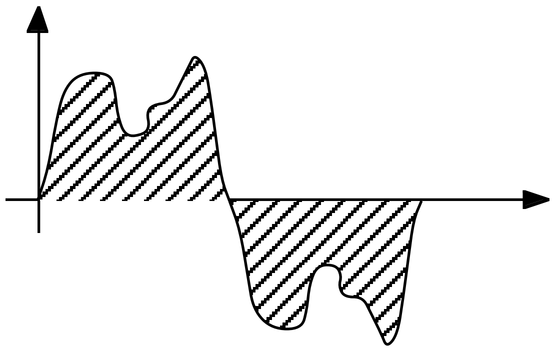
Chapter 3 - Waveforms, Power and Measurement

Recommended problems to study:

Reference formula

Part 1 - periodic signals

average values:



$$f_{avg} = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)dt = positive\_area - negative\_area$$

rms values:

$$f_{rms} = \sqrt{\frac{1}{T}\int_{t_1}^{t_1+T} f^2(t)dt}$$

Fourier series:

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n}{T}t\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n}{T}t\right)$$

Instantaneous power:

$p(t) = v(t)i(t) = V_{\max}\cos(\omega t + \theta)I_{\max}\cos(\omega t + \phi)$

Using the trig identity $2\cos(\alpha)\cos(\beta) = \cos(\alpha - \beta) + \cos(\alpha + \beta)$ this becomes

$$p(t) = \frac{V_{\max}I_{\max}}{2}\left[\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi)\right]$$

Average power

$$P = \frac{1}{T}\int_{t_1}^{t_1+T} p(t) = \frac{V_{\max}I_{\max}}{2}\cos(\theta - \phi)$$

For rms quantities

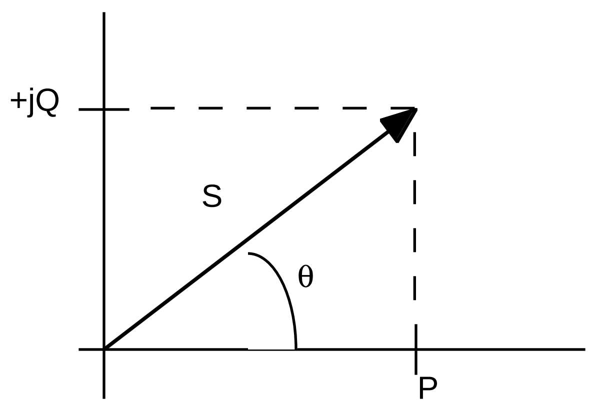
$P = V_{rms}I_{rms}\cos(\theta - \phi)$

where $\cos(\theta - \phi)$ is the power factor

Complex power:

$P = \mathrm{Re}\{VI^*\} = \mathrm{Re}\{V_{rms}\angle\theta I_{rms}\angle - \phi\} = V_{rms}I_{rms}\cos(\theta - \phi)$

Note the - sign for ϕ



apparent power	$S = VI^*$	VAs (Volt-amperes)
real power	$P = \mathrm{Re}\{VI^*\}$	Watts
reactive power	$Q = \mathrm{Im}\{VI^*\}$	VARs (Volt-amperes reactive)

$$P.F. = \frac{|P|}{|Q|} = \cos\theta$$

Part 2 - Measurement of DC and Periodic Signals
Part 3 - Power

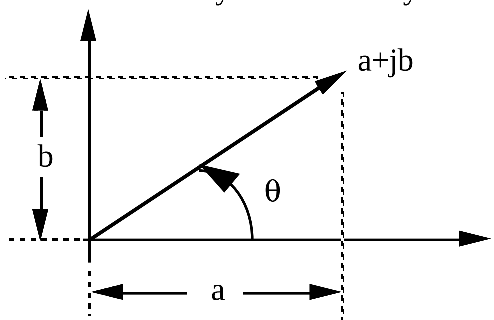
<u>Trigonometric relationships</u>

<u>Complex numbers</u>

The ability to convert complex numbers from a rectangular format (a+jb) to a polar form $Ae^{j\theta}$ is essential to describing the behavior of ac electrical networks. This conversion is provided by Euler's identity which states that

$a + jb = \sqrt{a^2 + b^2}\angle \mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$

and can be readily understood by the diagram shown below.



where

$a + jb = \sqrt{a^2 + b^2}\angle \mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$

and

$\tan\theta = \frac{b}{a}$

The rectangular form is a+jb; the polar (or phasor) form is $c\angle\theta$ where

$c = \sqrt{a^2 + b^2}$

<u>Phasors</u>

sinusoidal voltage $v(t) = V_{\max}\cos(\omega t + \phi)$

Euler's formula $e^{j\theta} = \cos\theta + j\sin\theta$

$$v(t) = \mathrm{Re}\left\{V_{\max}e^{j(\omega t+\theta)}\right\} = \mathrm{Re}\left\{V_{\max}e^{j\omega t}e^{j\theta}\right\}$$

For 60Hz power systems all voltages are at the same frequency so we ignore the $e^{j\omega t}$ term

$$v(t) = \mathrm{Re}\left\{V_{\max}e^{j\theta}\right\}, \text{ or } \hat{v} = V_{\max}\angle\theta$$

For power systems $\hat{v} = V_{rms}\angle\theta$ where $V_{rms} = \frac{V_{rms}}{\sqrt{2}}$

<u>Complex power, power factor, power factor correction</u>

For sinusoidal signals the power factor is defined by

pf=cosθ

where θ is the phase angle of the voltage or current relative to some reference. For power circuits, the generator (or line) voltage is usually taken as the reference since the loads are usually connected in parallel.

For phasors we can define complex (also called the reactive) power as:

CP = v×i* = real power ± j reactive power = P ± jQ

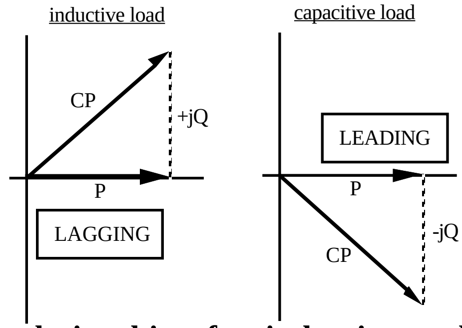
where

real power or P = v×i×cosθ

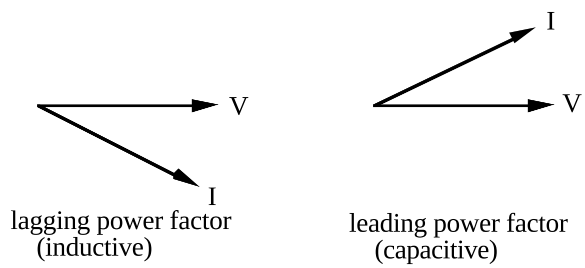
and

reactive power or $Q = v \times i \times \sin\theta$

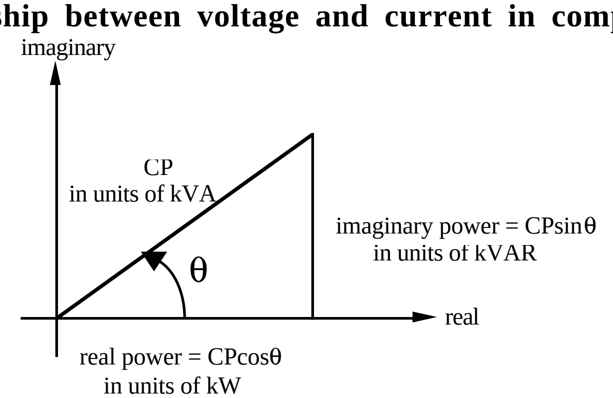
The relationship between P and Q determines the power factor as shown below.



