

## ENGR 210/EEAP 240 Lab 4

# Voltage Dividers and Sensor Calibration

### Objectives

- To construct a simple electrical circuit including a thermistor for temperature measurement.
- To calibrate a temperature measurement system
- To interface a thermistor probe to a computer-based DAQ system.

### Background

Chemical and polymer engineers use thermistors and other temperature probes to measure the temperature of chemical reactions. Mechanical and materials engineers use temperature probes to determine the thermal expansion of materials. Temperature sensors are used in thermostats for home and industrial heating and cooling systems. Many engineers use temperature sensors in a variety of ways. There are many types of temperature sensors such as RTD (resistance temperature detector) probes, thermocouples and thermistors. The thermistor is probably the cheapest and simplest to use temperature sensor.

In this lab you will study the behavior of a thermistor temperature sensor by building a simple electrical circuit which uses a thermistor, and the calibration of this thermistor temperature sensor. The temperature calibration method used is a simple method that you can apply to almost any measurement system where a sensor can be compared to a known (reference) sensor. In this case, you will be applying the calibration method to thermistor probes, so information on the operation of thermistor probes is also included here. Supplemental references on thermistor sensors are also included on the course Web page. Finally, you will look at how to build circuits designed to make the thermistor temperature measurements linear as a function of temperature.

### Calibration of Temperature Probes

Calibration of temperature measurement systems is very similar to a modeling problem. You need to find a model such that given a measured temperature, you can find a better estimate of the actual temperature. If you provide enough known temperature values to the system and note the measured temperature, you can determine the function from the measured temperature to actual temperature.

In this lab you will make no assumptions about the behavior of the thermistor sensor provided in the lab. However, you will be given a temperature sensor whose resistance vs. temperature behavior is known. This is commonly called a reference standard. In the real world the most accurate standards may be directly traceable to the National Institute of Standards and Technology (NIST).

Unlike the resistors you have seen in class (with the exception of a few homework problems) the resistor has had a constant value. To develop a model for this temperature sensor you will be required to fit a polynomial to your experimental data. [You can use Excel (or any other program with similar capabilities) to do this. Such curve fitting will give you an analytical expression for the resistance of the thermistor as a function of temperature.

### Temperature Probes



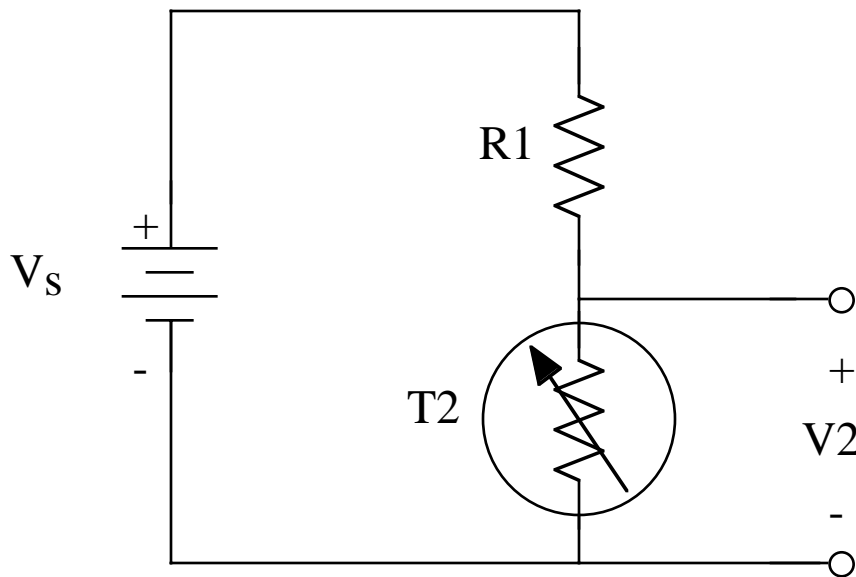


Figure 4-2. A Simple Voltage Divider

Figure 4-2 shows a simple voltage divider used to measure the change in resistance of the thermistor,  $T2$ . The symbol for  $T2$  indicates that it is a variable resistance sensor. At this point, you need to find the voltage across  $T2$  given the known parameters of the resistances of  $R1$  and  $T2$  and the voltage source,  $V_s$ . Because the current through  $T2$  is the same current that flows through both  $R1$  and  $T2$ , you can find the voltage by using two methods of calculating the current. First, the current through  $R1$  and  $T2$  is:

$$I_{R1} = I_{T2} = \frac{V_s}{R1 + T2}$$

where  $V_s$  is the supply voltage. The current through  $T2$  also is:

$$I_{T2} = \frac{V2}{T2}$$

where  $V2$  is the voltage across  $T2$ . Remember, both currents are the same, not just the same value, but the same current. So, you can find  $T2$  by setting both equations for current equal to each other and solving for  $V2$ :

$$\frac{V2}{T2} = \frac{V_s}{R1 + T2}$$

Thus:

$$V2 = \frac{T2 \cdot V_s}{R1 + T2}$$

The astute reader will notice that when making measurements, you are reading  $V2$  but you really want to find  $T2$ . So, do the algebra and find  $T2$ . Again, you know that the current through  $T2$  is the same current that flows through  $R1$ . You then set the currents equal to one another as before:

$$\frac{V2}{T2} = \frac{V_s - V2}{R1}$$

and solve for  $T2$ :

$$T2 = \frac{V2}{V_s - V2} R1$$

This particular circuit yields the temperature vs. voltage curve similar to that shown in Figure 4-3. This was plotted using data which from a DIFFERENT thermistor than the one you will use in the lab. Note that the function is almost linear for this thermistor. The dotted line indicates a best linear fit to the data. The worst case error for the line for estimating temperature from voltage occurs at the highest temperature on the chart. At a voltage of 0.34 V, the estimate is 94.5 °C whereas the actual temperature is 100 °C. For some systems, this error may be acceptable. When the system requires more accuracy from the calculations, you can use an equation or a table to more closely model the actual response shown in the graph. There are also techniques such as using a different type of voltage divider to provide a linear voltage-to-temperature conversion.

