ENGR 210 Lab 8
Frequency Response of Passive RC Filters

The objective of this lab is to introduce you to the frequency-dependent nature of the impedance of a capacitor and the impact of that frequency dependence on the input-output characteristics of capacitive circuits. The lab consists of making a number of measurements on some simple RC circuits, then analyzing those results.

Using an oscilloscope, you will measure the amplitudes of sinusoidal input and output signals for three different RC circuits. You will also measure the phase difference between these signals. Finally, you will examine the effect of each circuit on a simulated signal — for simplicity of generation, a square wave.

A. BACKGROUND (Required Pre Lab calculations are indicated by a "bullet," •)

1. Frequency and relative phase measurement

In this laboratory you will characterize several passive filters by measuring peak amplitudes of sinusoidal input and output signals and by indirectly measuring the phase shift of the output wave relative to the input wave. You will also indirectly measure the frequency of these signals.

In previous labs you have measured peak-to-peak voltages of sinusoidal waveforms. In this lab it is the peak amplitude, i.e., the maximum amplitude of the signal relative to 0 V, that is of interest, as shown schematically in Figure 1. To measure v_{peak}, set the voltage cursors to the signal maxima and minima as shown and measure the peak-to-peak voltage, $v_{p\text{-}p}$. As shown in the figure, $v_{peak} = 0.5v_{p\text{-}p}$.

The signal frequency is measured indirectly by measuring its period, T, as shown in Figure 1. To measure the period, set the time cursors to successive points where the signal crosses the time axis with a negative (or positive) slope. The frequency can then be calculated from the relationship

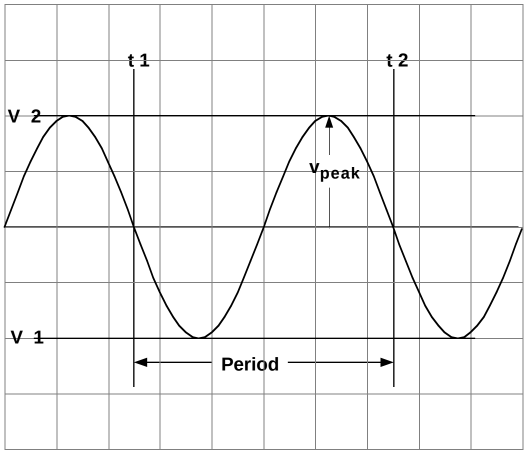

Figure 1. Measurement of wave period and peak amplitude.

$$f(\text{Hz}) = \frac{1}{T(\text{sec})}, \text{ or } \omega(\text{radians/sec}) = \frac{2\pi}{T(\text{sec})}. \tag{1}$$

The phase of the output wave relative to the input wave can be determined using a procedure similar to that used for measuring the period of a wave. Figure 2 shows two signals displayed on an oscilloscope. These represent the input to and output from a circuit. Note first that the two signals have the same period and, therefore, the same frequency. You will measure the time shift between the two signals, then

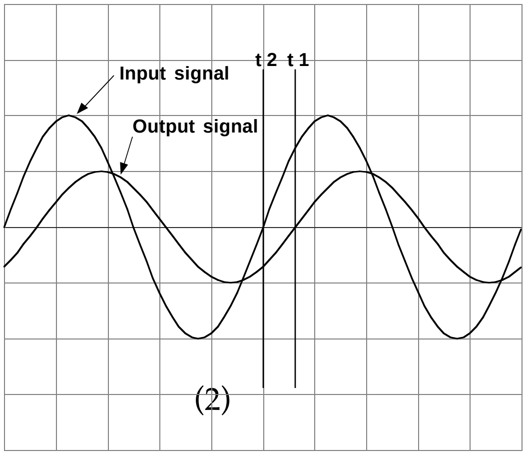

Figure 2. Measurement of phase shift.

calculate the phase shift. As shown in Figure 2, set the t1 time cursor at a location where the output signal crosses the time axis with a positive slope. Then set the t2 time cursor at the closest point at which the input signal crosses the time axis with a positive slope. Since in one period the phase of a periodic wave changes by 2π radians, or 360°, the phase shift, $\Delta\phi$, is given by

$$\Delta\phi(\text{radians}) = \frac{t2-t1}{T}\bullet 2\pi \tag{2}$$

or

$$\Delta\phi(\text{deg rees}) = \frac{t2-t1}{T}\bullet 360^\circ. \tag{3}$$

2. Transfer function

The circuits that you will study in this lab treat sinusoidal signals of different frequencies differently. Such circuits are said to be dispersive. Since they enable one to change the frequency characteristics of a signal they are called ***filters***. A given filter has a different effect on both the magnitude and the phase of each input frequency. For example, a television antenna produces a voltage composed of signals from many transmitters. By using a filter that eliminates signals of all frequencies except those between 82 MHz and 88 MHz one is able to select the Channel 6 signal and reject all signals that would interfere with the extraction of the appropriate video and audio information.

