to the phase shift due to the filter. Explain what this statement means. (HINT: Compare the phase of your circuit to that of a passive low pass filter.) Does this change how you calculate the $\Delta\phi$ (deg) column in Data Table 3 and Data Table 4.

5. Consider the buffered bandpass filter shown below.

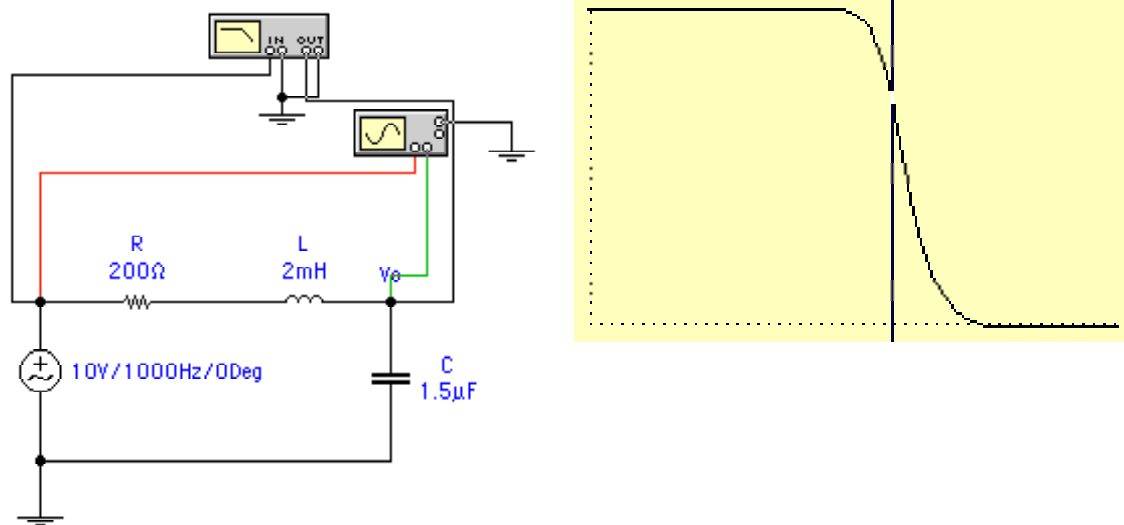


Construct an EWB simulation of this circuit. Using the Bode plotter instrument in EWB determine the magnitude transfer function of this circuit. Using the cursors determine the -3db filter cutoff frequencies. Replace the operational amplifier by a short from C1 to R2 and again determine the magnitude transfer function of this circuit. Use the cursors to determine the -3db filter cutoff frequencies. Explain why there is a difference, i.e., what is the function of the operational amplifier.

Appendix I

Frequency Analysis in EWB Using the Bode Plotter

The Bode plotter is a really neat instrument but does not really exist as a general purpose electronic instrument that you can use in the lab. To use it we have to attach it to a circuit as shown in Figure 12(a).



(a) RLC circuit with Bode plotter attached

(b) Bode plotter display of circuit frequency response

Figure 12. Frequency response of series RLC circuit using Bode plotter

Note that the Bode plotter IN gets directly connected to the input (V_s) of the circuit. Similarly the Bode plotter OUT gets connected to the output (V_o) of the circuit. You MUST also connect the grounds of the Bode plotter or it may not give correct results.

To use the Bode plotter double-click on it's icon to open the control panel. The plot in Figure 12 (b) is for a **Steady State** analysis. The Bode plotter settings are magnitude, gain on LIN ($I = 0$, $F = 1$), and frequency on LOG ($I = 1$ Hz, $F = 10$ kHz). A few notes on the Bode plotter axes are in order. You can use either a logarithmic (LOG) or linear (LIN) vertical scale for plotting the amplitude response of a filter. The phase response is best plotted using a linear scale. The I and F settings are very important in getting a understandable display and easy to incorrectly set. For the horizontal axis, I is the left most value and F is the right most value of the horizontal axis. On the vertical axis, I is the top of the vertical axis and F is bottom of the axis. For example, you would typically want $I=180^\circ$ (or some other positive angle) and $F=-180^\circ$ (or some other negative angle) for a phase response which will be similar to those in your textbook. If you can't see a plot when you finish your simulation then you need to adjust these axes to get a visible plot.

The vertical line on the Bode plotter screen is a cursor which can be used to measure amplitude and frequency. Since you can't read the Bode plot very well you should always use the Bode plotter's cursor to read out values. You can either drag the cursor to where you want it with your mouse, or you can use the two horizontal arrows to move it (this is usually better when you are trying to determine a cutoff frequency). The value of the point on the plot which intersects the vertical cursor is read out to the right of the cursor positioning arrows. The upper box is the vertical coordinate, and the lower box is the horizontal axis.

For example, in Figure 13 you can use the cursor to determine the break frequency (the frequency where the response of the filter is $A_{v(\text{mid})} = V_{\text{out}} / V_{\text{min}} = 0.71$) to be approximately 525 Hz. You could never read this number out any other way.

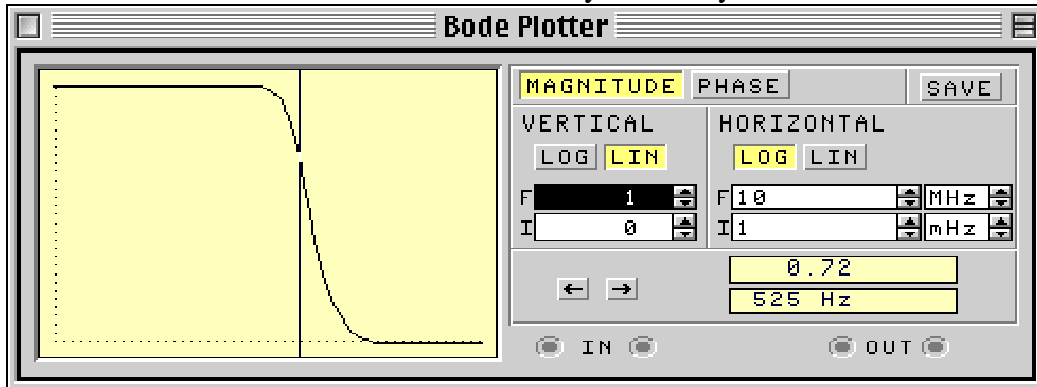


Figure 13. Bode Plotter instrument panel

Finally for some lab assignments you may want to get a table of your data. This can be done by pressing the SAVE button which will create a text file of the data used to obtain the Bode plot. For example, for the plot shown in Figure 13 you would get a text file similar to that shown in Figure 14. This is a very useful way to get data for lab reports, nice looking plots, and the like.

```
Bode data for
column 1  frequency (Hz)
column 2  gain (dB)
column 3  gain (linear)
column 4  phase (degrees)
```

Frequency	Gain (dB)	Gain	Phase
1.00000e-03	-1.71452e-10	1.00000e+00	-3.60000e-04
1.34896e-03	-3.11993e-10	1.00000e+00	-4.85627e-04
1.81970e-03	-5.67732e-10	1.00000e+00	-6.55092e-04
2.45471e-03	-1.03310e-09	1.00000e+00	-8.83695e-04
3.31131e-03	-1.87994e-09	1.00000e+00	-1.19207e-03

Figure 14. Sample Bode Plotter Data File