Complex power relationships for inductive and capacitive loads



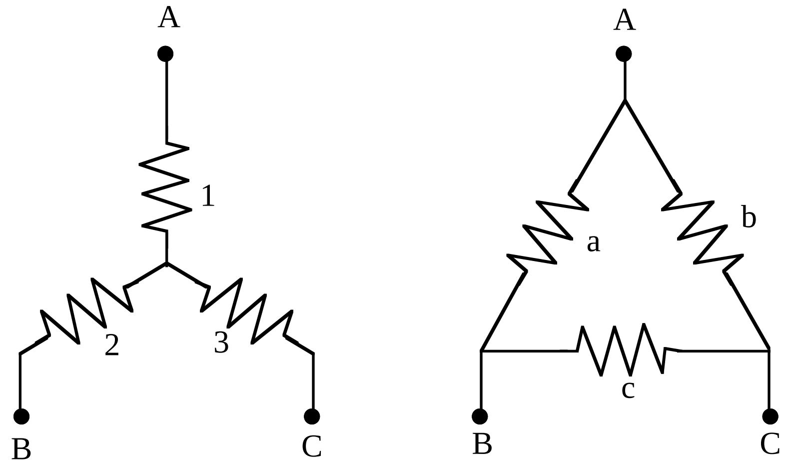
Relationship between voltage and current in complex loads



Relationship between real and imaginary power in a complex load. Note that the diagram is drawn for an inductive circuit. It would be reversed for a capacitive load.

Delta/wye conversions

Delta circuits can be transformed into wye circuits and vice versa to simplify circuit analysis.



With reference to the circuits shown above (Note that that A corresponds to A, B to B and C to C in the two circuits.) the conversion formula are:

Delta to Wye

$R^* = R_a + R_b + R_c$

$R_1 = \dfrac{R_a R_c}{R^*}$

$R_2 = \dfrac{R_a R_b}{R^*}$

$R_3 = \dfrac{R_b R_c}{R^*}$

Wye to Delta

$R^* = R_1 R_2 + R_1 R_3 + R_2 R_3$

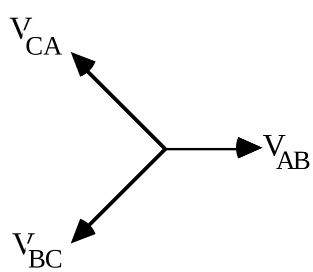
$R_a = \dfrac{R^*}{R_3}$

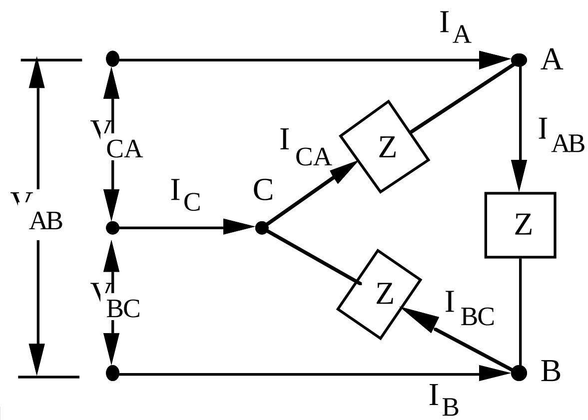
$R_b = \dfrac{R^*}{R_1}$

$R_c = \dfrac{R^*}{R_2}$

Three phase circuits

Three phase power is very complex for non-electrical engineers and usually accounts for several questions in the morning and afternoon sections of the exam. Do not attempt to understand how these expressions are derived—simply use them—and you should do well on that part of the exam. A three-phase load always has three terminals which I have labeled A, B and C in the drawings below. The connections to the voltage sources are called lines and the voltages between the terminals (lines) are called line-to-line voltages. The current passing through each terminal is called the line current. Other voltages and currents can be defined internal to the different loads possible, i.e. wye and delta. The line-to-line voltages are 120° out of phase relative to each other as shown in the diagram below.





Balanced Delta-connected Load

The delta load connects to the three-phase voltage source through the terminals A, B and C as described above. The delta load is balanced when all three load impedances are identical. The current through each load is called the phase current.

line-to-line voltages:	phase currents:	line currents:
$\hat{V}_{AB} = \lvert\hat{V}_{AB}\rvert \angle 0°$	$\hat{I}_{AB} = \lvert\hat{I}_{AB}\rvert \angle -\theta$	$\hat{I}_{A} = \lvert\hat{I}_{A}\rvert \angle -\theta -30°$
$\hat{V}_{BC} = \lvert\hat{V}_{BC}\rvert \angle -120°$	$\hat{I}_{BC} = \lvert\hat{I}_{BC}\rvert \angle -\theta -120°$	$\hat{I}_{B} = \lvert\hat{I}_{B}\rvert \angle -\theta -150°$
$\hat{V}_{CA} = \lvert\hat{V}_{CA}\rvert \angle +120°$	$\hat{I}_{CA} = \lvert\hat{I}_{CA}\rvert \angle -\theta +120°$	$\hat{I}_{C} = \lvert\hat{I}_{C}\rvert \angle -\theta +90°$

where θ is defined by

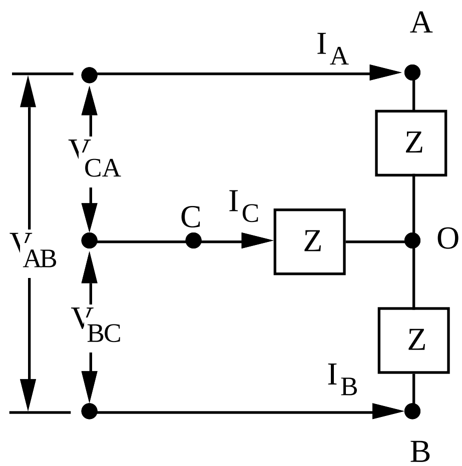
$\hat{Z} = \lvert\hat{Z}\rvert \angle\theta$

The magnitudes of the phase and line current are related to each other for a balanced system.

$$\lvert\hat{I}_{phase}\rvert = \frac{\lvert\hat{I}_{line}\rvert}{\sqrt{3}}$$

The total power consumed by a balanced load is

$$P_{total} = \sqrt{3}\,\lvert\hat{V}_{line\text{-}to\text{-}line}\rvert\,\lvert\hat{I}_{line}\rvert \cos\theta$$



Balanced Wye-connected Load

The wye load connects to the three-phase voltage source through terminals A, B and C as shown above. The wye load is called balanced when all three load impedances are identical as shown above. A unique characteristic of the wye load is that the three load impedances are connected together at a common node labeled O. The voltage across each load impedance, i.e. the voltage betwwen the terminal A, B or C and the common node O is called a phase-to-phase voltage. The phase relationships between the voltages and currents using the line-to-line voltage V_{AB} as the reference is then given by:

line-to-line voltages:

$\hat{V}_{AB} = |\hat{V}_{AB}| \angle 0°$

$\hat{V}_{BC} = |\hat{V}_{BC}| \angle -120°$

$\hat{V}_{CA} = |\hat{V}_{CA}| \angle +120°$

phase-to-phase voltages:

$\hat{V}_{AO} = |\hat{V}_{AO}| \angle -30°$

$\hat{V}_{BO} = |\hat{V}_{BO}| \angle -150°$

$\hat{V}_{CO} = |\hat{V}_{CO}| \angle +90°$

In a balanced system the three load impedances are identical in magnitude and phase resulting in the magnitudes of the different line-to-line voltages being equal.

$|\hat{V}_{AB}| = |\hat{V}_{BC}| = |\hat{V}_{CA}|$

The same result is true for the phase-to-phase voltages.

$|\hat{V}_{AO}| = |\hat{V}_{BO}| = |\hat{V}_{CO}|$

The magnitude of the phase-to-phase and line-to-line voltages are related to each other for a balanced system.

$$|\hat{V}_{\text{phase-to-phase}}| = \frac{|\hat{V}_{\text{line-to-line}}|}{\sqrt{3}}$$

The total power consumed by a balanced load is

$P_{total} = \sqrt{3}\,|\hat{V}_{\text{line-to-line}}|\,|\hat{I}_{line}| \cos\theta$

where θ is defined by

$\hat{Z} = |\hat{Z}| \angle\theta$

Note: for a wye load the line current is equal to the phase current.

Complex power, power factor, power factor correction

For sinusoidal signals the power factor is defined by

$pf = \cos\theta$

where θ is the phase angle of the voltage or current relative to some reference. For power circuits, the generator (or line) voltage is usually taken as the reference since the loads are usually connected in parallel.