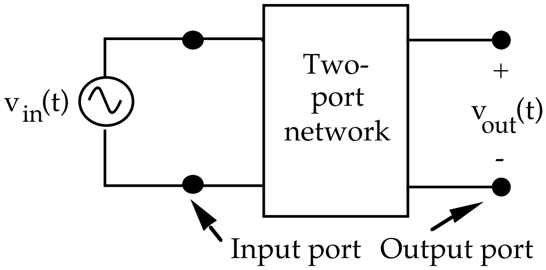

Figure 3. Two-port filter network.

Consider the filter network shown schematically in Figure 3. Networks like this are called ***two-port networks*** because they have an input port and an output port. Suppose that a sinusoidal signal of angular frequency ω and phasor $\mathbf{V}_{in}$ is input to the filter. In the steady state the output is sinusoidal, also with angular frequency ω. The output phasor is $\mathbf{V}_{out}$. The effect of the two-port on signals of all frequencies is described by its ***transfer function***, H(ω). H(ω) is defined as the ratio of the output phasor voltage to the input phasor voltage, as a function of frequency

$$H(\omega) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{V_{out}e^{j\phi_{out}}}{V_{in}e^{j\phi_{in}}}. \tag{4}$$

Since both phasors are complex, the transfer function is also complex and contains both phase and amplitude information. In this laboratory you will determine the transfer characteristics of several filter circuits.

3. Low pass filter

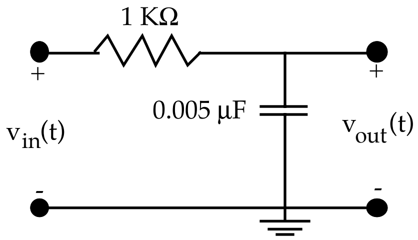

Figure 4. Low pass RC filter.

The first circuit that you will study is the low pass filter shown in Figure 4. As you can see from the figure, this circuit is a complex voltage divider. Its (complex) transfer function is

$$H(\omega) = \frac{1}{1+j\omega RC}. \tag{5}$$

Since H(ω) is a complex quantity it can be described in terms of a magnitude and an angle in the complex plane. The magnitude of H(ω), |H(ω)| gives the transfer characteristic of the amplitude and the angle, ∠H(ω) is the phase difference between the input and the output. The equations for these (which you already derived in class) are

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\omega/\omega_o\right)^2}} \tag{6}$$

$$\underline{/\,H(\omega)} = -\tan^{-1}\left(\omega/\omega_o\right), \tag{7}$$

where the characteristic angular frequency is $\omega_o = 1/RC$.

- Obtain Eqs. (6) and (7) from Eq. (5). Using the values for R and C from Figure 4, calculate the expected characteristic frequency for the low pass filter.
- Calculate values for |H(ω)| and ∠H(ω) for the following circular frequencies input to the low pass filter: 800 Hz, 10 kHz, 20 kHz, 35 kHz, and 80 kHz.

4. High pass filter

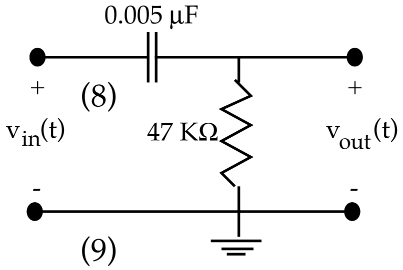

Figure 5. High pass RC filter.

The circuit for the high pass filter that you will study is shown in Figure 5. Its transfer function is

$$H(\omega) = \frac{j\omega RC}{1+j\omega RC} \tag{8}$$

and the magnitude and angle are

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\omega_o/\omega\right)^2}} \tag{9}$$

$$\underline{/\,H(\omega)} = \tan^{-1}\left(\omega_o/\omega\right). \tag{10}$$

- Obtain Eqs. (9) and (10) from Eq. (8). Using the values for R and C from Figure 5, calculate the expected characteristic frequency for the high pass filter.

- Calculate values for $|H(\omega)|$ and $\angle H(\omega)$ for the following circular frequencies input to the high pass filter: 200 Hz, 500 Hz, 800 Hz, 1 kHz, and 3 kHz.

5. Band pass filter

The final circuit that you will study is a band pass filter that is a combination of the high pass and low pass filters examined previously, as shown in Figure 6.