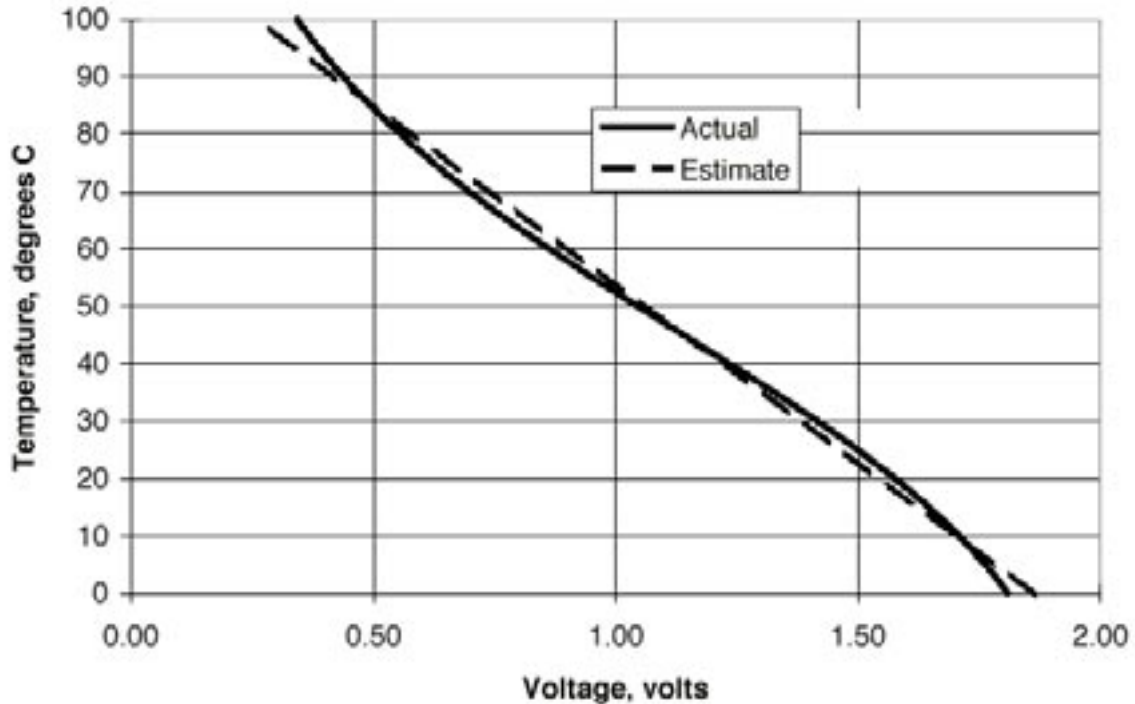


Figure 4-3. Temperature vs. Voltage for a Thermistor Voltage Divider

A linearizing voltage divider is shown in Figure 4-4. This circuit requires the use of two thermistors in a single probe case. By placing the two thermistors in the same probe (thus measuring the same temperature) and then placing the two thermistors in the circuit in parallel, the temperature-to-voltage relationship becomes very linear over the range of 0 to 100 °C. A major manufacturer of thermistors, Omega, Inc., specifies this particular circuit for their thermistors.

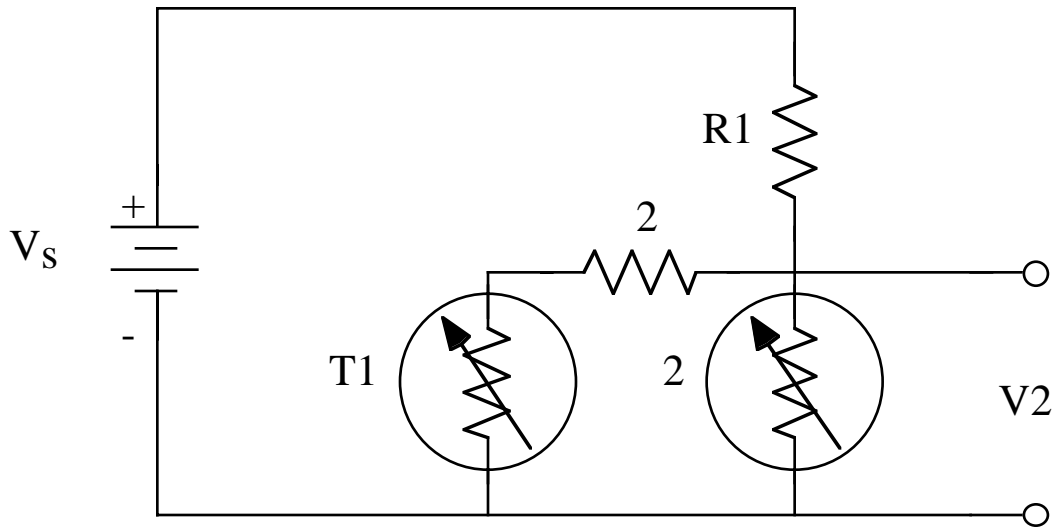


Figure 4-4. Linearizing Voltage Divider

### Building Circuits

Now that you have looked at different circuits to use as voltage dividers for thermistors, you can look at how to build those circuits. Two important items to understand are components and nodes. Tables 4-1 and 4-2 give descriptions of the components and nodes in Figure 4-4, respectively. A *component* is a resistor, thermistor, connector, voltage supply, or any other element that might be connected into a circuit. So, a component is some electrical device that you can place in the circuit. Note that some components, such as  $V_{in}$  (power supply), must be connected in a certain direction shown by the positive and negative terminals. Components such as  $R1$  may be connected without concern for polarity.

Component	Type	Description
R1	Resistor	3200 Ohms*
R2	Resistor	6250 Ohms*
T1	Thermistor	Resistance that varies with temperature, 30000 $\Omega$ at 25 $^{\circ}\text{C}$
T2	Thermistor	Resistance that varies with temperature, 6000 $\Omega$ at 25 $^{\circ}\text{C}$
Pigtail	Connector	Connects T1 and T2 to circuit board
$V_s$	Voltage Supply	Can be a battery or a power supply
$V_2$	Connections	Connects to appropriate analog voltage input channel using connection screws

Table 4-1. List of Components

(\* Specified by Omega for these particular T1 and T2)