For phasors we can define complex (also called the reactive) power as:

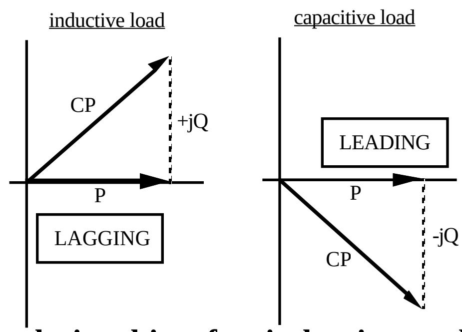
$CP = v \times i^* = \text{real power} \pm j \text{ reactive power} = P \pm jQ$

where

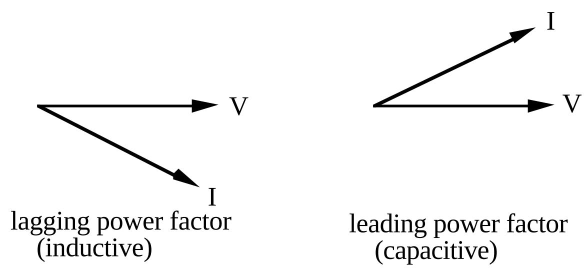
real power or $P = v \times i \times \cos\theta$

and reactive power or $Q = v \times i \times \sin\theta$

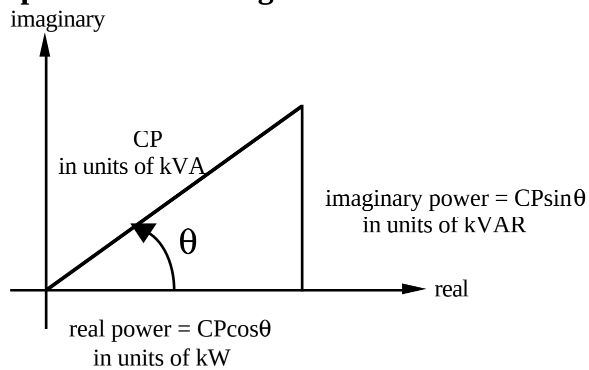
The relationship between P and Q determines the power factor as shown below.



Complex power relationships for inductive and capacitive loads



Relationship between voltage and current in complex loads



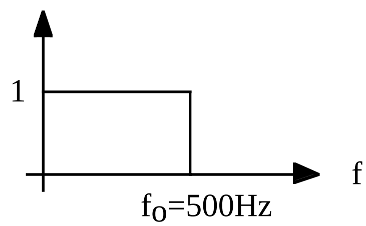
Relationship between real and imaginary power in a complex load. Note that the diagram is drawn for an inductive circuit. It would be reversed for a capacitive load.

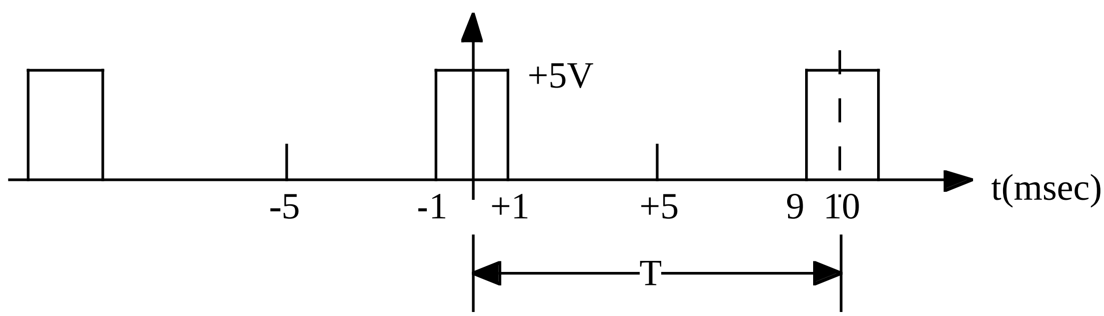
Chapter 3

Concentrates

3. A train of rectangular pulses is applied to an ideal low-pass filter circuit. The pulse height is 5 volts, and the duration of each pulse is 2ms. The repetition period is 10ms. The low-pass filter has a cut-off frequency of 500 Hz. What percentage of the signal power is available at the output of the filter?

The ideal filter





As drawn this pulse train is an even function. That was my option since I prefer even function series.

$$f(t)=\frac{1}{2}a_o+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi n}{T}t\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{2\pi n}{T}t\right)$$

The last term of this expression for $f(t)$ can be ignored. All $b_n=0$ since $f(t)$ is an even function and sine is odd.

$$v(t)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi n}{T}t\right)$$

$$a_o=2f_{avg}=2\frac{1}{10m\sec}\left[(5V)(2m\sec)\right]=2V$$

$$a_n=\frac{2}{T}\int_{t_1}^{t_1+T}f(t)\cos\left(\frac{2\pi n}{T}t\right)dt$$

Do the integral from t=-0.005 to t=+0.005 seconds. For that case, the integral reduces to

$$a_n = 2\frac{2}{0.01}\int_0^{0.001} 5\cos\left(\frac{2\pi n}{0.01}t\right)dt = \frac{4\times 5}{0.01}\left(\frac{0.01}{2\pi n}\right)\sin\left(\frac{2\pi}{0.01}nt\right)\Bigg|_0^{0.001} = \frac{10}{\pi n}\sin\left(\frac{\pi}{5}n\right)$$

$$\omega_n = \frac{2\pi n}{T} = \frac{2\pi}{0.01}n$$

$$f_n = \frac{n}{T} = \frac{n}{0.01} = 100n \text{ Hz}$$

The ideal filter will pass $a_o,\ a_1,\ a_2,\ a_3,\ a_4$ and a_5

Power is rms voltage squared.

$$P_{out} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\left(\frac{a_o}{2}\right)^2 + \frac{a_1^2}{2} + \frac{a_2^2}{2} + \frac{a_3^2}{2} + \frac{a_4^2}{2} + \frac{a_5^2}{2}\right]$$

The first term is different since the rms value of the dc term is the dc term $\frac{a_o}{2}$. For all other terms the rms voltage is given by $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$. Computing the terms we get

$$a_0 = 2 \qquad a_3 = \frac{10}{3\pi}\sin\left(\frac{3\pi}{5}\right) = 1.01$$

$$a_1 = \frac{10}{\pi}\sin\left(\frac{\pi}{5}\right) = 1.87 \qquad a_4 = \frac{10}{4\pi}\sin\left(\frac{4\pi}{5}\right) = 0.47$$

$$a_2 = \frac{10}{2\pi}\sin\left(\frac{2\pi}{5}\right) = 1.51 \qquad a_5 = \frac{10}{5\pi}\sin\left(\frac{5\pi}{5}\right) = 0$$

The output power is then

$$P_{out} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\left(\frac{2}{2}\right)^2 + \frac{(1.87)^2}{2} + \frac{(1.51)^2}{2} + \frac{(1.01)^2}{2} + \frac{(0.47)^2}{2} + 0\right] = \frac{4.51}{R}$$

