THREE PHASE POWER:
- 3 VOLTAGES, 120° APART.

WYE:

DELTA:

VOLTAGE SEQUENCES IN TEXT
LOOK AT PHASOR RELATIONSHIP:
HOW TO CONVERT LINE-LINE TO LINE-NEUTRAL

V_{an} IS REFERENCE (REF)

V_{ab} = \frac{1}{\sqrt{3}} V_{an} = \frac{1}{\sqrt{3}} V_{bn}

UNDER CONDITIONS OF BALANCE, (ASSUME BALANCE UNLESS
1) ALL 3 CURRENTS ARE EQUAL REASON TO RELIEVE OTHER
2) " " VOLTAGES " "
3) " " COMPLETE ANALYSIS CONSISTS OF
   LOOKING AT 1 LINE.

TYPES OF POWER:
APARENT (S) = GRAND TOTAL (VA)
REACTIVE (Q) - QUADRATURE PART OF S. (VA)
TRUE (P) = IN PHASE PART OF S. (W)

QUICK CHECK ➔ ALWAYS |S|^2 = P^2 + Q^2, SINCE

COS\theta = P.F.
**Quick Problem:**

3 φ, 11.5 kV generator drives 500 kW, 0.866 lagging.

**Find**

Line current: \[ \frac{11500V}{\sqrt{3}} = 6640V \text{ per phase} \]

\[ P = \frac{500kW}{3} = 167 \text{ kW/φ} \]

Now \[ S = \frac{P}{\text{P.F.}} = \frac{167 \text{ kW}}{0.866} = 192 \text{ kVA/φ} \]

\[ I = \frac{192 \text{ kVA/φ}}{6640V/\phi} = 29A \]

Generator rating: Just \[ 3\sqrt{3}/5 = 576 \text{ kVA} \]

**Compensator:**

\[ I_C = 14.5A / 90° \]

\[ V_{in} = 6640V \]

\[ \cos^{-1}(0.866) = 30° \]

\[ V_{in} = 6640V \]

\[ I = 29A \]

It is a capacitor, \[ \frac{1}{j} C = X_C \cdot \frac{2\pi}{f} \]

\[ C = 458 \cdot 2 \cdot \pi \cdot 60 \]

\[ C = 5.8 \text{ MF/line} \]

**Power Transformers:**

**Equivalent Ckt:**

\[ L_1, R_1, L_2, R_2 \text{ are connected as in the diagram.} \]
TRANSFORMER TESTS: (LOW VOLT SIDE - SAFER)

OPEN Ckt: \( V_{oc} = G_c - jB_c \), SERIES ELEMENTS USUALLY NEGLIGIBLE.

(ASSUME RATED VOLTAGE)

EX: TRANSFORMER RATED SOMUA 115kV: 13.8kV

\[ P_{oc} = 250kW \quad I_{oc} = 350A \]

Tests on Secondary (MAKE SURE YOU USE RIGHT VOLTAGE!) PREV:

\[ G_c = \frac{P_{oc}}{V_{oc}^2} = \frac{250kW}{(115kV)^2} = 18.9\text{mS} \]

\[ B_c = \frac{Q_{oc}}{V_{oc}^2} = \frac{(13.8kV)^2 - P_{oc}^2}{V_{oc}^2} = 0.36\text{mS} \]

SHORT Ckt: (ASSUME RATED CURRENT)

SHORTED WINDING SHORTS OUT CORE ELEMENTS.

FROM PRIMARY: \( E_{sc} = R_1 + jX_1 + a_{sp}^2 (R_2 + jX_2) \)

SECONDARY: \( E_{sc} = R_2 + jX_2 + a_{sp}^2 (R_4 + jX_4) \)

ASSUME \( R_1 = a_{sp}^2 R_2 \), \( X_1 = a_{sp}^2 X_2 \), AND TEST @ RATED CURRENT.

EX: SAME TRANSFORMER.

\[ P_{sc} = 300kW \quad V_{sc} = 1.5kV \]

\[ a_{ps} = 8.33 \]

\[ (R_1 + a_{ps}^2 R_2) = \frac{P_{sc}}{(I_{sc})^2} = 1.585 \Omega \]

REAL PUR VIEWER \( I_1 = a_{ps}^2 I_2 \)

\[ R_4 = 0.793\Omega \]

\[ R_2 = 11.4\text{m}\Omega \]

Q POWER YIELDS X:

\[ |S_{sc}| = I_{sc} \cdot [a_{ps} V_{sc}] = 5.44\text{MVA} \]

\[ X_{1,2} = \frac{Q}{I_{1,2}^2} = \frac{\sqrt{1s^2 - p^2}}{I_{1,2}^2} = 28.7\Omega \]

IF X_1 = aX_2,

\[ X_1 = 143\Omega \]

\[ X_2 = .207\Omega \]
ABCD PARAMETERS: TRANSFORMER AS Z-PORT NETWORK

ABCD BECOME LIKE Z-PORT PARAMETERS,

MATRICES MULTIPLY FOR SERIES CONNECTIONS

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\rightarrow
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\]

EX: SEE CLASS HANDOUT FOR OUR TRANSFORMER WORKED OUT IN ABCD PARAMS.

NOTICE UNITS!

\( B \) IS RESISTANCE
\( C \) IS CONDUCTANCE
\( A, D \) ARE DIMENSIONLESS CONSTANTS OF PROPORTIONALITY.
PER UNIT SYSTEM.
SIMPLY PERCENT (DECIMAL) OF BASE RATINGS.

EX: 100 KVA TRANS-LINE OPERATING AT 60 KVA CARRIES
POWER .6 P.U.

BE CAREFUL NOT TO MIX BASE RATINGS WITH \( Z \).

\[
\text{BASE } Z = \frac{\text{BASE } V_A}{\text{BASE } I} = \frac{\text{BASE } (VA)}{\text{BASE } V_A}
\]

SO \( Z \text{ (P.U.)} = \frac{\text{ACTUAL } Z}{\text{BASE } Z} \).

TO CONVERT BASES FOR \( Z \):

\[
Z_{\text{ACTUAL}} = Z_{\text{(P.U., OLD)}} \times \frac{(\text{BASE } V_A)^2}{\text{BASE } V_A}
\]

\[
Z_{\text{(P.U., NEW)}} = \frac{Z_{\text{ACTUAL}}}{Z_{\text{NEW BASE}}} = Z_{\text{(P.U., OLD)}} \times \left( \frac{\text{OLD BASE } V_A}{\text{NEW BASE } V_A} \right)^2 \times \frac{\text{NEW BASE } V_A}{\text{NEW BASE } V_A}
\]

IN BOOK, BUT CONFUSING.

EX: 3 \( \Phi \) SYNCHRONOUS MOTOR 10 KVA, 440V, \( X = .8 \text{ (P.U.)} \)

IN SYSTEM 100 KVA SYSTEM.

\[
\text{ACTUAL REACTANCE: FIND } Z \text{ BASE FIRST } \frac{\text{BASE } (VA^2)}{\text{BASE } V_A} = \frac{440^2}{10000} = \frac{19360}{3} \]

\[
= 19.36 \Omega
\]

\[
\text{ACTUAL } X = X \text{ (P.U.)} \times X \text{ (BASE)}
\]

\[
= 15.49 \Omega
\]

NEW SYSTEM:

\[
x_{\text{(P.U., NEW)}} = \frac{\text{ACTUAL } X}{\text{(440/3)}^2} = \frac{8 \text{ (P.U.)}}{\frac{4000000}{3}}
\]
ONE LINE DIAGRAMS

- Symbols in the book - should know them. (P 5-11)
- Just like schematics (Ex. P5-12)

DISTRIBUTION TRANSFORMERS

- Assume infinite bus (5.92, 5.95; P 5-11)
- Use formulas in text to set 25% P.U. tap impedance

VOLTAGE REGULATION

- Refer input V to secondary, call it constant (infinite bus)
- Difference in P.U. of \( V_{in} \) and \( V_{out} \) is

VOLTAGE REGULATION, when \( V_{in} = 1 \) P.U.