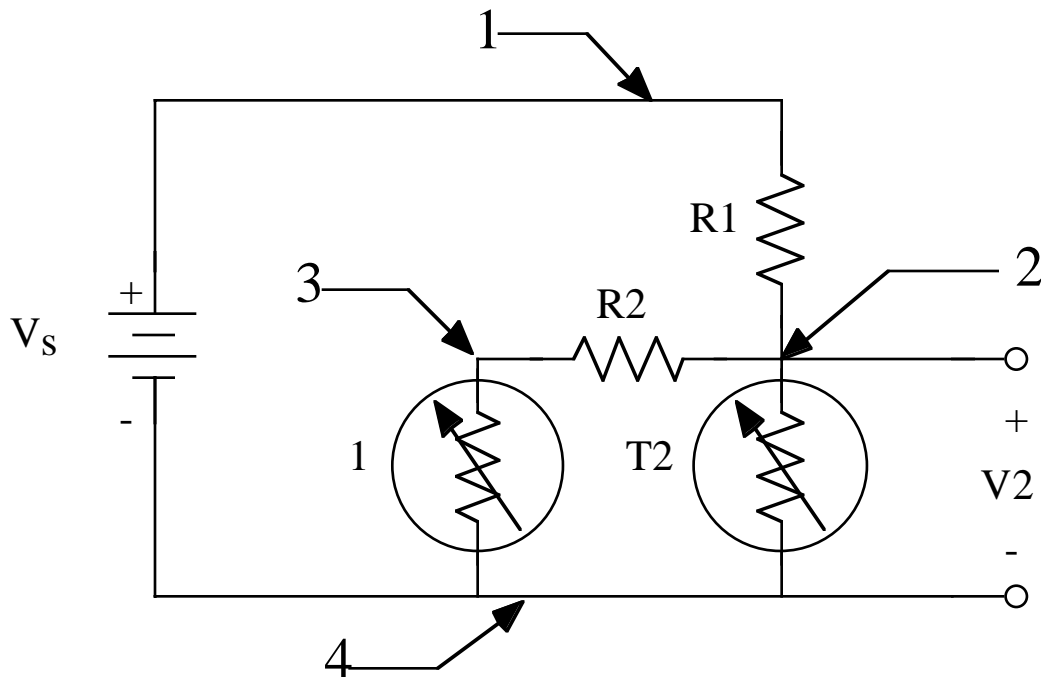


Figure 4-5. Nodes of an Electrical Circuit

Next, you need to understand the notion of an electrical node. An electrical node is any point used to connect two or more electrical components together. Figure 4-5 has labels for all four nodes of the linearizing voltage divider circuit (1, 2, 3, and 4). Note that node 2 has four different components connected to it: R1, R2, T2, and the positive terminal of V2 . A more subtle connection node is 4. Node 4 is connected to T1, T2, the negative terminal of V2 , and the negative terminal of Vs . The line between T1 and T2 is part of node 4 and is not a separate component. In fact, in the lab, the connection between T1 and T2 is already complete as part of the probe. But that does not mean node 4 is complete; connections to Vs and V2 must be made.

Node	Description
1	Connects R1 to positive terminal of Vs (the voltage supply)
2	Connects together the following: R1, R2, T2, and positive terminal of V2 (voltage to be measured)
3	Connects R2 to T1
4	Connects together the following: T1, T2, negative terminal of Vs, and negative terminal of V2

Table 4-2. List of Nodes

The following are tools and components you will use for constructing these circuits:

Breadboard.

A power supply set to output 2-10 V DC output.

Banana jacks for connection to the power supply.

Wire (stripped).

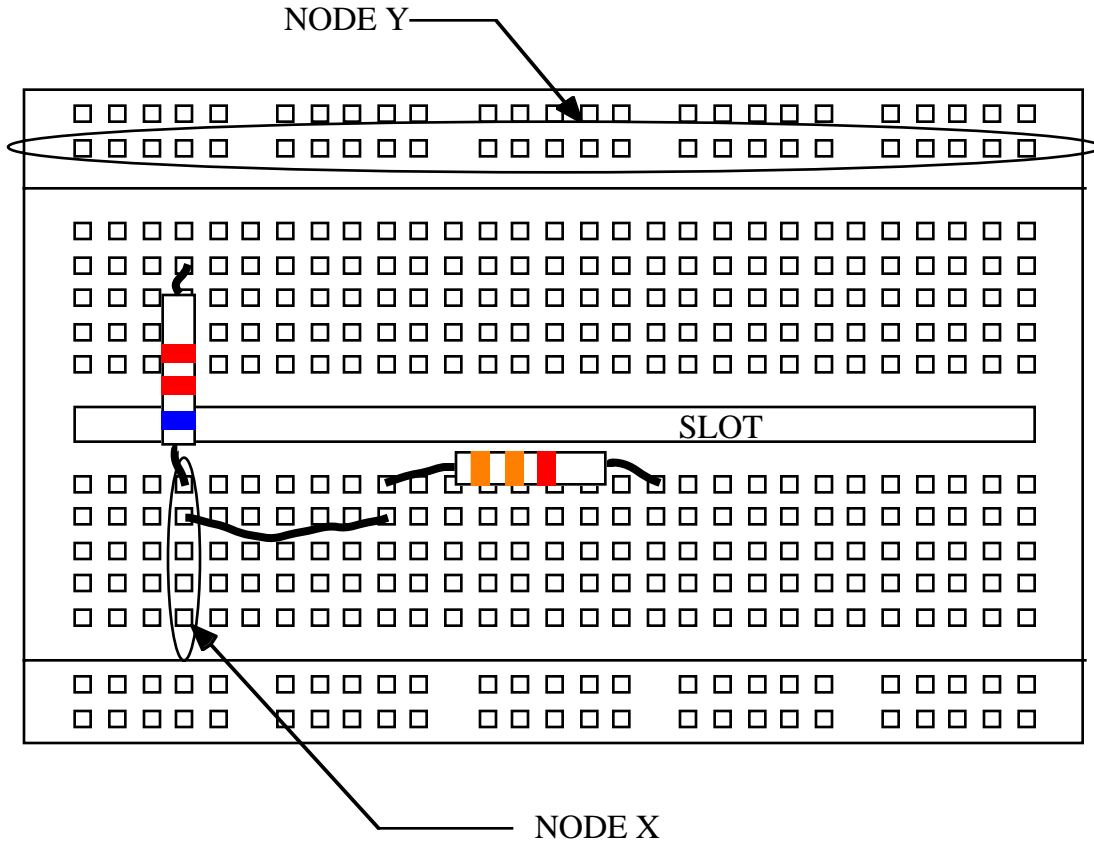


Figure 4-6. Breadboard with Examples of Both Types of Nodes Labeled

All of the above tools and components are supplied to you in the lab. You can sign out a toolbox with a protoboard from the lab instrument room. Figure 4-6 is a drawing of the breadboard. One wire fits into each hole. On the breadboard, the following are single nodes for connecting components. In the middle of the board (either side of the horizontal slot through the middle) are columns of five holes each. Each column of five holes is considered a node and can connect two to five components. Node X in Figure 4-6 is an example. Along the top and bottom of the breadboard are five groups of five holes each. The five groups circled in the figure are all a single node useful for power and ground nodes. In Figure 4-6, there are four groups of these 25 holes, making four different nodes.

Figure 4-7 shows an example of resistors connected in series. This means current flowing into R1 from the top will continue through the breadboard connection and then on to the resistor R2 on the left.