Using the definition of rms to determine the power for the input pulse waveform

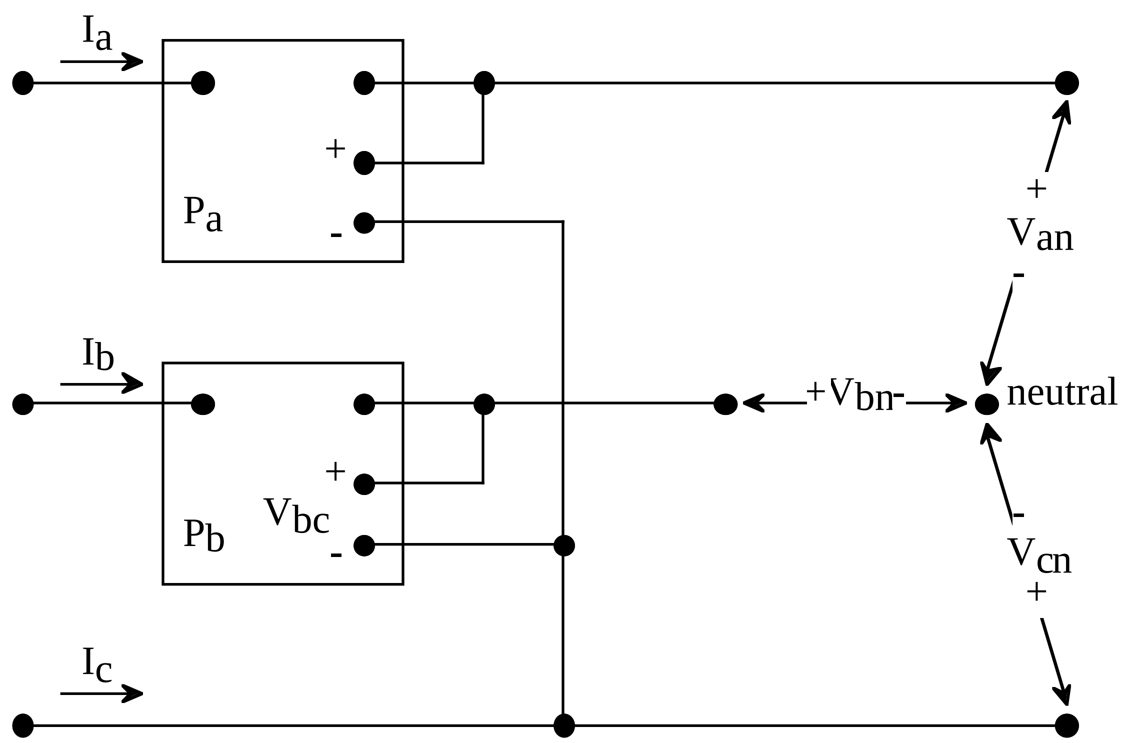
$$P_{in} = \frac{V_{rms}^2}{R} = \frac{1}{R}\left[\sqrt{\frac{1}{T}\int_{-0.001}^{+0.001} 25dt}\right]^2 = \frac{1}{R}\frac{1}{0.01}25(0.002) = \frac{5}{R}$$

The percentage power passed by the filter is then

$$\frac{P_{out}}{P_{in}} = \frac{\frac{4.51}{R}}{\frac{5}{R}} = \frac{4.51}{5} \cong 90\%$$

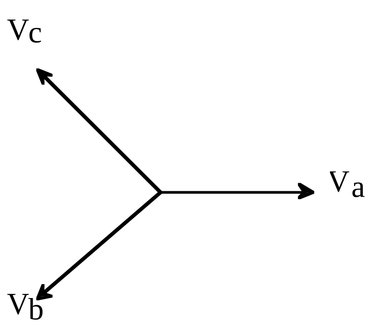
10. The measurement system shown below is used for a balanced load of 1 kW with a lagging power factor of 0.8. Determine the wattmeter readings.

See Section 3-16 for an explanation of the two-wattmeter method of three phase power measurement.



The neutral is an artificial point used to make the two-wattmeter analysis easier.

Since it is not specified assume an ABC phase sequence. For this problem $PF = \cos\theta = 0.8$ lagging so that $\theta = +36.87°$.



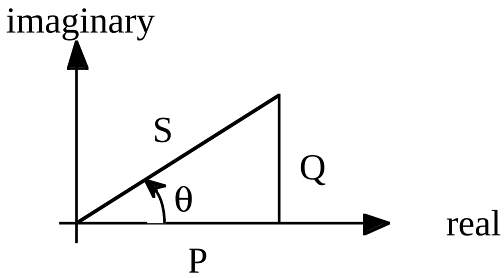
The voltages and currents are then specified by

$V_{an} = V\angle 0°$ $\quad$ $I_a = I\angle -\theta = I\angle -36.87°$

$V_{bn} = V\angle -120°$ $\quad$ $I_b = I\angle -\theta - 120° = I\angle -156.87°$

$V_{cn} = V\angle +120°$ $\quad$ $I_c = I\angle -\theta + 120° = I\angle +83.13°$

Recall the power triangle and write the expressions for the voltages and currents as seen by the wattmeters.



For wattmeter A:

$V_{ac} = V_{an} - V_{cn} = V\angle 0° - V\angle +120° = V(1.732\angle -30°)$

$I_a = I\angle -36.87°$

$S_{ac} = V_{ac} I_a^* = V(1.732\angle -30°)(I\angle -36.87°)^* = VI(1.732\angle -30°)(1\angle +36.87°) = VI(1.732\angle 6.87°)$

Or, in rectangular form

$S_{ac} = VI(1.72 + j0.207)$

For wattmeter B:

$V_{bc} = V_{bn} - V_{cn} = V\angle -120° - V\angle +120° = V(1.732\angle -90°)$

$I_b = I\angle -156.87°$

$S_{bc} = V_{bc} I_b^* = V(1.732\angle -90°)(I\angle -156.87°)^* = VI(1.732\angle -90°)(1\angle +156.87°) = VI(1.732\angle 66.87°)$

Or, in rectangular form

$S_{bc} = VI(0.68 + j1.59)$

For a balanced load $P = \sqrt{3} V_{line-line} I_{line} \cos\theta$ and $V_{phase} = \dfrac{V_{line-line}}{\sqrt{3}}$

In this problem V_{an}, V_{bn}, and V_{cn} are phase voltages. V_{ac} and V_{bc} are line-line voltages. Then

$P = \sqrt{3}\sqrt{3} V_{phase} I_{line} \cos\theta = 3 V_{phase} I_{line} \cos\theta$

Using the numbers for this problem

$1000 watts = 3 V_{phase} I_{line} (0.8)$

or VI=416.67 watts

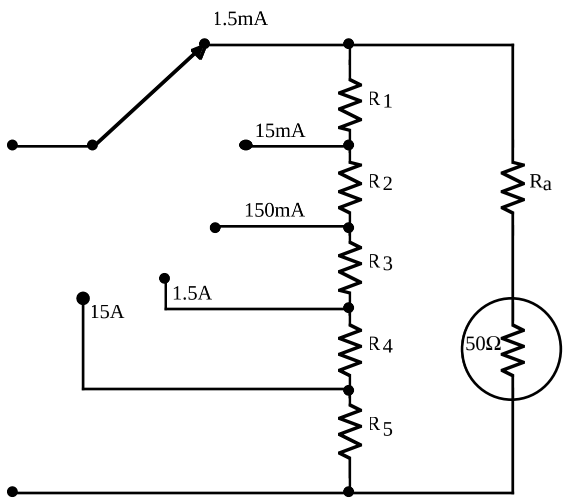
As a check on our calculations

$P_a = \mathrm{Re}\{S_{ac}\} = 1.72VI = 1.72(416.67) = 716.67\text{watts}$

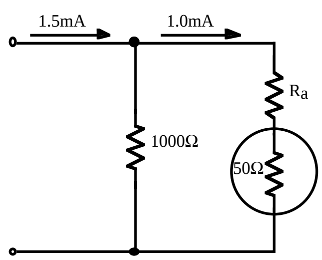
$P_b = \mathrm{Re}\{S_{bc}\} = 0.68VI = 0.68(416.67) = 283.33$

These powers add up to exactly 1000 watts so the answer looks good.

3. An ammeter is being designed to measure currents over the five ranges indicated in the accompanying illustration. The indicating meter is a 1.00 milliampere movement with an internal resistance of 50 ohms. The total resistance $(R_1 + R_2 + R_3 + R_4 + R_5)$ is to be 1000 ohms. Specify the resistances R_a and R_1 through R_5.