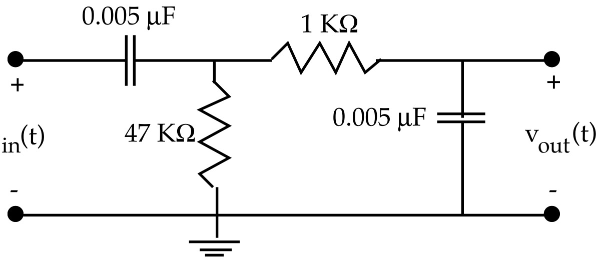

Figure 6. Band pass RC filter.

- Find an equation for the complex transfer function, $H(\omega)$, for this circuit. Is it simply an addition of the transfer functions of the low pass and high pass filters studied previously? Why or why not?

B. LAB INSTRUCTIONS

PART 1: A low pass filter

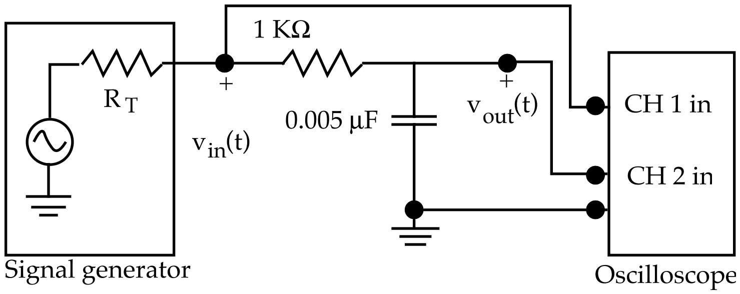

Figure 7. Low pass filter measurement circuit.

1. Build the low pass filter circuit shown in Figure 7. (Remember that the signal generator is connected to ground internally.)
2. Set up CH 1 and CH 2 of the oscilloscope for AC coupling and a x10 probe. Turn off CH 3 and CH 4. Initially set the time base to 20 µsec/div.
3. With the function generator connected to the circuit, set $v_{in}(t)$ to be sinusoidal at a frequency of ≈9 kHz and 10 $V_{p\text{-}p}$, with 0 V DC offset. (Note that all specified frequencies are approximate. You will measure the actual frequency that you are using.) Measure and record the peak amplitude of $v_{in}(t)$ and $v_{out}(t)$ in Data Table 3.
4. Measure the period of the input wave and record your result in Data Table 3. Measure the time shift between the input and output signals.
5. Complete Data Table 3 by making measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
6. Set the function generator to produce a 30 kHz, 10 $V_{p\text{-}p}$ square wave with 50% duty cycle and 0 V offset voltage. Sketch the input and output waveforms that appear on the oscilloscope in Data Table 4. Be sure to include the voltage (ordinate) and time (abscissa) scales for each waveform.

Part 2: A high pass filter

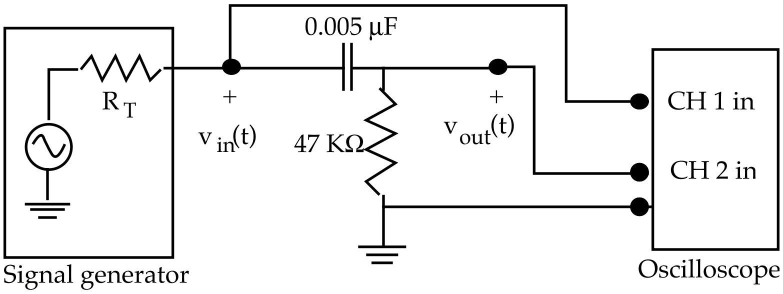

Figure 8. High pass filter measurement circuit.

1. Build the high pass filter circuit shown in Figure 8.
2. Set the time base of the oscilloscope to 2 ms/div initially. (Keep the CH 1 and CH 2 settings as before.) With the function generator connected to the circuit, set $v_{in}(t)$ to be sinusoidal at a frequency of ≈100 Hz and 10 V_{p-p}, with 0 V DC offset.
3. Measure the peak amplitude of $v_{in}(t)$, $v_{out}(t)$, T, and the time shift between input and output signals. Record this measurement in Data Table 5.
4. Complete Data Table 5 by making the measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
5. Set the function generator to produce a 500 Hz, 10 V_{p-p} square wave with 0 V offset voltage. Sketch the input and output waveforms that appear on the oscilloscope in Data Table 6. Be sure to record the voltage (ordinate) and time (abscissa) scales.