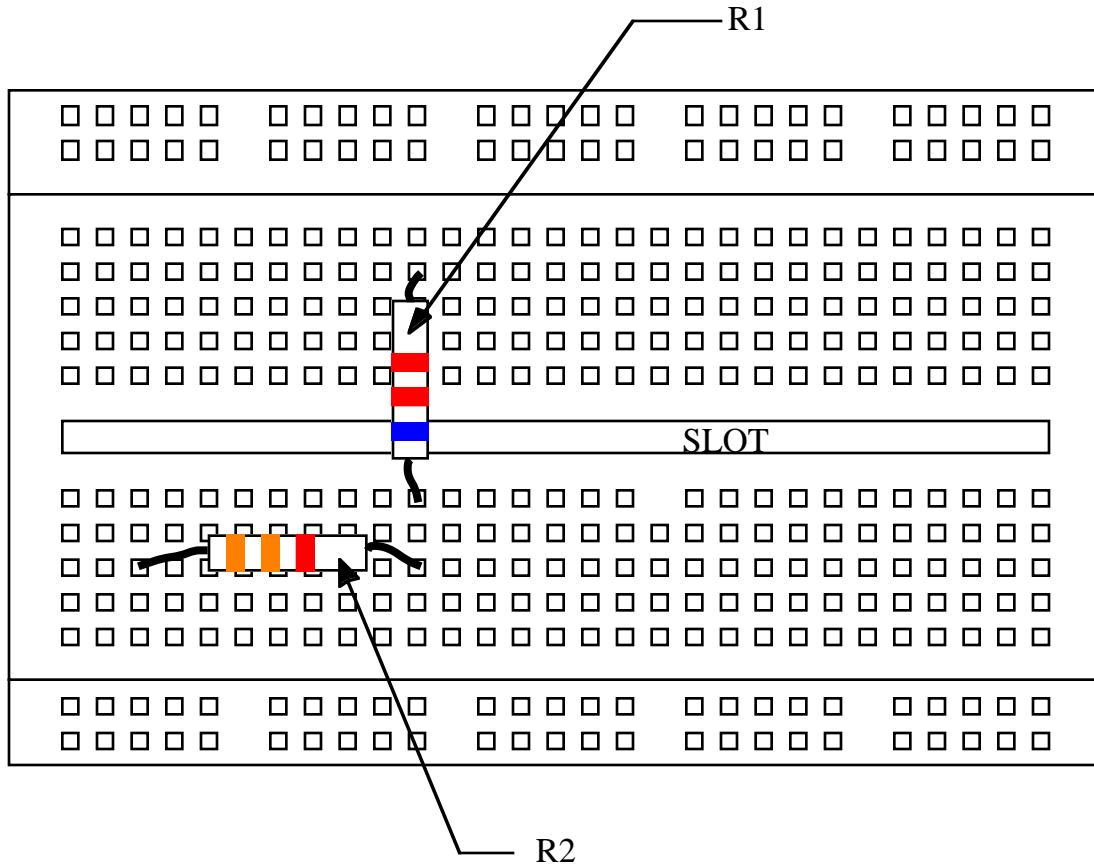


Figure 4-7. Breadboard with R1 and R2 Connected, as They Would Be in the Lab

The wire you will use in the lab and the resistors are sized such that they easily fit into the holes in the breadboard. You will be able to make connections between  $V_s$  and node 1 by placing one end of a connector wire into the red banana jack terminal of the power supply and connect the other end to a wire plugged into your protoboard. Then place one end of R1 in a hole in the same column as the wire from the red terminal.

### Pre-Lab Work

You must read through the background material about thermistors. Also review how to use the power supply and DMM from the simulated labs 1 and 2.

**IMPORTANT:** You should also write a multi-channel data acquisition VI for Lab View since you will need to simultaneously record data from at least two voltage dividers. You can modify your VI from Lab 3 for this purpose. See Appendices I and II for information on how to implement loops and do multi-channel analog input using LabView.

### Lab Procedure

You will need a computer workstation/data acquisition setup plus a hot plate. Once at your station, check to make sure you have the necessary equipment (listed below). When you are ready, you may complete the setup and begin.

### Equipment Needed



Standard workstation with a power supply.

Toolbox with breadboard.

Beaker for ice water

Unknown thermistor probe, these will be marked - record the identification label

Reference thermistor probe, these will be marked - record the identification label

20-gauge wire for making any electrical connections

In your tool kit you should have a needle-nose pliers (not absolutely necessary, but helpful for inserting components and wire into the breadboard).

Banana jacks from the power supply to the breadboard. One red wire and one black wire, each with banana jacks at both ends.

## Physical Setup

The first part of the lab is to build two copies of the circuit shown in Figure 4-2. In one of these T2 will be a previously calibrated thermistor — the reference thermistor — we will provide you with calibration data for these thermistors. In the other circuit T2 will be an unknown thermistor which you will be calibrating.

Insert both thermistors into a beaker of ice water placed on your hot plate. Be very careful not to break the thermistors or spill the water. Do NOT turn on the hot plate until you are ready to collect data!

Connect the thermistor leads to your protoboard. Be sure to make these leads long so that you will not accidentally knock the beaker over.

Connect the positive and negative terminals of the power supply to the red and black terminals on the breadboard.

When you have finished constructing the circuit **have a TA check your circuit.**

## Taking Measurements

5. Set the power supply set to between 2 and 10 volts. The recommended value is about 4 volts. The exact value is not critical but you should record this value. Keep this value constant for all measurements in this lab. You can also record this as one of your LabView inputs.
6. After the TA checks your circuit, use your Data Acquisition VI to start collecting data. This VI is similar to the VI you wrote in the last lab but should take two nearly simultaneous voltage measurements from the two different sensor circuits. If you wish you can also connect and use an additional channel to record the power supply voltage.
7. **Use the DMM to measure the power supply voltage and the voltage output from each voltage divider network just as you start. The water temperature is assumed to be at 32 degrees F for this measurement.**
8. Turn on the hot plate and slowly increase the temperature of the water. Our experience is that with a 300 ml beaker this will take about 10 minutes.
9. The VI is automatically recording the data while the water is heating. Do not worry if your hot plate will not heat the water to an actual boil. **Using the DMM record at least two additional voltage outputs from each divider network.** Ideally one of these would be at the boiling point of the water.
10. After your data acquisition run is over turn off your hot plate. Be very careful not to handle the hot beaker - let it cool.
11. Be sure to use a disk to copy your data file.

## Sensor Calibration and Analysis (To be done outside the lab)

Using the LabView data:

Read the data you collected into a spreadsheet or other data analysis program.