For 1.5mA



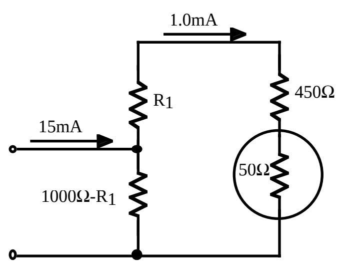
There is 0.5 milliampere through the 1kΩ resistor.
Since the voltages must be equal:

$$1000(0.5mA) = 1mA(R_a + 50\Omega)$$

or

$$R_a = 450\Omega$$

For 15mA



$$1mA(R_1 + 450\Omega + 50\Omega) = (15-1)mA(1000 - R_1)$$

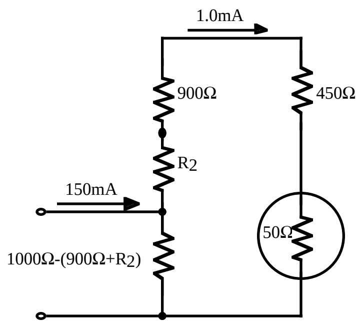
$$R_1 + 500\Omega = 14000 - 14R_1$$

$$15R_1 = 14000 - 500 = 13500$$

$$R_1 = 900$$

For 150mA

$$1mA\left(R_2+900\Omega+450\Omega+50\Omega\right)=(150-1)mA\left(1000-\left(900\Omega+R_2\right)\right)$$



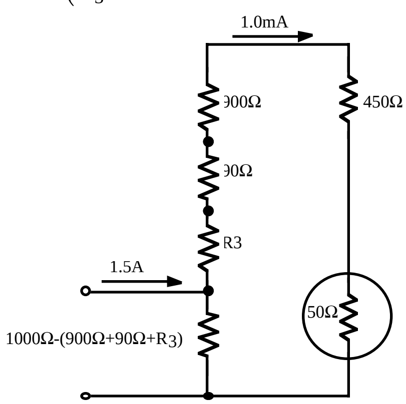
$$R_2+1400\Omega=149\left(100-R_2\right)$$

$$150R_2=14900-1400=13500$$

$$R_2=90$$

For 1.5A

$$1mA\left(R_3+900\Omega+90\Omega+450\Omega+50\Omega\right)=(1500-1)mA\left(1000-\left(900\Omega+90\Omega+R_3\right)\right)$$



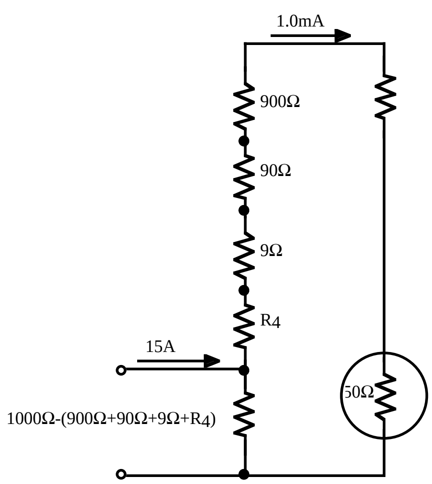
$$R_3+1490\Omega=1499\left(10-R_3\right)$$

$$1500R_3=13500$$

$$R_3=9\Omega$$

For 15A

$$1mA\left(R_4+9\Omega+90\Omega+900\Omega+450\Omega+50\Omega\right)=(15000-1)mA\left(1000-\left(900\Omega+90\Omega+9\Omega+R_4\right)\right)$$



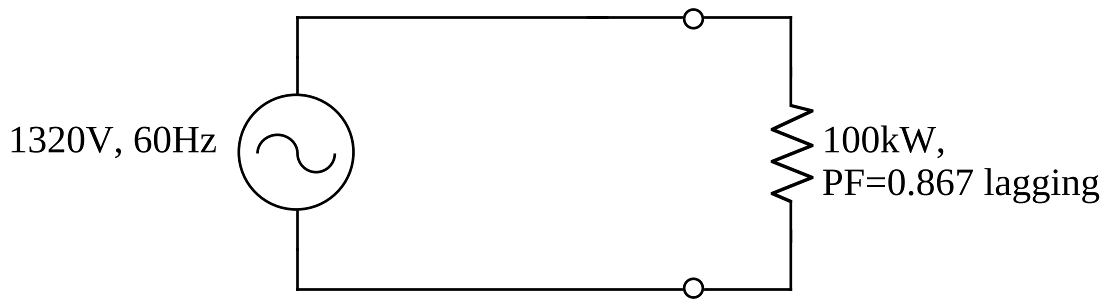
$$R_4+1499\Omega=14999\left(1-R_4\right)$$

$$15000R_4=13500$$

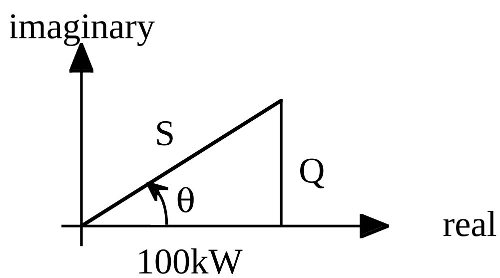
$$R_4=0.9\Omega$$

And finally $R_5=1000\Omega-\left(900\Omega+90\Omega+9\Omega+0.9\Omega\right)=0.1\Omega$

A load is connected to a voltage of 1320 volts at 60Hz. The load dissipates 100kW with a 0.867 lagging power factor. Specify the capacitance needed to correct the power factor to:
(a) 0.895 lagging
(b) 0.95 leading
(Give the voltage and volt-ampere-reactive ratings at 60Hz for the capacitors.)

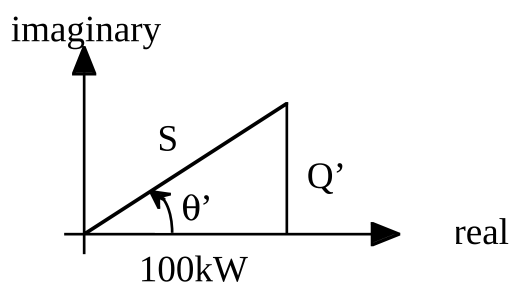


Determine the initial reactive power before correction.



$PF = 0.867 lagging = \cos\theta$ therefore $\theta = 29.89°$
From the power triangle
$Q = 100kW \tan\theta = 100\tan 29.89° = 100(0.5747) = +j57.47kVAR$

This reactive power must be corrected as per the problem specification. The desired power factor is $PF = 0.895 lagging = \cos\theta'$. Therefore, the new angle must be $\theta' = 26.49°$. The new reactive power for this angle comes from the new power triangle.



$Q' = 100kW \tan\theta' = 100\tan 26.49° = 100(0.4984) = +j49.84kVAR$
The difference in reactive powers must be supplied by the correction capacitor.

$Q + Q_{capacitor} = Q'$

$Q_{capacitor} = Q' - Q = +j49.84 - j57.47 = -j7.63kVAR$

$$P_{capacitor} = VI^*$$

$$-j7.63\times10^3 = (1320)\left(\frac{1320}{X_C}\right)^*$$

Solving for the capacitive reactance:

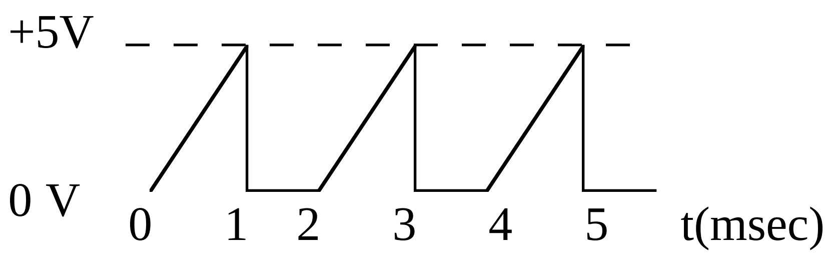
$$X_C^* = \frac{(1320)^2}{-j7.63\times10^3} = \frac{1}{-j2\pi(60)C}$$

Solving for the required capacitance

$$C = \frac{7630}{(1320)^2 2\pi(60)} = 1.16\times10^5 = 11.6\mu f$$

We already know the power rating to be 7.6kVAR and the voltage rating to be 1320 volts.

8. The signal shown below is measured with the following voltmeters:
(a) DC voltmeter,
(b) an RMS reading AC voltmeter using a d'Arsonval meter in a full-wave bridge in the feedback circuit of an opamp,
(c) an RMS reading AC voltmeter using a d'Arsonval meter in series with a diode in the feedback circuit of an opamp,
(d) a true RMS voltmeter such as an electrodynanometer,
(e) an RMS reading AC voltmeter using a peak detector, and
(f) an RMS reading AC voltmeter using a peak-to-peak detector.
Determine the reading on each meter.