(Example to follow)

P.F. CORRECTION. (Touched on before)

- Use vector (phasor diagrams to help)
- Remember: current magnitude (not phase) determines capacity of machines.

EX: 3Φ, 100 kVA TRANSFORMER, \( Z_0 = 0.03 + j0.04 \) P.U. \( = 0.05 \angle 55.1^\circ \) P.U.

- Full load at P.F. = 0.8 lagging.
- Assume rated current.
- What is change in regulation if P.F. is corrected to unity?

\[
|I| = 1 \text{ P.U.}, \quad \angle I = -\cos^{-1}0.8 = -39.6^\circ \\
I = 1 \angle -36.9 \text{ P.U.} \\
V_2 = 1 \text{ P.U.} \\
V_1 = V_2 + jZ_0 \\
V_1 = 1 + (0.048 + j0.014) \\
V_1 = 1.048 \angle 7.7^\circ \\
V_{28} = V_1 - V_2 = 1.048 - 1 = 0.048 \\
\Delta \text{regulation} = 4.8 - 3.1 = 1.77\% \text{ improved}.
\]
Transformer Connections

Already know Y & Δ.

-Look at open delta

Ex: Open Delta, 3Φ 440V consists of 2 1Φ machines,

1 leg missing.

Serve 50 kVA bal, 3Φ load, .8 lagging p.f.

Determine required ratings if

A 10kW resistive load added to a/d leg.

\[ V_{ab} = 440 \angle 30 \text{ V.} \]

\[ V_{bc} \]

\[ V_{ac} \]

\[ o \quad a \quad V_{an} \quad \text{is phase ref.} \]

\[ b \]

Do balanced load first:

\[ I_a = \frac{50 \text{kVA} \angle \cos 9.8^\circ}{440V} = 65.6 \angle -36.8^\circ \text{ A.} \]

Load is balanced, so \[ I_a = I_b = I_c \]

\[ I_b = 65.6 \angle -36.8^\circ = 65.6 \angle -156.8^\circ \text{ A.} \]

\[ I_c = 65.6 \angle 56.8^\circ = 65.6 \angle 83.2^\circ \text{ A.} \]

Add 10kW res to ab leg:

\[ I_a' = \frac{10 \text{kW}}{440V} = 22.7 \text{ A} \]

\[ \angle I_a' = \angle V_{ab} = 30^\circ, \text{ since in phase} \]

\[ S_{ab} = (I_a + I_a')V_{ab} \]

\[ = 34.1 \angle 51^\circ \text{ kVA} \]

\[ S_{bc} = I_b' V_{bc} = 28.9 \angle 7^\circ \text{ kVA} \]
FAULT CALCULATIONS

- 3 VOLTAGES PRESENT ON MACHINES DUE TO EFFECTS OF MECHANICAL INERTIA AND MAGNETIC SATURATION:

\[ E - \text{STANDARD OPERATING V (PRE FAULT)} \]
\[ E'' - \text{SUBTRANSIENT (FAULT HAS OCCURRED, BUT STS REACTING)} \]
\[ E' - \text{TRANSIENT (FULL BLOWN FAULT CONVX).} \]

EX: SYNCH GEN & SYNCH MOTOR, BOTH 15 MVA, 13.9 KV, 25% SUBTRANSIENT LEAKAGE, CONNECTED VIA .3+j.4 P.U. LINE, 15 MVA, 13.9 KV BASE MOTOR RUNS AT 12 MVA 13 KV 8 LAG P.F.

EQUIV. GENERATED VOLTAGES: (GO TO P.U. QUICKLY)

\[ E_m = \frac{13}{15} = 0.935 \text{ P.U.} \]
\[ I_L = \frac{12 \text{ MVA} \times \cos^2.8}{15 \text{ KV}} = 0.855 \angle -36.8^\circ \text{ P.U.} \]

\[ E_m' = E_m - I_L (jX_d) = 0.935 - (0.855 \angle -36.8^\circ)(0.25 \angle 90^\circ) \]
\[ = 0.807 - j1.171 \text{ P.U.} \]
\[ Z = Z_{line} + jX_d = 0.3 + j0.65 \text{ P.U.} \]
\[ \text{so} \ E_g' = E_m' + I_L Z = 1.47 + j2.9 \text{ P.U.} \]

SO MOTOR CURRENT WILL BE \[ \frac{E_g'}{Z} = \frac{0.807 - j1.171}{0.3 + j0.65} = 3.79 \angle -102^\circ \text{ P.U.} \]

SIMILARLY, GENERATOR CURRENT \[ \frac{E_g}{Z} = \frac{1.47 + j2.9}{0.3 + j0.65} = 5.48 \angle -54^\circ \text{ P.U.} \]

TOTAL FAULT CURRENT \[ I_g + I_m = 0.589 - j4.9 = 4.94 \angle -83^\circ \text{ P.U.} \]

THUS COMPONENTS MUST BE ABLE TO TAKE 5X RATED I UNDER FAULT.
ASYMMETRICAL FAULTS

- Can be done by pile-driver method, but very hard.
- Consider fault current as the sum of

SYMMETRICAL COMPONENTS

UNIT PHASORS

\[ a = 1 \angle 120^\circ \]
\[ a^2 = 1 \angle 240^\circ \]
\[ a^3 = 1 \]

\[ 1, a, a^2 \]

LET POSITIVE SEQUENCE

\[ I_{b1} = a^2 I_a \]
\[ I_{c1} = a I_a \]

\[ I_{a1} \]

\[ I_{b1} \]

\[ I_{c1} \]

AND NEGATIVE SEQUENCE

\[ I_{b2} = a I_a \]
\[ I_{c2} = a^2 I_a \]

\[ I_{a2} \]

\[ I_{b2} \]

\[ I_{c2} \]

ZERO SEQUENCE: COMMON MODE CURRENT \( I_{x0} \)

SO, NOW, BY SUPERPOSITION

\[ I_a = I_{a1} + I_{a2} + I_{x0} \]
\[ I_b = I_{b1} + I_{b2} + I_{x0} \]
\[ I_c = I_{c1} + I_{c2} + I_{x0} \]
\[ I_{x0} = \frac{1}{3}(I_a + I_b + I_c), \text{ since } I_{x0} \text{ is common.} \]
Thus

\[ I_{ao} = \frac{1}{3} (I_a + I_b + I_c) \]
\[ I_{a1} = \frac{1}{3} (I_a + \alpha I_b + \alpha^2 I_c) \]
\[ I_{a2} = \frac{1}{3} (I_a + \alpha^2 I_b + \alpha I_c) \]

AND

\[ V_{ao} = \frac{1}{3} (V_{an} + V_{bn} + V_{cn}) \]
\[ V_{a1} = \frac{1}{3} (V_{an} + \alpha V_{bn} + \alpha^2 V_{cn}) \]
\[ V_{a2} = \frac{1}{3} (V_{an} + \alpha^2 V_{bn} + \alpha V_{cn}) \]

Procedure for solving problems.

1) Follow rules in text for drawing
   Positive S.M. (Normal)
   Negative S.M. (No Generators, \( X_m = \frac{1}{3} (X_a + X_b + X_c) \))
   Zero S.M. (Use Fig 5.15, P 3-23)

2) Locate fault on models. Determine the Thevenin equivalent circuit of system at fault location

3) Determine how they combine from type of fault.