Using the calibration data (this will be available as a data file on the ENGR210 Web page) for the reference thermistor you used in your experiment determine the actual temperature corresponding to each one of your measurement points. If you took too many data points you only need to do this for 10-20 data points. There are many ways to do this. For example, you can manually (graphically) extrapolate the temperature from the provided data for these estimates or. A more accurate approach is to analytically fit a curve to the calibration data and evaluate the temperature for each data point.

Using your data from b) plot the resistance of your unknown thermistor as a function of temperature.

Using any curve fitting program or technique determine an expression for the resistance of your thermistor as a function of temperature. I recommend a polynomial curve fit.

Don't use too many powers — a quadratic or cubic should be enough.

Using this expression, compute the voltage output for the circuit of Figure 4.2 as a function of temperature.

Using this same expression, compute the predicted voltage output for the circuit of Figure 4.4 as a function of temperature. Use your calibrated data for both T1 and T2.

## Lab Report

Your lab report should include the following information:

Your LabView acquired data typed and formatted into tables with proper headings and units. This can take the form of a graph.

A detailed description of the calibration calculations. You must present the calculations such that another student in your major can repeat the process.

A completed data sheet for the DMM data (See below.).

Answers to the following questions:

(a) Why is the circuit of Figure 4.4 called a “linearization” circuit?

(b) Is the data obtained from LabView or the DMM more accurate? Explain your answer.

# Data Sheet

## LabView Voltage Measurements

<b>Measurement</b>	<b>Voltage Across Calibrated Thermistor LabView Reading</b>	<b>Voltage across Unknown Thermistor LabView Reading</b>	<b>Voltage across Calibrated Thermistor Digital Multimeter</b>	<b>Voltage across Unknown Thermistor Digital Multimeter</b>
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				
12.				

Note: You only need to take three DMM readings but feel free to take additional values. These can be useful for the extra-credit assignment. The LabView data will be in a data file. Do not forget to copy this from the machine!

## Extra Credit

An alternative method to calibrating thermistors is to use at least two reference temperature points, i.e., freezing and boiling, and an analytical model for the thermistor behavior. A commonly used analytical model for a thermistor is known as the Steinhart-Hart equation. Using the thermistor resistance values for freezing and boiling, fit the Steinhart-Hart equation to your data. Compare the results of this calibration approach to that you obtained using a calibrated thermistor. Which do you think is best? See Appendix III for an explanation of this method.

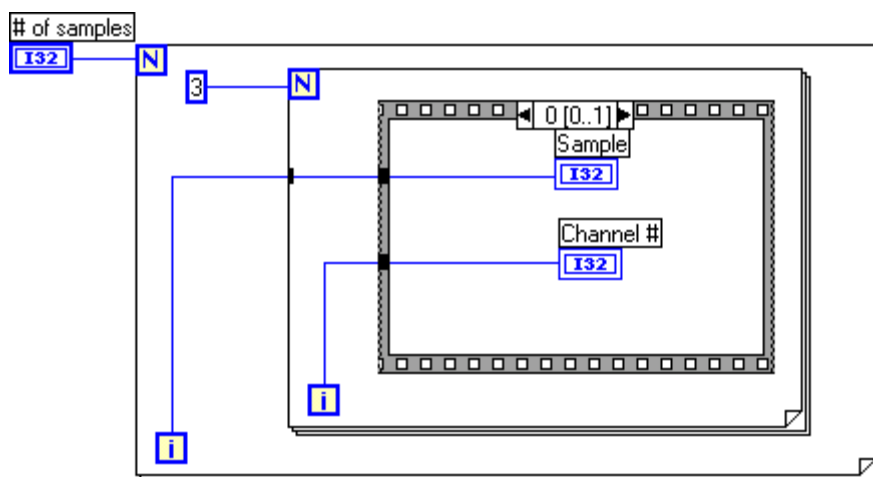
## Appendix I

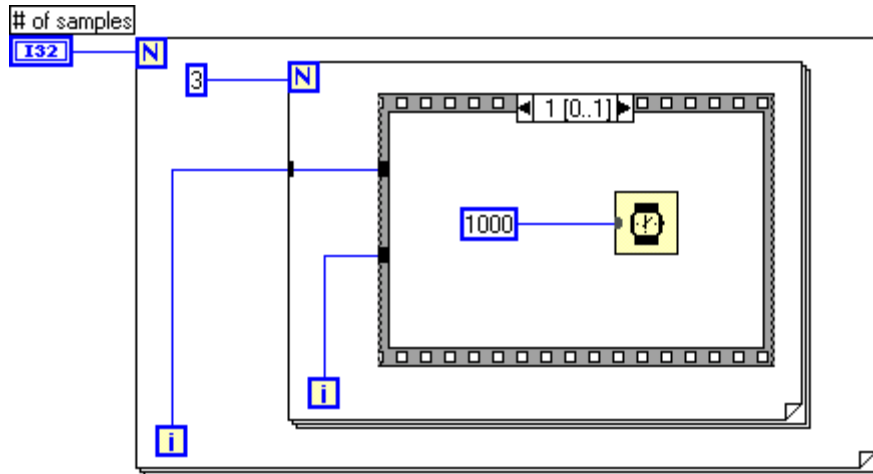
### Program Control in LabView

#### Program Loops

You used a program loop in Lab 3 to graph the data from the random number generator. LabView has two basic types of control structures: the “for” loop which can be used for continuous looping and the “sequence” which is used for time based looping. The basic “for” loop operates much like a for loop in a computer program — it simply repeats the commands inside the loop as quickly as it can. This is not satisfactory for many data acquisition applications where you need to collect data with a known time relationship. For example, you may want to collect a data sample every 15 seconds. This timed operation of a loop may be done using a sequence.

For loops are found in the Structures subpalette of the Functions palette. Their function is identical to that of a conventional For loop. They execute using a counter indicated as “i” until they reach a maximum count designated as “N”. N is an input to the loop and should be provided from a digital control of type integer as shown below. The “i” is a loop counter which is a variable that can be used by other functions. For loops can also be nested. In the example shown below, an outer loop executes “# of samples” times. Note that the loop count actually begins at zero and stops at N-1. Inside the For loop we have constructed another For loop. This one shows “3” being input as the loop counter. This means that the inner loop will execute for N=0, 1 and 2 for each iteration of the outer loop. Inside both loops we have a sequence (See Appendix II) which simply represents something which will be executed 0...3 times 0...”#-samples”-1 times.