The trick in this problem is to know that old style meters were always calibrated with sine waves. The meter always actually read the average value, but the scale was generated with the appropriate correction factor. Thus the procedure is calculate the scale for a sine wave as compared to the average value for a sine wave, and the calculate the average value of the waveform. The product of these two quantities will be the meter reading.

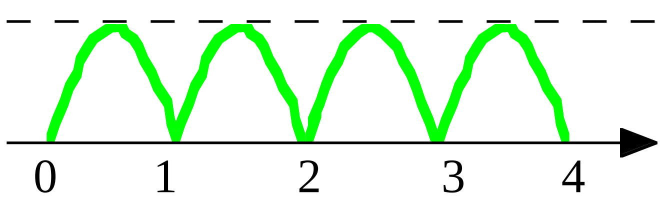
(a) a DC voltmeter reads the average value of a waveform

For this case the meter reading is

$$\frac{1}{T}\int_0^T f(t)dt = \frac{1}{2m\sec}\int_0^{1m\sec}\frac{5V}{1m\sec}tdt = \frac{1}{2}(5)\frac{t^2}{2}\bigg|_0^1 = \frac{5}{4}(1)^2 = 1.25volts$$

(b) a d'Arsonval meter reads the average value

The full wave bridge in the feedback loop of an opamp is a fancy way of telling you that it is a precision rectifier and you can neglect the voltage drop across the diode. The waveform that the meter will see for a sine wave is



The average value for this waveform is

$$V_{avg} = \frac{1}{2}\int_0^2 \left|\sin(\pi t)\right|dt = \frac{2}{2}\int_0^1 \sin(\pi t)dt = \frac{2}{2}\frac{-\cos(\pi t)}{\pi}\bigg|_0^1$$

$$V_{avg} = \frac{2}{2\pi}\left[-\cos\pi + \cos 0\right] = \frac{2}{2\pi}\left[-(-1)+1\right] = \frac{2}{\pi}$$

The rms value of this waveform (which is the units in which the meter is calibrated) is

$$V_{rms} = \sqrt{\frac{1}{2m\sec}\int_0^2 \sin^2(\pi t)dt} = \sqrt{\frac{2}{2}\int_0^1 \sin^2(\pi t)dt}$$

$$V_{rms} = \sqrt{\int_0^1 \frac{1}{2}(1-\cos(2\pi t))dt} = \sqrt{\frac{1}{2}\int_0^1 \frac{1}{2}dt - \int_0^1 \frac{\cos(2\pi t)}{2}dt} = \frac{1}{\sqrt{2}}$$

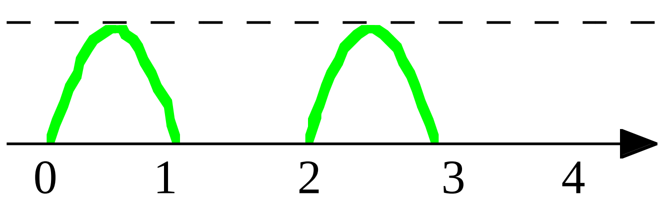
The calibration constant for the meter is then

$$\frac{V_{rms}}{V_{avg}} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\pi}} = \frac{\pi}{2\sqrt{2}}$$

The meter reading is given by multiplying the average value by the (sine wave) scaling factor:

$$\text{meter reading} = \frac{V_{rms}}{V_{avg}} V_{avg,waveform} = \frac{\pi}{2\sqrt{2}}(1.25) = 1.39Volts$$

(c) This is essentially the same as part (b) except that we have an ideal half wave rectifier and the sine wave calibration constant changes. Note that the ramp waveform is always positive and will give the same reading through either rectifier circuit.



The average value for this wave is half that of a full wave rectifier, so

$$V_{avg} = \frac{1}{2}V_{avg,\, full-wave} = \frac{1}{2}\left(\frac{2}{\pi}\right) = \frac{1}{\pi}$$

Computing the rms voltage for this waveform we get

$$V_{rms} = \sqrt{\frac{1}{2}\int_0^1 \sin^2(\pi t)dt} = \sqrt{\frac{1}{2}\int_0^1 \frac{1}{2}(1-\cos(2\pi t))dt} = \sqrt{\frac{1}{4}\int_0^1 dt - \frac{1}{4}\int_0^1 \cos(2\pi t)dt} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

The calibration constant for the meter is then

$$\frac{V_{rms}}{V_{avg}} = \frac{\frac{1}{2}}{\frac{1}{\pi}} = \frac{\pi}{2}$$

The meter reading is given by multiplying the average value by the (sine wave) scaling factor:

meter reading $= \frac{V_{rms}}{V_{avg}} V_{avg,waveform} = \frac{\pi}{2}(1.25) = 1.96 Volts$

(d) the true RMS is pretty easy. This is computed directly from the definition with no scaling factors.

$$V_{true-rms} = \sqrt{\frac{1}{2}\int_0^1 \left(\frac{5}{2}t\right)^2 dt} = \sqrt{\frac{1}{2}\int_0^1 25t^2 dt} = \sqrt{\frac{25}{2}\frac{t^3}{3}\bigg|_0^1} = \sqrt{\frac{25}{6}} = 2.04 Volts$$

(e) RMS using a peak detector is also pretty easy. This is just a different scaling factor based upon the peak value of the waveform.

The meter reads the peak value for the waveform of 5 Volts.

The calibration for sinusoids is $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$.

meter reading $= \frac{5}{\sqrt{2}} = 3.54 Volts$

(f) This is essentially the same as (e) except using peak-peak values.

The calibration for sinusoids is $V_{rms} = \frac{V_{peak-peak}/2}{\sqrt{2}} = \frac{V_{peak-peak}}{2\sqrt{2}}$.

Note that the meter still reads 5 Volts as the peak-peak value.

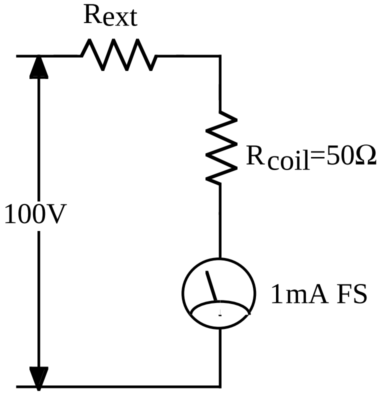
meter reading $= \frac{5}{2\sqrt{2}} = 1.77 Volts$

Concentrates

1. A voltmeter is being designed to measure voltages in the full-scale ranges of 3, 10, 30 and 100 volts DC. The meter movement to be used has an internal resistance of 50 ohms and a full-scale current of 1 mA. Using a four-pole, single-throw switch, design the voltmeter.

The meter circuit is easily designed using the equivalent circuit of the meter.

@100 volts



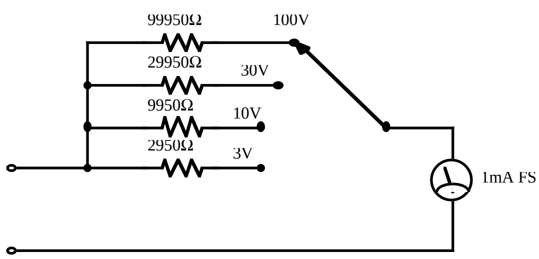
$$R = \frac{V}{I} = \frac{100volts}{1mA} = 10^5\Omega$$
$$R_{ext} = 10^5\Omega - 50 = 99950\Omega$$

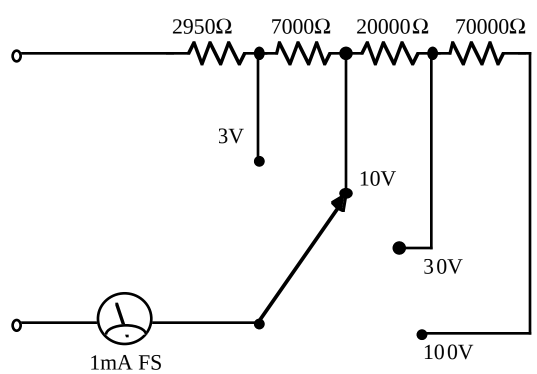
@30V $R = \frac{V}{I} = \frac{30volts}{1mA} = 30000\Omega$ and $R_{ext} = 30000 - 50 = 29950\Omega$

@10V $R = \frac{V}{I} = \frac{10volts}{1mA} = 10000\Omega$ and $R_{ext} = 10000 - 50 = 9950\Omega$

@3V $R = \frac{V}{I} = \frac{3volts}{1mA} = 3000\Omega$ and $R_{ext} = 3000 - 50 = 2950\Omega$

You can design several different types of meter circuits using this data.