Example

1) Line to Gnd
   I_b = I_c = 0, V_a = 0
   \[ I_{ao} = I_{a1} = I_{a3} = I_{a5} \]

2) Line-Line
   \( I_a = 0 \), \( I_b = I_c \)
   \[ V_{a1} = V_{a2} = V_{a4} \]

3) Line-Line-Gnd
   \( I_a = 0 \), \( V_{a1} = V_{a2} = V_{a4} = 0 \)
EXAMPLES:

1) LINE - GND

\[ I_b = I_c = 0 \]

SO \[ I_{ao} = I_{a1} + I_{a2} = \frac{I_a}{3} \]

CONNECT:

![Diagram of LINE - GND configuration]

 KNOW \[ I_{a1}, I_{a2}, I_{ao} \], so

\[ I_a = I_{a1} + I_{a2} + I_{ao} \]

2) LINE - LINE

\[ I_a = 0, I_b = -I_c \]

SO \[ I_{a1} = -I_{a2} \]

WE KNOW \[ I_{a1}, I_{a2}, I_{ao} \], so

\[ I_b = I_{bo} + I_{b1} + I_{b2} \]

\[ I_b = I_{ao} + a I_{a1} + a I_{a2} \]

\[ I_c = -I_b \]

3) LINE - LINE - GND

\[ I_a = 0, V_{bn} = V_{cn} = 0 \]

SO \[ V_{ao} = V_{bo} = V_{co} = \frac{V_{an}}{3} \]

SOLVE TO KNOW \[ I_{a1}, I_{a2}, I_{ao} \]

\[ I_b = I_{ao} + a^2 I_{a1} + a I_{a2} \] (FROM ABOVE)

\[ I_c = I_{co} + I_{ci} + I_{ci} \]

\[ I_c = I_{ao} + a I_{a1} + a^2 I_{a2} \]
CONCLUSIONS

1) IF FAULT IS SYMMETRICAL, ONLY P.S.M. IS NEEDED.

2) IF NO FAULT IS GOING TO occur
   DO NOT NEED C.S.M.
SUPPLEMENTARY NOTES: DERIVATION OF PHASE-SEQUENCE CONVERSIONS

**GIVEN**
\[ I_a = I_{a0} + I_{a1} + I_{a2}, \]
\[ I_b = I_{b0} + I_{b1} + I_{b2}, \]
\[ I_c = I_{c0} + I_{c1} + I_{c2}, \] are the phase currents,

**AND**
\[ I_{b1} = a^2 I_{a1}, \]
\[ I_{b2} = a I_{a2}, \]
\[ I_{c1} = a I_{a1}, \]
\[ I_{c2} = a^2 I_{a2}, \] are the sequence phase relationships, derive the sequence currents in terms of the phase currents.

**ZERO SEQUENCE CURRENT**

Note that \[ a + a^2 + 1 = 0. \] By definition, \[ I_{a0} = I_{b0} = I_{c0}. \]

So
\[ 3I_{a0} = I_a + I_b + I_c - (I_{a1} + I_{a2} + I_{b1} + I_{b2} + I_{c1} + I_{c2}), \]
\[ 3I_{a0} = I_a + I_b + I_c - (I_{a1} + a^2 I_{a1} + a I_{a2} + I_{a1} + a I_{a1} + a^2 I_{a2}), \]
\[ 3I_{a0} = I_a + I_b + I_c - (I_{a1} (1 + a + a^2) + I_{a2} (1 + a + a^2)), \]
\[ 3I_{a0} = I_a + I_b + I_c - (I_{a1} (0) + I_{a2} (0)), \]
\[ I_{a0} = \frac{1}{3} (I_a + I_b + I_c). \]

**POSITIVE SEQUENCE CURRENT**

\[ O = (1 + a + a^2) + (1 + a + a^2), \]
\[ O = I_{a2} (1 + a + a^2) + I_{a0} (1 + a + a^2), \]
\[ O = I_{a2} + a I_{a1} + a^2 I_{a2} + I_{a0} + a I_{a0} + a^2 I_{a0}, \]
\[ \text{SINCE } I_{a0} = I_{b0} = I_{c0}, \]
\[ O = I_{a2} + a I_{a1} + a^2 I_{a2} + I_{a0} + a I_{b0} + a^2 I_{c0}, \]
\[ \text{USING SEQUENCE PHASE RELATIONSHIPS,} \]
\[ O = I_{a2} + a^2 I_{c2} + a I_{b2} + I_{a0} + a I_{b0} + a^2 I_{c0}, \]
\[ O = (I_{a0} + I_{a2}) + a (I_{b0} + I_{b2}) + a^2 (I_{c0} + I_{c2}), \]

(continued)
\[ I_{a1} = a I_{b1} = a' I_{c1}, \]

\[ 3 I_{a1} = (I_{a0} + I_{a1} + I_{a2}) + a (I_{b0} + I_{b1} + I_{b2}) + a^2 (I_{c0} + I_{c1} + I_{c2}), \]

\[ 3 I_{a1} = (I_{a}) + a (I_{b}) + a^2 (I_{c}) \]

\[ I_{a1} = \frac{1}{3} (I_{a} + a I_{b} + a^2 I_{c}). \]

**NEGATIVE SEQUENCE CURRENT**

\[ O = (1 + a + a^2) + (1 + a + a^2), \]

\[ O = I_{a1} (1 + a + a^2) + I_{a0} (1 + a + a^2), \]

\[ O = I_{a1} + a I_{a1} + a^2 I_{a1} + I_{a0} + a I_{a0} + a^2 I_{a0}, \]

**SINCE**  

\[ I_{a0} = I_{b0} = I_{c0}, \]

\[ O = I_{a1} + a I_{a1} + a^2 I_{a1} + I_{a0} + a I_{c0} + a^2 I_{b0}, \]

**USING SEQUENCE PHASE RELATIONSHIPS,**

\[ O = (I_{a0} + I_{a1}) + a (I_{c0} + I_{c1}) + a^2 (I_{b0} + I_{b1}), \]

**SINCE**  

\[ I_{a2} = a^2 I_{b2} = a I_{c2}, \]

\[ 3 I_{a2} = (I_{a0} + I_{a1} + I_{a2}) + a (I_{c0} + I_{c1} + I_{c2}) + a^2 (I_{b0} + I_{b1} + I_{b2}), \]

\[ 3 I_{a2} = (I_{a}) + a (I_{c}) + a^2 (I_{b}), \]

\[ I_{a2} = \frac{1}{3} (I_{a} + a^2 I_{b} + a I_{c}). \]
SUPPLEMENTARY NOTES ON ABCD PARAMETERS

LIKE ALL Z-PORT NETWORK PARAMETERS, ABCD PARAMETERS ARE BASED ON THE LINEAR SEPARATION OF INDEPENDENT VARIABLES AND "SUMMING" THEM TOGETHER AGAIN (SUPERPOSITION IN ACTION).

FROM THE DEFINITION OF THE ABCD MATRIX,

\[
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}, \quad \text{OR} \quad V_1 = AV_2 + BI_2, \quad I_1 = CV_2 + DI_2,
\]

WHERE

WE MAY WRITE THE FORMULAS FOR THE ABCD PARAMETERS

\[
A = \left. \frac{V_1}{V_2} \right|_{I_2=0}, \quad B = -\left. \frac{V_1}{I_2} \right|_{V_2=0}, \\
C = \left. \frac{I_1}{V_2} \right|_{I_2=0}, \quad D = -\left. \frac{I_1}{I_2} \right|_{V_2=0}.
\]

GIVEN ANY Z-PORT NETWORK, WE MAY DERIVE THE ABCD PARAMETERS BY SIMPLY SOLVING THE ABOVE FOUR EQUATIONS.