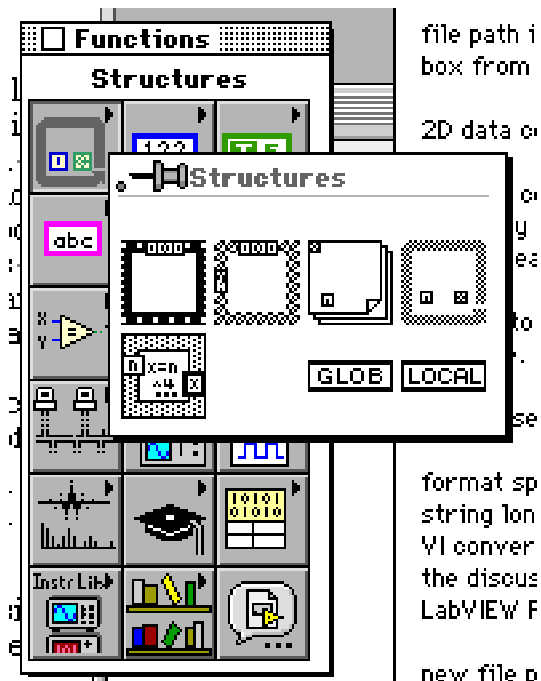




For those of you who are interested the inner procedure is a sequence which displays the loop counters and then simply waits 1000 milliseconds. This is repeated until the looping operations are completed.

## Sequences

Sequences are used in LabVIEW to develop a series of actions. The sequence is also found in the Structures subpalette of the Functions palette. You may insert a sequence using the normal LabVIEW procedure for inserting functions and indicators.

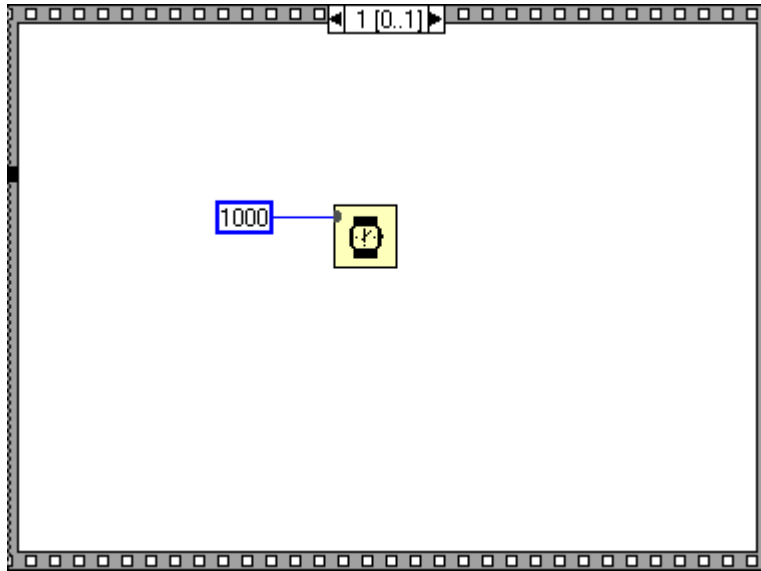


The sequence LabVIEW command looks like a film frame. It is found in the upper left most corner of the Structures subpalette of the Functions palette.

In general, each frame in a Sequence structure is an independent entity. It can communicate with other frames in the Structure and the Structure will only produce an output when the entire Sequence is completed.

As initially placed in your program the sequence consists of a single event called a frame and there will be no numbers at the top of the frame. However, the real function of the sequence is to allow you to sequence actions. Adding a sequence is done by right clicking







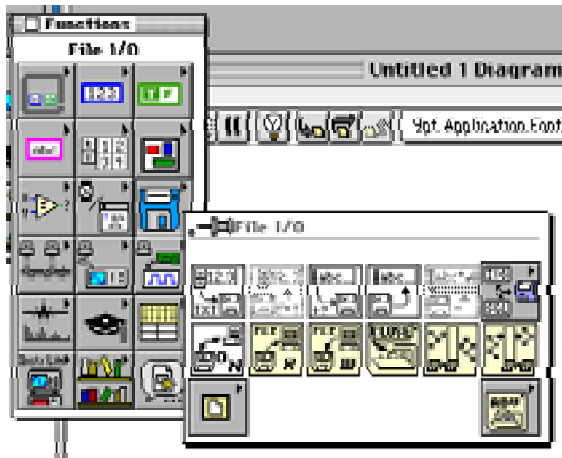
## Appendix II

### Multiple Channel Data Acquisition VI in LabView

Multiple channel data acquisition requires consideration of the program structure (i.e., using loops to time when to take a sample), how to take samples (the actual data acquisition VI), and how to save acquired data as a file. In this appendix we will only deal with the issues of how to use the Analog Input VI's to acquire data and output the data to a spreadsheet file.

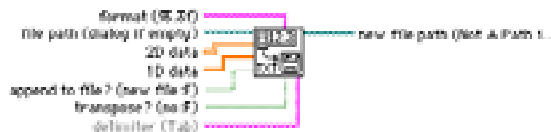
#### File Output

These are among the easiest LabView functions to use. They are found in the File I/O subpalette of the Functions palette. The one you want to use is the Write To Spreadsheet File VI.



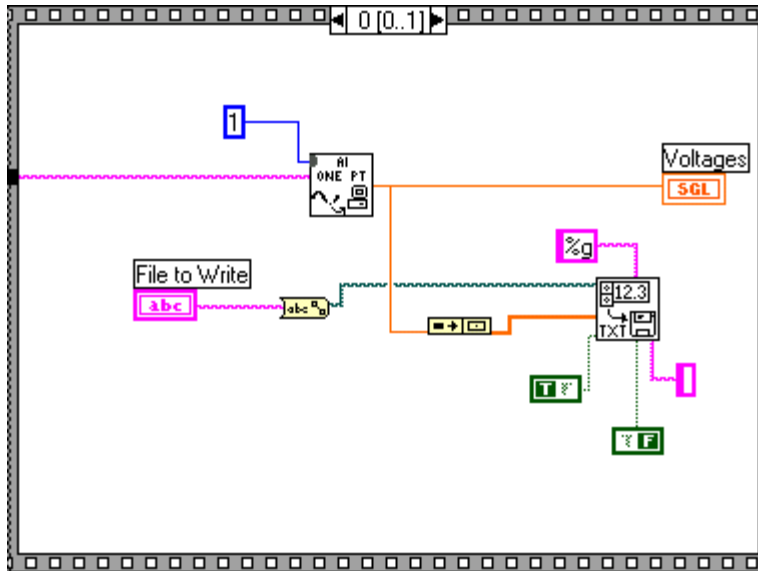
The File I/O subpalette is indicated by the floppy disk icon in the Functions palette. Remember that you have to be in the Diagram window to see the Functions palette, otherwise you will see the Controls palette.