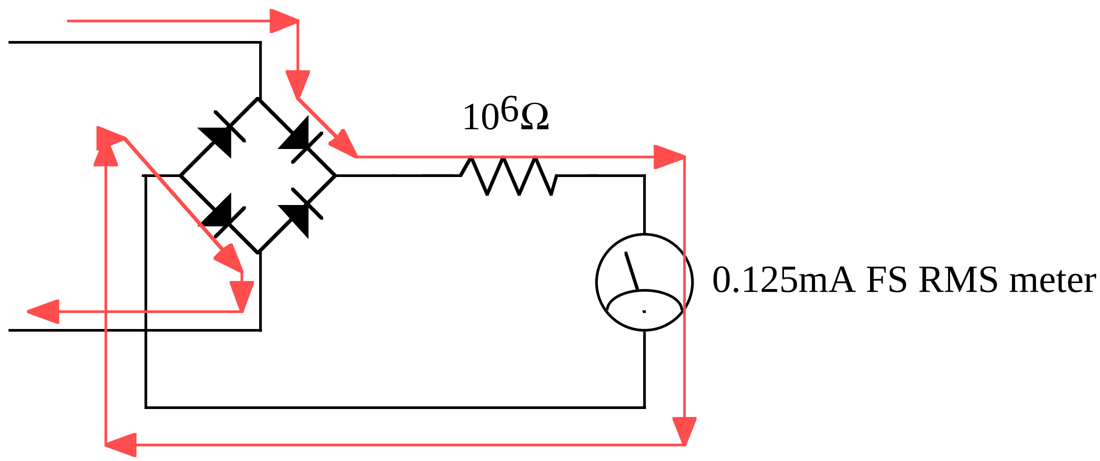
2. An AC voltmeter consists of a d'Arsonval meter with a full-scale current of 0.125mA and a series resistance of $10^6\Omega$. A full-wave bridge of silicon diodes (Vthr=0.6volts) is used to rectify the AC voltage. The full-scale needle deflection is 50 degrees. Give the scale increments in degrees from 0 to 100 volts in 10 volt increments for the RMS vaue of a pure sinusoid.

This is somewhat of a tedious problem. The crucial items to note are:

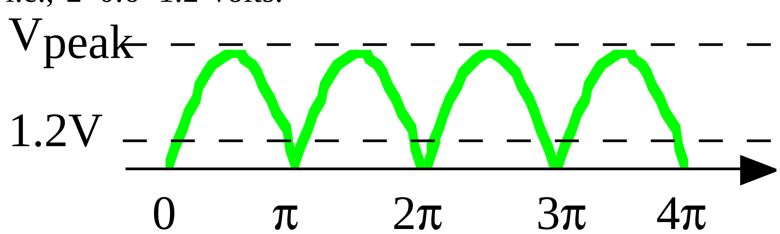
- full-wave bridge
- non-ideal diodes with a threshold
- RMS meter circuit

Otherwise the problem is fairly similar to the previous meter calibration problem.

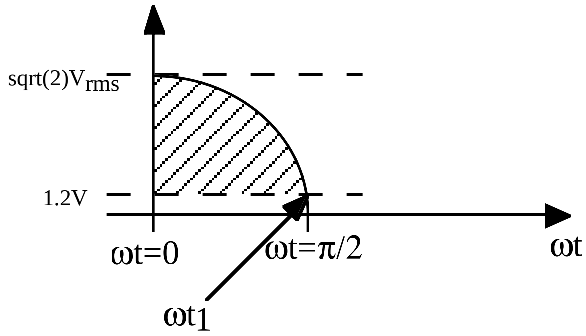
If you follow the current path through the meter circuit you see that the current flows through two diodes and we have two diode drops to include in our calculations.



Note that there can be no output when the AC input voltage is less than two diodes drops, i.e., 2*0.6=1.2 volts.



A d'Arsonval meter will read the average value of the voltage waveform shown below. Note that I want to write the peak value in terms of the RMS value for convenience.



This drawing shows what is happening. I have to subtract 1.2 volts from the sine wave. As a result I will only get a contribution to the average from the shaded region of the waveform. Only a quarter cycle is shown because of symmetry. The actual average will then be given by

$$V_{avg} = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{\pi/2}\int_0^{\omega t_1}\left[\sqrt{2}V_{rms}\cos\omega t - 1.2\right]d(\omega t)$$

ωt_1 is the point in time when the input voltage drops below the threshold of the diode bridge and there is no output. This can be solved for as

$$\sqrt{2}V_{rms}\cos\omega t_1 - 1.2 = 0$$

or

$$\omega t_1 = \cos^{-1}\left(\frac{1.2}{\sqrt{2}V_{rms}}\right)$$

Using this result the average voltage becomes

$$V_{avg} = \frac{2}{\pi}\left[\sqrt{2}V_{rms}\sin\omega t - 1.2\right]_{\omega t=0}^{\omega t=\omega t_1} = \frac{2}{\pi}\left[\sqrt{2}V_{rms}\sin\omega t_1 - 1.2\omega t_1\right]$$

The full scale deflection of the resistor-meter combination is

$$0.125\times10^{-3}\times10^{6} = 125 volts$$

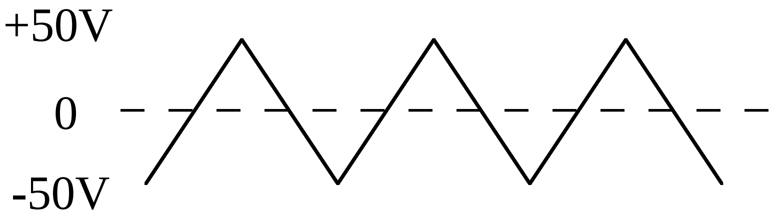
Assuming the meter deflection is linear we have an angular deflection sensitivity of $\frac{50°}{125 volts}$ and a meter deflection of $\theta = \frac{50°}{125 volts}V_{avg}$

This is not something I wanted to calculate by hand so I used a spreadsheet to compute it.

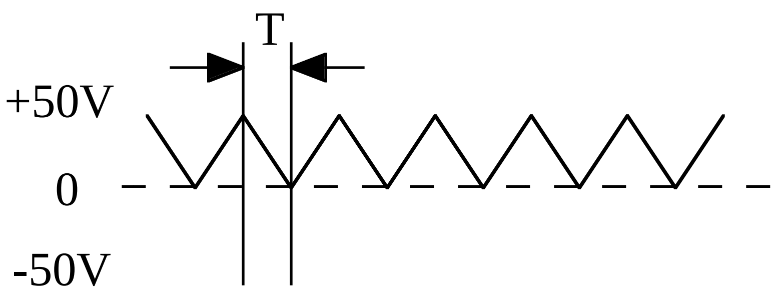
Vrms	ωt1	sqrt(2)*Vrms*sin(ωt1)	1.2*ωt1	difference	angular deflection (degrees)
10	1.4858	14.091	1.783	7.836	3.134
20	1.5284	28.259	1.834	16.823	6.729
30	1.5425	42.409	1.851	25.820	10.328
40	1.5496	56.556	1.859	34.821	13.928
50	1.5538	70.700	1.865	43.822	17.529
60	1.5567	84.844	1.868	52.824	21.130
70	1.5587	98.988	1.870	61.827	24.731
80	1.5602	113.131	1.872	70.829	28.332
90	1.5614	127.274	1.874	79.832	31.933
100	1.5623	141.416	1.875	88.835	35.534

4. A full wave rectifier type VTVM (vacuum tube voltmeter) is set to an RMS AC scale with a range of 50 volts. The meter is connected to a symmetrical (zero average) triangular waveform of 100 volts peak-to-peak. What does the meter read?