EXAMPLE #1 - SERIES RESISTOR

\[
V_1 = V_2 - RI_2, \quad I_1 = -I_2
\]

\[
A = \left. \frac{V_1}{V_2} \right|_{I_2=0} = 1, \quad B = -\left. \frac{V_1}{I_2} \right|_{V_2=0} = R, \\
C = \left. \frac{I_1}{V_2} \right|_{I_2=0} = 0, \quad D = -\left. \frac{I_1}{I_2} \right|_{V_2=0} = 1.
\]
EXAMPLE #2 - T NETWORK

\[
V_2 \bigg|_{I_2 = 0} = V_1 \frac{1}{1 + Y_1 Z_1} \bigg|_{I_2 = 0},
\]

\[
V_2 \bigg|_{I_1 = 0} = V_1 \frac{1}{1 + Y Z_2} \bigg|_{I_2 = 0},
\]

\[
A = \frac{V_1}{V_2} \bigg|_{I_2 = 0} = 1 + Y Z_1.
\]

\[
B = -\frac{V_1}{I_2} \bigg|_{V_2 = 0} = Z_1 + Z_2 + Y Z_1 Z_2.
\]

\[
C = \frac{I_1}{V_2} \bigg|_{I_2 = 0} = Y.
\]

\[
D = -\frac{I_1}{I_2} \bigg|_{V_1 = 0} = 1 + Y Z_2.
\]

EXAMPLE #3 - TRANSFORMER

LET \[ Z_1 = R_1 + jX_1, \]
\[ Z_2 = R_2 + jX_2, \]
\[ Y = G_c - jB_c. \]

NOTE THAT THE TRANSFORMER IS SIMPLY A T-NETWORK, WITH

\[ V_2 \rightarrow a_{ps} V_2, \]
\[ I_2 \rightarrow I_2 / a_{ps}, \]
\[ Z_2 \rightarrow a_{ps} Z_2. \]
SO NOW
\[
\frac{V_1}{I_{x=0} \cdot a_{ps} V_2} = 1 + Y Z_1,
\]

\[
A = \frac{V_1}{V_2} \bigg|_{I_{x=0}} = a_{ps} (1 + Y Z_1),
\]

\[
B = -\frac{V_1}{I_{x=0} \cdot a_{ps} V_2} = \frac{1}{a_{ps}} \left( Z_1 + a_{ps}^2 Z_2 (1 + Z_1 Y) \right),
\]

\[
C = \frac{I_1}{I_{x=0} \cdot a_{ps} V_2} = Y,
\]

\[
D = -\frac{I_1}{I_{x=0} \cdot a_{ps} V_2} = 1 + a_{ps}^2 Z_2 Y,
\]

IN A PROPERLY DESIGNED TRANSFORMER, WE MAY ASSUME THAT \( Z_1 = a_{ps}^2 Z_2 \), SO THAT

\[
A = a_{ps} (1 + Y Z_1), \quad B = \frac{1}{a_{ps}} Z_1 (Z + Z_1 Y),
\]

\[
C = a_{ps} Y, \quad D = \frac{1}{a_{ps}} (1 + Z_1 Y).
\]

THE TRANSFORMER TESTED IN CLASS PROVIDED THE PARAMETERS:

\[a_{ps} = 8.33, \quad Z_1 = 0.793 + j 143 \, \Omega, \quad Y = 0.0189 - j 0.36 \, mS, \quad Y Z_1 = (5.16 - j 0.015) \times 10^{-3}\]

SO

\[
A = 8.33 (1 + Y Z_1)
\]

\[
A = 8.37 + j 0.
\]

\[
C = 8.33 Y
\]

\[
C = 0.157 - j 3.00 \, mS
\]

\[
B = \frac{1}{8.33} Z_1 (Z + Z_1 Y)
\]

\[
B = 0.191 + j 3.45 \, \Omega.
\]

\[
D = \frac{A}{a_{ps}^2} = \frac{8.37}{8.33^2} = 0.121 + j 0.
\]
**Warmup 1.**

\[ I_{\text{phase}} = \frac{151/3}{V} = \frac{2 \times 10^8}{3 \times 230 \times 10^3} = 289.9 \text{ A} \]

The line current is \( \sqrt{3} \times \text{phase current} \)

\[ |I| = \sqrt{3} \times 289.9 = 502 \text{ A} \]

The power factor correction is on a phase basis. For unity power factor the lagging part of the current is cancelled.

\[ I = I_p + jI_q \]

\[ I_q = |I| \text{phase sin } \theta \]

\[ I_q = 289.9 \sin (-25.8^\circ) = -126.4 \]

\[ I_c = 126.4 = \frac{230 \times 10^3}{-jX_c} \]

\[ X_c = \frac{230 \times 10^3}{126.4} = 1820 \Omega \]

**Warmup 2.**

The open circuit test obtains the core parameters \( G \) and \( B \):

\[ G = \frac{P_{\text{o.c.}}}{V_{\text{o.c.}}} = \frac{900}{13800^2} = 4.73 \times 10^{-4} \text{ mhos} \]

\[ S = P + jQ : Q = S^2 - P^2 \]

\[ |Q| = I_1^2 (X_p + a^2 X_s) \]

\[ Q^2 = S^2 - P^2 = (13800 \times 2)^2 - 900^2 \]

\[ Q = 260.9 \]

\[ R = \frac{Q}{V^2} = \frac{260.9 \times 13.7 \times 10^{-6}}{13800^2} \text{ mhos} \]

The turns ratio:

\[ \frac{V_p}{N_p} = \frac{V_s}{N_s} \]

\[ \frac{N_p}{N_s} = \frac{13800}{460} = 30 \]

**Warmup 3.**

\[ N_p I_p = N_s I_s \]

\[ a = \frac{N_p}{N_s} = \frac{I_s}{I_p} \]

\[ a = \frac{100}{20} = 5 \]

\[ P_{\text{sc}} = I_p^2 R_{wp} + I_s^2 R_{ws} = 1000 \text{ W} \]

\[ I_p^2 (R_{wp} + a^2 R_{ws}) = 1000 \]

\[ R_{wp} + a^2 R_{ws} = \frac{1000}{20^2} = 2.5 \Omega \]

Assume \( R_{wp} = a^2 R_{ws} = 1.25 \Omega \)

\[ R_{ws} = \frac{1.25}{5} = 0.25 \Omega \]

\[ |S| = V_p I_p = 80 \times 20 = 1600 \text{ W} \]

\[ S = P + jQ \]

\[ Q = S^2 - P^2 \]

\[ |Q| = 1249 \text{ VAR} \]
Warmup 3 (continued)

\[ X_p + \alpha^2 X_s = \frac{124.9}{20^2} = 3.12 \Omega \]

\[ X_p = \frac{3.12}{2} = 1.56 \Omega \]

\[ X_s = \frac{1.56}{\alpha^2} = 0.0624 \Omega \]

\[ P = I^2 R \quad R = \frac{435}{(4.35)^2} = 23 \Omega \]

\[ |Q| = I^2 X \quad X = \frac{10.870}{(4.35)^2} = 574 \Omega \]

\[ Z_{base} = \frac{115 \text{ KV}}{4.35 \text{ A}} = 26,440 \]

\[ Z_p = \frac{23 + j574}{2 \times 440} = 0.0007 + j0.022 \]

\[ Z_p \approx 0.001 + j0.022 \]

Warmup 4.

\[ Z_{base} = \frac{V_{base}}{I_{base}} = \frac{440^2}{VA} = 10,000 \]

\[ Z_{base} = 19.36 \Omega \]

\[ Z = Z_p \times Z_{base} = j \times 19.36 \]

\[ Z = 15.49 \Omega \]

**SYSTEM BASE:**

\[ Z_{base-2} = \frac{440^2}{100 \text{ K}} = 1.936 \Omega \]

\[ Z_{pu-2} = \frac{Z}{Z_{base-2}} = 8 \text{ p.u.} \]

Warmup 5.