The Write to Spreadsheet VI is shown in the upper



You can place this spreadsheet output VI in your diagram and, if the VI is properly configured, it will write collected data to a tab delimited file which can be read by a spreadsheet program such as Excel and Lotus 123.

The actual data acquisition is slightly more complex. You can input data using either AI Sample Channel or AI Sample Channels. Each of these collects data. AI Sample Channel will read the specified channel and output a single number which is the value of the input voltage. AI Sample Channels will read each specified channel and output a vector of the voltage readings from all the channels. The waveform AI's are more complex to use and will acquire a series of readings from the specified channels. Only the Sample Channel AI's are discussed here.



This frame shows a procedure which can be used for acquiring a single point from a specified channel. In this case, the channel number is an input from outside the sequence.

This example looks complicated but only two VI's are of importance here: (1) AI One Point, and (2) Write to Spreadsheet File. Essentially this is a procedure which reads one data point from a specified channel, converts the data point into an array, and appends the data point to a tab delimited (spreadsheet) file. We have previously seen the inputs to the AI One Point VI. The "1" simply indicates device "1" which is in reality the number of the PCI slot in the PC. There is also an input to AI One Point from somewhere outside the sequence - in this case it is the input channel number for AI One Point.

Because the output from AI One Point ( a single number) and the input to Write to Spread Sheet (a 1D or 2D array) do not match we need to perform a data type conversion. We have not used arrays before but an array is exactly what an array would be in another computer language — an array of numbers. Since LabView is very sensitive to matching data types we need to convert the output data from the AI One Point into an array before it can be appended to the output file. This conversion can be performed by the Build Array function found in the Array subpalette of the Tools palette. The small rectangular box between the AI One Point VI output and the Write to Spreadsheet File input is a Build Array. This function assumes that AI One Point operates repetitively (perhaps done by putting the frame in a For loop). What Build Array does is take each output point from the AI One Point output and append them into a one dimensional array. This one dimensional array of data is passed to the input of the file output VI.

The requirement for the Build Array function can be completely bypassed by using the AI Sample Channels VI. This VI will sequentially sample the specified range of channels and output a vector of reads. For example if you would specify 2,1,0 channels 2, 1, and 0 would be sampled and their values output as a vector (1D array) to the Write to Spread Sheet VI.

Which VI you use depends upon your application and how you wish to implement your program.

The Write to Spread Sheet File VI inputs appear very complex with many more specifications; however, for this course we do not need to understand them all. The %g

tells the VI how to convert the input numbers into characters for file output. In this case, it stands for general format. The name of the file must be a string, i.e., text. This is another type of LabView variable — be sure that your input is of type string. There is an entire subpalette in the Functions palette and in the Controls palette for string functions; however, all you really need is a string constant.

In this case we show you another example of required data type conversion. The file name is specified as text but the String to Path converter converts the name into a complete file description of the type LabView wants. Append to File simply means that we should add this data to an existing file and not create a new file for each data point. The Transpose input is peculiar to the way Lab View represents numbers. Arrays are typically represented as rows of a spreadsheet. However, depending upon the order in which you collected the data you might want the data to be represented as columns. If your data is transposed you have set this control variable wrong.

## Appendix III

### Thermistor Theory

NTC (Negative temperature coefficient, i.e., the resistance decreases with temperature) thermistor materials are prepared by heating mixtures of metal oxides to high temperatures so that the oxides combine chemically to form the spinel crystallographic structure. The name derives from the mineral spinel,  $MgAl_2O_4$ , which has this structure. In this structure Mg occupies tetrahedral, or A sites, in the crystal lattice and Al occupies octahedral, or B sites. This is a normal spinel, with one 2+ metal ion on the A site, two 3+ metal ions on the B sites and four oxygens. This is commonly written  $Mg[A1_2]_{10}$ , where the elements in the bracket represent the B sites.

An inverse spinel has half the trivalent ion on the A sites and the divalent ion on the B sites, such as nickel ferrite,  $Fe[NiFe]O_4$ . Various degrees of inversion can occur depending on the metal ions, the temperature of reaction, and any annealing cycles to which the material is subjected. A common thermistor material is nickel manganite, a partially inverse spinel with manganese present on the B sites in 3+ and 4+ states.

These types of materials are referred to as valence-controlled semiconductors. Conduction occurs when ions having multiple valence states occupy equivalent crystallographic sites. They must be the same element and differ in valence by one unit and occupy B sites. The conduction mechanism is a thermally activated electron hopping process, in which the electrons hop from one cation ( $Mn^{3+}$ ) to another ( $Mn^{4+}$ ) in the B lattice sites under the influence of a potential gradient across the material.

The conductivity is a product of charge density and mobility. Charge density is determined by the number of charge carries, the density of B sites and the probability of a B site being active. The mobility is determined by the distance between the nearest neighbor B sites, the activation energy (needed for the electron to move from one site to another) and a frequency factor (how often it tries to jump). Charge carries are also produced by other defects such as non-stoichiometry and grain boundaries.

By considering the effects of all the above factors, an expression for conductivity can be derived:

$$\sigma = \sigma_{\infty} e^{-q/kT}$$

where  $\sigma_{\infty}$  is the infinite temperature conductivity (which includes consideration of charge density and mobility),  $-q$  is the activation energy,  $k$  is Boltzmann's constant, and  $T$  the absolute temperature. For thermistors, the resistivity  $s$  (and hence resistance) is of more interest and the above becomes

$$\sigma = \sigma_{\infty} e^{+q/kT}$$

#### Beta Constant

By replacing resistivity with resistance values and combining the activation energy and Boltzmann's constant terms, the familiar thermistor expression is obtained

$R = Ae^{\frac{\beta}{T}}$  where  $A$  includes dimensional factors and infinite temperature resistance,  $\beta$  is the material constant beta and  $T$  is the absolute temperature.

One can determine the beta constant by measuring the resistance at two temperatures and using the above equation,

$$\frac{R_1}{R_2} = e \exp\left(\frac{\beta}{T_1} - \frac{\beta}{T_2}\right)$$

$$\ln\left(\frac{R_1}{R_2}\right) = \beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\beta = \frac{\ln\left(\frac{R_1}{R_2}\right)}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

### Alpha Temperature Coefficient of Resistance

The temperature coefficient of resistance  $\alpha$  is determined by

$$\alpha = \frac{1}{R} \frac{dR}{dT}$$

and is usually expressed in terms of % change in resistance per degree.