Part 3: A band pass filter

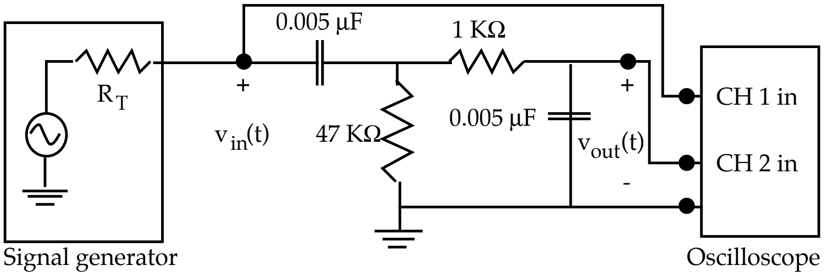

Figure 9. Band pass filter measurement circuit.

1. Build the band pass filter circuit shown in Figure 9. With the function generator connected to the circuit, set $v_{in}(t)$ to be sinusoidal at a frequency of ≈200 Hz and 10 V_{p-p}, with 0 V DC offset.
2. Measure the peak amplitude of $v_{in}(t)$, $v_{out}(t)$, T, and the time shift between input and output signals. Record these measurements in Data Table 7.
3. Complete Data Table 7 by making measurements at the remaining (approximate) frequencies. Be sure to use appropriate time base and voltage scales for each measurement.
4. Set the function generator to produce a 20 kHz, 10 V_{p-p} square wave with 0 V offset voltage. Sketch the input and output waveforms that appear on the oscilloscope in Data Table 8. Be sure to record the voltage (ordinate) and time (abscissa) scales. Repeat this procedure with a 1 kHz square wave in Data Table 9.

DATA AND REPORT SHEETS FOR LAB 8

Student Name (Print): _______________ Student ID: ______________________

Student Signature: ___________________ Date: ____________________________

Student Name (Print): _______________ Student ID: ______________________

Student Signature: ___________________ Date: ____________________________

Lab Group: __________________

Data Table 1. Prelab Calculation.

Calculated low pass filter characteristic frequency _ _ _ _ _ _ _ _ _ _

Data Table 2. Calculated Low Pass Filter Response

Nominal Frequency	$\lvert H(\omega) \rvert$	$\angle H(\omega)$
800 Hz		
10 kHz		
20		
35		
80		

Data Table 3. Low Pass Filter

Nominal Frequency	v_{in}	v_{out}	v_{out}/v_{in}	Period	Calculated Frequency	t2 - t1	$\Delta\phi$ (deg) (calculated)
1 kHz							
9							
17							
30							
50							
150							

Data Table 4. Low Pass Filter Square Wave Response

Volts/div:

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Volts/div:

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Time/div

____

Data Table 5. High pass filter

Nominal Frequency	v_{in}	v_{out}	v_{out}/v_{in}	Period	Calculated Frequency	t2 - t1	$\Delta\phi$ (deg) (calculated)
100 Hz							
400							
700							
1200							
3500							
11 kHz							

Data Table 6. High Pass Filter Square Wave Response

Volts/div:

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Volts/div:

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Time/div

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Data Table 7. Band pass filter.

Nominal Frequency	v_{in}	v_{out}	v_{out}/v_{in}	Period	Calculated Frequency	t2 - t1	$\Delta\phi$ (deg) (calculated)
200 Hz							
500							
1 kHz							
4							
20							
40							
70							
200							

Data Table 8. Band Pass Filter Square Wave Response (20kHz)

Volts/div:

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Volts/div:

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Time/div

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Data Table 9. Band Pass Filter Square Wave Response (1kHz)

Volts/div:

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Volts/div:

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Time/div

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QUESTIONS

1. Perform all required calculations to complete all the Data Tables. For each filter, plot your experimental results for voltage gain, |H(w)|, as a function of $\log_{10}$(frequency). (Plot each on a separate graph.) On separate graphs, plot your experimental results for phase shift (in degrees) as a function of $\log_{10}$(frequency). Draw a **smooth** curve that fits your data points. Do **not** simply connect the points in a "dot-to-dot" fashion. You may have to do this by hand. (These plots must be computer-generated. If you use MS Excel, be sure to choose the "Scatter plot" format since the frequencies at which measurements have been made are not uniformly spaced.)
2. By **hand**, enter the data points that you calculated in the Pre Lab for the low pass and high pass filters on the appropriate points. Do these points fall reasonably close to the smooth curve that you generated in 1, above? ("Reasonably" should take into consideration the tolerances on the components only.) If not, why?
3. The cutoff frequency, ω_o, of a filter is defined as the frequency at which the voltage gain is equal to 0.707 of its maximum value. For each filter, what is the maximum voltage gain and what is the measured cutoff frequency? How does this compare with the predicted value of $\omega_o = 1/RC$? Is your measured value within the tolerances of the components?
4. Discuss the differences between the input square waves and the output waves for each filter. Why do you think that they are so different?