The short circuit test is done at rated current:

\[ I_{rated} = \frac{500 \text{ KVA}}{115 \text{ KV}} = 4.35 \text{ A} \]

\[ |S_{sc}| = 4.35 \text{ A} \times 2.5 \text{ KV} = 10.88 \text{ KVA} \]

\[ = P + jQ = 435 + jQ \]

\[ |Q| = \sqrt{10.88^2 - 435^2} = 10.870 \]

\[ P = I^2 R \quad R = \frac{435}{(4.35)^2} = 23 \Omega \]

\[ |Q| = I^2 X \quad X = \frac{10.870}{(4.35)^2} = 574 \Omega \]

\[ Z_{base} = \frac{115 \text{ KV}}{4.35 \text{ A}} = 26,440 \]

\[ Z_p = \frac{23 + j574}{2 \times 440} = 0.0007 + j0.022 \]

\[ Z_p \approx 0.001 + j0.022 \]

**System BASE:**

\[ Z_{base-2} = \frac{440^2}{100 \text{ K}} = 1.936 \Omega \]

\[ Z_{pu-2} = \frac{Z}{Z_{base-2}} = 8 \text{ p.u.} \]

\[ V_s = 1 + i \angle 0 = 1 + 0.014 + j0.024 = 1.014 + j0.024 \]

\[ |V_s| = \sqrt{1.014^2 + 0.024^2} = 1.014 \quad \text{so} \quad |V_s|^2 = 1.014^2 \]

\[ V_s = V_L + 0.5 \left[ 0.014 + j0.024 \right] = V_L + 0.007 + j0.012 \]

\[ V_s = V_L + 0.007 + j0.012 \]

\[ 1.014^2 = (V_L + 0.007)^2 + 0.012^2 \]

Solving for \( V_L \):
Warmup 6. Continued:
\[ V_{\text{half-load}} = 1.007 \]
\[ V_{\text{no-load}} = 1.014 \]
\[ V_{\text{full-load}} = 1.000 \]
Regulation:
\[ \text{half to full load: } 1.007 - 1.001 = 0.006 \quad \text{or } 0.7\% \]
\[ \text{no load to full load: } 1.014 - 1.001 = 0.013 \quad \text{or } 1.4\% \]

Warmup 8.
\[ V_L = 1 \text{ p.u.} \]
\[ I_L = 1 \angle 36.9^\circ \]
\[ Z_{\text{lin}} = 0.03 + j0.04 = 0.05 \angle 53.1^\circ \]
\[ V_s = 1 + 0.03 \angle 26.9^\circ \]
\[ = 1.048 + j0.014 \]
\[ |V_s| = 1.048 \quad \text{so the regulation is } 0.048 \text{ or } 4.8\% \]
For unity PF \[ I_L = 1 \text{ p.u.} \]
\[ V_s = 1 + 0.03 + j0.04 = 1.03 + j0.04 \]
\[ |V_s| = 1.03 \quad \text{and the regulation is } 0.03 \text{ or } 3\% \]
The regulation is improved from 4.8\% to 3\%, a percent of a percent.
\[ \text{Class of } 4.8 - 3 = 1.8\% \text{ or } 0.018 \]

Warmup 7.
\[ |V_s| = 1 \angle 66.4^\circ \]
\[ V_s = \frac{0.014 - j0.024}{0.014 + j0.024} \]
\[ = 0.014 + j0.024 = 0.0278 \angle 59.7^\circ \]
\[ Z_I = 0.0278 \angle 59.7^\circ \times 1 \angle -66.4^\circ \]
\[ = 0.0276 \angle -6.7^\circ = 0.0276 - j0.0032 \]
\[ V_s = 1 + Z_I = 1.0276 - j0.0032 \]
\[ |V_s| = 1.0276 \]
At half load
\[ V_s = V_L + 0.0138 - j0.0016 \]
\[ |V_s| = 1.0276, \quad V_L = 1.014 \]
Regulation:
\[ \text{half to full load: } 1.014 - 1 = 0.014 \text{ or } 1.4\% \]
\[ \text{no load to full load: } 1.027 - 1 = 0.028 \text{ or } 2.8\% \]

Warmup 9.
\[ V_{an} = 254\angle 60^\circ \]
\[ V_{bn} = 254\angle 120^\circ \]
\[ V_{cn} = 254\angle 180^\circ \]
\[ \angle an = 65.6 \angle 0^\circ \]
\[ \angle bn = 65.6 \angle -120^\circ \]
\[ V_{ac} = V_{an} - V_{cn} = 440 \angle -30^\circ \]
**Warmup 9 Continued**

\[ S_{dc} = V_{dc} I_{dc}^* = 440 \angle -30^\circ \times 65.6 \]
\[ = 29.9 \text{ KVA @ } -30^\circ \]

\[ V_{bc} = V_{bn} - V_{cn} = 440 \angle -90^\circ \]

\[ S_{bc} = V_{bc} I_{bc}^* = 440 \angle -90^\circ \times 65.6 \angle 20^\circ \]
\[ = 29.9 \text{ KVA @ } +20^\circ \]

**Concentrate 1.**

(a) \[ I_{line} = \frac{500 \text{ kW} / 3 \text{ phases}}{\frac{11.5 \text{ KV}}{\sqrt{3}} \times 0.866} = 29 \text{ A} \]

(b) \[ |S| = \frac{500 \text{ KVAR}}{0.866} = 577 \text{ KVAR} \]

(c) \[ |I_b| = 29 \sqrt{1 - 0.866^2} = 14.5 \text{ A} \]

\[ X_{comp} = -\frac{11.5 \text{ KV} / \sqrt{3}}{14.5 \text{ A}} = -458 \Omega \]

**Concentrate 2.**

The turns ratio \( a \left( \frac{N_p}{N_s} \right) \)

is found so the low-voltage secondary measurements can be referred to the primary:

\[ a = \frac{115 \text{ KV}}{13.8 \text{ KV}} = 8.33 \]

**Assumption:** open circuit measurements are made at rated voltage.

\[ I_{p, oc} = \frac{I_{s, oc}}{\alpha} = \frac{350}{8.33} = 42.0 \text{ A} \]

\[ |S_{oc}| = |V_s I_s| = 13.8 \times 350 = 4.83 \text{ MVA} \]

\[ |Q_{oc}| = \sqrt{S_{oc}^2 - P_{oc}^2} = 4.82 \text{ MVAR} \]

\[ G = \frac{P}{V^2} = \frac{250\text{ KW}}{(115 \text{ KV})^2} = 18.9 \times 10^{-6} \text{ mhos} \]

\[ B = \frac{Q}{V^2} = \frac{4.82 \text{ MVAR}}{(115 \text{ KV})^2} = 364 \times 10^{-6} \text{ mhos} \]
CONCENTRATE 2 CONTINUED

ASSUMPTION: Short circuit test is at rated current.

\[ I_{\text{pri.-rated}} = \frac{50 \text{ MVA}}{115 \text{ KV}} = 435 \text{ A} \]

\[ V_{p-\text{oc}} = 12.5 \text{ kV} \]

\[ Q_{\text{oc}} = 12.5 \text{ kV} \times 435 \text{ A} = 5.44 \text{ MVA} \]

\[ R_p + a^2 R_3 = \frac{P}{I^2} = \frac{300 \text{ kW}}{435^2} = 1.585 \Omega \]

\[ (Q_a) = \sqrt{S_{\text{oc}}^2 - P_{\text{oc}}^2} = 5.43 \text{ MVAR} \]

\[ X_p + a^2 X_5 = \frac{Q_{\text{oc}}}{I^2} = 28.7 \Omega \]