The input waveform is input to an AC VTVM with a full wave bridge We we assume an ideal rectifier circuit.



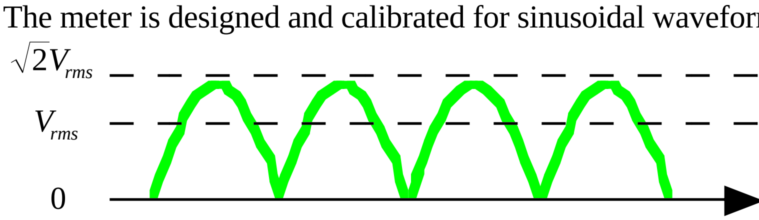
The output of the full wave rectifier will look superficially the same but with different voltage levels.



Compute the input to the meter.

$$V_{avg} = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{T}\frac{1}{2}(50)T = 25Volts$$

The meter is designed and calibrated for sinusoidal waveforms.



Computing the calibration relationship for the meter.

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} \sqrt{2}V_{rms}\sin t dt = \left.\frac{-\sqrt{2}V_{rms}\cos t}{\pi}\right|_0^{\pi} = \frac{2\sqrt{2}}{\pi}V_{rms}$$

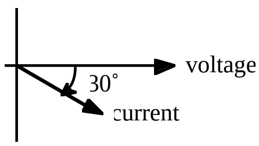
The meter will read an average (actual) of 25 volts and display it as the appropriate RMS value, i.e., $25V = \frac{2\sqrt{2}}{\pi}V_{rms}$ or

$$V_{rms} = \frac{25\pi}{2\sqrt{2}} = 27.77volts$$

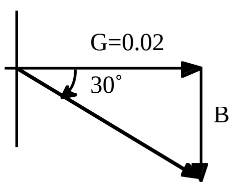
5. An impedance receives a line current which lags the voltage by 30˚. When the voltage across the impedance is 100 volts (RMS), the impedance dissipates 200 watts. Specify the reactance of a capacitance to be places in parallel with the impedance which would make the line current be in phase with the voltage.

The given circuit is

The current lags the voltage

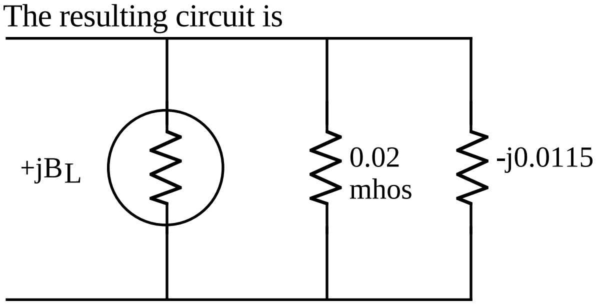


From the power specification $P = \frac{V^2}{R}$ or $R = \frac{V^2}{P} = \frac{(100)^2}{200} = 50$. This gives G=0.02mhos. At this point we have $I = YV = (G + jB)V = (0.02 + jB)V$. Since the phase angle is known to be 30˚ we can use the relationship between voltage and current to find B.



$\tan 30° = \frac{B}{G}$ or $B = G \tan 30° = 0.02 \tan 30° = 0.0115$. Note that B is actually negativ e.

The resulting circuit is

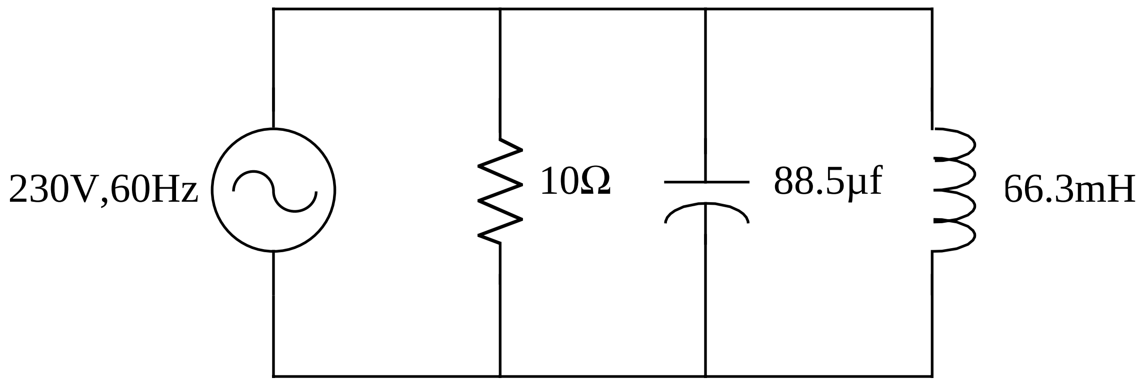


Having the line current being in phase means that the reactance is zero. This requires

$$jB_C + jB_L = 0$$
$$B_C = -B_L = -(-0.0115) = 0.0115$$
$X_C = -86.6\Omega$ for the desired power factor correction

6. A parallel combination of a resistance (10Ω), capacitance (88.5μf), and inductance (66.3 mH) has 60Hz, 230 volt (RMS) applied. Obtain the:
(a) reactances of C and L.
(b) admittance of each circuit element.
(c) phasor diagram for the currents, using the applied voltage as the reference.
(d) admittance diagram for the circuits, including the total admittance.
(e) input current as a sine function, taking the applied voltage as a reference. (Is the circuit inductive or capacitive?)
(f) power factor.
(g) power triangle.

The circuit is



The angular frequency is $\omega = 2\pi(60) = 376.99\,{}^{rad}\!/_{\sec}$

(a)

$$X_L = j\omega L = j(377)\left(66.3\times10^{-3}\right) = j25$$

$$X_C = \frac{1}{j\omega C} = -j\frac{1}{(377)\left(88.5\times10^{-6}\right)} = -j30$$

(b)

$$B_L = \frac{1}{j25} = -j0.04mhos$$

$$B_C = \frac{1}{-j30} = +j0.0334mhos$$

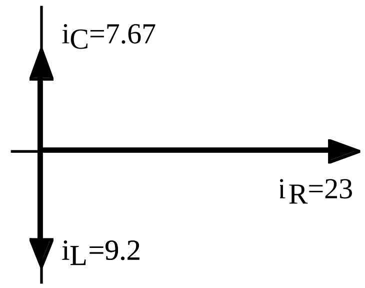
$$G = \frac{1}{R} = \frac{1}{10} = 0.1mhos$$

(c)

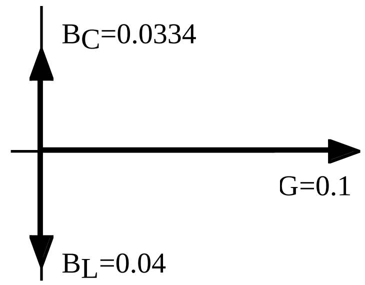
$$i_R = \frac{230\angle0^\circ}{10} = 23\angle0^\circ amps$$

$$i_C = \frac{230\angle0^\circ}{-j30} = 7.67\angle+90^\circ amps$$

$$i_L = \frac{230\angle0^\circ}{j25} = 9.2\angle-90^\circ amps$$



(d)

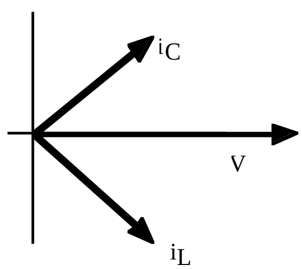


$Y_t = 0.1 + j0.0334 - j0.04 = 0.1 - j0.0066 = 0.1002\angle -3.776°$

(e)

$i = YV = (0.1002\angle -3.776°)(230\angle 0°) = 23.00 - j1.518 = 23.05\angle -3.776°$

The circuit is inductive since the angle is negative. Recall the phasor diagram



(f)

$PF = \cos(-3.776°) = 0.9978$

(g)

$p = vi^* = (230\angle 0°)(23.05\angle -3.776°)^* = 5301.5\angle +3.776° = 5289.99 + j349.14$

The power triangle will look like this