The coefficient of resistance and the material constant  $\beta$  are related to each other by

$$\alpha = \frac{-\beta}{T^2}$$

Beta and  $\alpha$  are two different ways of expressing the same property.

### R versus T Approximation Methods

Although the expression  $R = Ae^{\frac{\beta}{T}}$  gives good agreement with empirical data over short temperature spans, a better method of interpolation over larger temperature ranges is necessary for accurate temperature measurements.

Thermistor accuracy depends on the precision and uncertainty of the calibration system used. The precision of the measurement statement, however, is in large part due to the method of approximation and interpolation. The approximation methods outlined below provide several choices for maximum simplicity while allowing for precise interpolation.

### Narrow Range Approximation Methods

The following table shows two approximation methods, the applicable temperature range and range of deviation from nominal resistance.

Equation	Temperature Range	Deviation
----------	-------------------	-----------

$\ln(R_T) = \frac{A}{T}$	very small	-
$R_T = Ae^{\frac{\beta}{T}}$	-20 to +120°C	+0.94, -0.82°C

### Steinhart and Hart

The Steinhart and Hart equation is an empirical expression that has been determined to be the best mathematical expression for the resistance-temperature relationship of a negative temperature coefficient thermistor. It is usually found explicit in T:

$$\frac{1}{T} = a + b \ln(R) + c(\ln(R))^3 \quad (1)$$

where:

T = Kelvin units (°C + 273.15)

a,b,c = coefficients derived from measurement

ln(R) = natural logarithm of resistance in ohms

To find a, b and c, measure a thermistor at three temperatures. The temperatures should be evenly spaced, and at least 10°C apart. Use the three temperatures and resistances to solve three simultaneous equations.

$$\frac{1}{T_1} = a + b \ln(R_1) + c(\ln(R_1))^3$$

$$\frac{1}{T_2} = a + b \ln(R_2) + c(\ln(R_2))^3$$

$$\frac{1}{T_3} = a + b \ln(R_3) + c(\ln(R_3))^3$$

The equations allow you to derive a, b and c for any temperature range. We have calculated these coefficients for the range 0 to 100°C with 50°C as the intermediate point. These are listed below for your use.

#### Coefficients derived from 0, 50 and 100°C catalog resistance

Thermistor	25°C	a	b	c
mix	resistance			
L	100 Ω	0.0017709	0.0003406	1.479E-07
L	3000	0.0015632	0.0003108	9.747E-08
L	1 KΩ	0.001313	0.0002906	1.023E-07
B	2252 Ω	0.0014733	0.0002372	1.074E-07
B	3 KΩ	0.0014051	0.0002369	1.019E-07
B	5 KΩ	0.001288	0.0002356	9.557E-08
B	6 KΩ	0.0012474	0.000235	9.466E-08
B	10 KΩ	0.0011303	0.0002339	8.863E-08

H	10 KΩ	0.0010295	0.0002391	1.568E-07
H	30 KΩ	0.0009354	0.0002211	1.275E-07
H	100 KΩ	0.0008253	0.0002045	1.144E-07
H	300 KΩ	0.0008207	0.0001848	1.014E-07
H	1 MEGΩ	0.0008142	0.000167	8.819E-08

Knowing a, b and c for the thermistor allows you to use the Steinhart and Hart equation in two ways. If resistance is known and temperature is desired, use equation (1) above. If the temperature is known and expected resistance is desired, use equation (2) below. Remember that T is in Kelvin units.

$$R = e \exp \left[ \left( \beta - \left( \frac{\alpha}{2} \right) \right)^{1/3} - \left( \beta + \left( \frac{\alpha}{2} \right) \right)^{1/3} \right] \quad (2)$$

where

$$\alpha = \frac{a - 1/T}{c}, \quad \beta = \left[ \left( \frac{b}{3c} \right)^3 + \frac{\alpha^2}{4} \right]^{1/2}$$

It should be noted that these values of alpha and beta are not related to the alpha and beta used with single term exponential equations.

The ability to precisely interpolate for a given temperature from measurements at known fixed points depends in part on the closeness of those points. Fixed-points such as the water triple point, mercury triple point, gallium melting point and indium freezing point provide a solid basis for the interpolation.

For practical reasons some of the R vs. T tables have small interpolation differences when random values from the tables are used in the above equations, particularly over large temperature spans.

### Spreadsheet Program

The following spreadsheet program (Lotus 123) allows calculation of the Steinhart and Hart coefficients, using three resistances at three temperatures. It calculates resistance, dR/dT or determines the temperature for a known resistance.

Labels start with an apostrophe ('). Brackets indicate data you must enter. Other cells are formulas.

```

B1: 'Temp. (C)
C1: 'Resistance
D1: 'T (K)
E1: 'Ln (R)
A2: 'Low
B2: [Input low temperature in °C]
C2: [Input low temp. resistance in ohms]
D2: +B2+273.15
E2: @LN(C2)
A3: 'Mid
B3: [Input mid temperature in °C]
C3: [Input mid temp. resistance in ohms]
D3: +B3+273.15

```

```

E3: @LN(C3)
A4: 'High
B4: [input high temperature in °C]
C4: [Input high temp. resistance in ohms]
D4: +B4+273.15
E4: @LN(C4)
A6: 'Ln(R1)-Ln(R2)
B6: +E2-E3
A7: 'Ln(R1)-Ln(R3)
B7: +E2-E4
A8: '(1/T1)-(1/T2)
B8: 1/D2-1/D3
A9: '(1/T1)-(1/T3)
B9: 1/D2-1/D4
A11: 'Coefficients: a=
B11: 1/D2-B13*E2^3-B12*E2
A12: 'b=
B12: B8-B13*(E2^3-E3^3)/B6
A13: 'c=
B13: (B8-B6*B9/B7)/((E2^3-E3^3)-B6*(E2^3-E4^3)/B7)
A15: 'Solving for R, given T
A16: 'Degrees C=
B16: [Input known temperature in °C]
C16: +B16+273.15
D16: (B11-(1/C16))/B13
E16: '=A
D17: @SQRT((B12/(3*B13))^3+(D16^2)/4)
E17: '=B
A18: 'Resistance (Ohm)=
B18: @EXP((D17-(D16/2))^(1/3)-(D17+(D16/2))^(1/3))
A19: 'dR/dT=
B19: -1*B18/(C16^2*(B12+3*B13*(@LN(B18))^2))
A20: '%dR/dT=
B20: +B19/B18*100
A23: 'Solving for T, given R:
A24: 'Ohms=
B24: [Input known resistance in ohms]
A26: 'Temperature (C)=
B26: 1/(B11+B12*@LN(B24)+B13*(@LN(B24))^3)-273.15

```