ASSUMPTIONS: \( R_p = a^2 R_S \) \( X_p = a^2 X_S \)

\[ R_p = a^2 R_S = 0.79, \quad X_p = a^2 X_S = 14.35 \]

\[ Y = Y_a = (18.9 - j364) \times 10^{-6} \]

\[ Y_Z = 5.24 \times 10^{-3} \angle -0.3^\circ = 0.0052 \]

From Figure 5.10(c)

\[ A = D = 1 + Y_Z \approx 1.0 \]

\[ B = 2 Z_i + Y Z_i^2 = 28.7 \angle 86.9^\circ \]

\[ C = Y = 364 \times 10^{-6} \angle -87^\circ \]

From Eq's 5.61, 5.62:

\[ V_i = \frac{Aa^2 Z_L + B}{Ca^2 Z_L + D} \]

\[ Z_i = \frac{0.5 + 28.7 \angle 86.9^\circ}{28.7 \angle 85.9^\circ + 1} = 28.7 \angle 85.9^\circ \]

CONCENTRATE 3.

REFER TO EQ's 5.65 THROUGH 5.68.

\[ Y_c Z_i = (0.0002 - j0.00001)(0.01 + j0.02) \]

\[ = 22.8 \times 10^{-6} \angle 15.3 \]

\[ = (22.0 - 16.0) \times 10^{-6} \]

\[ A = \frac{1}{x} + 2 Z_i Y_c = 1.00 \]

\[ B = 2 Z_i + 2 Z_i = 0.04 + j0.08 \]

\[ C = Y_i + Y_c = 0.0004 - j0.002 \]

\[ D = 2 Z_i Y_c + 1 = 1.00 \]

\[ a^4 Z_L = 0 \]

\[ V_i = \frac{Aa^4 Z_L + B}{Ca^4 Z_L + D} = 0.14 + j0.08 \]

CONCENTRATE 4.

\[ A_{ps} = \frac{6.8 \text{ KV}}{440 \text{ V}} = 15.45 \]

\[ A_{pt} = \frac{6.8 \text{ KV}}{1.38 \text{ KV}} = 4.93 \]

For the secondary base:

\[ Z_{\text{BASE-5}} = (440 \text{ V})^2/30 \text{ kVA} = 6.45 \Omega \]
CONCENTRATE 4 CONTINUED

REFERRED TO THE SECONDARY:

\[ Z_p = \frac{49.5 + j110}{(15.45)^2} \]

\[ Z_p (\text{p.u.}) = \frac{49.5 + j110}{15.45^2} = 0.0324 + j0.091 \]

\[ Z_s (\text{p.u.}) = \frac{70.5 + j90}{15.45^2} = 0.046 + j0.058 \]

\[ Z_t (\text{p.u.}) = \frac{50.5 + j70}{15.45^2} = 0.033 + j0.045 \]

FOR THE TERTIARY BASE:

\[ Z_p = 2.1 + j4.9 \% \]

\[ Z_s = 3.0 + j3.9 \% \]

\[ Z_t = 2.2 + j3.0 \% \]

CONCENTRATE 5.

USING THE TRANSFORMER BASE:

\[ V_{\text{base}} = \frac{440}{\sqrt{3}} = 254; I_{\text{base}} = \frac{250kVA}{3} \]

\[ I_{\text{base}} = 328A; Z_{\text{base}} = \frac{254}{328} = 0.744 \Omega \]

WIRE RESISTANCE (P.U.)

\[ 0.053 \times \frac{500 \Omega}{1000} = 0.0342 \text{ p.u.} \]

\[ Z_{\text{base}} + R = 0.0442 + j0.05 \]

\[ V_{\text{in}} \text{ is transformer input,} \]

\[ V_L \text{ is assumed 1 p.u. in all cases;} \]

\[ V_{\text{in}} = V_L + (0.0442 + j0.05) I_L \]

UNCOMPENSATED

P.F. = 0.6 lagging.

\[ \left| I \right| = \frac{120KW/3}{254 (0.6)} = 0.8 \text{ p.u.} \]

\[ V_{\text{in}} = V_L + (Z + R) 0.8 \angle -60^\circ \]

\[ = 1 + (0.067 \angle 48^\circ)(0.8 \angle -53.1^\circ) \]

\[ V_{\text{in}} = 1.053 \angle -0.2^\circ \]
Concentrate 5 continued

**Compensation**

Rated wire current: \( \frac{225}{328} = 0.686 \) p.u.

The compensation will be just sufficient to deliver the same load power at 0.686 p.u. amps:

\[
\cos \theta = \frac{0.48}{0.686} = 0.7
\]

\[ I_{\text{line}} = 0.686 \angle 45.6^\circ \]

\[ V_{\text{in}} = 1 + (0.067 \angle 45.5^\circ) I_{\text{line}} \]

\[ V_{\text{in}} = 1.046 \]

The input voltage decreases from 1.053 to 1.046 keeping the primary voltage at 1.053, the load voltage increases by the same percentage:

\[ V_{\text{L}} = \frac{1.053}{1.046} = 1.007 \text{ volts} \]

Concentrate 6.

(a) \( I_L = 0.855 \angle 0^\circ, \quad Z = 0.716 \angle 65.2^\circ \)

\[
E_{gm}^* = 0.935 - 0.855 (j0.25) = 0.959 \angle 12.9^\circ
\]

\[
I_m = \frac{E_g^*}{j0.25} = 3.84 \angle 102.9^\circ
\]

\[
E_{g'g} = 0.935 + 0.855 Z = 1.315 \angle 25^\circ
\]

\[ I_g = \frac{E_{g'g}}{Z} = 1.84 \angle 40.2^\circ \]

**Breaker Ratings:**

**GEN:** 1.84 x 15 MVA = 27.6 MVA @ 13.9 KV

**Mot:** 3.84 x 15 MVA = 57.5 MVA @ 13.9 KV

(b)

\[ I_L = 0.855 \angle 36.9^\circ \]

\[ I_m = \frac{0.935 - j0.25 I_L}{j0.25} = 4.308 \angle 99.1^\circ \]

\[ I_g = \frac{0.935 - Z I_L}{Z} = 1.403 \angle 28.6^\circ \]

**Breaker Ratings:**

**GEN:** 21 MVA @ 13.9 KV

**Mot:** 64.6 MVA @ 13.9 KV

(c)

\[ P_F = 1, \quad I_{\text{fault}} = 4.96 \angle 83.6^\circ \]

\[ P_F = 0.8, \quad I_{\text{fault}} = 4.96 \angle 83.5^\circ \]

Total fault current not affected by power factor.
Concentrate 7.
As in example 5.20, the pre-fault current is \(0.76 \angle 2.3^\circ\) at point A:

\[ V_{th} = 0.9 + (0.76 \angle 2.3^\circ)(0.02+j0.25) = 0.927 \angle 11.85^\circ \]

The impedance seen from point A is:

\[ Z_{th} = (0.09+j0.25) \parallel (1.6+j1.2) = 0.075 \angle 89.2^\circ \]

Then:

\[ I_{a1} = \frac{V_{th}}{Z_{th}} = 12.4 \angle -77.3^\circ \text{ p.u.} \]

For a balanced fault,

\[ I_{a0} = I_{ar} = 0 \text{ so } I_{\text{fault}} = I_{a1} \]

\[ |I_{\text{fault}}| = 12.4 \text{ p.u.} \]


Concentrate 8.
From problem 7:

\[ E_{th,a} = 0.927 \angle 11.85^\circ \]

\[ Z_{th,2} = 0.075 \angle 89.2^\circ = Z_1 \]

\[ Z_2 = 0.075 \angle 89.2^\circ = 0.085 \angle 89.3^\circ \]

\[ Z_0 = 3(10.6) + 10.03 = j1.83 \]

Then:

\[ I_{a1} = \frac{E_{th,a}}{Z_1 + Z_2} = \frac{0.927 \angle 11.85^\circ}{1.967 \angle 89.9^\circ} \]

\[ I_{a1} = 0.469 \angle -78.1^\circ \]

\[ I_a = I_{a1} + I_{ar} + I_{a0} \]

but \( I_{ar} = I_{a0} = 0 \)

then

\[ I_a = 1.41 \angle -78.1^\circ \]


Concentrate 9.
From diagram:

\[ V_1 = \frac{V_2}{N_2} \]

\[ I_1 = I_2 = \frac{N_1}{N_2} I_2 \]

\[ N_1(I_1 - I_2) = N_2 I_2 \]

RQD: \( V_1 I_1 = V_2 I_2 = \frac{750 \text{ MVA}}{3} \)

**Common:**

\[ V_1 (I_1 - I_2) = \text{RATING} \]

Series: \( (V_2 - V_1) I_2 = \text{RATING} \)

\[ V_1 = 345 \text{ KV} \quad I_1 = \frac{250 \text{ MVA}}{345 \text{ KV}} \]

\[ V_2 = 500 \text{ KV} \quad I_2 = \frac{250 \text{ MVA}}{500 \text{ KV}} \]

Common

\[ \text{RATING} = 345 \left( \frac{250}{345} - \frac{250}{500} \right) = 77.5 \text{ MVA} \]

Series

\[ \text{RATING} = (500 - 345) \left( \frac{250}{800} \right) = 77.5 \text{ MVA} \]
Concentrate 10. Using motor base, assume 90% efficiency & p.f. = 0.85 lag.

\[ P_{in} = 0.9 \times P_{in} \quad V_{in} = \frac{P_{in}}{0.85} \]

\[ V_{in} = \frac{500 \times 0.746}{0.9 \times 0.85} = 488 \text{ KVA} \]

Converting system ζ to motor base:

\[ Z_{s} = (0.05 + j0.2) \frac{488 \text{ KVA}}{100 \text{ MVA}} \]

\[ Z_{s} = 0.0078 + j0.0544 \text{ p.u.} \]

Converting transformer ζ to motor base:

\[ Z_{t} = 0.05[\cos 81.9° + j \sin 81.9°] \frac{488}{2500} \]

\[ = 0.0014 + j0.0097 \text{ p.u.} \]

Estimating motor starting reactance:

\[ I_{\text{start}} = 6 \times \text{p.f.} = 5.1 \text{ p.u.} \]

\[ X_{\text{start}} = \frac{1}{5.1} = 0.1961 \]

Assumption

At full-load steady state the motor voltage is 1 p.u. and the current is 1 p.u. Then the source voltage is found:

\[ V_{s} = V_{t} + I_{t} Z_{t} \quad I_{t} = 1 \angle -31.8° \]

\[ V_{s} = 1 + (1 \angle -31.8°) (0.0092 + j0.0064) \]

\[ = 1.043 \angle 2.7° \]

By line starting the motor initially the voltage is determined by voltage division:

\[ V_{m} = \frac{V_{s}}{Z_{\text{start}} + Z_{t}} \]

\[ = 1.043 \frac{10.1961}{0.0092 + j0.0064} \]

\[ = 0.736 \angle 2° \]

so the voltage is 73.6% of the running voltage for a dip of 21.4%.

Timed 1.

The Transformer base is used.

(a) \( H_{\text{P, rated}} = \frac{0.9 \times 0.85 \times \text{KVA rated}}{0.746} \)

\[ = 1.025 \text{ KVA rated} \]

KVA rated = \( 3 \times \frac{0.44}{\sqrt{3}} I_{\text{rated}} \)

\[ = 0.762 I_{\text{rated}} \]

Maximum voltage drop - taking the starting current as 6 x p.f. x I_{\text{rated}}:

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So the impedance seen by the transformer primary would be \( \frac{1}{(0.4)^2} Z_L = \frac{Z_L}{0.16} \)

The starting current would then be \( \frac{I_{\text{rated}}}{0.16} \) and

\[ I_{\text{rated}} = 37.7 \times 0.16 = 235.6 \text{ HP} \]

**Timed 2.**

\[ V_{an} = 254 \angle 0^\circ \]
\[ V_{ab} = 440 \angle 30^\circ \]
\[ V_{cb} = 440 \angle 90^\circ \]

**Balanced Load:**

\[ I_a = \frac{50 \text{KVA} \angle 0^\circ}{254} \]

\[ I_a = 65.6 \angle -36.9^\circ \]

\[ I_{cn} = 65.6 \angle 88.1^\circ \]

(a) \[ V_{ab}^2 \]

\[ \frac{V_{ab}^2}{10 \text{ kw}} = 19.36 \angle 3^\circ \]

\[ I_{\phi} = \frac{440}{19.36} = 22.7 \text{ A} \]

in phase with \( V_{ab} \)

then

\[ I_a = I_{an} + 22.7 \angle 30^\circ \]

\[ = 77.4 \angle -21.2^\circ \]

\[ S_{ab} = (440 \angle 30^\circ)(77.4 \angle -21.2^\circ) \]
Timed 2 CONTINUED

\[ S_{AB} = 34.05 \angle 57.23 \]
\[ = 21.3 \text{ kW} + j26.5 \text{ kVAR} \]
\[ S_{CB} = (440 \angle 90^\circ)(65.6 \angle -82.15^\circ) \]
\[ = 28.87 \angle 6.87^\circ \]
\[ = 28.64 \text{ kW} + j3.45 \text{ kVAR} \]

(b) \[ I_C = 65.6 \angle 83.1^\circ + 22.7 \angle 90^\circ \]
\[ = 88.2 \angle 84.9^\circ \]
\[ S_{CB} = (440 \angle 90^\circ)(88.2 \angle -84.9^\circ) \]
\[ = 38.81 \angle 5.1^\circ \]
\[ = 38.65 \text{ kW} + j3.45 \text{ kVAR} \]
\[ S_{AB} = (440 \angle 30^\circ)(65.6 \angle 36.9^\circ) \]
\[ = 28.87 \angle 66.9 \]
\[ = 11.31 \text{ kW} + j26.56 \text{ kVAR} \]

Timed 3.
The 25 MVA Base is used. Converting 15 MVA impedance to 25 MVA Base

\[ Z_{15} = 10.06 \times \frac{25}{15} = 10.10 \]
So both windings have the same impedance.

(a) With equal reactances and equal voltages, the two transformers carry equal currents, i.e. 50% of the load current.

(b) The load voltage is taken as 1 p.u., and on the 25 MVA base,

\[ I_L = \frac{30 \text{ MVA}}{25 \text{ MVA}} \angle \frac{-18^\circ}{-36.87^\circ} \]

\[ 1.05 V_L + \frac{V_L}{1.05} + I_L \]

The Thevenin equivalent of the transformers:

\[ 1.025 V_L = V_{L} + j0.05 I_L \]
\[ = 1 + 0.037 + j0.048 \]
So \[ V_S = 1.012 \angle 2.65^\circ \]
then

\[ I_{25} = \frac{1.05 V_S - j0.11}{10.1} = 0.787 \angle 51.4^\circ \]
\[ = 0.491 - j0.615 \]
Timed 3. Continued

\[ I_{15} = \frac{V_{51}}{Z_{10.1}} = 0.481 \angle -13.1^\circ \]
\[ = 0.468 - j0.109 \]

and
\[ I_L = I_{25} + I_{15} = 0.959 - j0.724 \]
\[ = 1.202 \angle 37.0^\circ \]

which checks within rounding errors.

\[ I_{c1} = I_{25} + \frac{I_L}{2} \]
\[ I_{15} = \frac{I_L}{2} - I_{c1} \]
\[ I_{c1} = \frac{I_{25} - I_{15}}{2} \]
\[ = 0.023 - j0.505 \]
\[ 2 \]
\[ = 0.023 - j0.505 \]
\[ = 0.253 \angle -87.4^\circ \]

Timed 4.
Using the calculations of

Concentrate 8:

\[ E_{th,a} = 0.927 \angle 11.5^\circ \]
\[ Z_1 = 0.075 \angle 89.2^\circ \]
\[ Z_2 = 0.071 \angle 89.3^\circ \]

Then, from figure 5.18:

- \[ I_a = \frac{E_{th,a}}{Z_1 + Z_2} = \frac{0.635}{77.0^\circ} \]

Using eq. 5.150

\[ I_{fault} = \sqrt{3} I_a = 11.0 \angle 27.0^\circ \]

Using eq. 5.144 with \( V_a = 0 \)

\[ V_a = -2V_b \]

eq. 5.145:

\[ 3V_a = V_a + (a + a^2) V_b = -3V_b \]

Figure 5.18:

\[ V_{a1} = \frac{Z_2}{Z_1 + Z_2} E_{th,a} = 0.464 \angle 11.9^\circ \]

\[ V_b = -0.464 \angle 11.9^\circ \]

\[ V_a = 0.928 \angle 11.9^\circ \]

Timed 5.

As no current flows when bus #3 is open, \( E_{th,a} = 10^\circ \)

The Thévenin equivalent impedance seen from bus #3 is found by shorting the sources to obtain a bridge circuit.
Notice that the bridge is balanced, so that no current flows in the branch from bus 1 to bus 2. So buses 1 and 2 constitute a single node, and

\[ Z_{th} = \frac{1}{100} \left( \frac{42}{100} + \frac{j10}{100} \right) \frac{15}{100} \]

\[ = \frac{1}{100} \frac{1}{100} \frac{1}{100} \]

From figure 5.18:

\[ -I_{a2} = I_{a1} = \frac{E_{th}}{Z_{th}} = 12.5 \angle -90^\circ \]

From eq 5.150:

\[ I_{\text{fault}} = -j\sqrt{3} I_{a1} = -21.65 \angle 0^\circ \]

Because the voltages at buses 1 and 2 are equal, the current delivered by each is inversely proportional to the impedance.

To bus 3:

\[ I_{13} = \frac{2}{3} I_{\text{fault}}, \quad I_{23} = \frac{1}{3} I_{\text{fault}} \]

at bus #1:

\[ V_{a1} = 1 \angle 0 \text{ no fault current } \]

\[ V_{b1} = 1 \angle -120^\circ - \frac{1}{3} 0.05 \times \frac{2}{3} I_{\text{fault}} = 0.52 \angle -164^\circ \text{ p.u.} \]

\[ V_{c1} = 1 \angle 120^\circ - \frac{1}{3} 0.05 \times \frac{2}{3} I_{\text{fault}} = 1.66 \angle 107.5^\circ \text{ p.u.} \]

at bus #2

\[ V_{a2} = 1 \angle 0 \]

\[ V_{b2} = 1 \angle -120^\circ - \frac{1}{8} 0.1 \angle 0^\circ I_{\text{fault}} = 0.52 \angle -164^\circ \text{ p.u.} \]

\[ V_{c2} = V_{c1} = 1.66 \angle 107.5^\circ \text{ p.u.} \]

Timed 6.

The transformer primary is rated at 60MVA for the secondary and 21 MVA for the tertiary, so its total rating is 81 MVA.

Assuming all impedances are referred to the primary, and given in the primary base:

\[ Z_{\text{base}} = \left(\frac{138 \text{ kV}}{81 \text{ MVA}}\right)^2 = 235 \Omega \]
Timed 6. CONTINUED

\( Z_e = \frac{j31}{235} = 0.1328 \Omega \)

(a) For a 3-phase fault on the X winding, ignoring any T-current (which would be quite small)

\[ Z_e + Z_N = j0.0132 + j0.0077 = 0.111j \text{ p.u.} \]

Then the primary fault current is:

\[ I_{\text{h-fault}} = \frac{1}{0.111j} = -j9.02 \text{ p.u.} \]

The actual fault current is amplified by the turns ratio \( \frac{138kV}{34.5kV} = 4.0 \)

The primary current base is \( \frac{81 M/3}{138 kV/3} = 339 \text{ A} \)

Fault current: \( j339 \times 4 \times 9.02 = 12,231 \angle 90^\circ \text{ A} \)

(b) For a single-phase fault to ground (see fig. 5.15e) with both H and X in grounded neutral configurations:

\[ Z_0 = Z_1 = Z_2 = \frac{1}{0.111} \]

Then, from fig. 5.17(b)

\[ I_{a1} = \frac{E_{a1} - \alpha}{Z_1 + Z_2 + Z_0} = -j3 \]

From eq. 5.136:

\[ I_a = 3 I_{a1} = -j9 \text{ p.u.} \]

\[ I = 4 \times 339 \times (-19) = -j12,200 \text{ A} \]

Timed 7. \( \frac{I_1}{16-I1.2} \)

\( AV \) 
\( V \)

Transformer base is used

\[ I_1 = \frac{V(1+\delta) - 1}{10.04} \]

\[ I_2 = \frac{V \cos \delta + V \sin \delta - 1}{10.06} \]

Because \( I_1 + I_2 = 1.6 - j1.2 \)

and \( |I_1| = |I_2| = 1 \text{ p.u.} \)

it is necessary that

\( I_1 = I_2 = 0.8 - j0.6 \)
**Timed 9. continued**

Then:

\[ V(\Delta) = 0.04 \left[ \frac{0.8 - j0.6}{1.0} \right] + 1 \]

\[ V(\Delta) = 1.0 \angle 18^\circ \]

\[ V/\theta = 1.006 \left[ \frac{0.8 - j0.6}{1.0} \right] + 1 \]

\[ = 1.037 \angle 2.7^\circ \]

Then

\[ |V| = 1.037 \]

\[ \angle \theta = 2.7 \angle 1.8 = 0.9^\circ \]

\[ V = 1.037 \angle 1.8^\circ \]

\[ 1 + \Delta = \frac{1.026}{1.037} = 0.989 \]

\[ \Delta = -0.011 \]

\[ \Delta = \frac{1}{3} \angle \frac{\pi}{2} \]

\[ n = 1.8 \rightarrow 2 \]

\[ \theta = 0.9^\circ \]

**On the same base:**

\[ Z_L = \frac{1.565}{2.88} (0.02 + j0.1) \]

\[ = 0.011 + j0.054 \text{ p.u.} \]

\[ Z_{tr} = R (1 + j15) \quad R = \frac{0.75}{1 + j15} \quad 0.005 \text{ p.u.} \]

\[ Z_{tr} = 0.005 + j0.075 \text{ p.u.} \]

\[ V_{load} = \frac{12.16}{12} = 1.013 \text{ p.u.} \]

\[ I_{load} = \frac{40 \text{ MVA}/50 \text{ MVA}}{1.013} \angle 0.85 \]

\[ = 0.789 \angle -31.8^\circ \]

\[ Z = Z_{tr} + Z_L = 0.13 \angle 82.9^\circ \]

\[ V_5 = I_z + 1.013 = 1.08 \angle 42^\circ \]

**Regulation:** 1.08 - 1.013 = 0.067 or 6.7%